Round 1 **Babtha 1**

Question 1

Solve the following simultaneous equations:

$$x + \frac{y}{3} + \frac{z}{3} = 0$$

$$x - y + z = 2$$

$$\frac{x}{3} - \frac{y}{2} - \frac{z}{6} = \frac{3}{2}$$

$$3x + y + z = 0 \qquad -3x + 3y - 3z = -6$$

$$\underline{x - y + z = 2} \qquad \underline{2x - 3y - z = 9}$$

$$4x + 2z = 2 \qquad -x - 4z = 3$$

$$2x + z = 1$$

$$\underline{-2x - 8z = 6}$$

$$-7z = 7 \qquad z = -1$$

$$2x - 1 = 1 \qquad x = 1$$

$$1 - y - 1 = 2 \qquad y = -2$$

Question 2

Let $f(x) = \frac{x}{x-1}$, where $x \neq 1, x \in \mathbb{R}$.

Calculate $f'(x)$ giving your answer in the form $\frac{a}{(x+b)^c}$ where $a, b, c \in \mathbb{R}$, hence or otherwise, calculate how many tangents there are to $f(x)$ that are parallel to the x-axis.

You must give **both** $f'(x)$ in the correct form and the number of tangents in your answer.

$$f'(x) = \frac{(x-1)(1) - (x)(1)}{(x-1)^2} = \frac{-1}{(x-1)^2}$$

$\frac{-1}{(x-1)^2}$ cannot =0, as $-1 \neq 0$ and denominator cannot equal 0. Hence, there are no tangents, i.e. 0 tangents parallel to the x-axis.

Round 2 **Babtha 2**

Question 1

Find the value of $k \in \mathbb{Q}$ for which:

$$k \log_a x + 2k \log_a x^2 + 3k \log_a x^3 + 4k \log_a x^4 = \log_a x$$

$$\Rightarrow k \log_a x + 4k \log_a x + 9k \log_a x + 16k \log_a x = \log_a x$$

$$\Rightarrow \log_a x\,(k + 4k + 9k + 16k) = \log_a x$$

$$(k + 4k + 9k + 16k) = 1$$

$$k = \frac{1}{30}$$

Question 2

Given that:

$\int_a^b f(x)\,dx = 3p$, $\int_b^c f(x)\,dx = 4p + 1$ and $\int_c^d f(x)\,dx = 7p + 9$

Find, in terms of p, the value of:

$$\int_a^d f(x)\,dx$$

$$\int_a^d f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx + \int_c^d f(x)\,dx = 3p + 4p + 1 + 7p + 9 = 14p + 10$$

Round 3 **Babtha 3**

Question 1

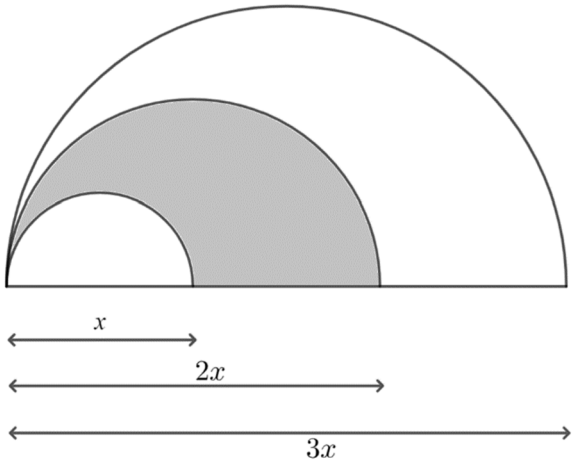
Find the exact value of

$$\frac{5^{702} - 5^{700} + 24}{5^{700} + 1}$$

$$\frac{5^{702} - 5^{700} + 24}{5^{700} + 1} = \frac{5^{700}(5^2 - 1) + 24}{5^{700} + 1} = \frac{5^{700}(24) + 24}{5^{700} + 1} = \frac{24(5^{700} + 1)}{5^{700} + 1} = 24$$

Question 2

The diagram shows three nested semicircles, the largest of which has diameter $3x$, the middle-sized semicircle has diameter $2x$ and the smallest semicircle has diameter x as shown. What fraction of the largest semicircle is shaded?



$$\frac{1}{2}\pi(x)^2 - \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 = \pi\left(\frac{3x^2}{8}\right)$$

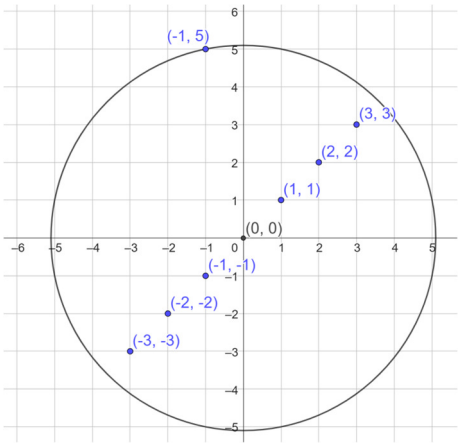
$$\Rightarrow \frac{\pi\left(\frac{3x^2}{8}\right)}{\pi\left(\frac{9x^2}{8}\right)} = \frac{1}{3}$$

