Round 1

Question 1

In a school every student studies **at least one** language: French, German, or Spanish. There are **115 students in total**. 70 study French, 65 study German and 60 study Spanish. $2x$ study French and German, x study German and Spanish and $2x$ study French and Spanish. x study all 3.
Find the value of x.

Solution

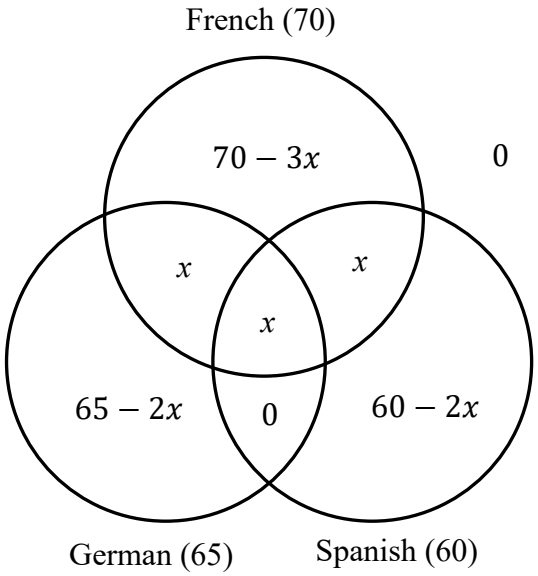
$70 - 3x + 65 - 2x + 60 - 2x + 3x = 115$

$195 - 4x = 115$

$195 - 115 = 4x$

$80 = 4x$

$x = 20$



Question 2

Write $\dfrac{(6x^5y^3)^2}{(3x^2y^7)(4xy^{-3})}$ in the form of ax^by^c where a, b and $c \in \mathbb{Z}$.

Solution

$$\frac{(6x^5y^3)^2}{(3x^2y^7)(4xy^{-3})} = \frac{36x^{10}y^6}{12x^3y^4} = 3x^7y^2$$

Round 2

Question 1

A bag contains only red, green and blue counters.

- 8 of the counters are red.
- 15 of the counters are green.
- The rest of the counters are blue.

Sam chooses one counter at random from the bag.

The probability that this is a green counter is $\frac{3}{8}$.

How many blue counters are in the bag?

Solution

$$\frac{15}{8+15+x} = \frac{3}{8}$$

$$\frac{15}{23+x} = \frac{3}{8}$$

$$15(8) = 3(23+x)$$

$$120 = 69 + 3x$$

$$3x = 31$$

$$x = 17$$

Question 2

In a magic square, the numbers in each of the three rows, each of the three columns, and each of the two diagonals have the same sum. This sum is called the magic constant. The figure shown below is a magic square. Find u.

	$u+3$	
		12
$u+2$	$u-5$	u

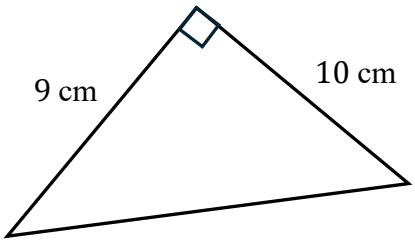
Solution

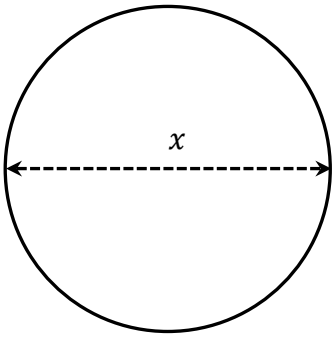
Row 3's total is $3u - 3$
Therefore, the middle box must be $u - 1$
Consider the diagonal and column 3, they share the same box top right, so
$u + 2 \ + u - 1 = 12 + u$
$2u + 1 = 12 + u$
$u = 11$

Round 3

Question 1

Shown below are a right-angled triangle and a circle.





The perimeter of the triangle is equal to the circumference of the circle.
Find the diameter of the circle and give your answer correct to 2 decimal places.

