Problem No. 1
N is a number between 1 and 20, such that the sum of the numbers from 1 to N equals the sum of the numbers from N+1 to 20. What is the value of N?

Solution to No. 1
The sum of numbers from 1 to 20 is 210. Half that sum is $\frac{210}{2} = 105$. The sum of numbers from 1 to 14 = 105 so N = 14.
Alternative: By trial and error: 1+2+3+4+5+6+7+8+9+10+11+12+13+14 = 105
20+19+18+17+16+15 = 105
Alternative: By formula: $\frac{n}{2}(n+1) = \frac{20}{2}(21) - \frac{n}{2}(n+1)$ leads to $n^2 + n - 210 = 0$ and hence $(n-14)(n+15) = 0$. Then n = 14 or n = – 15. Therefore, n = 14 since the answer is a positive number.
Answer: N = 14.

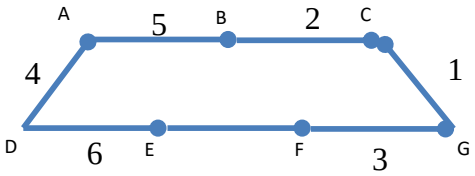
Problem No. 2
A list of 5 positive integers has mean 5, mode 5, median 5 and range 5. How many different ordered lists may be formed?

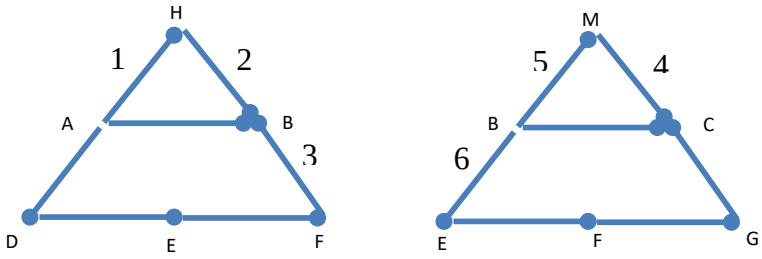
Solution to No. 2
Let the list be a, b, c, d, e. Since the median is 5 then c = 5. Since the mode is 5 there must be at least two 5's. We now have a, 5, 5, d, e or a, b, 5, 5, e.
Since the mean is 5 the total must be 25. If there were five 5's then the range would not be 5.
If there were four 5's then they add to 20 and the fifth number must also be 5 so that is disqualified as the range is not 5.
If there were three 5's then the other two numbers must add to 10. Since the range is 5 the maximum can be 10 or the minimum 0. For 0+10, 1+9 and 2+8 the range is too large. For 3+7 and 4+6 the range is too small. The only possibility is 2.5 + 7.5 but these are not integers.
Thus there must be only two 5's. The other three numbers must add to 15 without any repetition of numbers and a range of 5. The choices are 2+6+7 = 15 or 3+4+8 = 15.
When the list of numbers is in order we have 2, 5, 5, 6, 7 and 3, 4, 5, 5, 8.
Answer: 2 lists

Problem No. 3

Seven matchsticks of equal length are placed end-to-end, as illustrated, to form a rhombus.
By relocating 3 matchsticks, exactly 2 equilateral triangles may be formed (all 7 matchsticks are used).
In how many ways may this be done?

Solution to No. 3





One slant matchstick and one each from the top and bottom rows may be taken from the right-hand side (CG, BC, FG) and used to form the 2 triangles as shown (on the left diagram, above) which are Triangle HAB and Triangle HDF.
Similarly, one slant and one from the top and bottom rows may be taken from the left-hand side (AD, AB, DE) to form the 2 triangles Triangle MBC and Triangle MEG (on the right diagram, above).
Thus, initially, there are 2 ways of forming the triangles.

A consideration: Since each match has a head and tail there are 2 ways of placing each match giving 2x2x2 = 8 ways for each position. Thus there are ae 8x2 = 16 ways to form the triangles.

Another consideration: If the matches are considered to be distinguishable then there are 6 ways [(1-2-3); (1-3-2); (2-1-3); (2-3-1); (3-1-2); (3-2-1)] of placing the three from the right and similarly for the three on the left (for 4, 5 and 6) giving 6x2 =12 ways to form the triangles.

Furthermore, if both these considerations are taken together there are 8x6x2 = 96 ways to form the triangles.

The question is not clear whether these considerations should be taken into account or not and so different answers may be correctly given.

Answer: Either of 2, 12, 16, 96.

Problem No. 4

If you were to write down all of the numbers that could be produced by changing one digit in the number 300, how many of the numbers formed are prime?

Solution to No. 4

Changing the leading (1^{st}) digit will form numbers ending in 00. E.g. 100. None of these are prime.
Changing the middle (2^{nd}) digit will form numbers ending in 0. None of these are prime.
Changing the ending (3^{rd}) digit will give numbers from 301 to 309. The only possible primes are 301, 307 since the others are either even or divisible by 3.
301 is divisible by 7 (7x43 = 301) so it is not prime.
307 is not divisible by 2, 3, 5, 7, 11, 13, 17 ($\sqrt{307} = 17.52$ so no prime divisor above 17 need be tested).
Therefore 307 is prime.
There is 1 prime number.
Answer: 1.

