Finding Creativity, Diversity and Meaning in Maths

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GRADUATE SCHOOL OF
EDUCATION

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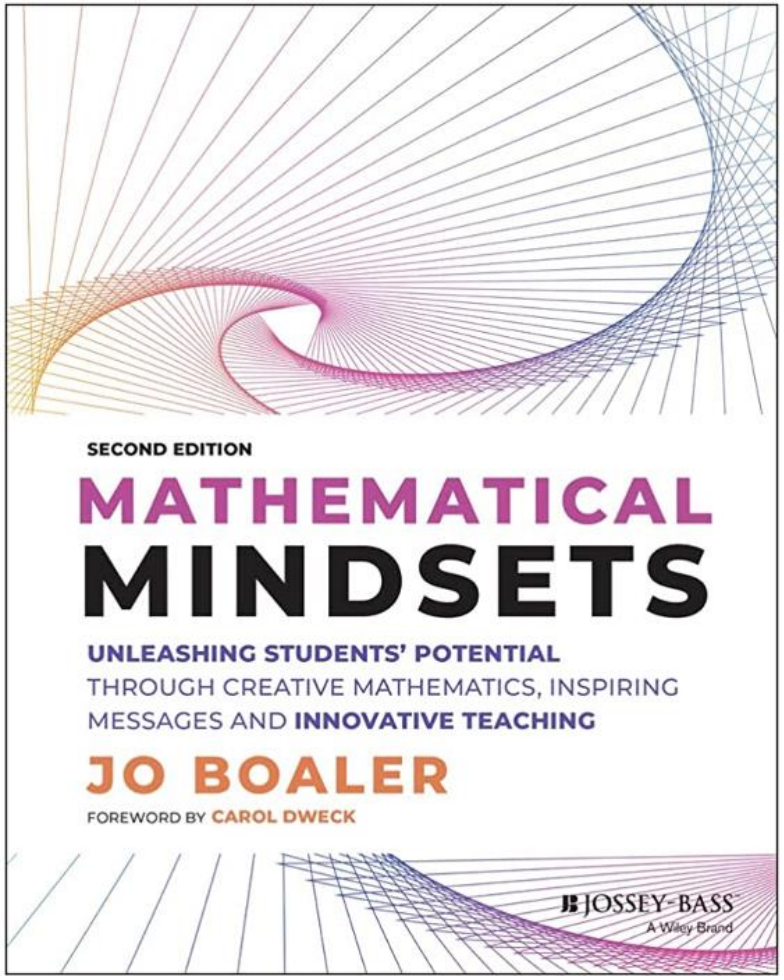
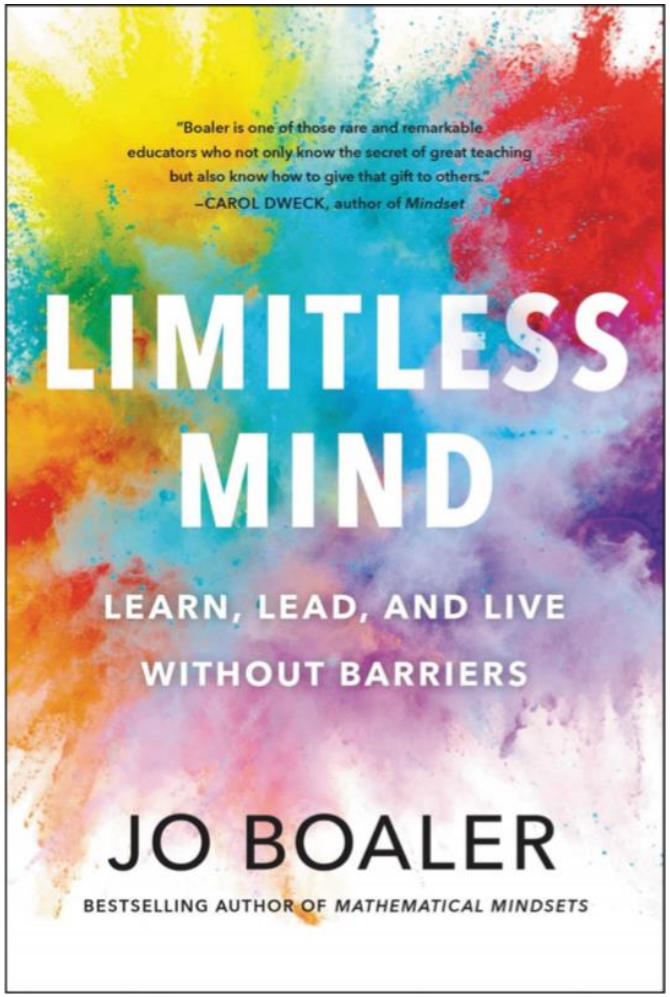
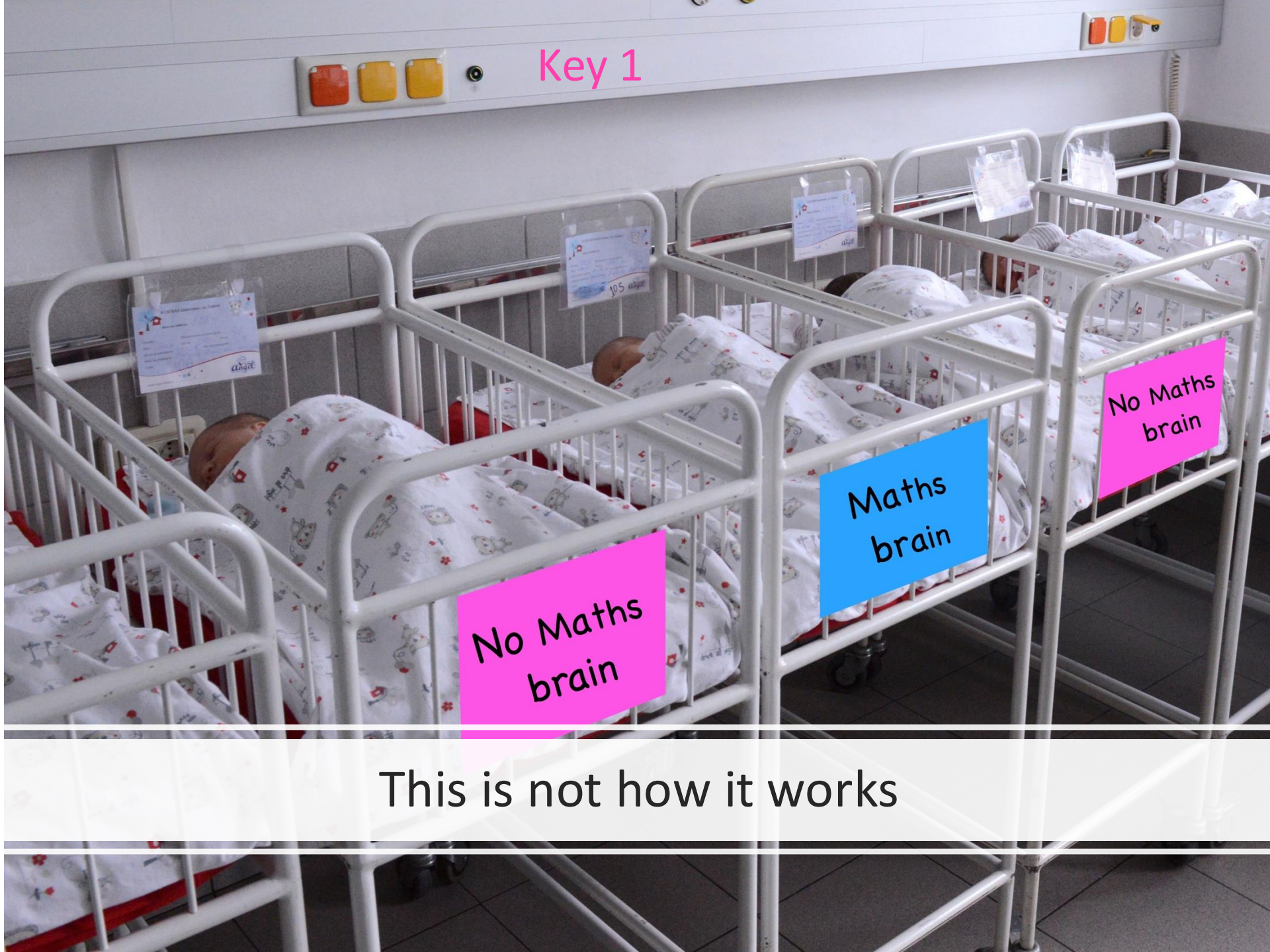
- Our goal is to share free resources for teachers & students showing the beauty and creativity in open mathematics – erasing maths anxiety
- Over 67 million visits to website in last 10 years
- Hundreds of free tasks & lessons TK-16
- Used in 2/3 of US Schools
- 145,000 newsletter subscribers
- Newsletters opened in over 160 countries
- New Data Science Initiative



