Round 1 Babtha 1

Question 1

Solve the following simultaneous equations:

$$x + \frac{y}{3} + \frac{z}{3} = 0$$

$$x - y + z = 2$$

$$\frac{x}{3} - \frac{y}{2} - \frac{z}{6} = \frac{3}{2}$$

Question 2

Let $f(x) = \frac{x}{x-1}$, where $x \neq 1, x \in \mathbb{R}$.

Calculate $f'(x)$ giving your answer in the form $\frac{a}{(x+b)^c}$ where $a, b, c \in \mathbb{R}$, hence or otherwise, calculate how many tangents there are to $f(x)$ that are parallel to the x-axis.

You must give **both** $f'(x)$ in the correct form and the number of tangents in your answer.

Round 2 Babtha 2

Question 1

Find the value of $k \in \mathbb{Q}$ for which:

$$k \log_a x + 2k \log_a x^2 + 3k \log_a x^3 + 4k \log_a x^4 = \log_a x$$

Question 2

Given that:

$\int_a^b f(x)\,dx = 3p$, $\int_b^c f(x)\,dx = 4p + 1$ and $\int_c^d f(x)\,dx = 7p + 9$

Find, in terms of p, the value of:

$$\int_a^d f(x)\,dx$$

Round 3 Babtha 3

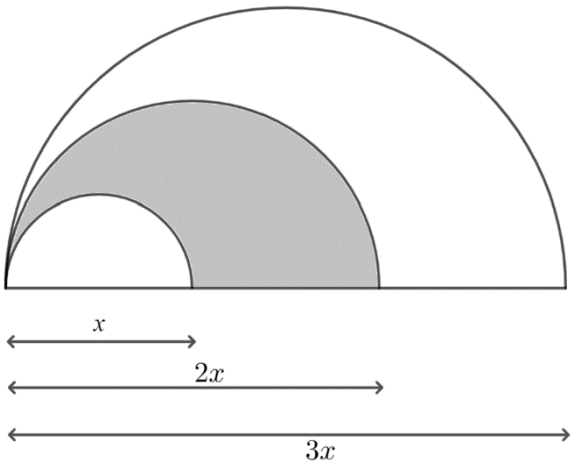
Question 1

Find the exact value of

$$\frac{5^{702} - 5^{700} + 24}{5^{700} + 1}$$

Question 2

The diagram shows three nested semicircles, the largest of which has diameter $3x$, the middle-sized semicircle has diameter $2x$ and the smallest semicircle has diameter x as shown. What fraction of the largest semicircle is shaded?



Round 4 Babtha 4

Question 1

The circle k has centre $(0,0)$ and contains the point $(-1,5)$. Find all possible points of the form $(p,p), p \in \mathbb{Z}$, such that (p,p) lies inside the circle.

Question 2

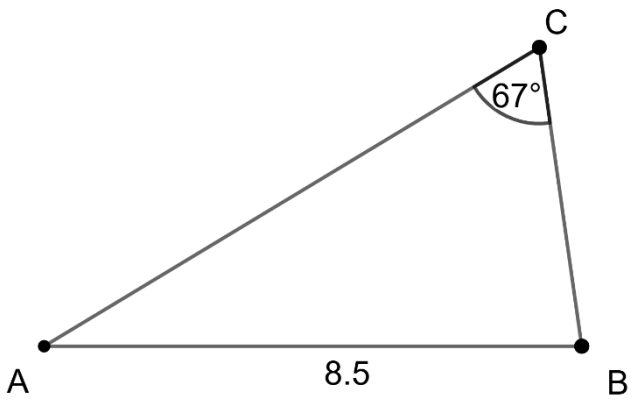
Solve for $x \in \mathbb{R}, x \neq 0, -1$ given that:

$$\left(\frac{2\sqrt{x}}{1+x}\right)\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right) = x - 3$$

Round 5 Babtha 5

Question 1

ABC is an acute, non-right-angled triangle, with $|AB| = 8.5$ and $|ACB| = 67^{\circ}$. Find the length of the radius of the circumcircle of triangle ABC. Give your answer correct to 2 decimal places.



Question 2

Let $i = \sqrt{-1}$.