Problem No. 5

$2025 = a^b.c^d$. What is the minimum value of a+b+c+d? (where a, b, c, d are natural numbers)

Solution to No. 5

2025 = 5.5.3.3.3.3.

	a	b	c	d	a+b+c+d
$5^2.3^4$	5	2	3	4	14
$5^2.9^2$	5	2	9	2	18
$15^2.3^2$	15	2	3	2	22
Other possibilities which could be considered					
$25^1.3^4$	25	1	3	4	33
$45^2.1^1$	45	2	1	1	49
$45^1.45^1$	45	1	45	1	92
$2025^1.1^1$	2025	1	1	1	2028

Obviously, using any other combinations which involve a power of 1 will have a larger base number and will not provide a minimum value.
The minimum value is 14.

Answer: 14

Problem No. 6

A school TY class decides to design and sell school hoodies at Christmas as their business venture. They can buy hoodies at €30 each which they can purchase in bundles which are multiples of 20. Their research tells them that they will sell 100 hoodies only if they are priced at €75 each. They also find that each time they increase the price by €5 twenty fewer hoodies will be sold and that each time they reduce the price by €5 twenty more hoodies will be sold.
How many hoodies should be bought in order to maximise profit if only one order is made?

Solution to No. 6

Buy bundle of	Cost @ €30ea	Sell for /Revenue	Profit = Revenue – Cost
80	2400	€80: 6400	4000
100	3000	€75:7500	4500
120	3600	€70: 8400	4800
140	**4200**	**€65: 9100**	**4900**
160	4800	€60: 9600	4800
180	5400	€55: 9900	4500

We see that maximum profit is made by buying 140 tracksuits and selling them at €65 each.

Note: Although this problem may be solved by setting up a continuous quadratic function the information on pricing indicates that separate steps are involved and so it may be solved as above.

Answer: 140

Problem No. 7
What is the maximum number of times that "Friday, the 13th of a month" can occur in a non-leap year?

Solution to No. 7
The number of days from a date in one month to the same date in the next month (e.g. from 13th Jan to 13th Feb) is given on the following list:

J-F	F-M	M-A	A-M	M-J	J-J	J-A	A-S	S-O	O-N	N-D
31	28	31	30	31	30	31	31	30	31	30

The day of the week which occurs one month later will be changed by the excess of division by 7 on these numbers. Thus we have the following list of movement of days of the week for each following month: (e.g. if Jan 13th is on Friday then Feb 13th will be on Monday, 3 days-of-week later)

J-F	F-M	M-A	A-M	M-J	J-J	J-A	A-S	S-O	O-N	N-D
+3	+0	+3	+2	+3	+2	+3	+3	+2	+3	+2

If there is a Friday 13th in some month then there will be another if the cumulative following numbers (of excess on division by 7) add to a multiple of 7.
Cumulative totals if Friday 13th occurs in a given month:

Friday on:	J-F	F-M	M-A	A-M	M-J	J-J	J-A	A-S	S-O	O-N	N-D
Jan 13; then Oct 13	3	3	6	8	11	13	16	19	**21**	24	26
Feb 13; then Mar 13; then Nov 13		**0**	3	5	8	10	13	16	18	**21**	23
Apr 13; then Jul 13				2	5	**7**	10	13	15	18	20
May 13					3	5	8	11	13	16	18
Jun 13						2	5	8	10	13	15
Aug 13								3	5	8	10
Sep 13; then Dec 13									2	5	**7**

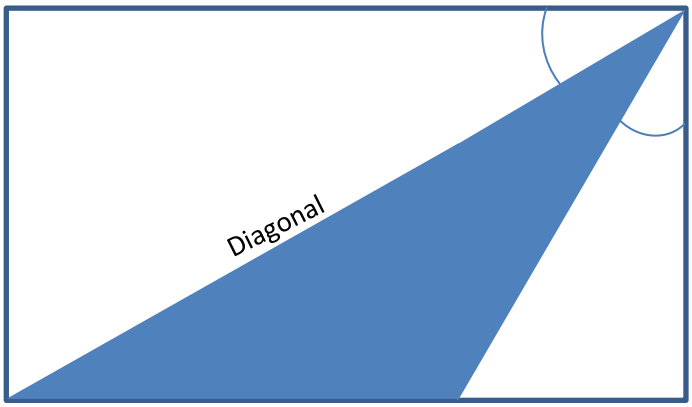
The multiples of 7 are highlighted.
The maximum number occurs when we have
Friday February 13th followed by Friday March 13th followed by Friday November 13th.

Thus, "Friday, the 13th of a month" can occur a maximum of 3 times in a non-leap year.

Answer: 3

Problem No. 8

A 90^o angle is trisected and a rectangle formed so that one of the trisectors is a diagonal (as shown). Find the ratio of the area of the shaded triangle to the area of the rectangle in its simplest form.