Solution

Missing side of triangle:

$h^2 = 9^2 + 10^2$

$h^2 = 181$

$h = \sqrt{181}$

Perimeter of triangle:

$9 + 10 + \sqrt{181} = 19 + \sqrt{181}$

Circumference of circle:

$\pi x = 19 + \sqrt{181}$

Diameter of circle:

$$x = \frac{19 + \sqrt{181}}{\pi}$$

$x = 10{\cdot}33$ cm

Question 2

Starting on the 22nd floor of their apartment building, Tara goes up the stairs and Jenna goes up with the lift. Beginning when Tara starts going up, Jenna waits for 2 minutes for the lift. Tara goes up from each floor to the next floor in 15 seconds. The lift goes up from each floor to the next floor in 3 seconds. Tara and Jenna arrive on the *n*th floor at the same time. What is the value of *n*?

Solution

Each go up $n - 22$ floors

Tara $= 15(n - 22)$

Jenna $120 + 3(n - 22)$

$15(n - 22) = 120 + 3(n - 22)$

$15n - 330 = 120 + 3n - 66$

$12n = 384$

$n = 32$

Tara time (s) stairs	Floor	Jenna time (s) lift
0	22	120
15	23	123
30	24	126
45	25	129
60	26	132
75	27	135
90	28	138
105	29	141
120	30	144
135	31	147
150	32	150

Round 4

Question 1

The islands of Nom and Dim have fewer than 2000 residents all together. Nom is bigger having at least 50 more residents than Dim. One third, one fourth, one fifth and one seventh of the population of Nom are all whole numbers. The sum of these whole numbers gives the population of Dim. What is the population of Dim?

Solution

$3 \times 4 \times 5 \times 7 = 420$

$\frac{1}{3} = \frac{140}{420}$, $\frac{1}{4} = \frac{105}{420}$, $\frac{1}{5} = \frac{84}{420}$, $\frac{1}{7} = \frac{60}{420}$

Population of Dim (sum of integers)

$140 + 105 + 84 + 60 = 389$

$420 - 389 = 31$ Not valid - Must be at least 50 people between Nom and Dim.

Population Dim $\neq 389$

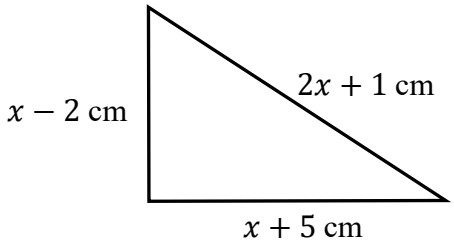
$\frac{1}{3} = \frac{280}{840}$, $\frac{1}{4} = \frac{210}{840}$, $\frac{1}{5} = \frac{168}{840}$, $\frac{1}{7} = \frac{120}{840}$

$280 + 210 + 168 + 120 = 778$

$840 - 778 = 62$ This is 50 larger than the population of Nom.

Population of Dim = 778

Question 2



Solve for x, correct to two decimal places.

Solution

$(2x + 1)^2 = (x - 2)^2 + (x + 5)^2$

$(4x^2 + 4x + 1) = (x^2 - 4x + 4) + (x^2 + 10x + 25)$

$4x^2 + 4x + 1 = 2x^2 + 6x + 29$

$2x^2 - 2x - 28 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{(-2)^2 - 4(2)(-28)}}{2(2)} = 4{\cdot}27 \text{ or } x = -3{\cdot}27 \text{ (not valid)}$$

$x = 4{\cdot}27$

Round 5

Question 1

A football squad has 3 goalkeepers, 8 defenders, 9 midfielders, and 5 forwards. The mean height of all 25 footballers is 181cm. The mean height of the goalkeepers is 188 cm. The mean height of the defenders is 187cm. The mean height of the midfielders is 176 cm.

Work out the mean height of the forwards.

Solution

Total
25 players = 181
$\text{Total mean} = 25 \times 181$
$= 4525$

Goalkeepers
$3 \times 188 = 564$
Defenders
$8 \times 187 = 1496$
Midfielders
$9 \times 176 = 1584$

$\text{Total} - \text{GK} - \text{D} - \text{MF}$
$4525 - 564 - 1496 - 1584 = 881$
$\frac{881}{5} = 176{\cdot}2$