Round 4 **Babtha 4**

Question 1

The circle k has centre $(0,0)$ and contains the point $(-1,5)$. Find all possible points of the form $(p,p), p \in \mathbb{Z}$, such that (p,p) lies inside the circle.

$$(-3,-3),(-2,-2),(-1,-1),(0,0),(1,1),(2,2),(3,3)$$



Question 2

Solve for $x \in \mathbb{R}, x \neq 0, -1$ given that:

$$\left(\frac{2\sqrt{x}}{1+x}\right)\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right) = x-3$$

$$\frac{2x}{1+x}+\frac{2}{1+x} = x-3$$

$$\frac{2x+2}{1+x} = x-3$$

$$\frac{2(x+1)}{1+x} = x-3$$

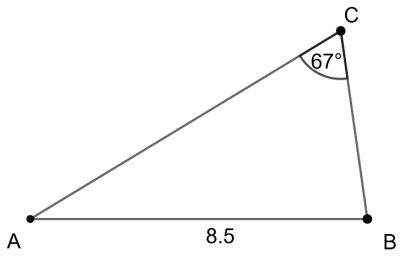
$$2 = x-3$$

$$x = 5$$

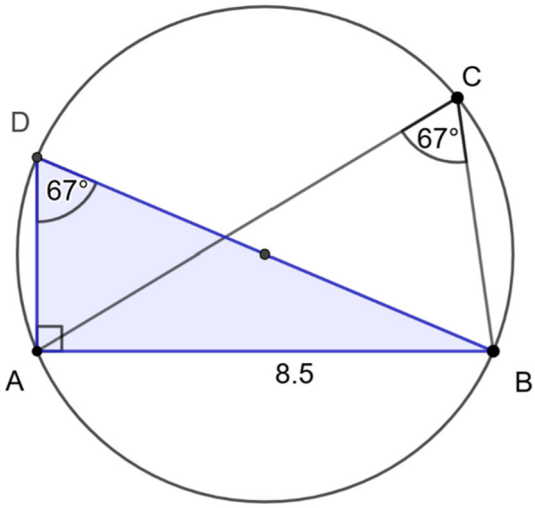
Round 5 **Babtha 5**

Question 1

ABC is an acute, non-right-angled triangle, with $|AB| = 8.5$ and $|ACB| = 67^o$. Find the length of the radius of the circumcircle of triangle ABC. Give your answer correct to 2 decimal places.



Construct the circumcircle of triangle ABC. With AB as base, construct triangle ABD so that BD is a diameter as shown. From the right-angled triangle formed we get:



$$\sin 67 = \frac{8.5}{2r}$$

$$r = \frac{4.25}{\sin 67} = 4.617 \approx 4.62$$

Question 2

Let $i = \sqrt{-1}$.

Find the value(s) of $\sqrt{i}$.

Give your answer(s) in rectangular form, $a + bi$, where $a, b \in \mathbb{R}$.

$$i = \cos(90 + n360) + i(\sin(90 + n360))$$

$$\sqrt{i} = \; i^{\frac{1}{2}} = (\cos\left(\frac{90}{2} + \frac{n360}{2}\right) + i(\sin(45 + n180))$$

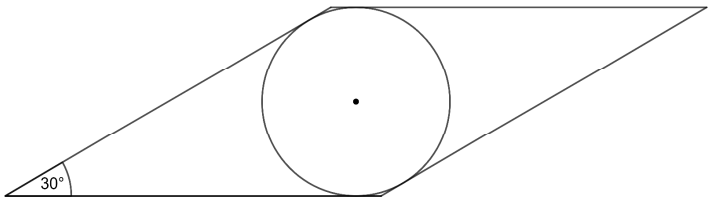
$$n = 0\text{: } \sqrt{i} = \cos 45 + i \sin 45 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$

$$n = 1\text{: } \sqrt{i} = \cos 225 + i \sin 225 = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$$

