Round 1 Babtha 1

Question 1

Suppose:

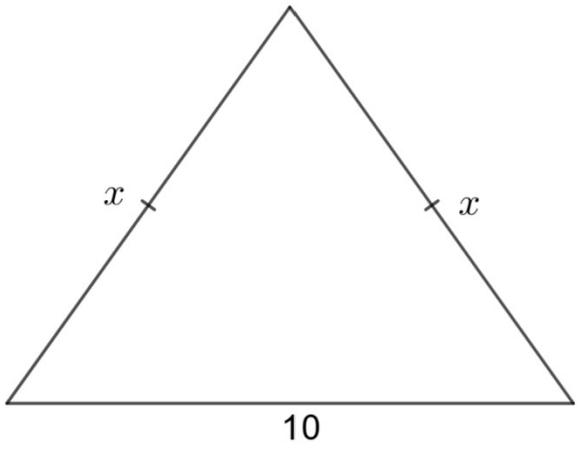
$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{5^2}\right)\ldots\left(1-\frac{1}{2026^2}\right)=\frac{a}{2b}$$

Find the values of a and b, where $a, b \in \mathbb{Z}$.

Question 2

An isosceles triangle has a **fixed** base of length 10 cm, with other sides of length x cm. The triangle is growing as time changes. The rate of change of x is 4 cm per second.

Find the rate of change of the area of the triangle when $x = 13$cm.



Round 2 Babtha 2

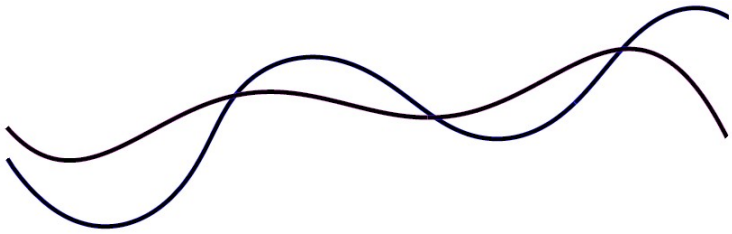
Question 1

Solve for all $x \in \mathbb{R}$, such that:

$$\frac{1}{x^2 - 10x - 29} + \frac{1}{x^2 - 10x - 45} = \frac{2}{x^2 - 10x - 69}$$

Question 2

The diagram shows the tyre tracks made by the front and back wheel of a bicycle. Was the bicycle:



1. Going from right to left ←.
2. Going from left to right →.
3. It doesn't make any difference – it's the same either way.

Round 3 Babtha 3

Question 1

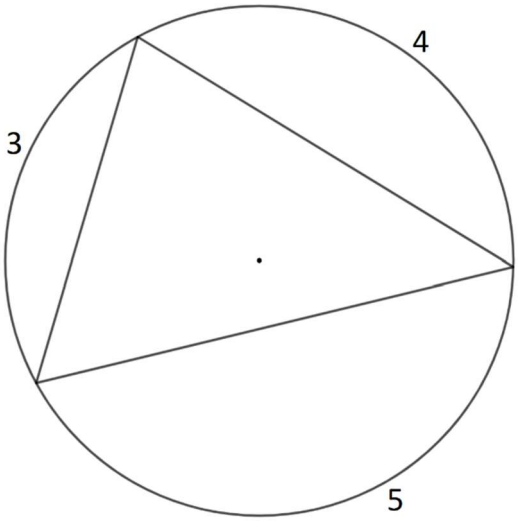
Solve the following equation for $x \in \mathbb{R}$:

$$3^x - \left(\sqrt{3}\right)^{x+4} + 20 = 0$$

Give your answer(s) in their simplest possible form of: $2\log_a b$ where $a, b \in \mathbb{N}$.

Question 2

A triangle is inscribed in a circle as shown. The vertices of the triangle divide the circle into three **arcs of lengths** 3, 4 and 5.



Find the area of the triangle in the form: $\frac{a}{\pi^2}(b + \sqrt{b})$ where $a, b \in \mathbb{N}$.

Round 4 Babtha 4

Question 1

A random sample of 250 schoolbags were weighed and the data was analysed.

Using only the data from the sample, a **95%** confidence interval (**LC HL method**) for the sample mean weight of the bags was found to be: [4.961kg, 4.999kg].

A school claimed that the population mean weight of schoolbags was 5kg. A hypothesis test was carried out at the **1%** level of significance.

1. Calculate the p-value, **as a percentage**, correct to **2 decimal places**.
2. Determine whether the null hypothesis was rejected or not at the **1%** level of significance.

Question 2

For certain **real** values of p, q, r and s, the equation

$$z^4 + pz^3 + qz^2 + rz + s = 0$$

has four **non-real** roots. The product of two of these roots is $13 + i$ and the sum of the other two roots is $3 + 4i$, where $i = \sqrt{-1}$.

Find the value of $q \in \mathbb{R}$.

Round 5 Babtha 5

Question 1

Let:

$$\frac{1}{x} - \frac{1}{y} = 2026$$

Find the value of:

$$\frac{x + 2027xy - y}{2y - 2026xy - 2x}$$

Question 2

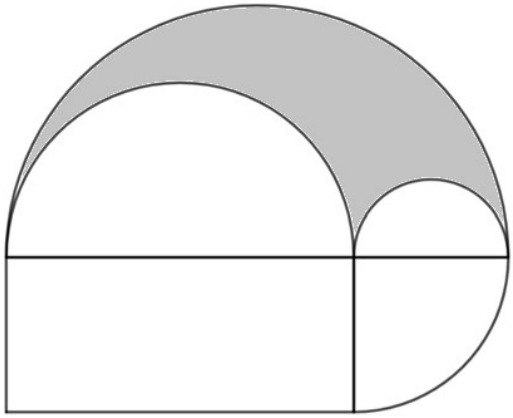
The sum of 18 consecutive positive integers is a perfect square. What is the smallest possible value of this sum?

