Problem No. 1

There is a set of numbers which have the property that the number is unchanged under a rotation of 180^o. A student was given a problem to find the next numbers in the following sequence (which shows all the possible 1-digit and 2-digit numbers)

0, 1, 8, 11, 69, 88, 96, …

How many 3-digit numbers are there which have this property?

Solution No. 1

The following digits qualify to belong to numbers in the sequence: 0, 1, 6, 8, 9.

However, the complete number must obey the property of the sequence. For 3-digit numbers the following 12 numbers are the only ones which qualify.

101, 111, 181, 609, 619, 689, 808, 818, 888, 906, 916, 986

Answer: 12

Problem No. 2

If you were to write down in ascending order all of the numbers that are not factors of 480, what is the 10^{th} number on the list?

Solution No. 2

The factor pairs of 480 are: (1, 480), (2, 240), (3, 160), (4, 120), (5, 96), (6, 80), (8, 60), (10, 48), (12, 40), (15, 32), (16, 30), (20, 24)

The factors taken in order are: 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30, 32, 40, 48, 60, 80, 96, 120, 160, 240, 480.

The numbers not on the list are: 7, 9, 11, 13, 14, 17, 18, 19, 21, 22, …

The 10^{th} number on this list is 22.

Answer: 22

Problem No. 3

The difference between an interior angle of a regular polygon and an exterior angle of the same polygon is 100^o.

How many sides does the polygon have?

Solution No. 3

Let x and y represent the sizes of the angles.

Then $x + y = 180$ and $x - y = 100$. Adding, we get $2x = 280 \Rightarrow x = 140, y = 40$. Since an internal angle of 40^o for a regular polygon is impossible we see that the internal angle is 140^o.

Since the internal angle is the sum of the base angles of an isosceles triangle formed by a side of the polygon with the centre we see that the triangle has a 40^o angle at the centre. The full angle at the centre of the polygon is 360^o and since $\frac{360}{40} = 9$ there are 9 such triangles which may be formed at the centre and so the polygon has 9 sides.

Answer: 9

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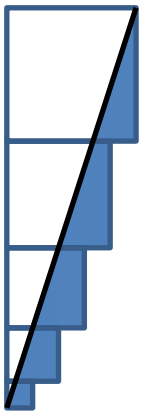
Problem No. 4
There are 10 boys in a mixed TY class of boys and girls. Two students are picked at random for the Student Council. If the probability that two boys are chosen is 0.15, how many girls are in the class? **Show all your work on a separate sheet.**

Solution No. 4
The probability of 0.15 may be written as $\frac{15}{100}$ or $\frac{3}{20}$. Let the number of students in the class be x. The probability that the first student chosen is a boy is $\frac{10}{x}$. The probability that the 2nd student chosen is a boy is $\frac{9}{x-1}$. The probability that the first is a boy and the second is a boy is $\frac{(10)(9)}{(x)(x-1)}$ which is equal to $\frac{3}{20}$. This leads to the equation $30(20) = x(x-1)$ which is $x^2 - x - 600 = 0$.
Solving, we get $(x-25)(x+24) = 0$ which gives x = 25 or x = -24 which is impossible.
Therefore, there are 25 students in the class. So, there must be 25 – 10 = 15 girls.

Answer: 15 girls

Alternative solution: The solution may be found by Trial and Error.
Suppose that there are 10 boys and 10 girls. Then the probability of choosing 2 boys is $\frac{10}{20} \times \frac{9}{19} = 0.236$. For 10 boys and 11 girls we get $\frac{10}{21} \times \frac{9}{20} = 0.214$. … For 10 boys and 15 girls we get $\frac{10}{25} \times \frac{9}{24} = 0.15$ which is the required value. Thus there are 15 girls in the class.

Problem No. 5
The shape shown consists of squares with sides of lengths 5, 4, 3, 2, 1 given in cm. Find the area of the shaded portion. Give your answer in cm^2.