Question 2

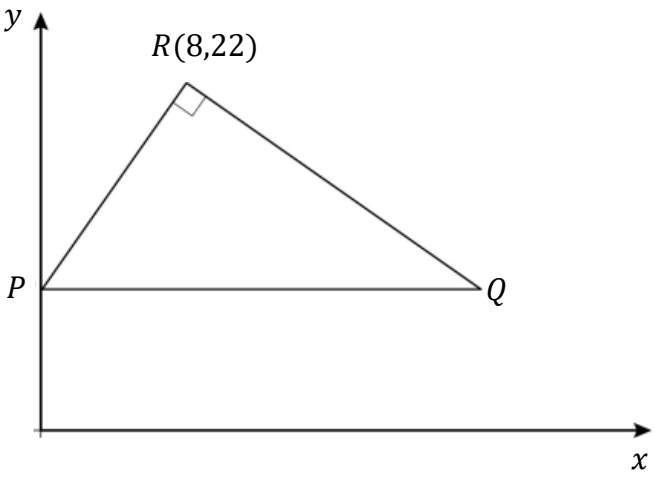
The points P, Q and R form a right-angled triangle.

$R = (8,22)$.

The slope of $[PR] = 2$.

$[PQ]$ is parallel to the x-axis.

Work out the coordinates of point Q.



Solution

Slope $P(0, y)$ and $R\ (8,22)$

Slope $[PR] = \frac{22 - y}{8 - 0} = 2$

$22 - y = 16$

$y = 6 \Longrightarrow P = (0,6)$

$[RQ]$ is perpendicular to $[PR] \Longrightarrow$ Slope $[RQ] = -\frac{1}{2}$

Slope $[PQ] = \frac{22 - 6}{8 - x} = -\frac{1}{2}$

$2(16) = -1\ (8 - x)$

$32 = -8 + x$

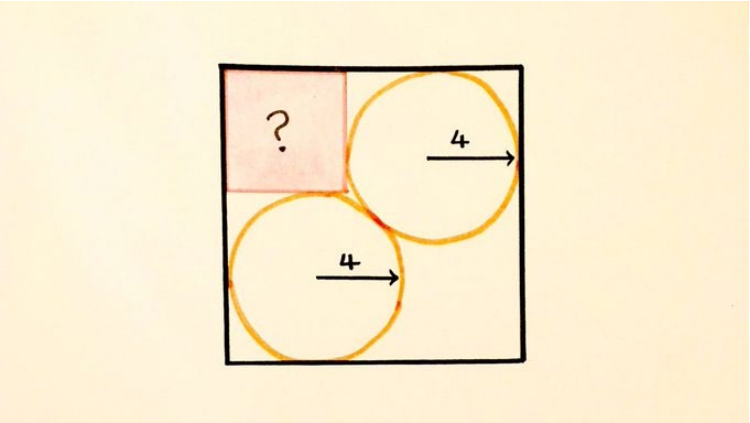
$x = 40$

$Q = (40,6)$

Round 6

Question 1

Find the area of the smaller shaded square to the nearest whole unit squared.



Solution

Diagonal from vertex of larger square to centre of circle:

$c^2 = a^2 + b^2$

$c^2 = (4)^2 + (4)^2 \Longrightarrow c = 4\sqrt{2}$

Diagonal of large square:

$s = 4\sqrt{2} + 4\sqrt{2} + 4 + 4 = 8 + 8\sqrt{2}$

Length of side of square(x):

$x^2 + x^2 = \left(8 + 8\sqrt{2}\right)^2$

$x = \sqrt{\dfrac{\left(8 + 8\sqrt{2}\right)^2}{2}}$

Area of shaded square:

$\left(\sqrt{\dfrac{\left(8 + 8\sqrt{2}\right)^2}{2}} - 8\right)^2 = 32 \text{ units}^2$

$2x^2 = \left(8 + 8\sqrt{2}\right)^2$

$2x^2 = 373{\cdot}019336$

$x = 13{\cdot}65685425$

Area of shaded square:

$(13{\cdot}65685425 - 8)^2$

$(5{\cdot}656854249)^2 = 32 \text{ units}^2$

Round 6

Question 2

On March 1st 2001, the ratio of Ian's age to his mother's age was 1:11.

On March 1st 2018, the ratio of Ian's age to his mother's age was 2:5.

Write the ratio of Ian's age to his mother's age on March 1st 2030.

Solution to be given in the form $p:q$ with p and $q \in \mathbb{N}$ and the HCF of $p, q = 1$ (i.e. p and q are in their lowest form).