Round 6 **Babtha 6**

Question 1

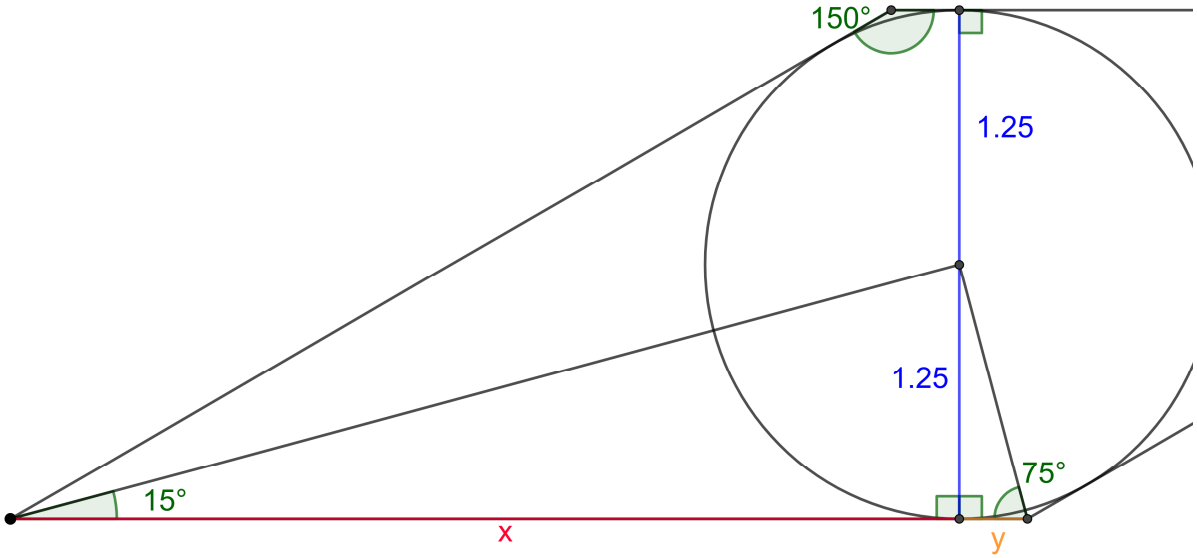
A circle of radius 1.25, is perfectly ***inscribed*** within a rhombus as shown below, with smaller interior angle of size 30^o as shown. I.e. the circle is tangential to all four sides at the point of contact. Find the area of the rhombus.



We need base length $x + y$, and perpendicular height $2(1.25) = 2.5$.

$\tan 15 = \frac{1.25}{x}$ => $x = \frac{1.25}{\tan 15}$; $\tan 75 = \frac{1.25}{y}$ => $y = \frac{1.25}{\tan 75}$ => $x + y = 5$

Area: $5(2.5) = 12.5$ units squared



Question 2

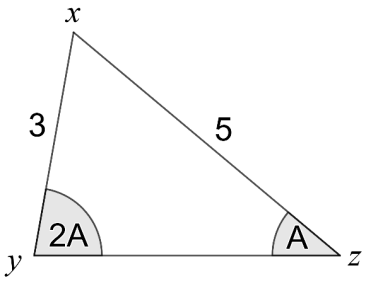
Given that the 31st December 2024 fell on a Tuesday, which day of the week is the most frequently occurring in 2025?

Wednesday. In short, there are more chances overall for days to land on Wednesday.

Round 7 **Babtha 7**

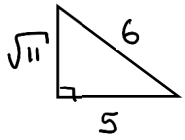
Question 1

In the triangle xyz, $|\angle xyz| = 2\text{A}$, $|\angle xzy| = \text{A}$, $|xy| = 3$ and $|xz| = 5$. Find the value of $\tan \text{A}$ in its simplest form, $\frac{\sqrt{c}}{d}$ where $c, d \in \mathbb{N}$.



$$\frac{\sin A}{3} = \frac{\sin 2A}{5} = \frac{2 \sin A \cos A}{5}$$

$\Rightarrow \cos A = \frac{5}{6}$;



$\Rightarrow \tan A = \frac{\sqrt{11}}{5}$

Question 2

Find the exact value of:

$$\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \ldots \left(1 - \frac{1}{2025}\right)$$

Give you answer in the form $\frac{a}{b}$; $a, b \in \mathbb{Z}, b \neq 0$.

$$\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \ldots \left(1 - \frac{1}{2025}\right) = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\left(\frac{4}{5}\right) \ldots \left(\frac{2023}{2024}\right)\left(\frac{2024}{2025}\right) = \frac{1}{2025}$$

Round 7 **Babtha 7**

Question 3

Let $y = \ln\sqrt{\frac{x-1}{x+1}}$. Find $a, b, c \in \mathbb{Z}$ such that $\frac{dy}{dx} = \frac{a}{x^b+c}$.

$$y = \ln\sqrt{\frac{x-1}{x+1}} = \frac{1}{2}\ln(x-1) - \frac{1}{2}\ln(x+1)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x-1}\right)(1) - \frac{1}{2}\left(\frac{1}{x+1}\right)(1) = \frac{1}{2}\left(\frac{x+1-(x-1)}{(x-1)(x+1)}\right) = \frac{1}{2}\left(\frac{2}{x^2-1}\right) = \frac{1}{x^2-1}$$