Solution No. 5
The total area of the squares is 25 + 16 + 9 + 4 + 1 = 55. The triangle in white has a base of 5 and a height of 5 + 4 + 3 + 2 + 1 = 15. The area of the triangle is $5 \times \frac{15}{2} = 37.5$.. Therefore the shaded area is 55 – 37.5 = 17.5.
Answer: 17.5 cm^2.

Problem No. 6
A seamstress charges a bride €2180 including 23% VAT for supplying her wedding dress. The seamstress purchased the materials including VAT at 23%. She then charged twice this amount for her labour. How much did she pay for the materials before tax?
Give the answer correct to the nearest euro.

Solution No. 6
€2180 ÷1.23 = €1772.36 was the charge before VAT. This is 3 times the price paid for the fabric so the fabric cost €1772.36 ÷ 3 = €590.79 including VAT. The fabric cost €590.79÷1.23 = €480.32 before VAT. This is €480 correct to the nearest euro.
Answer: €480

Problem No. 7
A train from Cork to Dublin travelling at an average speed of 100 km per hour takes three hours to complete the journey. A malfunction on the train caused a delay on the journey, reducing the average speed to 90 km per hour.
How long was the delay in minutes?

Solution No. 7
Three hours at 100km per hour means that the distance is 3x100 = 300km. At an average speed if 90km per hour the train would take $\frac{300}{90} = 3\frac{1}{3}$ hours = 3hrs 20 minutes. The delay was 20 minutes.
Answer: 20 minutes

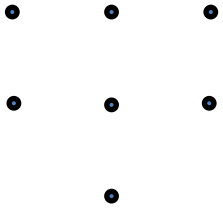
Problem No. 8
Three friends, Hans, Jake and Karl, train for a marathon by running on a dedicated circuit which has a fixed starting point. Hans takes 5 minutes to complete the circuit while Jake takes 7 minutes and Karl takes 11 minutes. Hans and Jake start together but Karl starts 1 minute later.
What is the number of minutes it takes so that the three runners cross the starting point together for the first time?

Solution No. 8
Hans completes 7 circuits in 35 minutes and Jake completes 5 circuits in 35 minutes so they will be together at the starting point every 35 minutes. That is, at 35, 70, 105, … minutes.
Karl passes the starting point after 1 minute (as he starts) and then every 11 minutes after that. That is at 12, 23, 34, … minutes. This is a number given by 11n + 1.
We can continue the sequences until they coincide.
The first sequence is 35, 70, 105, 140, 175, 210, …. To check which number satisfies 11n +1 we can find the remainders on division by 11 for each of these numbers which are: 2, 4, 6, 8, 10, 1. This means that they coincide after 210 minutes.
Or, we can (or simply continue the sequence 1, 12, 23, … 177, 188, 199, 210).
So, 210 minutes (or 3 hours and 30 minutes) is the shortest time until all three are together.
Note: Hans completes 42 circuits, Jake completes 30 circuits, Karl completes 19 circuits.
Answer: 210 minutes