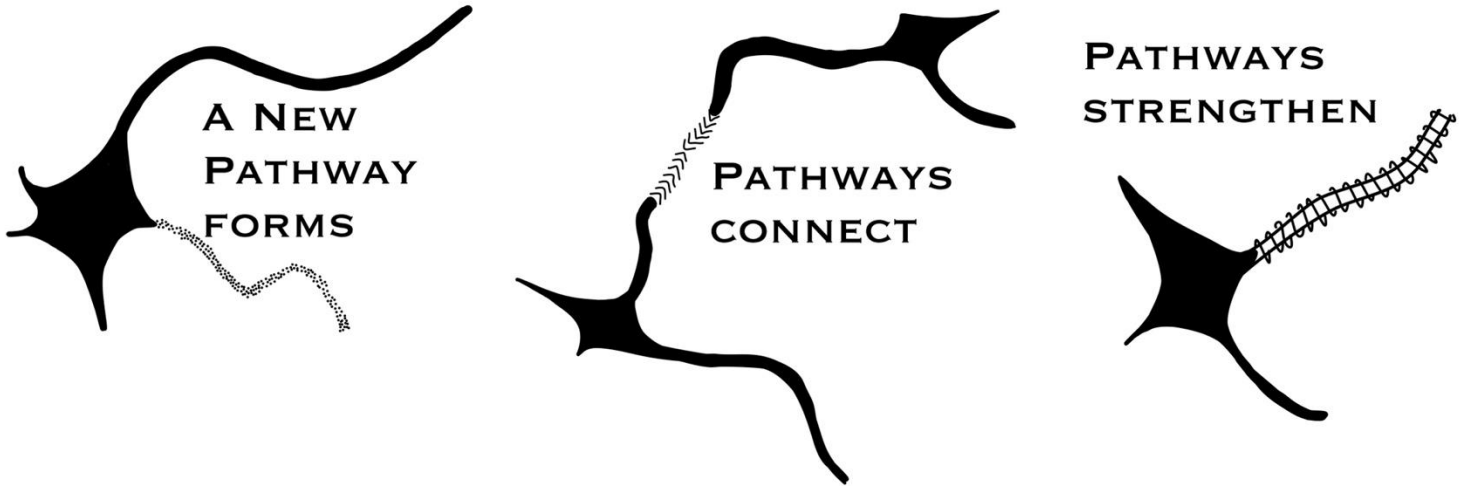


Learning Keys



This is not how it works

What happens when we learn?



We are not born with these pathways

- In 1st grade parents were told he was “learning disabled” “had a very low IQ”
- “The worst child teachers had seen in 20 years”
- Graduated in 2018 with a doctorate in Applied Mathematics from Oxford University.
- His mother (Lois Lechford) authored: Reversed: A Memoir

Nicholas Lechford

Key 2:

Evidence of the importance of times of struggle

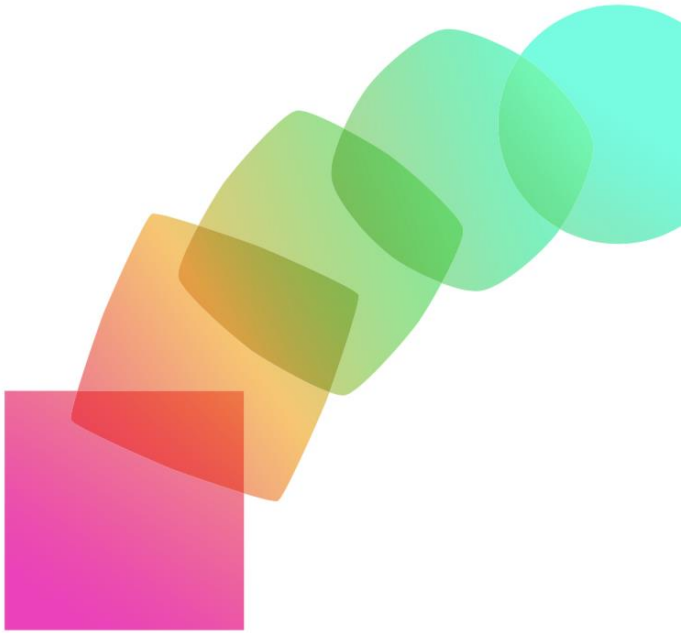
Clear evidence from neuroscience & education

Embracing Struggle

For learners – it is freeing, they keep going when they struggle, it breeds persistence

When learners say “this is hard” – I say, “great that is the feeling of your brain working hard”

Adults go into meetings differently



www.sciencenewsforstudents.org/article/secret-science-mistakes-boost-understanding

PSYCHOLOGY

A secret of science: Mistakes boost understanding

Failure can change how you learn — and propel you toward success

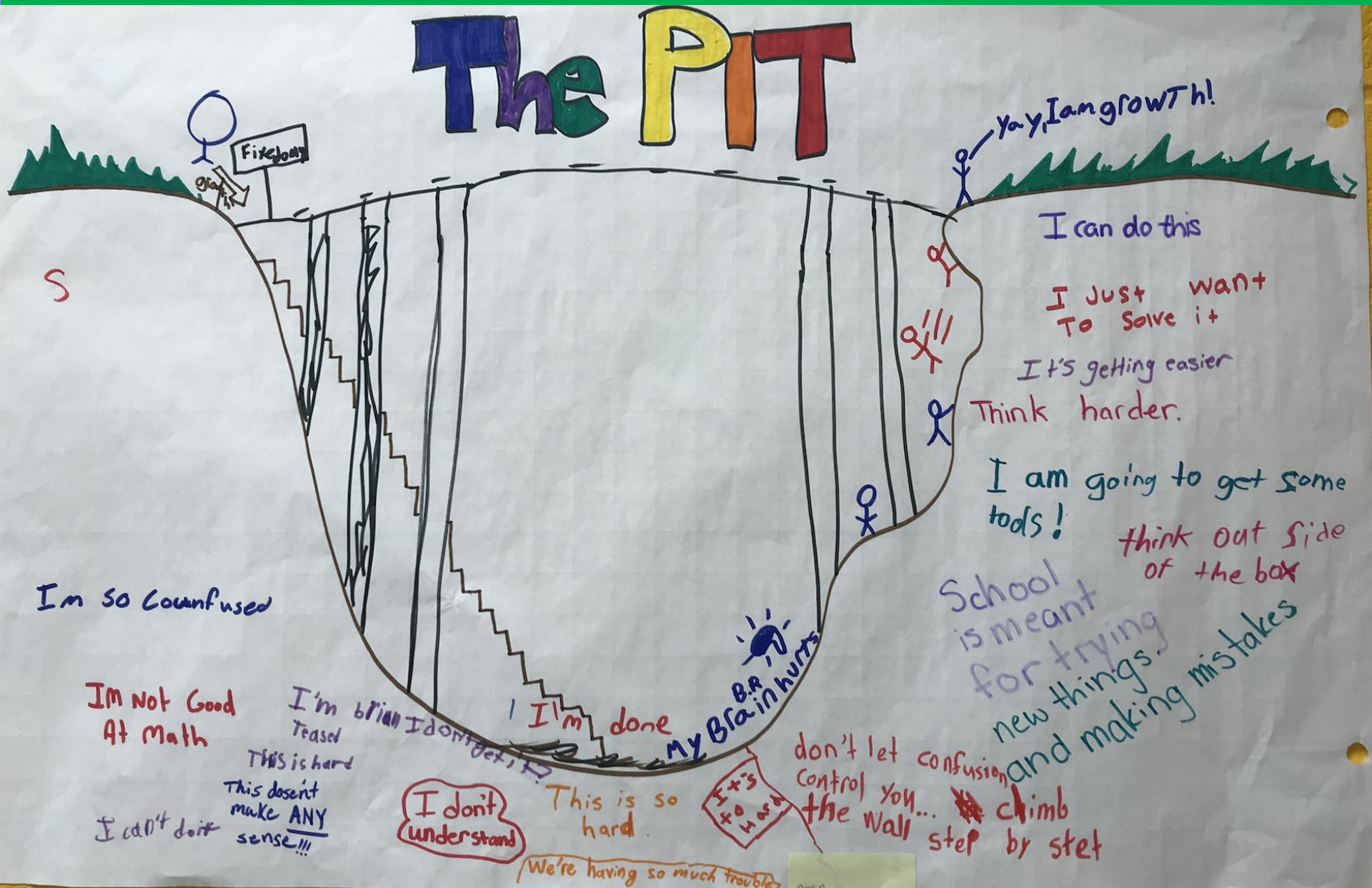
Many kids fear making a mistake will label them as a failure. In fact, science is built on a mountain of mistakes, many made by the greatest minds. The trick is to view each mistake as a step along the path to understanding something better.