Solution to No. 8

See Diagram: Let |BC| = 1. Since Angle CAB = 30^o. Tan $30^o = \frac{|BC|}{|AB|} = \frac{1}{\sqrt{3}}$ so |AB| = $\sqrt{3}$.

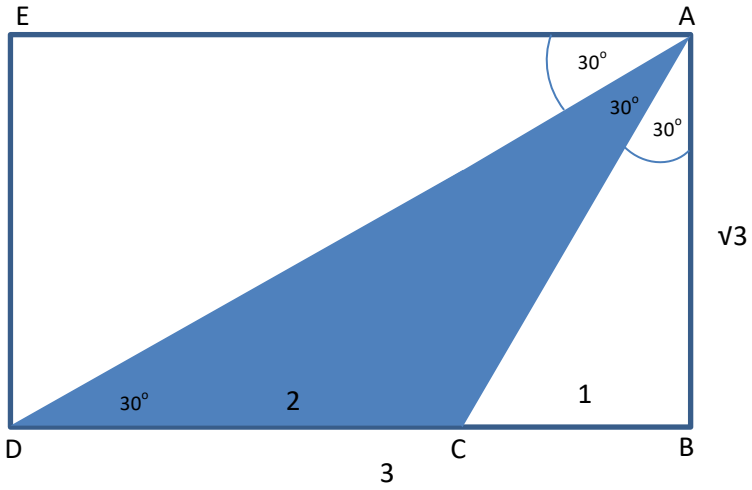
Angle BDA = 30^o so $\frac{|AB|}{|BD|} = \frac{\sqrt{3}}{|BD|} = Tan30^o = \frac{1}{\sqrt{3}}$. Therefore |BD| = 3 and so |CD| = 2.

Area Rectangle ABDE = |BD|.|AB|= $3\sqrt{3}$.

|CD| = 2 so Area Triangle ACD = $\frac{|CD|}{2}\sqrt{3} = \frac{2\sqrt{3}}{2} = \sqrt{3}$.

The ratio of areas is Area Triangle ACD : Area Rectangle ABDE = $\sqrt{3} : 3\sqrt{3}$ which is 1:3 in simplest form.

Answer: 1:3



Problem No. 9

The numbers on many phones which are not smart-phones (sometimes known as button phones) use a keypad with the **ITU E.161 standard**. This may allow text messages to be sent on these phones.
A student presses two different such keys (of those which produce letters) at random giving only 3 presses in total.
What is the probability that the combination of letters produced starts with a vowel?
Give your answer fully in decimal form.

Solution to No. 9

See Telephone keypad - Wikipedia or E.161 - Wikipedia

	Option 1	Option 2	Option 3	Option 4	Option 5	
1st key/ 2nd key/ 1st key	Single press/ Double press	Double press/ Single press	Single press-Single press/ Single press	Single press/ Single press-Single press	Single press/ Single press/ Single press 1st key again	Grand totals
	ae;ah;ak;an; aq;au;ax db;dh;dk;dn; dq;du;dx gb;ge;gk;gn; gq;gu;gx jb;je;jh;jn; jq;ju;jx mb;me;mh; mk;mq;mu;mx pb;pe;ph;pk; pn;pu;px tb;te;th;tk;tn; tq;tx wb;we;wh; wk;wn;wq;wu	bd;bg;bj; bm;bp;bt;bw **ea;eg;ej;em; ep;et;ew** ha;hd;hj;hm; hp;ht;hw kq;kd;kg;km; kp;kt;kw na;nd;ng;nj; np;nt;nw qa;qd;qg;qj; qm;qt;qw **ua;ud;ug;uj; um;up;uw** xa;xd;xg;xj; xm;xp;xt	**aad;aag;aaj;aam; aap;aat;aaw** dda;ddg;ddj;ddm; ddp;ddt;ddw gga;ggd;ggj;ggm; ggp;ggt;ggw jja;jjd;jjg;jjm jjjp;jjt;jjw mma;mmd;mmg;mmj; mmp;mmt;mmw ppa;ppd;ppg;ppj; ppm;ppt;ppw tta;ttd;ttg;ttj; ttm;ttp;ttw wwa;wwd;wwg;wwj; wwm;wwp;wwt	**add;agg;ajj;amm; app;att;aww** daa;dgg;djj;dmm; dpp;dtt;dww gaa;gdd;gjj;gmm; gpp;gtt;gww jaa;jdd;jgg;jmm; jpp;jtt;jww maa;mdd;mgg;mjj; mpp;mtt;mww paa;pdd;pgg;pjj; pmm;ptt;pww taa;tdd;tgg;tjj; tmm;tpp;tww waa;wdd;wgg;wjj; wmm;wpp;wtt	**ada;aga;aja;ama; apa;ata;awa** dad;dgd;djd;dmd; dpd;dtd;dwd gag;gdg;gjg;gmg; gpg;gtg;gwg jaj;jdj;jgj;jmj; jpj;jtj;jwj mam;mdm;mgm; mjm;mpm;mtm;mwm pap;pdp;pgp;pjp; pmp;ptp;pwp tat;tdt;tgt;tjt; tmt;tpt;twt waw;wdw;wgw;wjw; wmw;wpw;wtw	
Total	56	56	56	56	56	280
Vowel begins	7	14	7	7	7	42