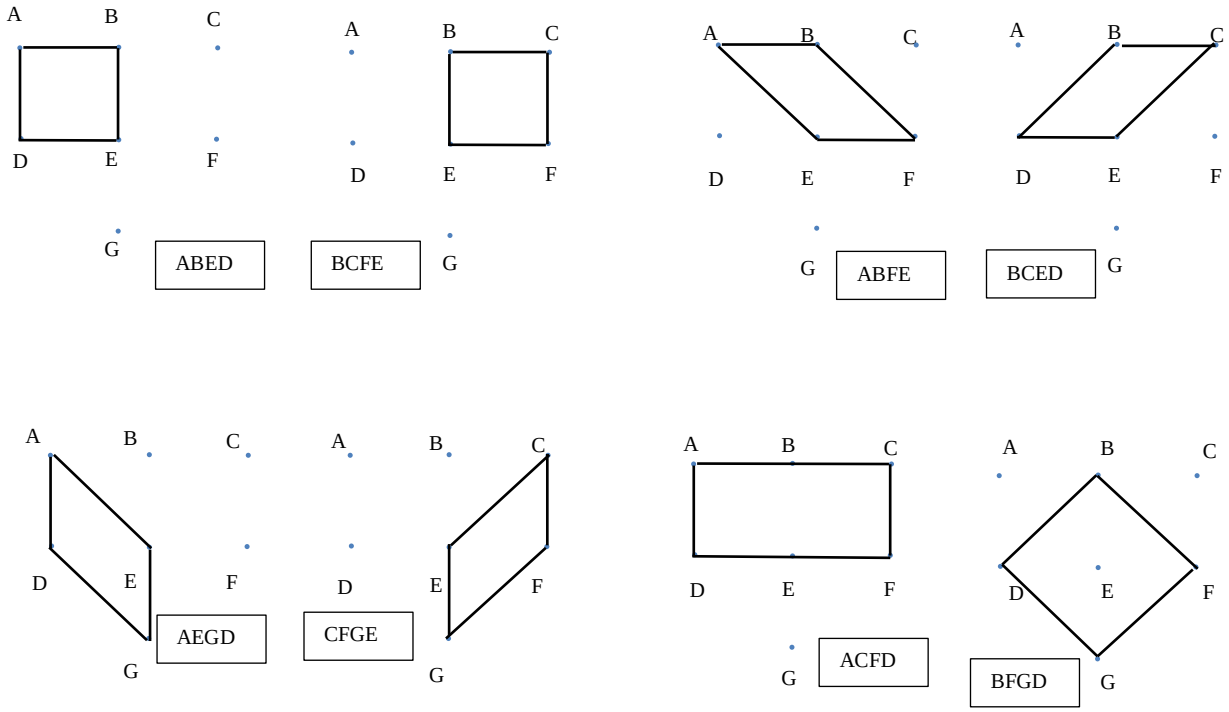
Problem No. 9

A diagram contains 7 points in two rows of 3 points and a centre column of 3 points. The points are equally spaced in both the rows and columns (as illustrated). The vertical spacing is the same as the horizontal spacing.

How many parallelograms may be formed by joining the points and using only these points as vertices of the parallelograms?



Solution No. 9



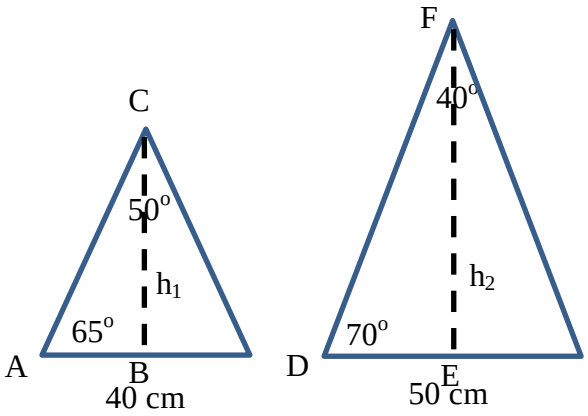
Answer: 8 parallelograms

Problem No. 10

Two isosceles triangles have apex angles of 50^o and 40^o respectively. Their bases are 40 cm and 50 cm respectively.

Which triangle (the first or the second) has the larger area and by how much? Give your answer in cm^2 correct to 1 place of decimals.

Solution No. 10



The base angles, respectively, are $\frac{180^o-50^o}{2} = 65^o$ and $\frac{180^o-40^o}{2} = 70^o$.

In triangle ABC, |AB| = 20 and $Tan65^o = \frac{h_1}{20}$ so $h_1 = 20Tan65^o$;

In triangle DEF, |DE| = 25 and $Tan70^o = \frac{h_2}{25}$ so $h_2 = 25Tan70^o$. Area of a triangle = $\frac{1}{2} basexheight$

The areas are, respectively, $20(20Tan65^o) = 857.8027682cm^2$ and $25(25Tan70^o) = 1717.173387cm^2$.