You should want to learn and work at the edge of your understanding

Jo Boaler, Mathish

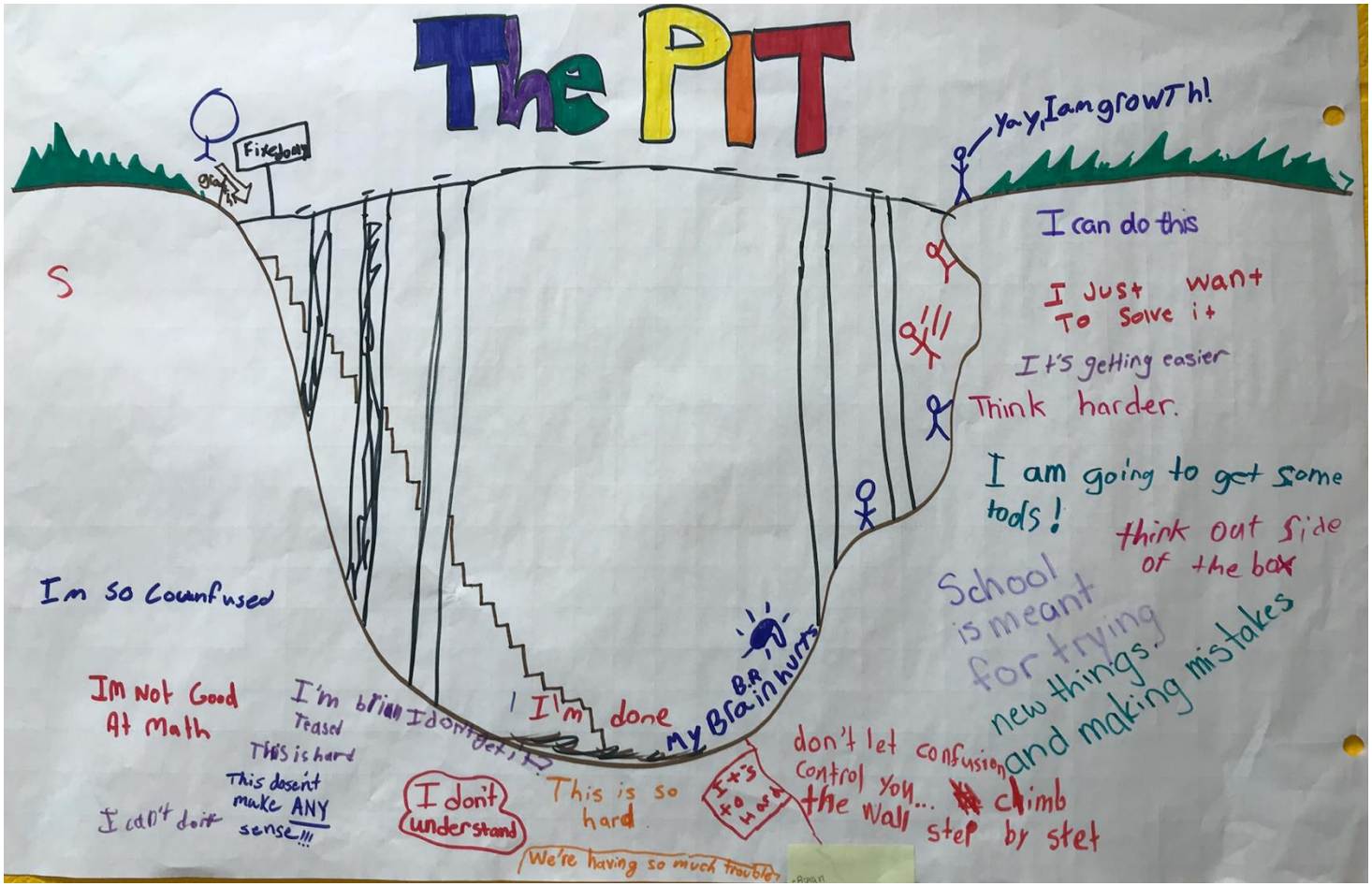
Encouraging Struggle

Encouraging Struggle – Jennifer Schaefer



“Ms Schaefer I am really in the pit”

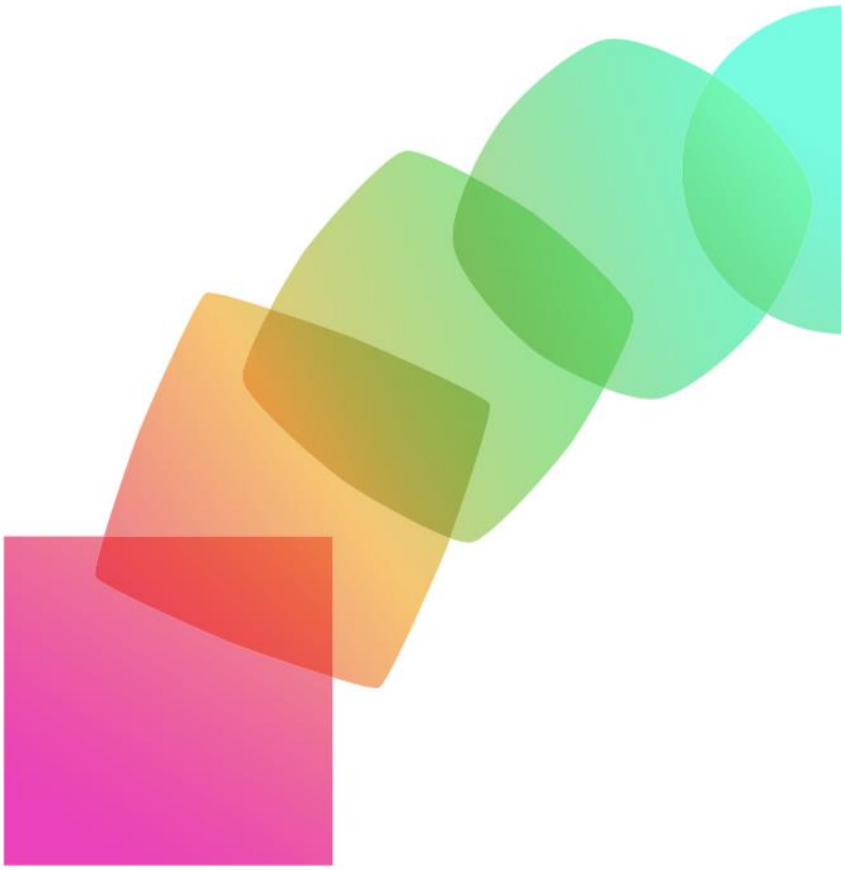
“Excellent - what classroom tools do you need?”



Key 3: Mindset

Letting go of Limits

Developing a growth mindset – believing that you can achieve



Extensive

Conducted with all age ranges

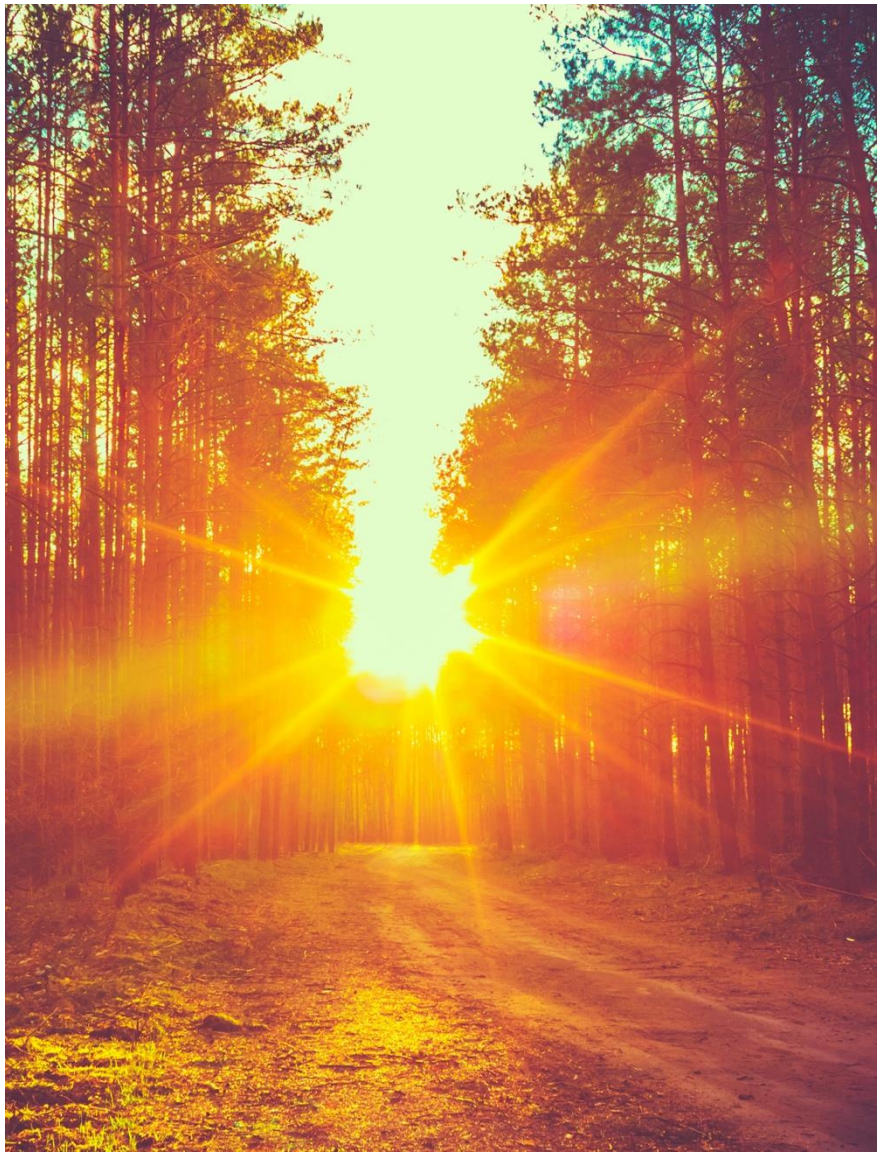
Improves learning, health, aggression

Mindset Evidence

Changing mindset

It is a journey not a switch

Once people become aware of the science, change starts to take place.