Round 6 Babtha 6

Question 1

The diagram shows a rectangle, with **area** of the **rectangle equal to 4** units squared, a quarter circle and three semicircles of various radii.

Find the ***exact*** area of the shaded region in the diagram.



Question 2

Simplify:

$$b^{\log_b(2a)!-2\log_b(a)!}$$

Give your answer in the form $\binom{n}{r}$, that is **"n choose r"**, where n, r are both given in terms of a.

Round 7 Babtha 7

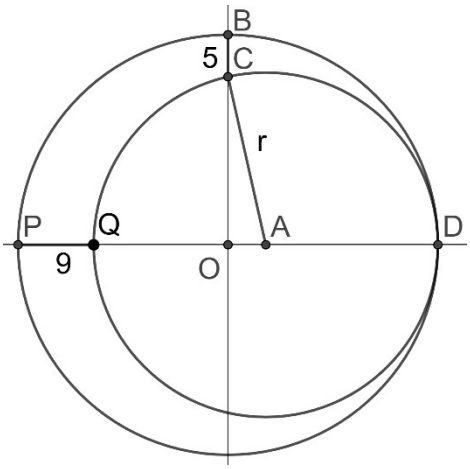
Question 1

For $x \in \mathbb{R}$, let:

$$f(x) = \begin{cases} (x-2)^2 + 1, & x \leq 3 \\ 2x - 4, & x > 3 \end{cases}$$

Solve for all values of x for which $f(x) > 2026$.

Question 2



The diagram shows a larger circle, with centre at the origin $O(0,0)$ and radius R, and a smaller circle with centre A and radius r. The circles touch at the point D. The length of the line segment BC is 5 and of PQ is 9.

Find the length of the sum of the radii i.e. $R + r$.

Question 3

Let z be a complex number satisfying the equation:

$2z^2 - 5z + 4 = 0$. Find the value of $\frac{z^3}{z^6 + 8}$.

Question 4

The Binomial Expansion (given below) is defined for powers other than positive integer powers i.e. n does not have to be a positive integer.

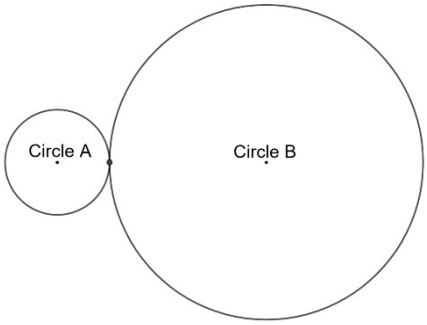
$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r = \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2 + \cdots + \binom{n}{r} x^{n-r} y^r + \cdots + \binom{n}{n} y^n$$

Hence, or otherwise, find the first three terms, in their simplest form, in ascending powers of x, of the expansion:

$$\left(1 - \frac{x}{4}\right)^{-\frac{1}{2}}$$

Round 8 Babtha 8

Question 1



The radius of circle B is three times the radius of circle A. Starting from the position shown in the figure, circle A rolls around circle B. When circle A returns to its starting point, how many rotations will it have completed?

Question 2

A coin is selected at random from a bag containing exactly two coins. One of the coins is a fair, unbiased coin with a head and a tail and the other coin has two heads. The selected coin is tossed three times in succession and lands heads up each time - HHH.

Given that HHH was tossed in succession, what is the probability that the coin that was thrown was the coin with two heads?

Question 3

The integers from 1 to 1000 are written in order around a circle. Starting at 1, every fifteenth number is marked (that is 1 ,16, 31, ...). This process is continued until a number is reached which has already been marked. Find the ***number*** of unmarked numbers.

Question 4

While finding the sine of a certain angle, an absent-minded student failed to notice that their calculator was not in the correct angular mode. They were lucky to get the right answer.

The two smallest positive real values of x for which the sine of x in degrees is equal to the sine of x in radians are: $\frac{a\pi}{b+\pi}$ and $\frac{c\pi}{d-\pi}$, where $a, b, c, d \in \mathbb{N}$.

Find $a + b + c + d$.

Answers

R1	Q1	$a = 2027; b = 2026$
	Q2	$\frac{65}{3}$
R2	Q1	$-3, 13$
	Q2	Option 2: Left to right.
R3	Q1	$2\log_3 4; 2\log_3 5$
	Q2	$\frac{9}{\pi^2}(3+\sqrt{3})$
R4	Q1	$p-value = 3.94\%$; Do Not Reject $\mathrm{H_o}$.
	Q2	51
R5	Q1	$\frac{1}{2026}$
	Q2	225
R6	Q1	π
	Q2	$\binom{2a}{a}$
R7	Q1	$\{x < -43\} \cup \{x > 1015\}$
	Q2	45.5
	Q3	$\frac{8}{5}$
	Q4	$1, \frac{x}{8}, \frac{3x^2}{128}$
R8	Q1	4
	Q2	$\frac{8}{9}$
	Q3	800
	Q4	900

Units not required.