Thus, the triangle with the 50cm base is larger by $1717.173387 - 857.8027682 = 859.3706188\ cm^2$ $= 859.4cm^2$ correct to 1 decimal place.

Answer: The second triangle is larger by 859.4 cm^2.

Problem No. 11

The addition of four numbers is shown where each of the letters represents a digit. Find the **lowest** value of A + B + C + D.

$$\begin{array}{r} ABCD \\ BCD \\ CD \\ +\quad D \\ \hline 6666 \end{array}$$

Solution No. 11

In the following the carry digits are denoted by x, y, z. Adding the right-hand column, we get 4D = x6. So, Case 1: (D = 4, x =1) or Case 2: (**D= 9**, x = 3). Then, in Case 1: 3C + 1 = y6 so 3C = y5 giving C = 5, y = 1. This leads to 2B + 1 = 6 which is impossible.

or in Case 2: 3C + 3 = y6 so 3C = y3 giving **C = 1**, y = 0.

When y = 0 then 2B = z6 so Case 3: (**B = 3**, z = 0) or Case 4: (**B = 8**, z = 1).

In Case 3: **B = 3 so A = 6** or in Case 4: **B = 8 so A = 5**.

So, we have the values for ABCD = Case 3: 6319 or Case 4: 5819.

Then A+B+C+D = 6+3+1+9 = 19 or 5+8+1+9 = 23. The lowest value is 19.

Answer: 19

Problem No. 12
In a sequence of increasing natural numbers, the 3rd and subsequent terms are made up of the sum of the previous two terms. The eighth term is 207.
What is the value of the ninth term?

Solution No. 12
The terms of the sequence may be set out as a, b, (a+b), (a+2b), (2a+3b), (3a+5b), (5a+8b), (8a+13b), … The 8th term is (8a+13b) = 207.
This is a combination of multiples of 8 with multiples of 13. Since multiples of 8 are even the required multiple of 13 must be odd and less than 207. The odd multiples of 13 are 13, 39, 65, 91, 117, 143, 169, 195.
The difference between each of these and 207 must be a multiple of 8. The differences are 194, 168, 142, 116, 90, 64, 38, 12.
The multiples of 8 are (207 – 39) = 168 = 8(21) giving b = 3 and (207 – 143) = 64 = 8(8) giving b = 11. But the sequence starting 21, 3, … is not an increasing sequence. Therefore a = 8, b = 11.
We have: 8, 11, 19, 30, 49, 79, 128, 207, **335**.
The value of the 9th term is 335.

Answer: 335

Problem No. 13
A clock was programmed to have 80 minutes in a 'new' hour and 20 'new' hours in a 'new' day. If the clock was started at midnight on Tuesday what time would the clock show when an ordinary clock showed 12:00 on the following Thursday?

Solution No. 13
Midnight Tuesday to midday Thursday is 36 hours on the ordinary clock which is 36x60 = 2160 minutes. Then $\frac{2160}{80} = 27$ 'new hours' on the clock. This is 20 'new hours' + 7 'new hours' which is 1 'new day' + 7 'new hours'. The programmed clock will show 7:00 on Thursday as the time.

Answer: 07:00

Problem No. 14

A tide-clock does not display numbers. At the time of High Tide the hand of a tide-clock may be set at the 12 o'clock position which is marked as 'High Tide'. The 6 o'clock position is marked as 'Low Tide' which may also be set. The hand of the clock rotates once every 12h 25min 14sec. On a given day at 05:32 the pointer is set at the Low Tide position.

At what time will the pointer be at the High Tide position for the first time on the following day if it is not reset?

Give the answer on the 24-hour clock correct to the nearest minute.