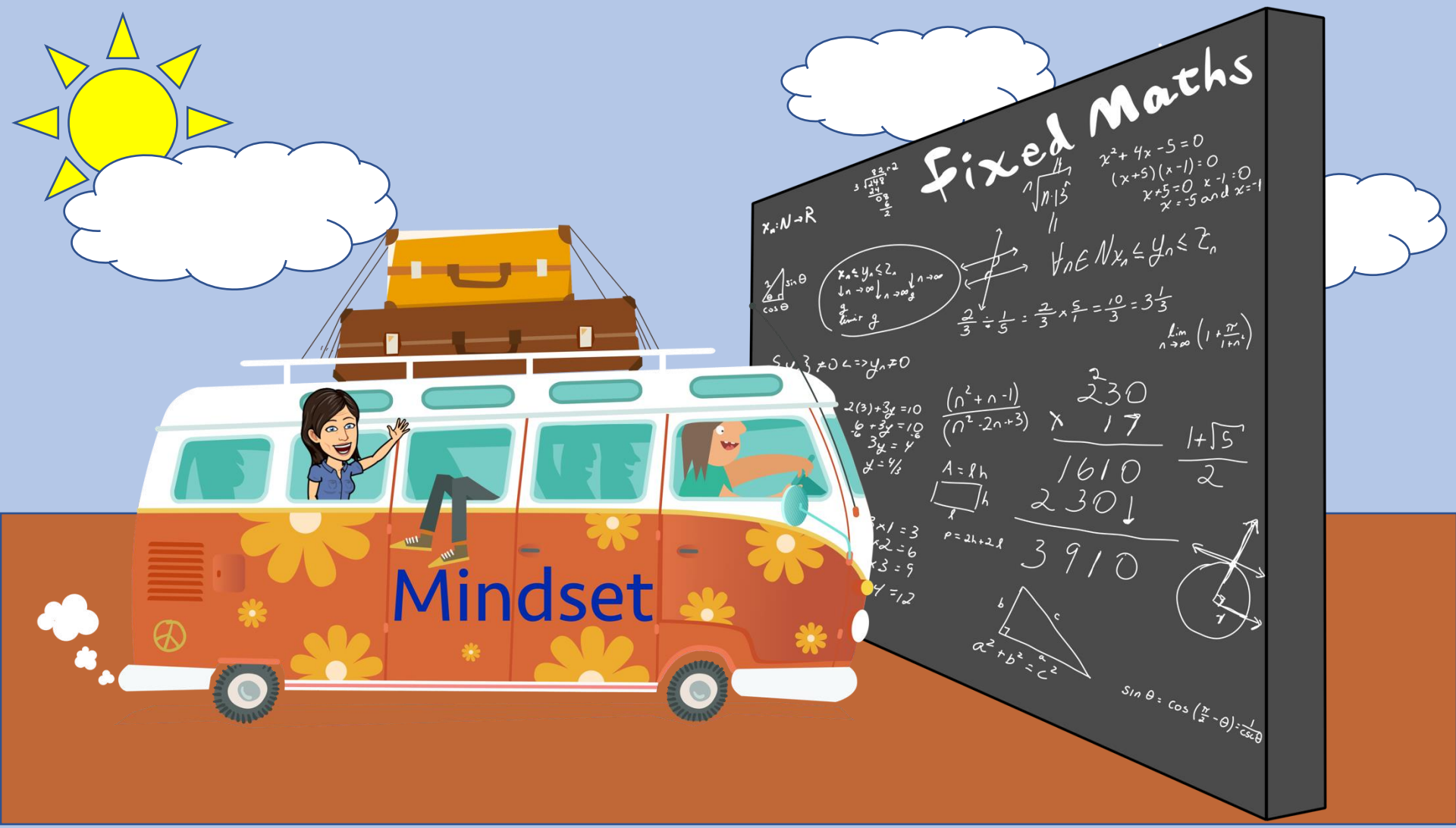


What happens when students change their mindsets?

Online class – English – Spanish - Portuguese
½ million people have taken it

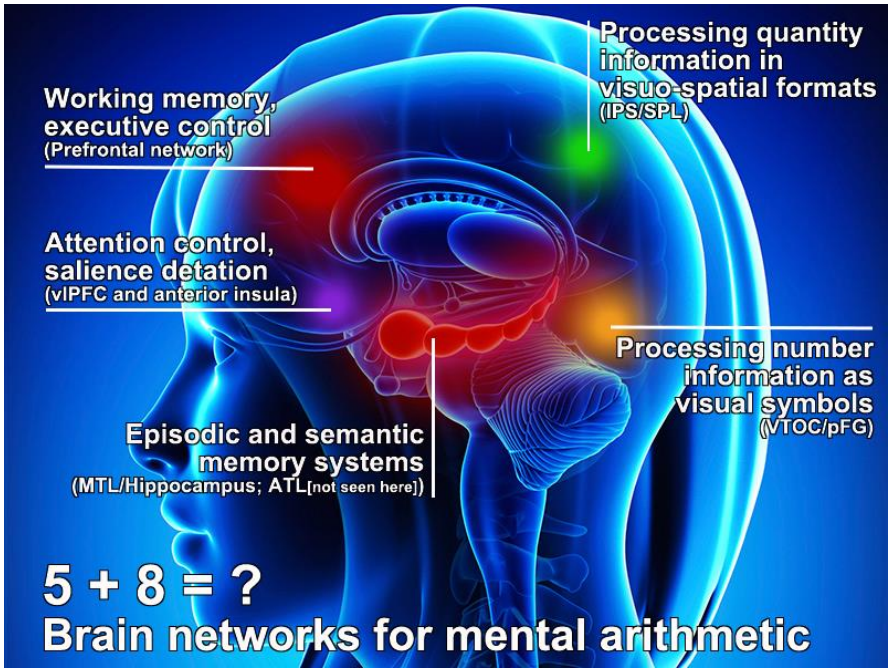
- Randomized Controlled Trial
- Middle school teachers who taught 2 classes gave it to one of their classes, (10 schools, 4 districts, 1090 students)
- At the end of the school year the students who took the class achieved at significantly higher levels on mathematics standardized tests (SBAC) & Math GPA.
- During the school year the students who took the class were 68% more engaged in math class.
- *Boaler, Dieckmann, Pérez-Núñez, Sun & Williams, 2018*

Key 4. Engage with content with a lens of multiplicity – valuing different ways of thinking & seeing content


Fixed Maths
Mindset

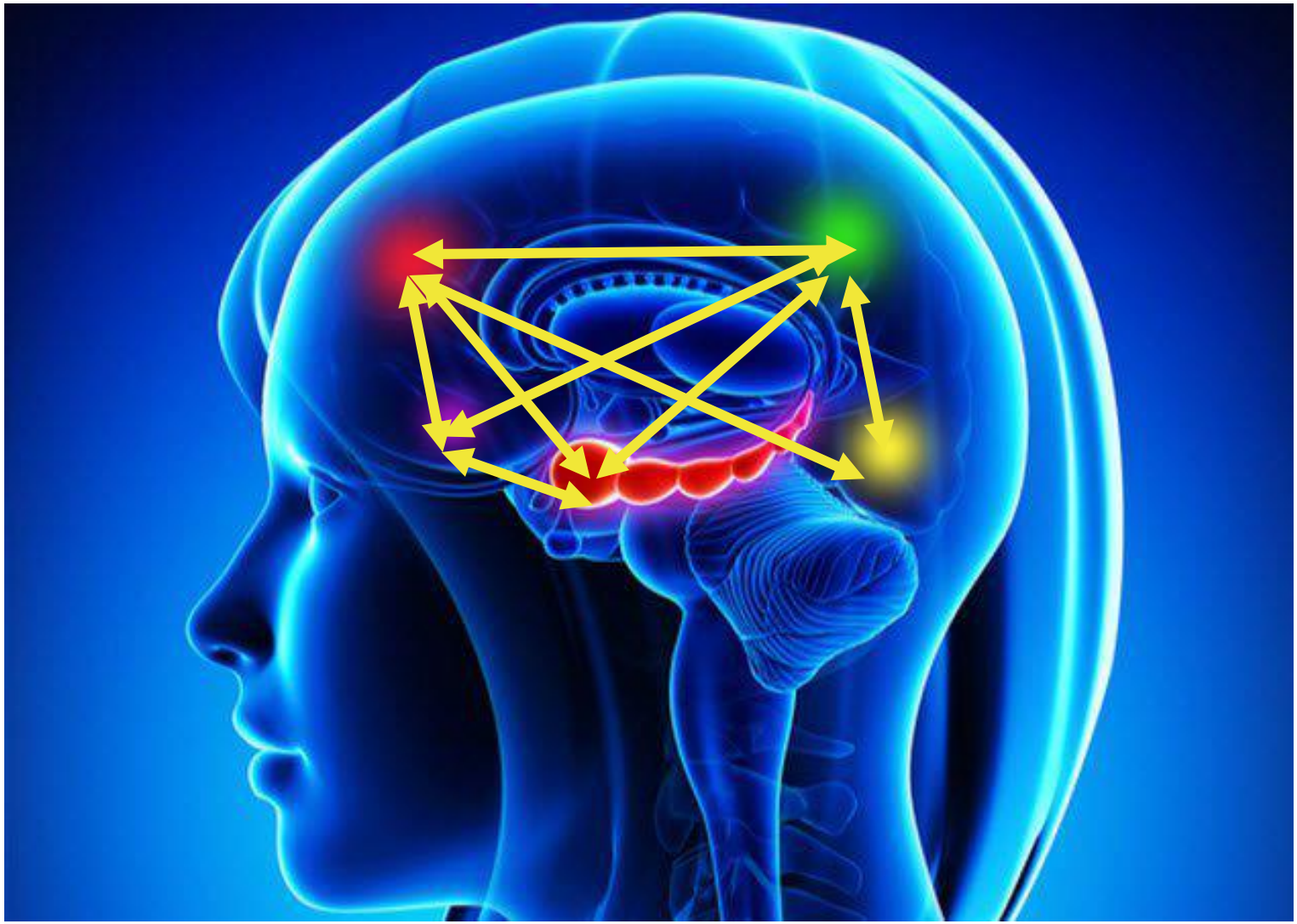
Multiple Brain Pathways

When we work mathematically different areas of the brain light up and communicate, including 2 visual pathways – the ventral and dorsal pathways.