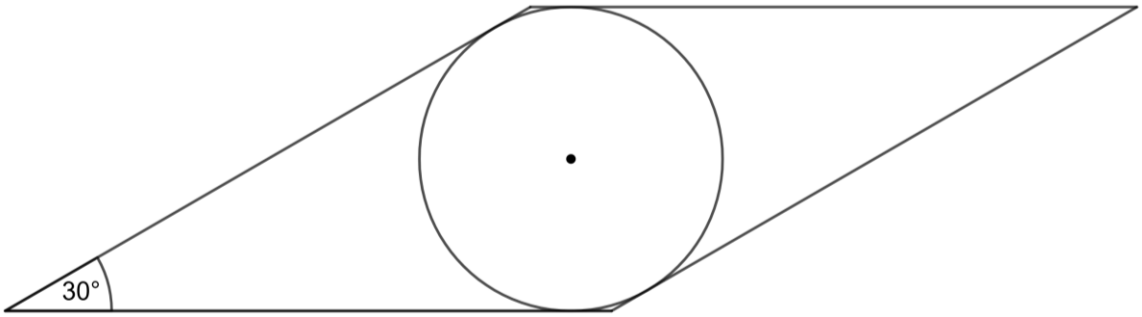
Find the value(s) of $\sqrt{i}$.

Give your answer(s) in rectangular form, $a + bi$, where $a, b \in \mathbb{R}$.

Round 6 Babtha 6

Question 1

A circle of radius 1.25, is perfectly ***inscribed*** within a rhombus as shown below, with smaller interior angle of size 30^o as shown. I.e. the circle is tangential to all four sides at the point of contact. Find the area of the rhombus.



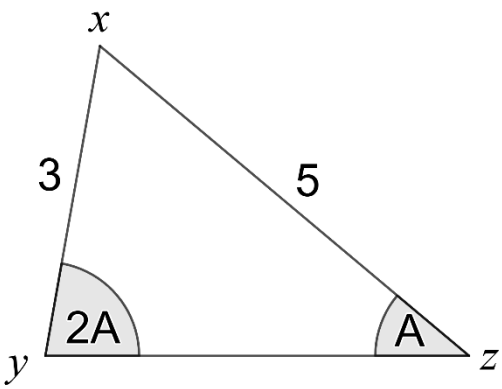
Question 2

Given that the 31st December 2024 fell on a Tuesday, which day of the week is the most frequently occurring in 2025?

Round 7 Babtha 7

Question 1

In the triangle xyz, $|\angle xyz| = 2\text{A}$, $|\angle xzy| = \text{A}$, $|xy| = 3$ and $|xz| = 5$. Find the value of $\tan \text{A}$ in its simplest form, $\frac{\sqrt{c}}{d}$ where $c, d \in \mathbb{N}$.



Question 2

Find the exact value of:

$$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\ldots\left(1-\frac{1}{2025}\right)$$

Give you answer in the form $\frac{a}{b}$; $a, b \in \mathbb{Z}, b \neq 0$.

Question 3

Let $y = \ln\sqrt{\frac{x-1}{x+1}}$. Find $a, b, c \in Z$ such that $\frac{dy}{dx} = \frac{a}{x^b+c}$.

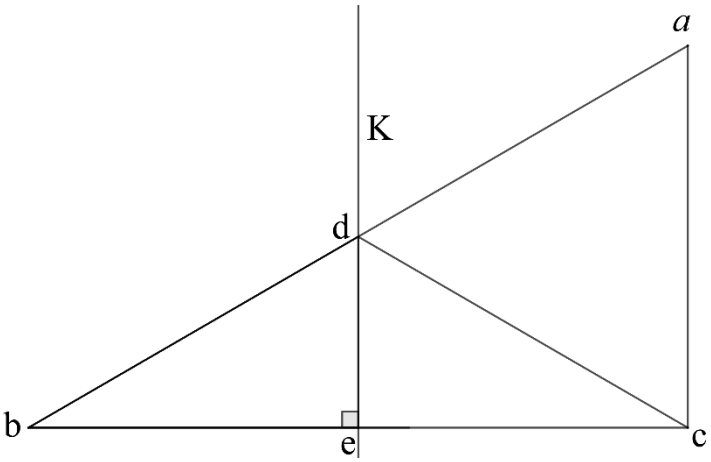
Question 4

In the triangle abc, $ac \perp bc$ and $|\angle abc| = 30°$.