Since two keys are pressed no single key is pressed three times. The key-pad may be set to lower case or upper case but cannot be switched between cases. The options can be set out as in the table.
The letters which can be accessed using either one or two key presses are shown in the picture given. There are 8 letters in each case. One key press (8 letters): a, d, g, j, m, p, t, w. This set contains one vowel, 'a'.
Two (quick) key presses (8 letters): b, e, h, k, n, q, u, x. This set contains two vowels, 'e' and 'u'.
There are 280 possibilities in the sample space with 42 favourable outcomes.
The probability of a favourable outcome happening at random is $\frac{42}{280} = \frac{3}{20} = 0.15$
Answer: 0.15.
Note: Having no time-delay for a double key-press allows for the inclusion of Options 1 and 2 only. This leads to the answer $\frac{21}{112} = \frac{3}{16} = 0.1875$. By including a time-delay Options 3, 4 and 5 may be included giving the result above.

Problem No. 10
Hugo and Naoise are members of their local youth club. Both take part in a chess competition where they win a number of league points for a win, a lesser number for a draw and no points for a loss. After 10 games, Hugo has scored 44 points for winning 7 and drawing 3 of his games. Meanwhile, Naoise has won 5, drawn 3 and lost 2 of her games.
How many points has Naoise scored?

Solution to No. 10
Let x = no. of points for a win and y = no. of points for a draw. To reach 44 we have $7x + 3y = 44$. The possibilities are $7(5) + 3(3)$ or $7(2) + 3(10)$. Since a win obtains more points than a draw the only possibility is a win is worth 5 points and a draw is worth 3 points.
Therefore Naoise scored $5(5) + 3(3) = 25 + 9 = 34$.

Answer: 34 points

Problem No. 11

In a football (soccer) game the home team won by 5 goals to 4. The home team scored first and were never behind in the game. In how many ways could the goals have been scored?

Solution to No. 11

1	2	3	4	5	6	7	8	9
H	H	H	H	H	A	A	A	A
				A	H	A	A	A
					A	H	A	A
						A	H	A
							A	H
			A	H	H	A	A	A
					A	H	A	A
						A	H	A
							A	H
				A	H	H	A	A
						A	H	A
							A	H
					A	H	H	A
							A	H
		A	H	H	H	A	A	A
					A	H	A	A
						A	H	A
							A	H
				A	H	H	A	A
						A	H	A
							A	H
					A	H	H	A
							A	H
			A	H	H	H	A	A
					A	H	A	A
						A	H	A
							A	H
				A	H	H	A	A
						A	H	A
							A	H
					A	H	H	A
							A	H
	A	H	H	H	H	A	A	A
					A	H	A	A
						A	H	A
							A	H
				A	H	H	A	A
						A	H	A
							A	H
					A	H	H	A
							A	H
			A	H	H	H	A	A
					A	H	A	A
						A	H	A
							A	H
				A	H	H	A	A
						A	H	A
							A	H
					A	H	H	A
							A	H

H = Home team score
A = Away team score

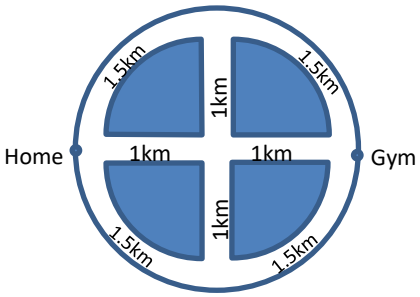
1 score	2 scores	3 scores	4 scores	5 scores	6 scores	7 scores	8 scores	9 scores
1-0 →	2-0 →	3-0 →	4-0 →	5-0 →	5-1 →	5-2 →	5-3 →	5-4
↘	1-1 →	2-1 →	3-1 →	4-1 →	4-2 →	4-3 →	4-4	
		↘	2-2 →	3-2 →	3-3			

There are 19 different scores which could appear on the scoreboard. There are 42 different ways of going through the scores from 1-0 to 5-4.

For instance, to get to 2-1 we could have 1-0, 2-0; 2-1 where 1-1 does not appear or 1-0, 1-1, 2-1where 2-0 does not appear.