Menon, V. (2015) Salience Network in Arthur Toga *Brain Mapping: An Encyclopedic Reference*

The most powerful brains are interconnected



Numbers, words, pictures, graphs, algorithms, tables, models, movement

youcubed
at Stanford University

Changing questions in maths class so that there is :

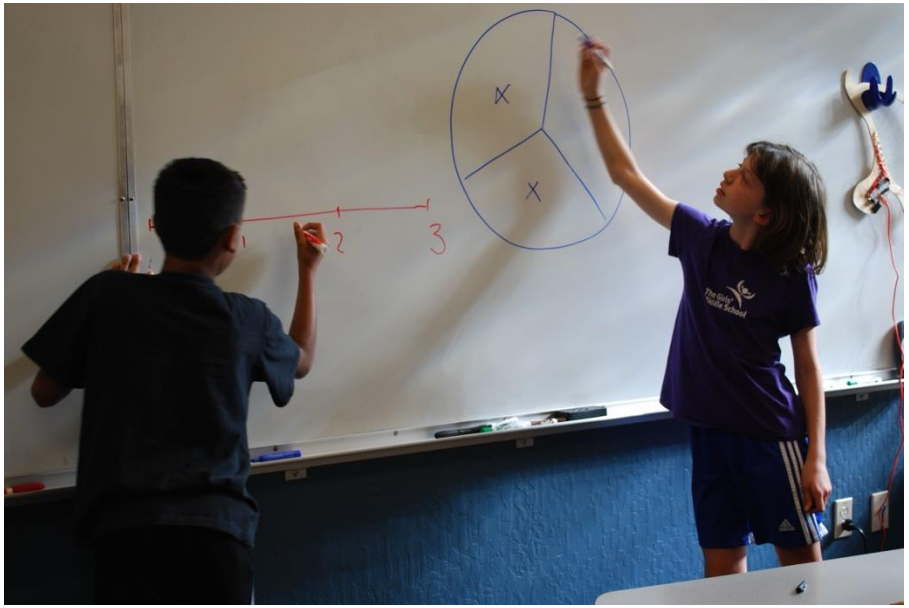
Opening up maths

- Space inside them to learn
- Opportunities for brain connections

fractions 1 ÷ 2/3

Fixed Maths

What is the answer?



Growth Maths

"You may know a rule but I don't care about the rule today" Cathy Humphreys

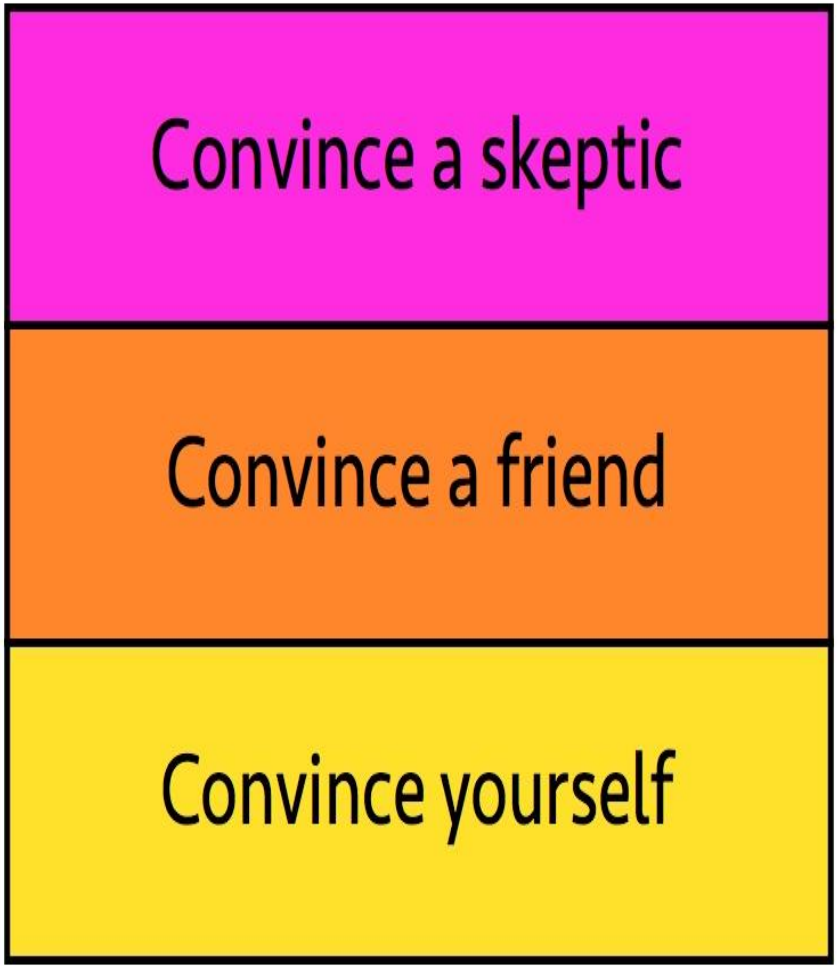
You need to make sense of your answer

Show your thinking visually

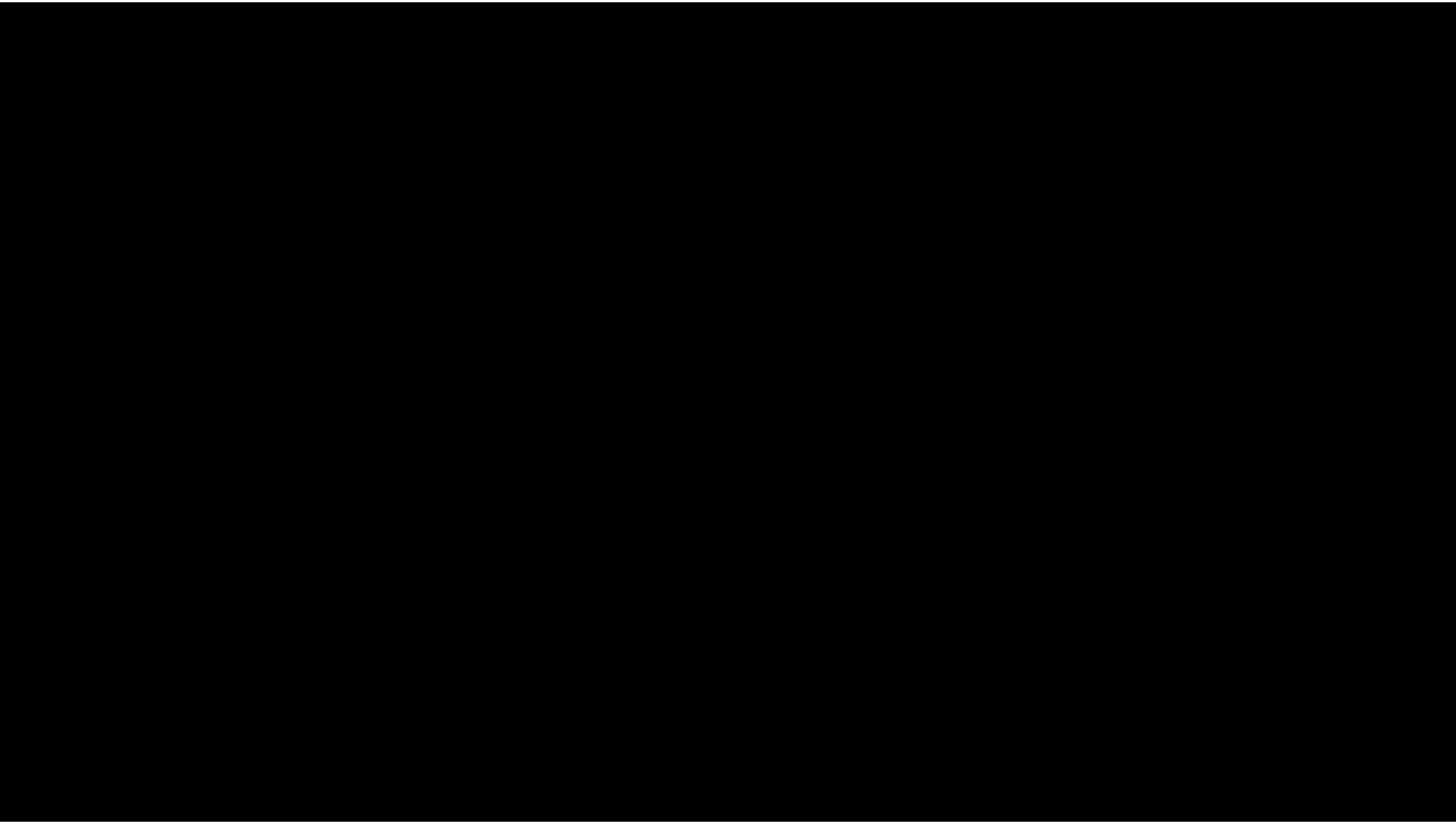
The task now has space inside it for learning

Encouraging collaboration

Reasoning is the essence of mathematics



Josh’s proof



Mathematics Teaching in the Middle School (2019)

Prove It to Me!

Jo Boaler

Engage your learners through tasks proven to significantly promote reasoning and problem solving, which touch on many of the Mathematics Teaching Practices in *Principles to Actions: Ensuring Mathematical Success for All*. These tasks are discussed in this article, another installment in the series.

Reasoning and problem solving are both critical acts in mathematics learning. I have met many teachers who tell me they would love to highlight them more but feel they cannot because of the pressures of content coverage, pacing guides, and district tests. In this article, I describe a teaching intervention that highlighted problem solving and reasoning and resulted in changed mathematics pathways for students, whether they were at the low, middle, or high end of the achievement spectrum. Two summers ago, my Stanford youcubed team and I invited 81 middle school students to the Stanford campus to learn mathematics differently. I knew from many research studies that students would be helped if they were actively engaged in mathematics, solving creative and rich problems, and if they believed in their potential (Boaler 2016; 2015). I did not, however, expect the huge positive change the students made in their achievement, their beliefs about themselves, and their relationship with mathematics.