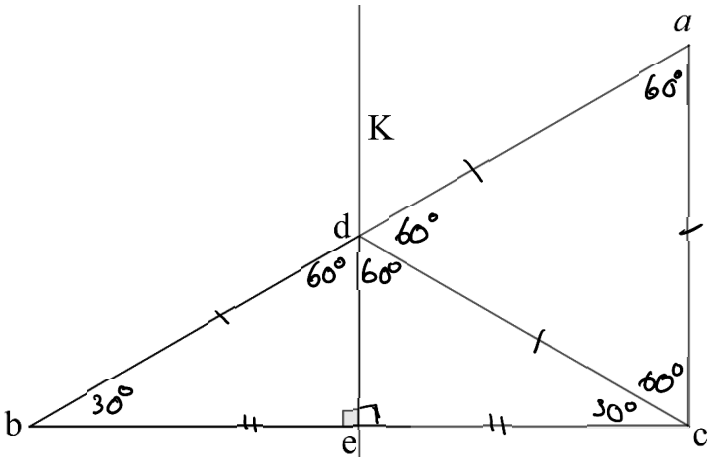
Question 4

In the triangle abc, $ac \perp bc$ and $|\angle abc| = 30°$.

K is the perpendicular bisector of $[bc]$ and K intersects $[ab]$ at d.

Find $\frac{area\ \Delta bed}{area\ \Delta bca}$.

Give your answer in its simplest form.



K bisects => $\frac{|be|}{|bc|} = \frac{1}{2}$ i.e. Δbed is a reduction by scale factor ½ of Δbca.

$$\frac{area\ \Delta bed}{area\ \Delta bca} = \frac{\left(\frac{1}{2}\right)^2 area\ \Delta bca}{area\ \Delta bca} = \frac{1}{4}$$

Or similar - there are many, many methods.

Round 8 **Babtha 8**

Question 1

Find the values of $a, b \in \mathbb{N}, b < 10$, such that:

$$\left(\sqrt{5}\right)\left(\sqrt[3]{4}\right) = \sqrt[b]{a}$$

$$\left(\sqrt{5}\right)\left(\sqrt[3]{4}\right) = (5)^{\frac{1}{2}}(4)^{\frac{1}{3}} = (5)^{\frac{3}{6}}(4)^{\frac{2}{6}} = [5^3(4^2)]^{\frac{1}{6}} = (2000)^{\frac{1}{6}} = \sqrt[6]{2000}$$

Question 2

An equilateral triangle is formed by the vertices $A(0,0)$, $B(10,0)$ and $C(5, 5\sqrt{3})$. Find the ***exact*** coordinates of the incentre of the triangle (i.e. in surd form where necessary).

Equilateral triangle => incentre = centroid. $\left(\frac{0+10+5}{3}, \frac{0+0+5\sqrt{3}}{3}\right) = \left(5, \frac{5\sqrt{3}}{3}\right)$

Question 3

A drawer contains a randomly mixed collection of socks, all identical except for colour. There are 10 blue, 17 yellow, 20 white and 53 black socks. A number of socks will be selected at random and removed together. How many socks must be removed to ***guarantee*** having a pair of the same-coloured socks?

There are only 4 different colours. The worst-case scenario is that you first pull out one of each colour sock. Once you have these 4 socks, you have run out of possibilities; hence, your next sock must match one of the types already chosen. Choosing 5 different socks guarantees at least one pair of matching socks.

Question 4

Evaluate:

$$\left[\binom{25}{0}+\binom{25}{1}+\binom{25}{2}+\binom{25}{3}+\cdots\binom{25}{24}+\binom{25}{25}\right] - \left[\binom{20}{0}+\binom{20}{1}+\binom{20}{2}+\binom{20}{3}+\cdots\binom{20}{19}+\binom{20}{20}\right]$$

Write your answer in its **simplest** form as $a^1(b)^k$, where $a, b, k \in \mathbb{N}$; a, b both prime.

Since $1^{anything} = 1$, then using the binomial expansion:

$$\binom{25}{0}+\binom{25}{1}+\binom{25}{2}+\binom{25}{3}+\cdots\binom{25}{24}+\binom{25}{25} = (1+1)^{25} = 2^{25}$$

Similarly:

$$\binom{20}{0}+\binom{20}{1}+\binom{20}{2}+\binom{20}{3}+\cdots\binom{20}{19}+\binom{20}{20} = (1+1)^{20} = 2^{20}$$

Thus, we have:

$$2^{25} - 2^{20} = 2^{20}(2^5 - 1) = 2^{20}(31) = 31^1(2)^{20}$$