Solution No. 14

Low Tide is set for 05:32 of Day 1. The tide clock will indicate the next High Tide at 05:32 + $\frac{12h25\ \text{min}\ 14sec}{2} = 5hr32\,\text{min} + 6hr12\,\text{min}\,37\,\text{sec} = 11hr44\,\text{min}\,37\ sec$ on Day 1. The next High Tide will be indicated for 11hr 44 min 37 sec + 12hr 25min 14sec = 24hr 9min 51sec of Day 1. But this has gone into Day 2 and is 00:10 on Day 2 to the nearest minute.

Answer: 00:10

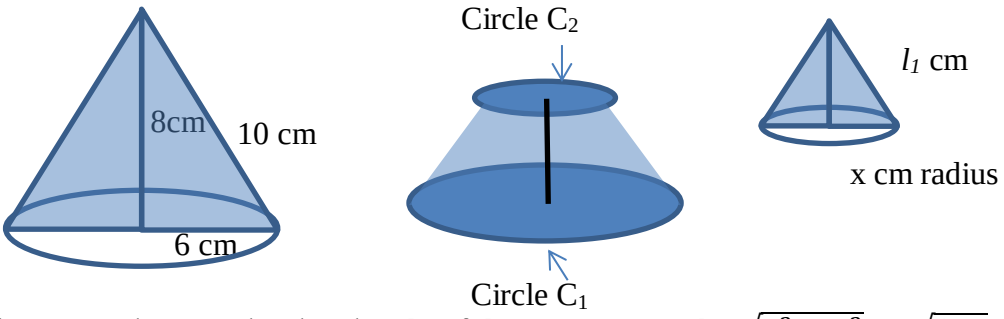
Problem No. 15

A right circular cone has radius 6cm and height 8cm. A frustum is obtained by slicing the cone perpendicular to its axis, forming a shape with two circles and a curved side.

If the area of the curved surface of the frustum is equal to the area of the two circles formed, what is the height of the frustum in cm given that it is an integer?

Show all your work on a separate sheet.

Solution No. 15



Using Pythagoras' Theorem the slant height of the cone is given by $\sqrt{6^2+8^2} = \sqrt{100}$ = 10 cm for the large cone.The slice removes a small cone from the top with radius x cm and slant height $l_1$cm.

The curved surface area of a cone is πrl which is $\pi(6)(10) = 60\pi$ cm^2 for the large cone (*). The area of circle C_1 is $\pi r^2 = \pi 6^2 = 36\pi$ cm^2; The area of circle C_2 is πx^2 cm^2 (**).

The CSA of the small cone is $\pi x l_1$ cm^2;

Then CSA of the large cone less CSA of the small cone = the sum of the areas of the two circles.

$$60\pi - \pi x l_1 = 36\pi + \pi x^2 \Rightarrow 24\pi = \pi x(x + l_1)$$

$$24 = x(x + l_1)$$

Since the lengths are integers we require factors of 24 which are 1(24); 2(12); 3(8); 4(6).

These may be written as 1(1+23); 2(2+10); 3(3+5); 4(4+2) to represent x(x+l_1).

Since x<6 and l_1 <10 we only consider (x=3, l_1 = 5) or (x = 4, l_1 = 2).

A radius of x = 4cm cannot combine with a slant height of l_1 = 2cm so the only possibility is x = 3, l_1 = 5. By Pythagoras' Theorem the height of the small cone is $\sqrt{5^2 - 3^2} = \sqrt{16} = 4$ cm.

Therefore, the height of the frustum is 8 – 4 = 4 cm.

Alternative solution: Let h be the height of the samall cone. By proportion $h = \frac{8}{6}x = \frac{4}{3}x$ and the slant height $l_1 = \frac{10}{6}x = \frac{5}{3}x$. The CSA of small cone = $\pi x \frac{5}{3}x = \frac{5}{3}\pi x^2$.

The CSA of the frustum = $60\pi - \frac{5}{3}x^2$ (using * above).