STRUCTURE OF THE YOUCUBED SUMMER CAMP

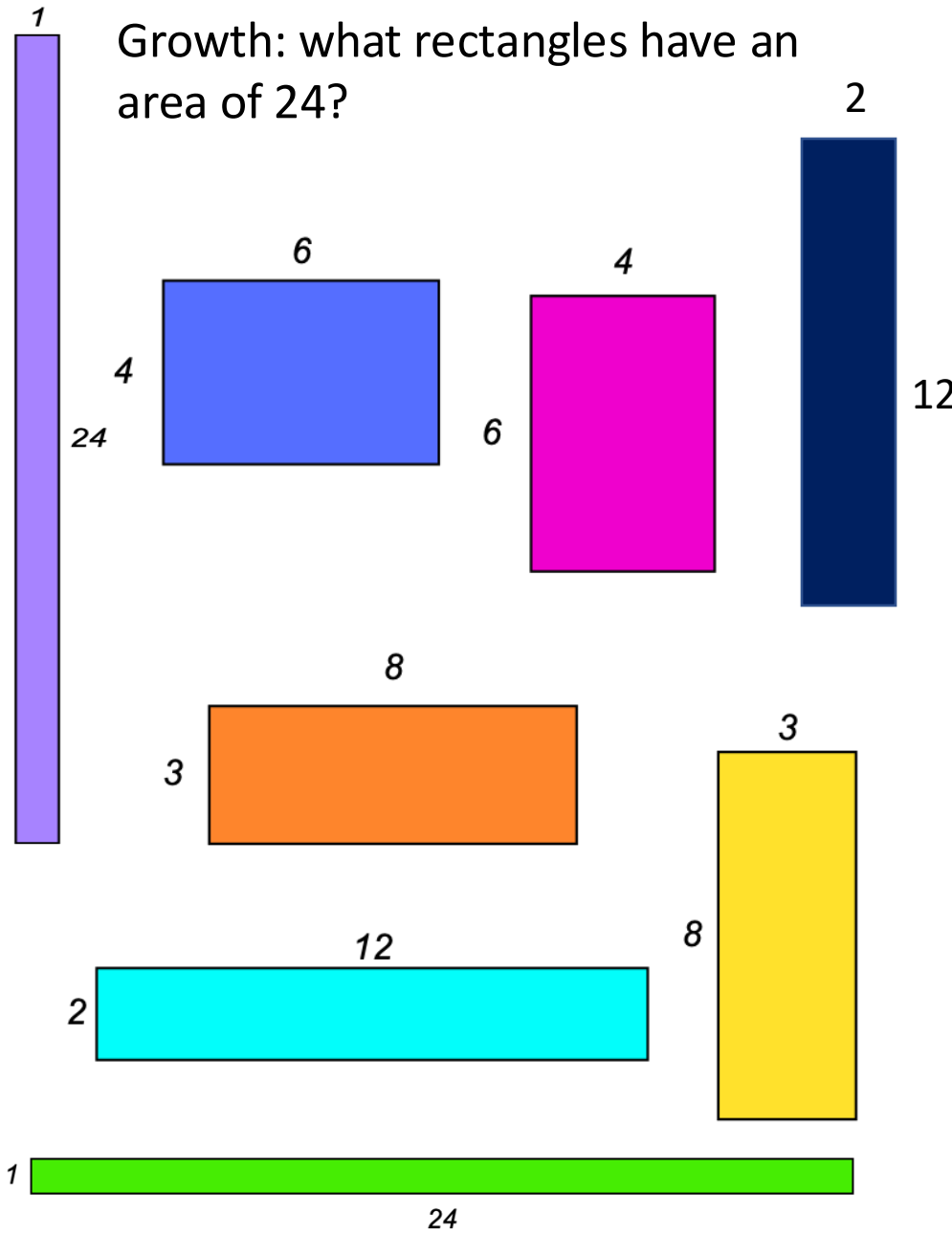
In preparation for the youcubed summer camp, I asked district math leaders if they could find students who had developed negative ideas about themselves and about math. They assured me that they could and sent the details of the camp to teachers

Rectangle Area

Fixed:

What is the area of an 8 x 3 rectangle?

Growth: what rectangles have an area of 24?



Work this out in your head, with no paper and pencil:

18 x 5


4
18
x5

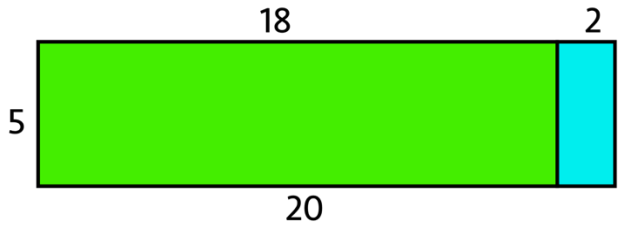
20 x 5 = 100
2 x 5 = 10
100-10 = 90

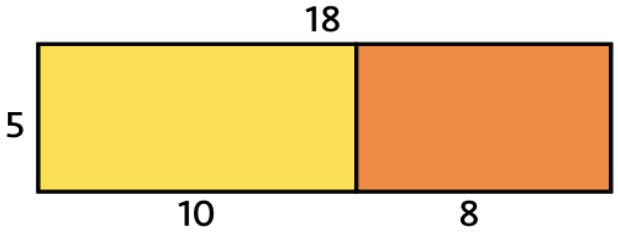
10 x 5 = 50
8 x 5 = 40
50 +40 = 90

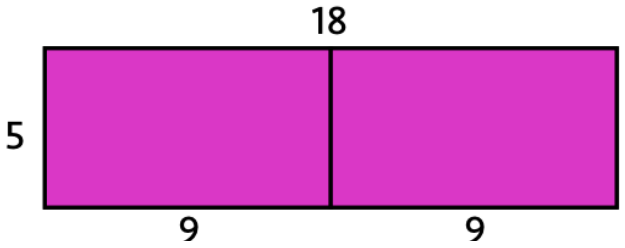
9 x 5 = 45
9 x 5 = 45
45 + 45 = 90

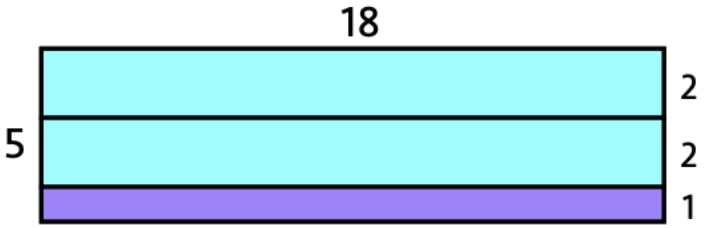
18 x 2 = 36
18 x 2 = 36
18 x 1 = 18
36 + 36 + 18 = 90

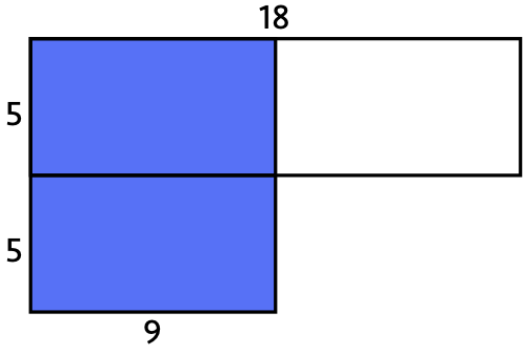
18 x 5 = 9 x 10











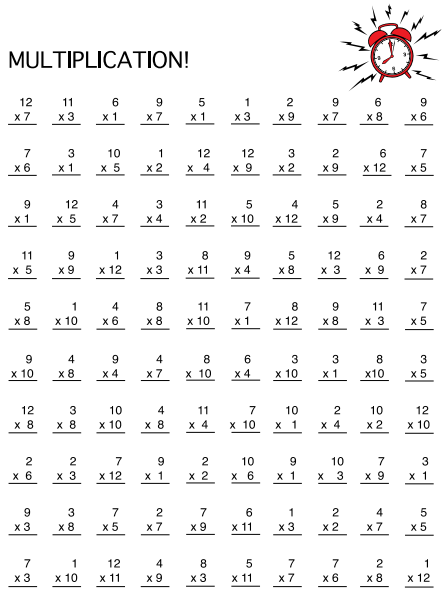
Key 5. Flexibility & Depth – over speed

Dissociating Mathematics from Speed

"I was always deeply uncertain about my own intellectual capacity; I thought I was unintelligent. And it is true that I was, and still am, rather slow. I need time to seize things because I always need to understand them fully. Towards the end of the eleventh grade, I secretly thought of myself as stupid. I worried about this for a long time.

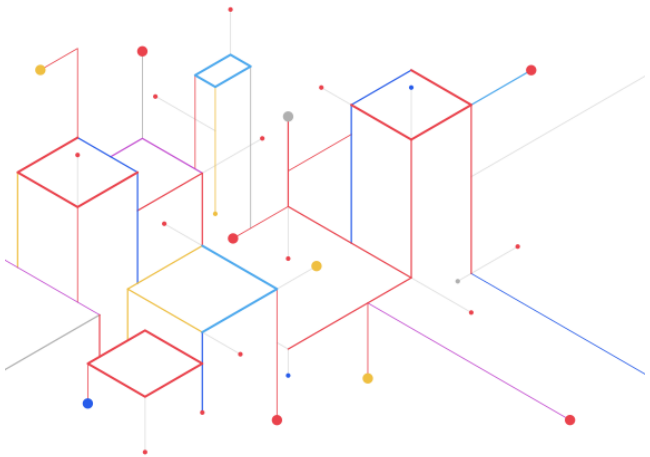
I'm still just as slow. (...)At the end of the eleventh grade, I took the measure of the situation, and came to the conclusion that rapidity doesn't have a precise relation to intelligence. What is important is to deeply understand things and their relations to each other. This is where intelligence lies. The fact of being quick or slow isn't really relevant."