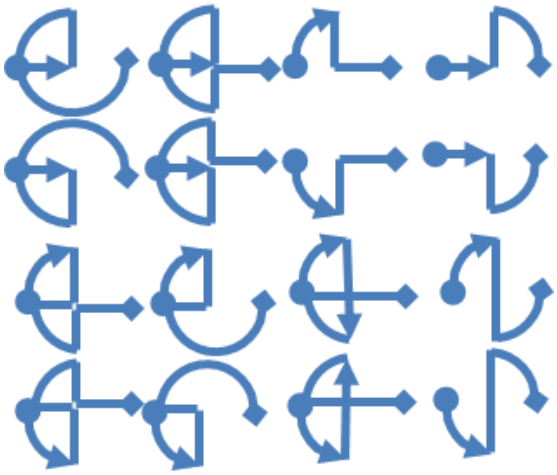
Answer: 42

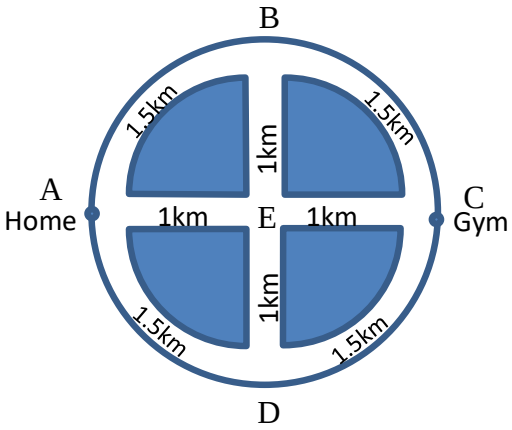
Problem No. 12



Every day a jogger runs from Home to the Gym (see Diagram). There are 4 straight roads, each 1km in length, and 4 curved roads, each 1.5km in length.
His coach requires him to run **more** than 3km each day.
He decides to run a different path each day but never travelling twice on the same stretch of road on a given day. Any intersection point may be passed more than once except that when he reaches the Gym he must stop and enter.
On how many days can he run using a different route each day?

Solution to No. 12
He can run a different route each day for 16 days.





Label each of the intersection points as A, B, C, D, E as shown.
The paths which can be taken are as illustrated in the arrow graph:

AEBADC;	AEDABC;	ABEADEC;	ADEABEC
AEBADEC;	AEDABEC;	ABEADC;	ADEABC
ABEC;	ADEC;	ABEDAEC;	ADEBAEC
AEBC;	AEDC;	ABEDC;	ADEBC

Answer: 16

Problem No. 13
There are 20 sweets on a table. In a game two players take turns to choose some sweets, but must choose at least one but never more than half of what remains. The loser is the one who has no valid move. How many sweets should the first player choose on the first move to ensure that they may always win the game?
Give your explanation on a separate sheet.

Solution to No. 13
No valid move can be made when 1 sweet remains on the table since if 1 sweet is taken then it is more than half of what remains and the move is invalid. The winner must aim to leave just one sweet for their opponent.
This may be achieved by following the strategy shown in the table below.
If any number other than 5 is chosen by the first player then their opponent has the opportunity to follow the winning strategy. E.g. if 10 sweets are taken by the first player then the second player can choose 3 to leave 7 on the table and hence be in a position to win.
Note: With 15 on the table (Player B + Player A) total take = 8; with 7 on the table (Player B + Player A) total take = 4.

On the table	Player A can choose	Remaining on the table allowing Player A to win	Player B can choose
START 20	5	15	1, 2, 3, 4, 5, 6, 7
14, 13, 12, 11, 10, 9, 8	7, 6, 5, 4, 3, 2, 1	7	1, 2, 3
6, 5, 4	3, 2, 1	3	1
2	1	1	None – lose the game

Answer: 5

Problem No. 14
At a fast-food outlet chicken nuggets are sold in boxes of either 6, 9 or 20 nuggets. What is the largest number of nuggets that cannot be purchased using these boxes?

Solution to No. 14
Many small numbers of nuggets cannot be purchased. E.g. 7 nuggets. Testing numbers individually will identify which can or cannot be purchased using multiples of 6, 9 or 20 in combination.
E.g 42 = 7 boxes of 6;
Note that 43 cannot be purchased;
Then 44 = 4 boxes of 6 and 1 box of 20. Furthermore, once it is established that the subsequent set of 6 numbers after 43 (44 to 49) may be purchased it may be seen that any higher numbers may be formed by the addition of 6 with these numbers and thus the highest number which cannot be purchased is 43.
44 = 1(20) + 4(6); 45 = 5(9); 46 = 2(20) + 1(6); 47 = 1(20) + 3(9); 48 = 8(6); 49 = 2(20) + 1(9).

Answer: 43

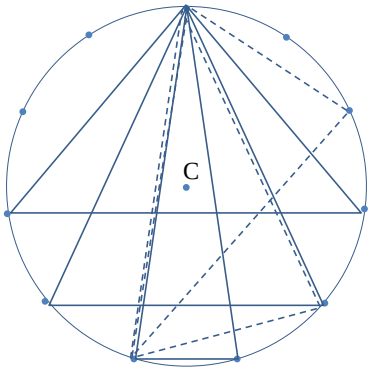
Problem No. 15
Use each of the numbers 2, 3, 5, 7, 7, 9 once only and any mathematical symbols to make a total of 185.

Solution to No. 15
Some possibilities:
9x7x3 -7 – 2 + 5 = 185. or 9(7+7+3x2)+5 = 185 etc.

Problem No. 16
A regular 11- sided polygon is inscribed in a circle, centre C. Triangles are drawn using 3 of the polygon's vertices to be vertices of each of the triangles. How many of the triangles formed will have the centre, C, within the triangle?