Solution

$2001 \ldots\ldots 1:11 = 2:22 = 3:33 = 4:44$

$2018 \ldots\ldots 17:28 = 19:39 = 20:50 = 21:61$ (Adding 17)

But only 20:50 is in the ratio 2:5

$2030 \ldots\ldots \ldots$ Ian $= 32$ years

$2030 \ldots\ldots \ldots$ Mother $= 62$ years

Ratio $32:62 = 16:31$

Round 7

Question 1

Sean is training for a marathon. He runs either route A or route B.

During April, he runs route A nine times and route B five times. Route B is 8 miles longer than route A. In total, he runs 89 miles in April.

In May, he will start running route C. Route C is 20% longer than route B.

Work out the length of route C.

Solution

$9A + 5B = 89$

$B - A = 8 \quad (\times 9)$

$$\begin{array}{r} 9A + 5B = 89 \\ -9A + 9B = 72 \\ \hline 14B = 161 \end{array}$$

$B = 11{\cdot}5$

$C = 11{\cdot}5 \times 120\% = 13{\cdot}8$ miles

Round 7

Question 2

The first 3 terms in an arithmetic sequence are $2k-1, 5k+2, \ 8k+5$.

Find in terms of k, the 20th term in the sequence.

Solution

$d = 8k+5-(5k+2) = 3k+3$

$T_n = a+(n-1)d$

$T_{20} = 2k-1+(20-1)(3k+3)$

$T_{20} = 2k-1+57k+57$

$T_{20} = 59k+56$

Question 3

Solve for $x, x \in \mathbb{N}$

$$x+\frac{1}{3+\frac{1}{x}} = \frac{16}{7}$$

Solution

$$x+\frac{1}{\frac{3(x)+1}{x}} = \frac{16}{7}$$

$$x+\frac{x}{3x+1} = \frac{16}{7}$$

$$\frac{x(3x+1)+x}{3x+1} = \frac{16}{7}$$

$$\frac{3x^2+x+x}{3x+1} = \frac{16}{7}$$

$7(3x^2+2x) = 16(3x+1)$

$21x^2+14x-48x-16 = 0$

$21x^2-34x-16 = 0$

$(21x+8)(x-2) = 0$

$x = -\frac{21}{8} = -2{\cdot}625$ and $x = 2 \ x \in \mathbb{N} \ \therefore x = 2$

Round 7

Question 4

Cian deposits €8000 into a savings account for 4 years, earning compound interest. The interest rate is 6% for the first year, and 2.5% for each of the remaining years. Calculate the total interest earned as a percentage of his original investment.

Solution

$F = P(1 + i)^t$

$F = P(1 + 0{\cdot}06)^1 = 8480$

$F = P(1 + 0{\cdot}025)^3 = 9132{\cdot}03$

$9132{\cdot}03 - 8000 = 1132{\cdot}03$

$\frac{1132{\cdot}03}{8000} \times 100 = 14{\cdot}15\%$

Round 8

Question 1

Two opposite sides of a square are increased by 40% while the other two sides are decreased by 50%.
By what percentage is the area of the square decreased?

Solution

New sides of shape (rectangle): $1{\cdot}4x$ and $0{\cdot}5x$

Area new rectangle :$1{\cdot}4(0{\cdot}5)x^2 = 0{\cdot}7x^2$

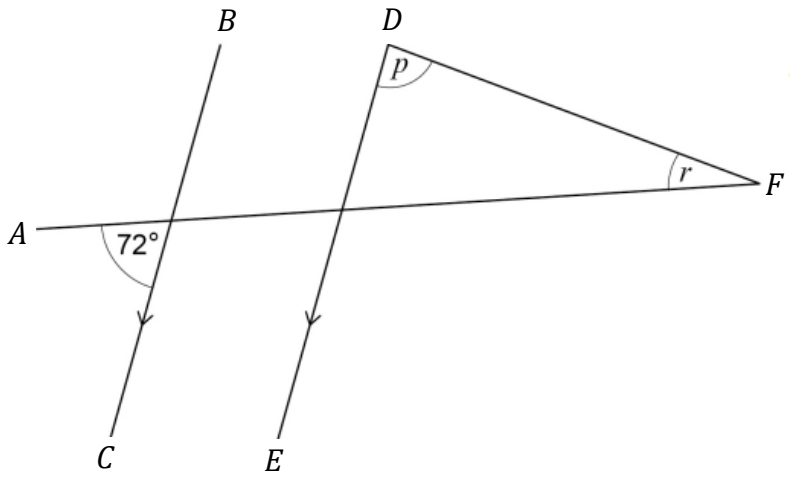
Original area: x^2

Decrease: $1 - 0{\cdot}7 = 0{\cdot}3$=30%

Round 8

Question 2

The angle $\angle p$ is three times the angle $\angle r$. Calculate the size of angle $\angle p$.