K is the perpendicular bisector of $[bc]$ and K intersects $[ab]$ at d.

Find the ratio of $\frac{area\ \Delta bed}{area\ \Delta bca}$.

Give your answer in its simplest form.



Round 8 Babtha 8

Question 1

Find the values of $a, b \in \mathbb{N}, b < 10$, such that:

$$\left(\sqrt{5}\right)\left(\sqrt[3]{4}\right) = \sqrt[b]{a}$$

Question 2

An equilateral triangle is formed by the vertices $A(0,0)$, $B(10,0)$ and $C(5, 5\sqrt{3})$. Find the ***exact*** coordinates of the incentre of the triangle (i.e. in surd form where necessary).

Question 3

A drawer contains a randomly mixed collection of socks, all identical except for colour. There are 10 blue, 17 yellow, 20 white and 53 black socks. A number of socks will be selected at random and removed together. How many socks must be removed to ***guarantee*** having a pair of the same-coloured socks?

Question 4

Evaluate:

$$\left[\binom{25}{0}+\binom{25}{1}+\binom{25}{2}+\binom{25}{3}+\cdots\binom{25}{24}+\binom{25}{25}\right] - \left[\binom{20}{0}+\binom{20}{1}+\binom{20}{2}+\binom{20}{3}+\cdots\binom{20}{19}+\binom{20}{20}\right]$$

Write your answer in its **simplest** form as $a^1(b)^k$, where $a, b, k \in \mathbb{N}$; a, b both prime.

Tie Break

1	What is the value of 2025 plus 2025 thousandths?
2	Find: $\lim_{h\to 0} \frac{e^{x+h}-e^{x}}{h}$
3	Calculate the p-value, correct to 4 decimal places, for a test statistic of $z = 2.32$.
4	Find the exact coordinates of the centroid of the triangle with vertices $(-4,3), (5,1)$ and $(-2,-6)$.
5	Find the least $n \in \mathbb{N}$ such that $S_n > 1.99$, where: $$S_n = 1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^n}$$
6	If $x * y = x^2 + 2y + 3$, find the two values of a for which $a * a = 6$.
7	An empty bath can be filled using the cold tap alone in 2.5 hours. It will take the hot tap 2.75 hours to fill the empty bath alone. How long will it take to fill the empty bath if both taps are used to fill it together. Give your answer in hours and minutes, correct to the nearest minute.
8	In a particular experiment, the probability of success on a single Bernoulli trial is 0.6. Find the probability that the twentieth success occurs on the twenty-fifth trial. Give your answer correct to three significant figures.
9	A total of n points are equally spaced around a circle and are labelled with the integers 1 to n, in order. Two points are called diametrically opposite if the line segment joining them is a diameter of the circle. If the points labelled 7 and 35 are diametrically opposite, find the value of n.
10	The y-intercepts of three parallel lines are $2, 3$ and 4. The sum of the x-intercepts of the three lines is 36. What is the slope of these parallel lines?

ANSWERS

R1	Q1	$(1,-2,-1)$
	Q2	$f'(x) = -\frac{1}{(x-1)^2}$ or equivalent. 0 tangents.
R2	Q1	$\frac{1}{30}$
	Q2	$14p + 10$
R3	Q1	24
	Q2	$\frac{1}{3}$
R4	Q1	$(0,0), (1,1), (-1,-1), (2,2), (-2,-2), (3,3), (-3,-3)$
	Q2	5
R5	Q1	4.62
	Q2	$\pm\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$ or $\pm\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$
R6	Q1	12.5 units squared
	Q2	Wednesday
R7	Q1	$\frac{\sqrt{11}}{5}$
	Q2	$\frac{1}{2025}$
	Q3	$a = 1, b = 2, c = -1.$
	Q4	$\frac{1}{4}$
R8	Q1	$a = 2000, b = 6$
	Q2	$\left(5, \frac{5\sqrt{3}}{3}\right)$
	Q3	5
	Q4	$31^1(2)^{20}$
Tiebreak	1	2027.025
	2	e^x
	3	0.0204
	4	$\left(-\frac{1}{3}, -\frac{2}{3}\right)$
	5	7
	6	$a = 1, a = -3.$
	7	1 hour 19 minutes
	8	0.0159
	9	56
	10	$-\frac{1}{4}$