Solution to No. 16

The vertices of the polygon lie on a circle. To aid viewing the edges of the polygon are not shown here.



Using one fixed vertex we can establish how many triangles may be formed from this point and which contain the centre. As illustrated, 5 different triangles may be formed. This may be seen by counting the number of segments separating the vertices in each case. Thus, we have (3, 5, 3); (4, 3, 4); (5, 1, 5); (5, 2, 4); (5, 4, 2). Other choices are merely rotations of these to use another point as the fixed vertex. Since this can be done in 11 ways there is a total of 11x5 = 55 triangles which may be formed.

Alternative method: In order to form a triangle which contains the centre two vertices of the triangle must lie in the same semicircle. For an 11-gon 5 segments can lie in the same semicircle. Thus, to form triangles we can have two vertices of the triangle base separated by either 1, 2, 3, 4 or 5 segments.
For a separation of 1 segment for the first two vertices both points can be separated by 5 segments from the third vertex allowing for 1 triangle to be formed. Since there are 11 segments this may be done in 11 ways.
For a separation of 2 segments a separation of 5 from one vertex and 4 from the other allows 2 triangles to be formed. This may be done in 11 ways allowing for 2x11 = 22 triangles.
For a separation of 3 segments the other separations must add to 8 (4+4; 5+3; 3+5) forming 3 triangles, each to be done in 11 ways allowing for 3x11 = 33 triangles.
Similarly, separations of 4 and 5segments allow for 44 and 55 triangles, respectively.
This gives a total of 11+22+33+44+55 = 165 triangles. But the triangles have been counted three times too many since each side has been used as a base in turn. Therefore, the number of distinct triangles is $\frac{165}{3} = 55$.

Alternative: A system of direct counting gives 55.

Answer: 55

Problem No. 17

Place the numbers $\frac{1}{6}, \frac{1}{3,}, \frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, 2, 3, 6$ in a 3 by 3 grid such that the products of the three numbers the rows, columns and both diagonals results in 1 in each case.

Solution to No. 17

2	$\frac{3}{2}$	$\frac{1}{3}$
$\frac{1}{6}$	1	6
3	$\frac{2}{3}$	$\frac{1}{2}$

Checking multiplications shows that the result is 1 for each row, column and both diagonals. The placement of the values may vary from this grid.

Problem No. 18

A train travelling at constant speed takes 12 seconds to pass completely through a tunnel 110 meters long and 15 seconds to pass completely through a tunnel 215 meters long. What is the speed of the train in kilometres per hour?

Solution to No. 18

Let t be the length of the train in metres.
The train travels (110+t)m in 12 seconds and (215+t)m in 15 seconds.
Subtracting, we see that it travels 105m in 3 seconds which is 35m/sec.
Converting to km per hour we get $35x\frac{60x60}{1000} = 126$ km/hr.

Answer: 126 km/hr

Problem No. 19

At the 2024 Paris Olympics Arshad Nadeem from Pakistan won a gold medal for his javelin throw. He also set a new Olympic Record for the distance thrown which was 92.97m.
See video
RTÉ Sport on X:
Assuming

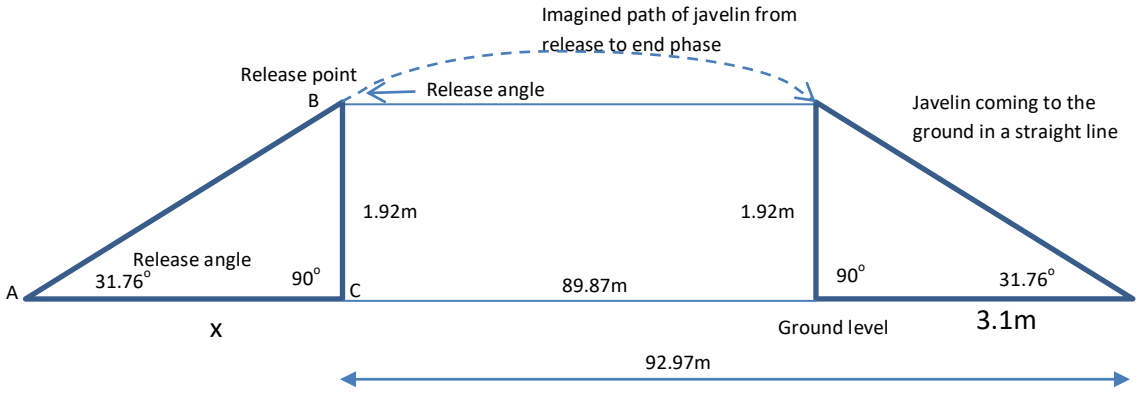
1) that the acute angle at which the javelin landed was the same as the angle at release (the Release Angle is shown on the video at timestamp 0:25 – 00:29)
2) that the Release Height was the same as Nadeem's height and
3) that the javelin travels in a straight line for the short final part of its flight

how much of the distance thrown may be attributed to Nadeem's height?
Give the answer in metres correct to one decimal place.