Solution

The third angle in the triangle is 72° (vertically opposite angles)

$180° - 72° = 108°$

$r + 3r = 108°$

$4r = 108°$

$r = 27°$

$p = 3(27°)$

$p = 81°$

Question 3

Find the value of $5^a + 5^{a+1} = \sqrt{4500}$

Solution

$5^a(1 + 5^1) = 30\sqrt{5}$ Common factor

$5^a(6) = 30\sqrt{5}$

$5^a = 5\sqrt{5}$ Divide both sides by 6

$5^a = 5 \times 5^{\frac{1}{2}}$

$a = 1{\cdot}5 = \frac{3}{2}$

Round 8

Question 4

Travelling to a pop concert I know that if I average 30 km/h for the journey I will arrive 1 hour early, but if I average only 20 km/h for the journey I will arrive 1 hour late.
How fast do I need to travel to arrive exactly on time?

Solution

Let d = distance to the concert

$$\frac{d}{20} - \frac{d}{30} = 2$$

$$30d - 20d = 1200$$

$$10d = 1200 \Longrightarrow d = 120$$

Journey travel times are $x - 1$ and $x + 1$

$$\frac{120}{x-1} = 30$$

$$120 = 30(x - 1)$$

$$30x - 30 = 120$$

$$30x = 150 \Longrightarrow x = 5 \text{ hours}$$

$$\text{Speed} = \frac{120}{5} = 24 \text{ km/h}$$

Tiebreaker

1 Two locations A and B are connected by a 5 km trail which features a lookout C. A group of 15 hikers started at A and walked along the trail to C. Another group of 10 hikers started at B and walked along the trail to C.

The total distance travelled to C by all hikers from the group that started in A was equal to the total distance travelled to C by all hikers from the group that started in B.

Find the distance (in kilometres) from A to C along the trail.

Solution:

Let the distance from A to C be x. Then the distance from B to C is $5 - x$.
The total distance travelled to C by all hikers from the group that started in A is $15x$.
The total distance travelled to C by all hikers from the group that started in B is $10(5 - x)$.
$15x = 10(5 - x)$
$15x = 50 - 10x$
$25x = 50$
$x = 2$ km (units not required)

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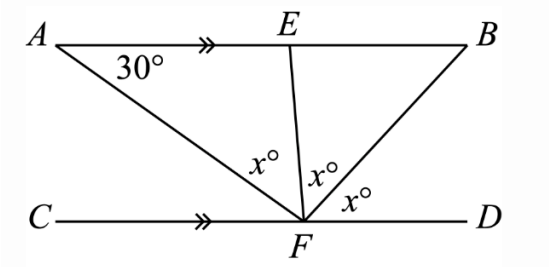
2 In the diagram AB is parallel to CD. Points E and F are on AB and CD, respectively, so that $\angle FAB = 30°$ and $\angle AFE = \angle EFB = \angle BFD = x°$. What is the value of x?



Solution:

Since AB and CD are parallel, then to $\angle FAB$ and $\angle AFD$ are co-interior angles.
Therefore $\angle FAB + \angle AFD = 180° \Longrightarrow 30° + 3x° = 180°$.
$3x° = 150° \Longrightarrow x = 50$

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3 A ball of wool makes one winter glove. If the diameter is doubled, how many gloves could be made?

Solution:

Volume $= \frac{4}{3}\pi r^3 = 1$ glove. If the diameter is doubled the radius is doubled

New volume $= \frac{4}{3}\pi(2r)^3 = \frac{4}{3}\pi 8r^3 = 8$ gloves.

Eight gloves can be made when the diameter is doubled.

