Round 1 Babtha 1

Question 1

Solve the following inequality where $x \in \mathbb{R}, x \neq 0$

$$\frac{4x+5}{x} \geq 3$$

Question 2

Find the equation of the circle with the following properties:

- Its centre is in the 1st quadrant
- The origin, $(0,0)$, is on the circle
- The circle cuts the y-axis at points that are 4 units apart
- The circle cuts the x-axis at points that are 8 units apart

Round 2 Babtha 2

Question 1

Find the value of the term which is independent of x in the expansion of

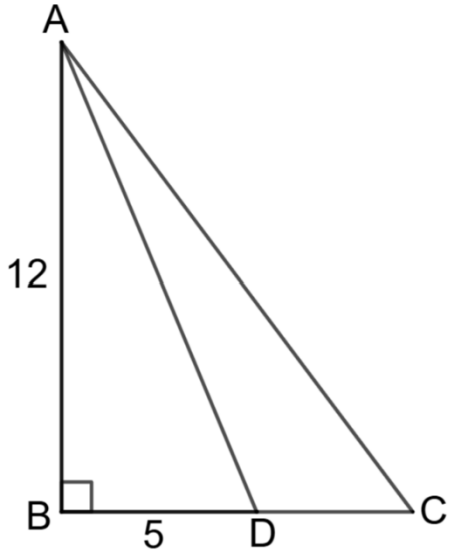
$$\left(x^2 - \frac{2}{x}\right)^9$$

Question 2

The triangle ABC is right angled at B, with $|AB| = 12$. Point D, on BC, is such that $|BD| = 5$.

The area of ΔADC is 80% of the area of ΔABD.

Determine the perimeter of triangle ADC.



Round 3 Babtha 3

Question 1

Express z in the form $2(a) + ai$, where $a \in \mathbb{Z}$, given that

$$z = \frac{30}{3 - i} - 3$$

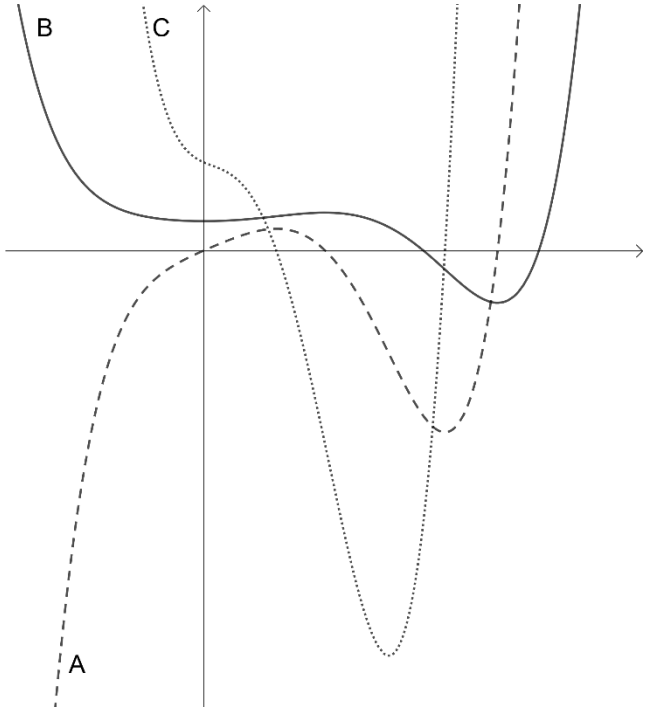
Question 2

A fair coin is tossed three times. What is the probability that the second toss was heads ***given that*** the first toss was heads?