Then $36\pi + \pi x^2 = 60\pi - \frac{5}{3}x^2$ (using ** above) giving $36 + x^2 = 60 - \frac{5}{3}x^2$

$\Rightarrow x^2 + \frac{5}{3}x^2 = 60 - 36 = 24$. Hence, $\frac{8}{3}x^2 = 24 \Rightarrow x^2 = 9 \Rightarrow x = 3$ (taking positive measure).

Then the small cone height, $h = \frac{4}{3}(3) = 4$. The height of the frustum is 8 – 4 = 4cm.

Answer: 4 cm.

Problem No. 16

If it takes 4 gardeners 4 hours to dig 4 circular flower beds each of diameter 4 meters, how long will it take 6 gardeners to dig 6 circular beds each of diameter 6 metres, working at the same rate?

Solution No. 16

Each gardener digs 1 bed of flowers with diameter 4m in 4 hours. The bed has a radius of 2m and an area of 4π m^2. Thus, each gardener digs at a rate of 1π m^2 per hour.

Beds with a diameter of 6m have an area of 9π m^2. Working at the same rate a gardener will take 9 hours to dig this garden. Six gardeners will dig 6 of these gardens in 9 hours working at the same rate.

Answer: 9 hours.

Problem No. 17

Using all the digits 1 to 9 (only once for each) complete the table so that the totals given are correct. The calculations are done across from left to right and down from top to bottom. The symbols for calculation are used as they occur.

What three numbers fill the diagonal going from Top Left to Bottom Right?

	+		-		= 4
×		+		×	
	×		+		= 27
÷		-		-	
	÷		×		= 3
= 45		= 5		= 50	

Solution No. 17

9	+	3	-	8	= 4
×		+		×	
5	×	4	+	7	= 27
÷		-		-	
1	÷	2	×	6	= 3
= 45		= 5		= 50	

We examine the possibilities for forming each total.

45: 9x5÷1

50: 9x6-4; 8x7-6

From these two calculations we can fill in the left-most and right-most columns using 9x5÷1 and 8x7-6 or 7x8-6.

3: 1÷2x6; 1÷3x9 – we must use 1÷2x6

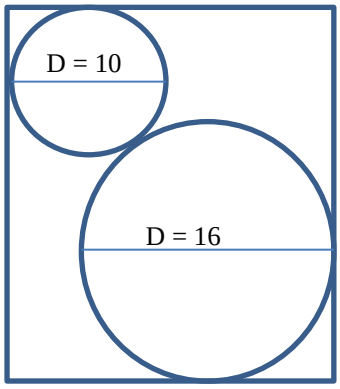
27: Using 5 only allows 5x4+7 so we can complete the middle row..

The final digit to use is 3 in the top middle cell which agrees with the totals of 4 and 5

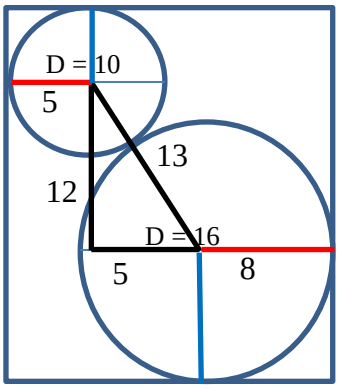
Answer: The three numbers on the diagonal from top left to bottom right are 9, 4, 6.

Problem No. 18

Find the height, h, of the rectangle. D is the diameter of the circle in each case and 'w' is the width of the rectangle. (Note: the given diagram is not to scale).



