Round 1 Babtha 1

Question 1

Find the ***exact*** value of the following infinite product:

$$\sqrt[3]{2025}\left(\sqrt[3]{\sqrt[3]{2025}}\right)\left(\sqrt[3]{\sqrt[3]{\sqrt[3]{2025}}}\right)\left(\sqrt[3]{\sqrt[3]{\sqrt[3]{\sqrt[3]{2025}}}}\right)\ldots$$

Question 2

When $x^3 + px^2 + qx + 1$ is divided by $x - 1$, the remainder is 9.

When $x^3 + px^2 + qx + 1$ is divided by $x + 2$, the remainder is -9.

Find the values of $p, q \in \mathbb{Z}$.

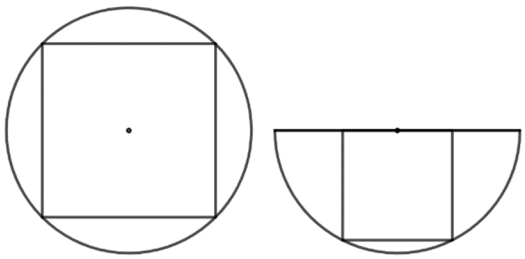
Round 2 Babtha 2

Question 1

In the decimal expansion of $\frac{1}{13}$, what is the 2025th digit?

Question 2

A circle and semicircle with equal radii are shown. Squares, of different sizes, are inscribed in each shape.



The area of the small square in the semicircle is x.

The area of the large square in the full circle is y.

Find the ratio $x : y$.

Round 3 Babtha 3

Question 1

Four-digit numbers are made using the numbers 1, 2, 3, 4 such that each number is used only once in the four-digit number e.g. 4213 or 3124. What is the sum of all such four-digit numbers?

Question 2

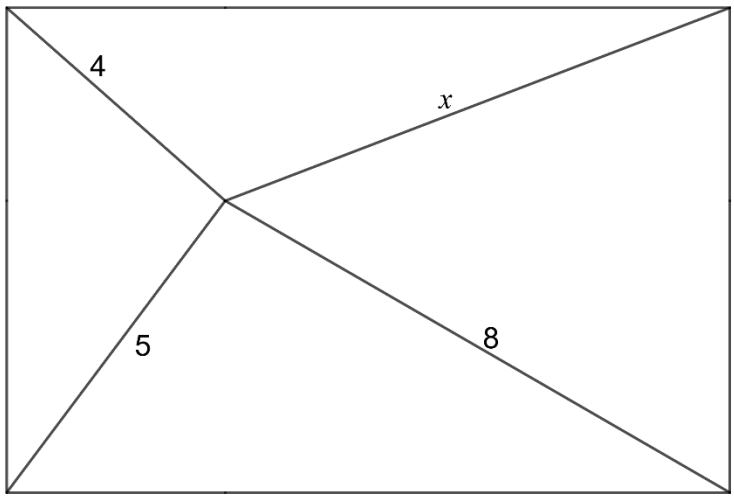
$1, \omega, \omega^2$ are the three roots of $z^3 - 1 = 0$, with $\omega, \omega^2 \neq 1$; $\omega, \omega^2 \in \mathbb{C}$. Evaluate:

$$(1(\omega)(\omega^2))^{2025}$$

Round 4 Babtha 4

Question 1

The figure shown is a rectangle. Find the value of x, giving your answer in surd form.



Question 2

Maryam tells only the truth on Mondays, Tuesdays, Wednesdays and Thursdays. She tells only lies on all the other days.

William tells only the truth on Fridays, Saturdays, Sundays and Mondays. He tells only lies on all the other days. On one day last week, they both said, "Yesterday I told lies."

On which day of the week was that?

Round 5 Babtha 5

Question 1

Find the values of $a, b \in \mathbb{N}, a < b$, such that:

$$\log_3 6! = a + a^a \log_3 a + \log_3 b$$

Question 2

Find the range of values of $k \in \mathbb{R}$ for which the following functions ***do not*** intersect:

$$f(x) = 3x^2 + kx - 2$$

$$g(x) = x^2 + 2x - 10$$

Round 6 Babtha 6

Question 1

Find the value of $x^2 \in \mathbb{R}$ such that $\cos(\tan^{-1} x) = x$.

Give your answer in the form:

$$x^2 = \frac{a + \sqrt{b}}{c}, \qquad a, b, c \in \mathbb{Z}.$$

Question 2

Let $f(x) = x^3 + kx^2 - \frac{1}{2}$.

Find the value of $k \in \mathbb{Q}$, for which $f(x) = 0$ has three real roots, two of which are equal.