Round 4 Babtha 4

Question 1

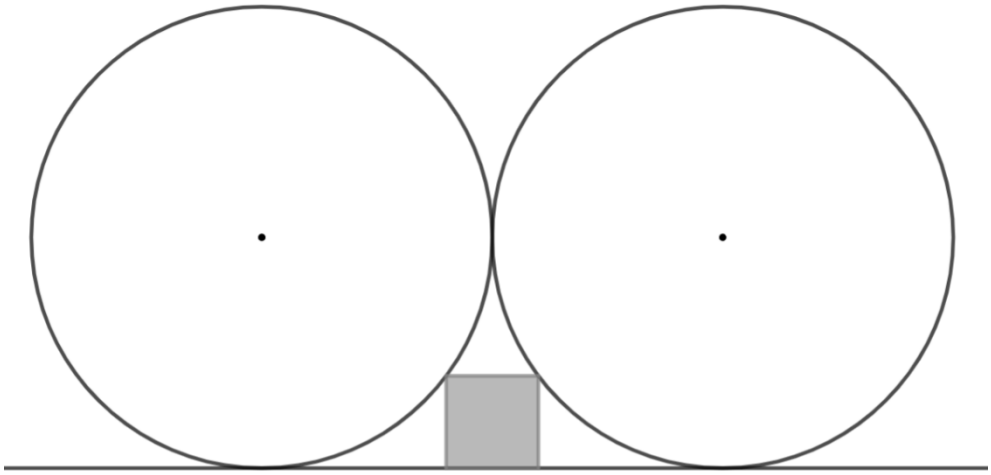
Part of the graph of a function $f(x)$, along with the graphs of both its first and second derivatives $f'(x)$ and $f''(x)$ are shown below.



The graphs are labelled A, B and C. Put these labels in order of $f(x)$, $f'(x)$ and $f''(x)$.

Question 2

Two tangential circles with equal length radius, r, and area 100π, are shown in the diagram.



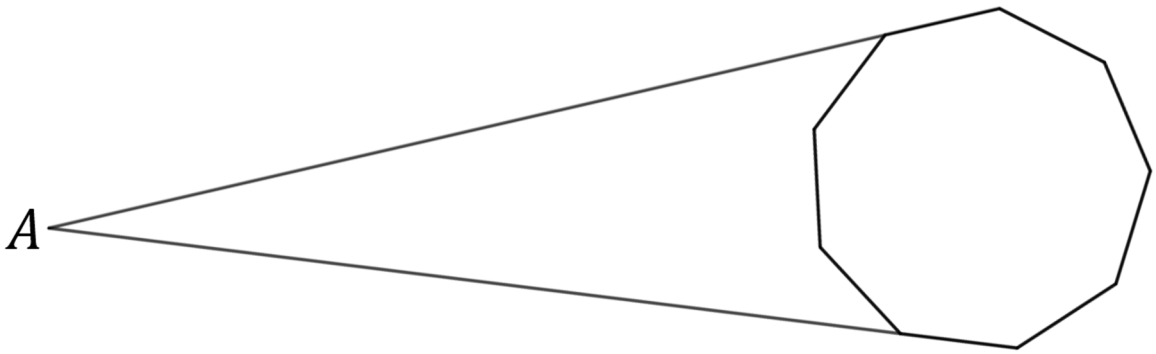
A square is drawn between the circles as shown, touching each circle at one point only.

Find the area of the square.

Round 5 Babtha 5

Question 1

The diagram shows a regular nonagon (nine-sided polygon) with two of the sides extended to meet at the point A. What is the size, in degrees, of the acute angle at A?



Question 2

Find the values of $x \in \mathbb{R}$ such that $|x|^2 + 4|x| - 5 = 0$.

Round 6 Babtha 6

Question 1

Let $f(x) = \sin^{2026} x + \cos^{2026} x$

$f'(x)$ can be written in the form:

$$\frac{2026}{k} \sin kx \left((\sin^b x)^k - (\cos^b x)^k\right)$$

Find the values of $b, k \in \mathbb{N}\backslash\{1\}$, such that b is as large as possible and k is as small as possible.

Question 2

Ciarán has been making two types of puzzles with wooden blocks: a smaller puzzle that contains exactly ***six*** blocks and a larger puzzle that contains exactly ***seven*** blocks.

Exactly 167 blocks in total were used to create a certain number of smaller and larger puzzles.

Determine ***all*** possibilities for how many of each type of puzzle Ciarán has made.

Round 7 Babtha 7

Question 1

Find the value of $n \in \mathbb{R}$ for which:

$$3 = -\log_n\left(\log_n\left(\sqrt{\sqrt{\sqrt{n}}}\right)\right)$$

Question 2

Ms White says that their friend, Mr Green, walks from his house into town at the rate of 5 km/h but walks back at only 3 km/h. The total journey – there and back – takes exactly 7 hours. What is the distance from Mr Green's house to the town, in km?

Question 3

The points $P(2,3)$ and $Q(4,3)$ are two vertices of the triangle PQR. The angle bisector at vertex Q is $x - 3y + 5 = 0$. The equation of the line through QR can be written in the form $ax - 4y + c = 0$. Find the value of $a^2 + c^2$.