Solution to No. 19

From the video
RTÉ Sport on X:
we see that the Release Angle is given as 31.76^o.

On Arshad Nadeem - Wikipedia Nadeem's height is given as 1.92m (other sites give 1.9m).



The javelin comes to the ground below the level of the release point due to the height of the thrower. Using the assumption given of the angles the extra distance it travels due to this may be calculated from the triangle ABC in the diagram since the triangles on each end are symmetrically equal.
$\text{Tan}(31.76^o) = \frac{|BC|}{|AC|}$. Therefore, $|AC| = \frac{|1.92|}{\tan 31.76°} = 3.1014$
This is 3.1m correct to 1 place of decimals.
(If Nadeem's height is taken as 1.9m then the value is 3.0691 which is 3.1m correct to 1 decimal place)

Answer: 3.1m

Problem No. 20
A, B and C are three positive whole numbers such that

$$A.B.C = 240$$
$$A.C + B = 46$$
$$A + C.B = 64$$

Write down the value of A + B + C

Solution to No. 20
A.B.C = 240. The factors of 240 are 1.2.2.2.2.3.5. Using three of these for A, B and C we have the possibilities for A.B.C and A.C + B and B.C + A
We can have: 1.2.120; 2.2.60; 2.4.30; 4.4.15; 4.5.12
Of this list, combinations of these numbers as A.C + B give 64 but no combination of these as B.C + A gives 46.
Also we can have 8.5.6; A combination of these as A.C + B gives 46 but no combination of these as A.C + B gives 64
Also we can have 4.10.6;
With A = 4; B = 6; C = 10 we get A.C + B = 4.10 + 6 = 46 and B.C + A = 6.10 + 4 = 64.
There is no other possible combination.
Thus, A = 4; B = 6; C = 10. Then A+B+C = 4+6+10 = 20.

Answer: 20

Problem No. 21
Four ingredients – Honey, Olive oil, Vinegar and Lemon juice - are used to make a salad dressing with the following instructions:

18 parts must not contain honey
16 parts must not contain olive oil
14 parts must not contain vinegar
15 parts must not contain lemon juice

In what ratio of parts are the ingredients used? Give the ratio numbers in the form
Honey : Olive oil : Vinegar : Lemon juice.

Solution to No. 21
Let the number of parts in the salad dressing be x. Then x-18 parts are honey; x-16 parts are olive oil; x-14 parts are vinegar; x-15 parts are lemon juice.
Then x-18 + x-16 + x-14 + x-15 = x
Thus 3x = 63 and x = 21.
Thus the salad dressing is 21-18 = 3 parts honey; 21-16 = 5 parts olive oil; 21-14 = 7 parts vinegar; 21-15 = 6 parts lemon juice.

Answer: The ratios are 3:5:7:6

Problem No. 22

A signpost on a motorway indicates that there are two towns at different distances further ahead. Both of the distances are square numbers. Having travelled a number of kilometres, the next signpost shows the distances to both towns are, again, square numbers. Later again, having travelled 120 km from the first signpost a third signpost shows the distances ahead to the two towns to be square numbers. What distances are displayed on the first signpost?

Give your explanation on a separate sheet.

Solution to No. 22

The distance between the first and third signpost has to be the difference between two squares. Therefore, $a^2 - b^2 = 120$ or $(a + b)(a - b) = 120$. We can examine all the product pairs which give 120 and see which may be written in the form (a+b)(a-b) where a and b are integers and then as the difference of two squares. The pairs of even factors may be written in this manner.

We have the factor pairs: (1.120);(2.60); (3.40); (4.30); (5.24); (6.20); (8.15); (10.12).

10.12	6.20	4.30	2.60
(11-1)(11+1)	(13-7)(13+7)	(17-13)(17+13)	(31-29)(31+29)
$11^2 - 1^2$	$13^2 - 7^2$	$17^2 - 13^2$	31^2-29^2
121 - 1	169 - 49	289 - 169	961 - 841

Between some pair of these we must find another pair of squares which satisfy the requirements. We can examine the distances which may be formed to cover the total distance in each of the 4 cases.

121 – 100 – 81 – 64 – 49 – 36 – 25 – 16 – 9 – 4 - 1	169 – 144 121 – 100 – 81 – 64 - 49	289 – 256 – 225 – 196 - 169	961 – 900 - 841
21+99 40 + 80 57 + 63 72 + 48 85 + 35 96 + 24 **105 + 15** 112 + 8 117 + 3	25 + 95 48 + 72 69 + 51 88 + 32 **105 + 15**	33 + 87 64 + 56 93 + 27	61 + 59

The only pair which matches is 105 + 15.

For 105 + 15 = 120 we have the first signpost: **Town A 169 km; Town B 121 km**. Travel 105 to the second signpost which shows Town A (169-105) 64 km; Town B (121-105) 16 km. Travel 15 to the third signpost which shows Town A (64 -15) 49 km and Town B (16-15) 1 km. This satisfies the requirements of the problem

Alternative solution: By observation: It may be noticed that the distance from 1 to 16 is 15 and from 49 to 64 is also 15. To have 120 from the first to the last signpost we need another 105. Checking, we get 16 + 105 = 121 which is a square number and also 64 + 105 = 169 is also a square number. Note: This does not show that it is the only solution.