Lauren Schwartz,
Fields Medal Winning
Mathematician



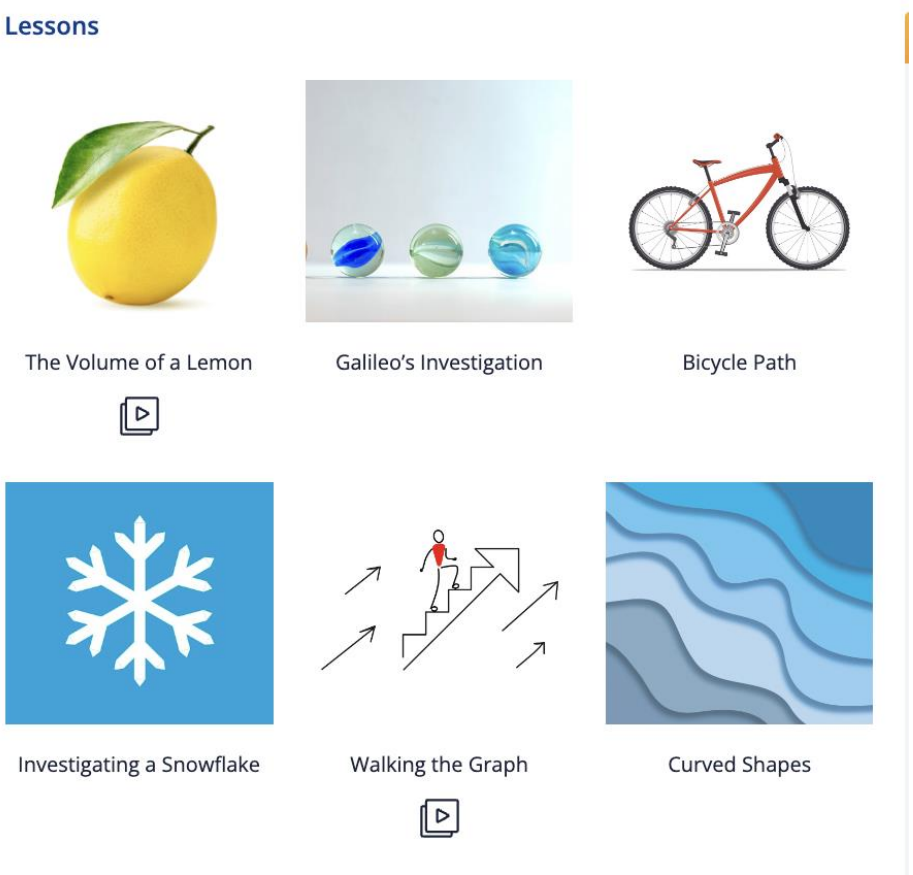
Moving to a Limitless Perspective: From Reel Link Films:

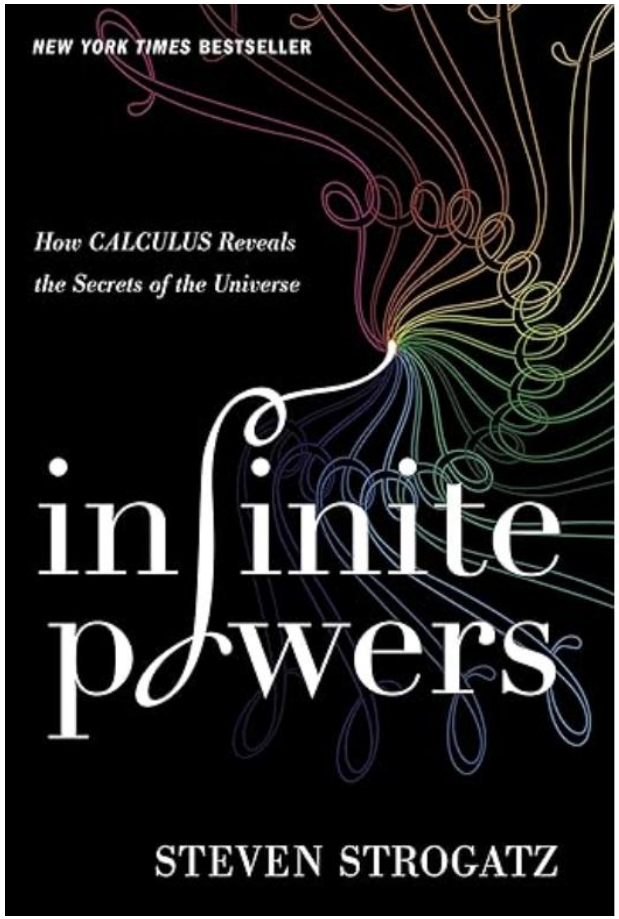
conceptual learning



- Deeper
- Asking questions, being curious
- Reasoning
- Being skeptical
- Visualizing
- Building mental representations

Calculus – through visuals & physical models



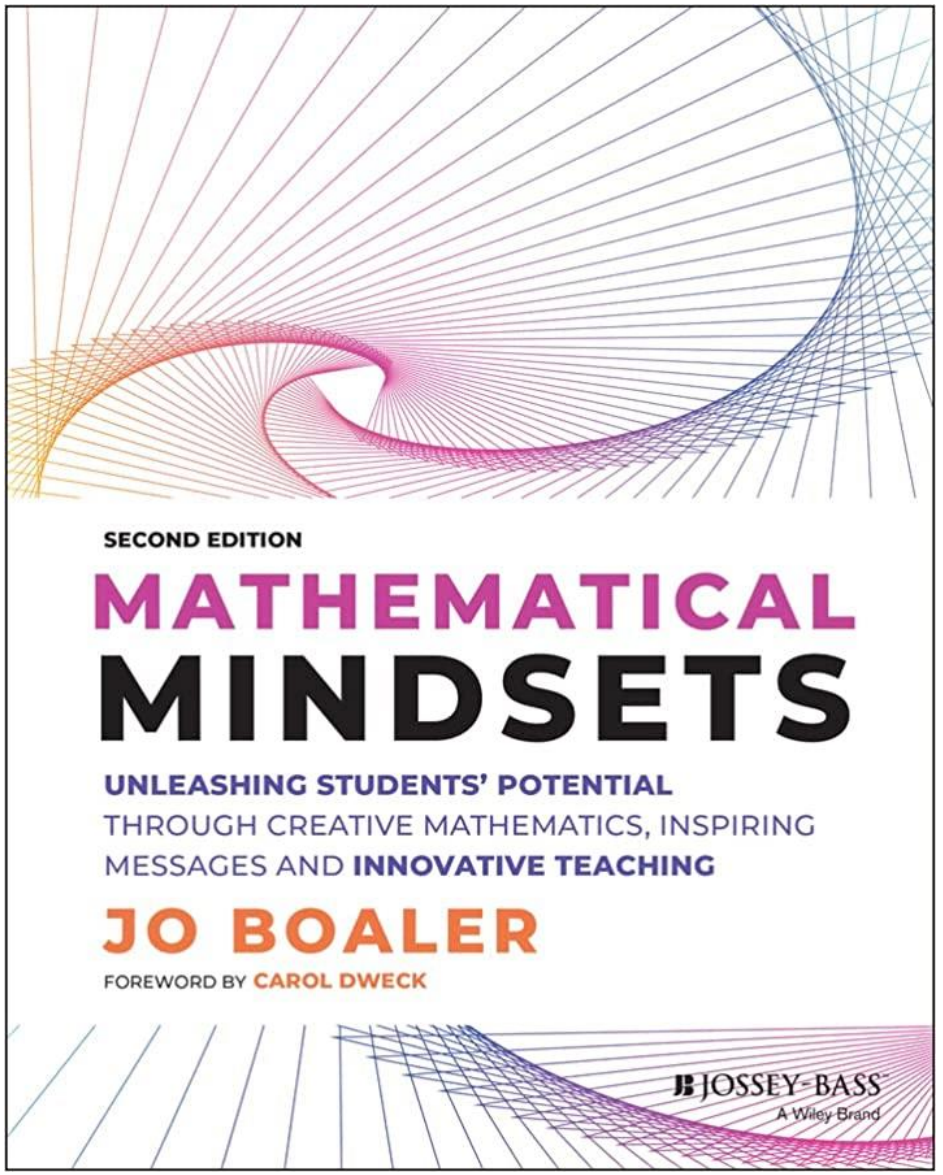


Stanford calculus class

Calc Student reflection

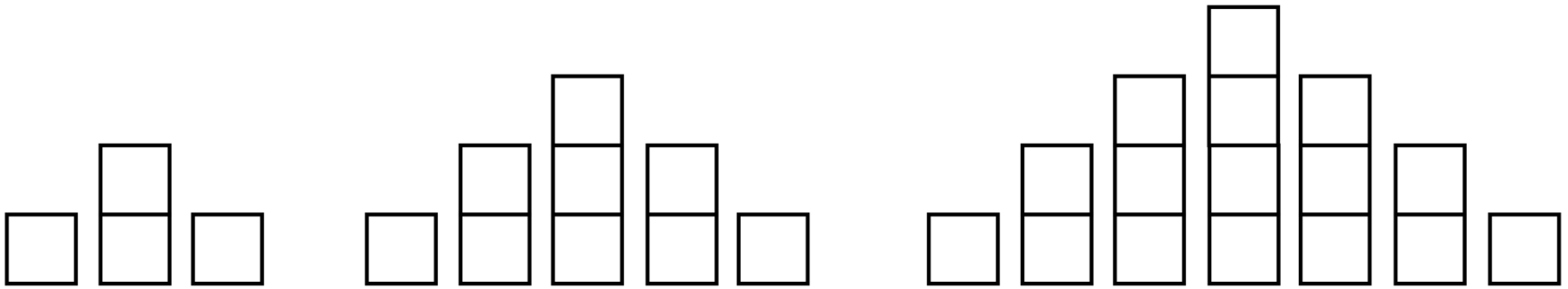
I feel as though my relationship with maths has slowly shifted from abusive to healthy over the past month. I know that we still have a ways to go, but maths and I are on the "right track". I feel like I am now able to see the light at the end of the tunnel so to speak. This course gave maths and me a chance to really sit down and understand each other. (Final Reflection)

Can we open up algebra?