Tiebreaker

4 The mean, the median and the mode of the five numbers 12, 9, 11, 16, x are all equal. What is the value of x?

Solution:

Mean $= \dfrac{12 + 9 + 11 + 16 + x}{5} = \dfrac{48 + x}{5}$

If $x = 9$ the numbers become $9, 9, 11, 12, 16$ $-$ Mode $= 9$, Median $= 11$ ⊗

If $x = 11$ the numbers become $9, 11, 11, 12, 16$ $-$ Mode $= 11$, Median $= 11$, Mean $= \dfrac{59}{5}$

$= 11{\cdot}8$ ⊗

If $x = 12$ the numbers become $9, 11, 12, 12, 16$ $-$ Mode $= 12$, Median $= 12$, Mean $= \dfrac{60}{5} = 12$

If $x = 16$ the numbers become $9, 11, 12, 16, 16$ $-$ Mode $= 16$, Median $= 12$ ⊗

Therefore $x = 12$

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5 Solve for x, $1 - \dfrac{13}{x^2} = -5199$

Solution:

$x^2 - 13 + 5199x^2 = 0 \Longrightarrow 5200x^2 = 13$

$x^2 = \dfrac{13}{5200} = \dfrac{1}{400} \Longrightarrow x = \pm\dfrac{1}{20} = \pm 0{\cdot}05$

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6 A standard die with six faces is tossed onto a table. Michelle counts the total number of dots on the five faces that are not lying on the table. What is the probability that this total is at least 19?

Solution:

The total of all six faces is 21.

The total of the five faces without 6 is 15.

The total of the five faces without 8 is 16.

The total of the five faces without 4 is 17.

The total of the five faces without 3 is 18.

The total of the five faces without 2 is 19.

The total of the five faces without 1 is 20.

Therefore the $P(19 \text{ or more}) = \dfrac{2}{6} = \dfrac{1}{3}$

Tiebreaker

7 Simplify $\frac{(5^{16}-5^{15})}{4}$. Write your answer in the form a^b, where a is as small as possible and where a and b are natural numbers. The answer must be in its simplist form.

Solution:

$$\frac{(5^{16}-5^{15})}{4} = \frac{5^{15}(5-1)}{4} = \frac{5^{15}(4)}{4} = 5^{15}$$

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8 The length of the diagonal of a square is 10. What is the area of this square?

Solution:

Let the side length be x. The diagonal is the hypotenuse of a right-angled triangle.

$2x^2 = 100 \Longrightarrow x^2 = 50.$

The area of the square is 50 units^2.

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9 The year 2023 is not prime since $7 \times 17 \times 17 = 2023$.

What is the next numerically prime year?

Solution:

2027

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10 $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$

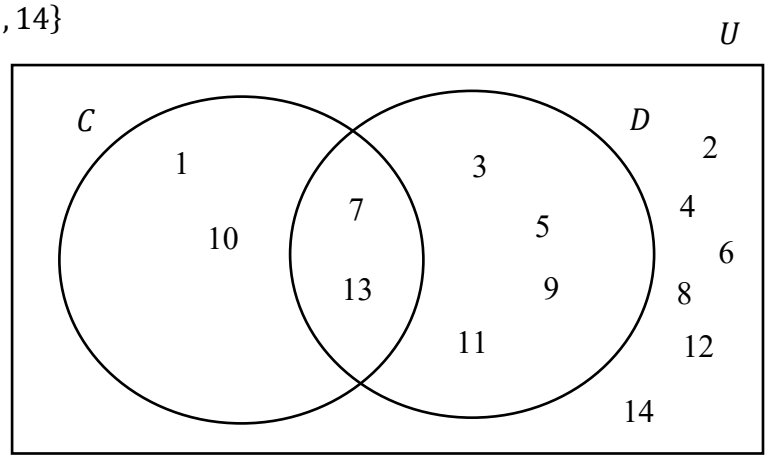
$C = \{1, 7, 10, 13\}$

$D = \{3, 5, 7, 9, 11, 13\}$

List the elements of $C' \cap D'$.

Solution:

$C' = \{2, 3, 4, 5, 6, 8, 9, 11, 12, 14\}$

$D' = \{1, 2, 4, 6, 8, 10, 12, 14\}$

$C' \cap D' = \{2, 4, 6, 8, 12, 14\}$