Question 4

Let $3^a + 3^b = p$ and $3^a - 3^b = q$.

Express 3^{a+b} in terms of p and q.

Give your answer in its simplest form.

Round 8 Babtha 8

Question 1

Solve for $x \in \mathbb{R}$, giving your answer in its simplest form:

$$3^x = x^9$$

Question 2

Find all positive integers n, for $1 \leq n \leq 10$, such that:

$$n^{2025} + n^{2026}$$

is divisible by 5.

Question 3

Evaluate the following, where the angles are in degree measure, giving your answer in its simplest form:

$$(\tan 1°)(\tan 2°)(\tan 3°)(\tan 4°) \ldots . (\tan 89°)$$

Question 4

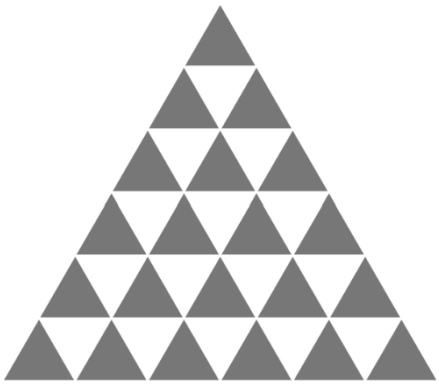
Consider the letters of the word COFFEE.

How many ***distinct*** arrangements of any size less than or equal to 6 letters can be made using each letter at most once, i.e. the total number of distinct 1 letter, 2 letter, 3 letter, 4 letter, 5 letter and 6 letter arrangements.

E.g. FOC, OFC, COF, FFE, FEF, EFF are all examples of possible 3 letter distinct arrangements.

Tie Break

1	What is common name of the number 10 raised to the power of 100?
2	A list of 5 natural numbers has mean 5, mode 5, median 5 and range 5. How many such lists are possible?
3	Evaluate $\lim_{x\to 9}\left(\frac{\sqrt{x}-3}{x-9}\right)$ giving your answer in its most simplified form.
4	A "micro-century" is one millionth of a century. How long, to the nearest minute, is a micro-century?
5	Find the image of the point $(3,4)$ under axial symmetry in the line $2x+4y-7=0$
6	Solve $4\sin^2 x + 2\cos x - 3 = 0$ where $0 \leq x \leq \pi$. Give your answers in their most simplified form.
7	If $2a+3, 2b+3$ are the roots of the equation $x^2-5x-2=0$, construct an equation with roots $a+2, b+2$. Give your answer in its simplest form of $px^2+qx+r=0$ with $p,q,r \in \mathbb{Z}$.
8	Consider the 4 integers: $2, 14, p, 2p-2$. If the standard deviation $\sigma=\sqrt{20}$, find the value of $p \in \mathbb{N}$.
9	A bowl contains raspberries and blackberries. 13 more blackberries are added to the bowl, resulting in making $\frac{4}{5}$ of the bowl ***now*** blackberries. 25 raspberries are *then* added to the bowl resulting in $\frac{3}{5}$ ***now*** raspberries. What fraction of the original berries in the bowl were raspberries?
10	How many regular hexagons are in the following shape:



ANSWERS

R1	Q1	$\{x \leq -5\} \cup \{x > 0\}$ OR $x \leq -5, x > 0$ Do NOT accept $0 > x \geq -5$ (AND indicated which is incorrect)
	Q2	$(x-4)^2 + (y-2)^2 = 20$ OR $x^2 + y^2 - 8x - 4y = 0$
R2	Q1	5376
	Q2	32
R3	Q1	$2(3) + 3i$
	Q2	$\frac{1}{2}$
R4	Q1	B, A, C
	Q2	16
R5	Q1	$20°$
	Q2	± 1
R6	Q1	$k = 2, b = 1012$
	Q2	(S, L): $(22,5)$; $(15, 11)$; $(8,17)$; $(1,23)$
R7	Q1	2
	Q2	$13.125\ km$
	Q3	9
	Q4	$\frac{p^2 - q^2}{4}$
R8	Q1	27
	Q2	$n = 4, 5, 9, 10$
	Q3	1
	Q4	522
Tiebreak	1	A googol.
	2	2
	3	$\frac{1}{6}$
	4	53
	5	$(0, -2)$
	6	$\frac{\pi}{5}, \frac{3\pi}{5}$
	7	$2x^2 - 7x + 2 = 0$
	8	6
	9	$\frac{5}{12}$
	10	11