Solution No. 18

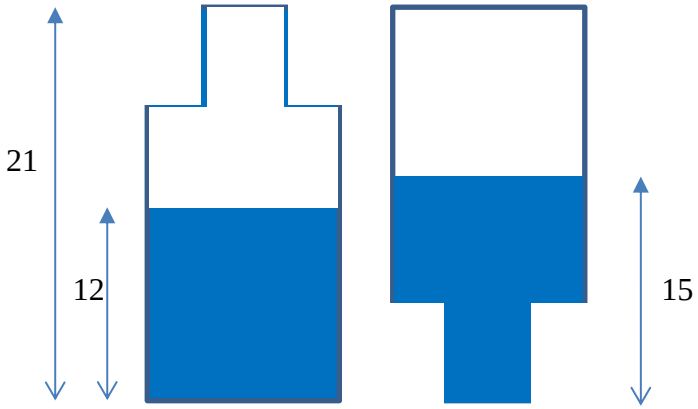


The line joining the two centres is the sum of the radii and has length 5 + 8 = 13. This is the hypotenuse of a right angled triangle. The width of the rectangle is composed of Radius 5 + Radius 8 + Base of the triangle. The total is 18. Therefore the base of the triangle is 18 – 5 – 8 = 5.
This shows us that we have the 5-12-13 right-angled triangle (by Pythagoras' Theorem) whose height is 12.
Therefore the height of the rectangle is the sum of the two radii + the height of the triangle.
That is, 5 + 8 + 12 = 25.

Answer: h = 25

Problem No. 19

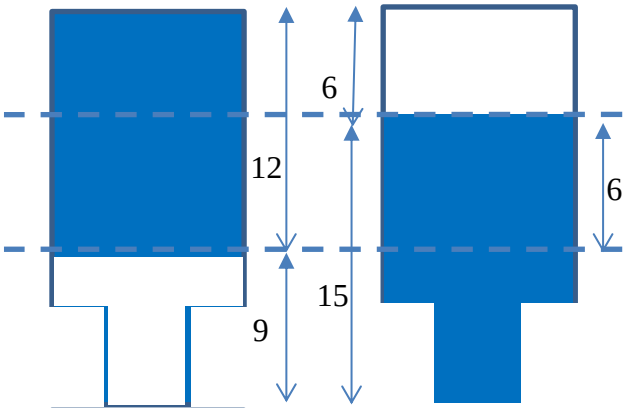
A container of water as shown is partially filled. What fraction of the container is filled?
The measurements are given in cm.



Solution No. 19

By inverting the container but not allowing the water to fall at first (left hand diagram) and then viewing it after it has fallen (right hand diagram) we can see that the volume of the 9cm section not filled on the bottom of the left hand diagram is equal to the volume of the empty 6cm section at the top of the right hand diagram. The volume of the 6cm section in the centre remains the same. Thus the container contains three sections with equal volumes to that in the 6cm section in the middle. Since the upright container has the volume of two of the 6cm sections it must be 2/3 filled.

Answer: $\frac{2}{3}$



Problem No. 20

In a high-rise car park the ramp angle may be up to 15^o. A small transition ramp with an angle of 6^o to its foundation is required at the beginning of the ramp in order that cars gradually reach this angle.

The length of the foundation of the transition ramp is given by the maximum wheelbase of a car (distance between the axels) which it can accommodate.

The wheelbase of a car may be related to its length by the following approximation formula:

$L = \frac{3}{4}(W + 2.3)$ where W = wheelbase length in m, L = Length of car in m.

An SUV has a Length of 4.8m. What is the minimum surface length of the transition ramp built on that foundation required to accommodate the SUV?

Give your answer in cm correct to the nearest cm.

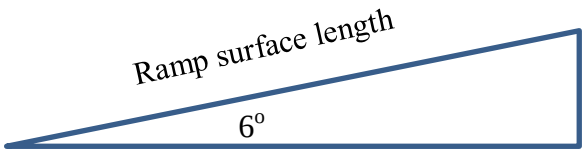
Solution No. 20

From $L = \frac{3}{4}(W + 2.3)$ we have $4.8 = \frac{3}{4}(W + 2.3)$ which gives $W = 4.1$.

Thus, the length of the foundation for building the transition ramp is 4.1m.

The ramp is at an angle of 6^o. So, we have $Cos(6^o) = \frac{4.1}{Ramp\ length}$.

This gives the ramp length as $\frac{4.1}{Cos(6^o)} = 4.1225\text{m} = 412\text{cm}$ correct to the nearest cm.



Foundation length = 4.1m

Answer: 412cm