Answer: 169; 121 (or 121; 169)

Problem No. 23
The letters x, y, z stand for non-zero digits. The positive integer xyz is a multiple of 3, the positive integer zyxyz is a multiple of 15 and the positive integer xyzyx is a multiple of 8. What is the integer xyz?

Solution to No. 23
zyxyz is a multiple of 15 and thus a multiple of 5. No digit is 0 so therefore z = 5.
zyxyz is a also a multiple of 3 and so 10 + x + 2y is a multiple of 3. So, x + 2y must leave remainder 2 on division by 3 (ie. 2Mod3). That is, x + 2y = one of 2, 5, 8, 11, 14, 17, 20, 23,…

xyzyx is a multiple of 8 and must be even. So, x is even.

x + y + z is a multiple of 3 since xyz is a multiple of 3. So, x + y + 5 is a multiple of 3 which means x + y must leave a remainder of 1 on division by 3 (ie. 1Mod3).
That is. x+ y = one of 1, 4, 7, 10, 13, 16…

x (even)	y	x+y	x+2y	xyz =xy5	xyzyx	zyxyz
2	2	4	6			
2	5	7	12			
4	3	7	10			
6	1	7	**8**	615	61516: not divisible by 8	
2	8	10	18			
4	6	10	16			
6	4	10	**14**	645	64546: not divisible by 8	
8	2	10	12			
4	9	13	22			
6	7	13	**20**	675	67576: divisible by 8	57675: divisible by 15
8	5	13	18			
8	8	16	24			

Thus, x = 6; y = 7; z = 5

Answer: 675

Problem No. 24
I want to paint each face of a cube in a different colour. I can select from 6 colours.
How many different outcomes may I have?

Solution to No. 24
Choose a side and its opposite. Paint one side in a colour. This may be done in 1 way. The opposite side may be painted in any of the 5 remaining colours. Thus this pair of opposite sides may be painted in 5x1 = 5 ways. The 4 remaining sides lie in a circle between these two. The 4 remaining colours may be arranged in a circle in 3x2x1 ways. I.e 6 ways. Thus the total number of ways of painting the cube is 5x6 = 30 ways.

Alternatively, to see how many choices are required for the 4 remaining sides, choose a colour for one side and any of the 3 other colours for the opposite. The 2 remaining colours for the other two opposite sides may be swopped and so have 2 ways of being painted. Thus these 4 sides may be painted in 3x2 = 6 ways.
The total number of ways is 30.

Answer: 30

Problem No. 25

I have four digits but no zero will you find.
One score is their total when in that frame of mind.
My fourth, for example, though greater than the third
Is still small by half when to my second compared.
Minimal difference between first and second persists
Though no repetition of digits in my number exists.

What number am I?

Solution to No. 25

1) Digits: 1, 2, 3, 4, 5, 6, 7, 8, 9
2) Four digits sum to 20 – no repetitions. 9+8+2+1; 9+7+3+1; 9+6+4+1; 9+6+3+2; 9+5+4+2; 8+7+4+1; 8+7+3+2; 8+6+5+1; 8+6+4+2; 8+5+4+3; 7+6+4+3; 7+6+5+2;
3) Fourth > Third so 4th cannot be 1
4) Fourth = Half of Second: Pairs required with 2nd even (2nd, 4th) = (4,2); (6,3); (8,4): Eliminate as follows:

~~9+8+2+1~~; ~~9+7+3+1~~; ~~9+6+4+1~~; 9+6+3+2; 9+5+4+2; 8+7+4+1; ~~8+7+3+2~~; ~~8+6+5+1~~; 8+6+4+2; 8+5+4+3; 7+6+4+3; ~~7+6+5+2~~;

We have remaining: a) 9+6+3+2; b**) 9+5+4+2**; c) **8+7+4+1**; d) 8+6+4+2 e) 8+5+4+3 f) 7+6+4+3

5) First and Second differ by 1: From remaining we have: (1st, 2nd, x, 4th) = (3,2, x, 1); (**5,4, x, 2**); (**7,8, x, 4**); (3, 4, x, 2). This eliminates a), d), e), f) leaving b) and c).
6) Rearranging: Second must be even; Fourth = Half Second: Fourth >Third

From b) we have: 5-4-9-2 – Eliminate since 4th is not greater than 3rd
From c) we have 7-8-1-4 – This satisfies the requirements

Answer: The number is 7814

Problem No. 26

On the Engineers Ireland website what is Justine Butler's essential skill that can inhibit or boost a career?

Solution to No. 26

On the webpage: https://www.engineersireland.ie/News/my-engineering-life-qa-justine-butler-jacobs-2 the question to Justine was at number 8 and her answer was as follows:

> ***8) What is the one piece of advice you would give to somebody starting out as an engineer?***
> *Attitude is an essential skill that can inhibit or boost your career.*

Answer: Attitude