Round 7 Babtha 7

Question 1

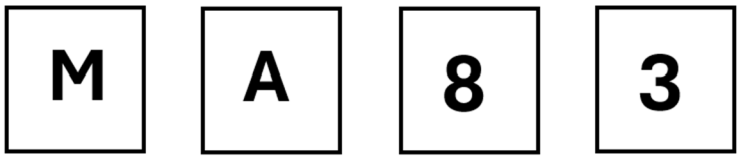
Solve the simultaneous equations for $x > 0, y > 0 \in \mathbb{R}$, giving your solutions as integers and/or surd form, if needed:

$$x^{\frac{1}{2}} + y^{\frac{2}{3}} = 6$$

$$x^{-\frac{1}{2}} + y^{-\frac{2}{3}} = \frac{3}{4}$$

Question 2

Each of the cards shown has a letter on one side and a number on the other.



Which cards **must** you turn over to test the following rule:

"Any card with an A on one side has a 3 on the other side. "

Question 3

The points $A(-4, 3)$ and $B(-3, -4)$ are points on the circle k: $x^2 + y^2 = 25$.

The perpendicular bisector of AB cuts the circle k at points P, in the first quadrant and Q, in the third quadrant.

Calculate $|PQ|$.

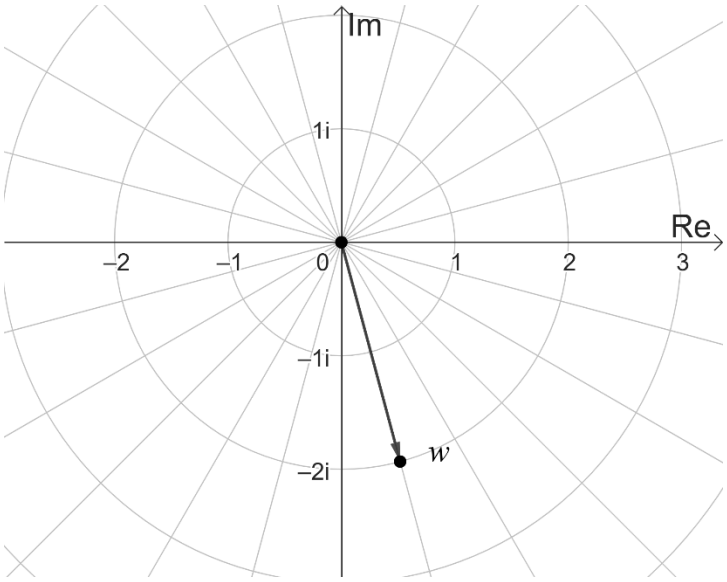
Question 4

One of the three roots, w, of a complex equation:

$z^n = a + bi$, where $a, b \in \mathbb{R}$

is plotted and shown on the diagram, which **is to scale**.

Find the ***original equation in surd form***.



Round 8 Babtha 8

Question 1

The roots of the equation $x^2 + (4p - 5)x + p = 0$ are consecutive integers. Find all possible values of p.

Question 2

How many different 4-digit codes are there if you cannot have the digit 1 **immediately** followed by the digit 3 (with repeated digits allowed). E.g. 0055 allowed but 8139 not allowed.

Question 3

A six-digit number is formed by taking a three-digit number and repeating it. E.g. 376 generates 376376 and 211 generates 211211. List any prime numbers that divide ***all*** such possible six-digit numbers.

Question 4

Find the exact value of:

$$\frac{1}{\sqrt{2025 + \sqrt{2025^2 - 1}}}$$

Give your answer in the form:

$\sqrt{b} - \sqrt{a}$, where $b > a;\ a, b\ \in \mathbb{N}$

Answers

R1	Q1	45
	Q2	$p = 2, q = 5$
R2	Q1	6 Or 7
	Q2	$2:5$
R3	Q1	66,660
	Q2	1
R4	Q1	$\sqrt{55}$
	Q2	Friday
R5	Q1	$a = 2, b = 5;$ Or $a = 1, b = 240;$
	Q2	$-6 < k < 10$
R6	Q1	$\frac{-1 \pm \sqrt{5}}{2}$
	Q2	$\frac{3}{2} = 1.5$
R7	Q1	$(16, 2\sqrt{2}); (4,8)$
	Q2	A; 8
	Q3	10
	Q4	$z^3 = -4\sqrt{2} + 4\sqrt{2}i$
R8	Q1	2
	Q2	9701
	Q3	$7, 11, 13$
	Q4	$\sqrt{1013} - \sqrt{1012}$