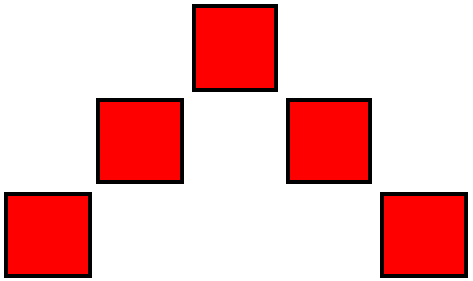
Ruth
Parker

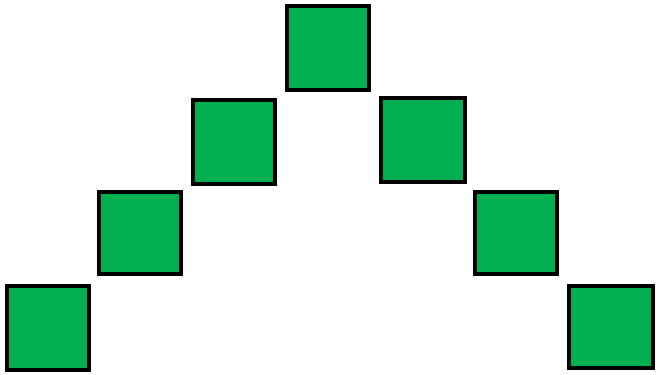
The pattern is growing –
Where do you see the extra squares?

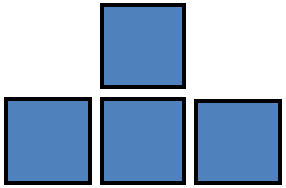


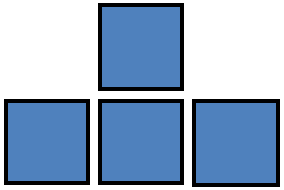
Case 1 Case 2 Case 3

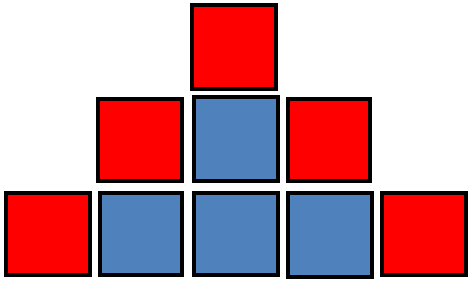
The Raindrop Method



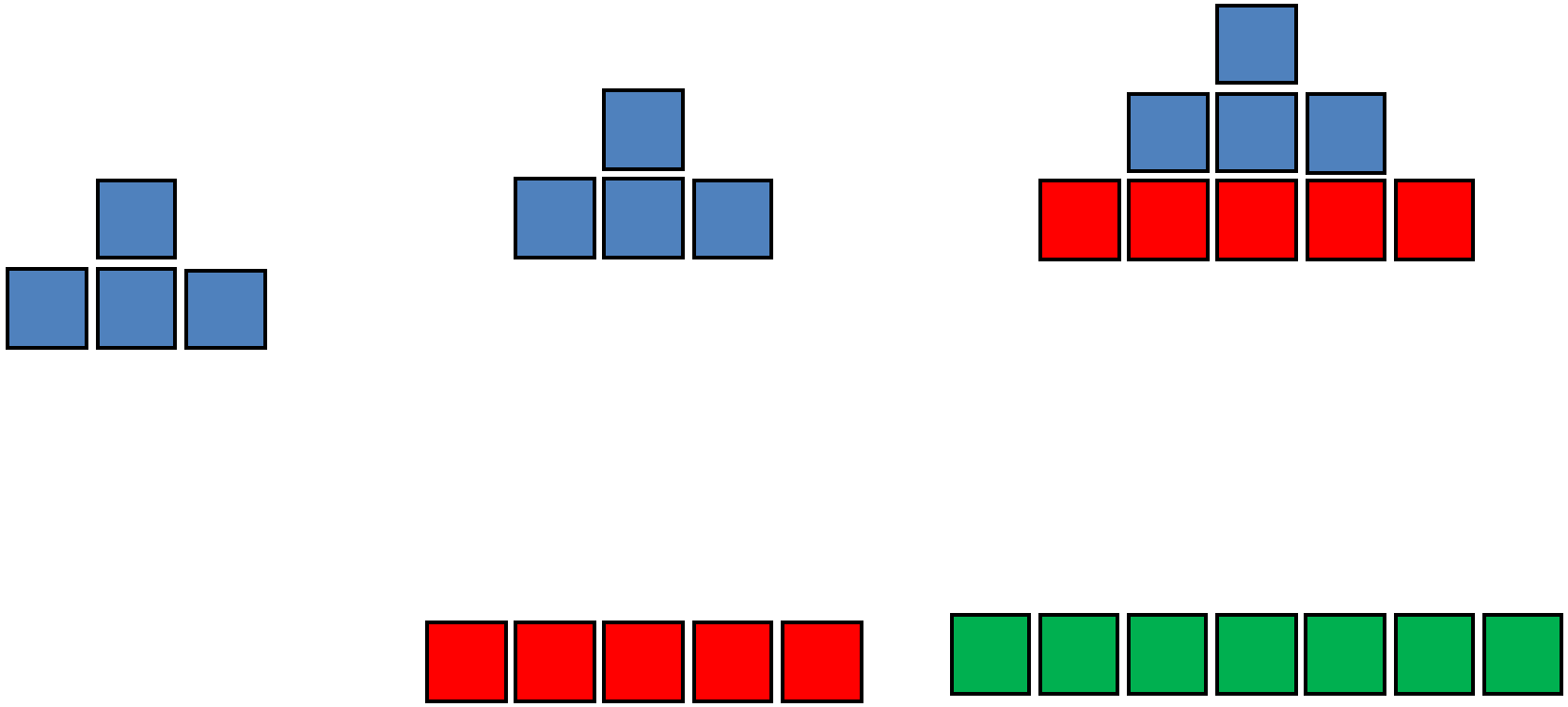




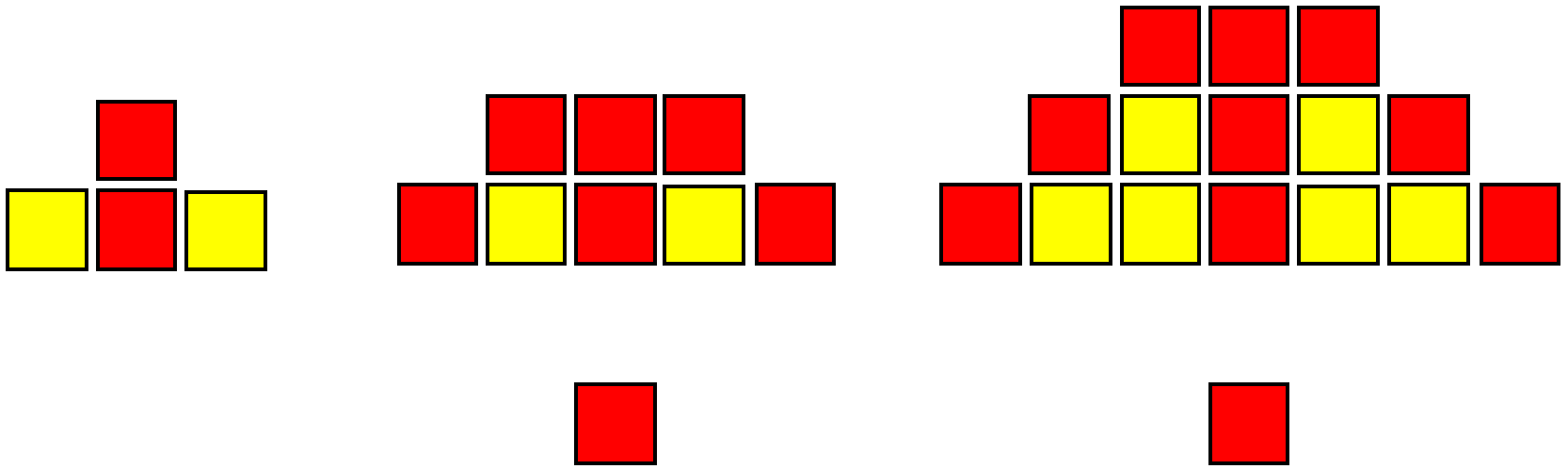




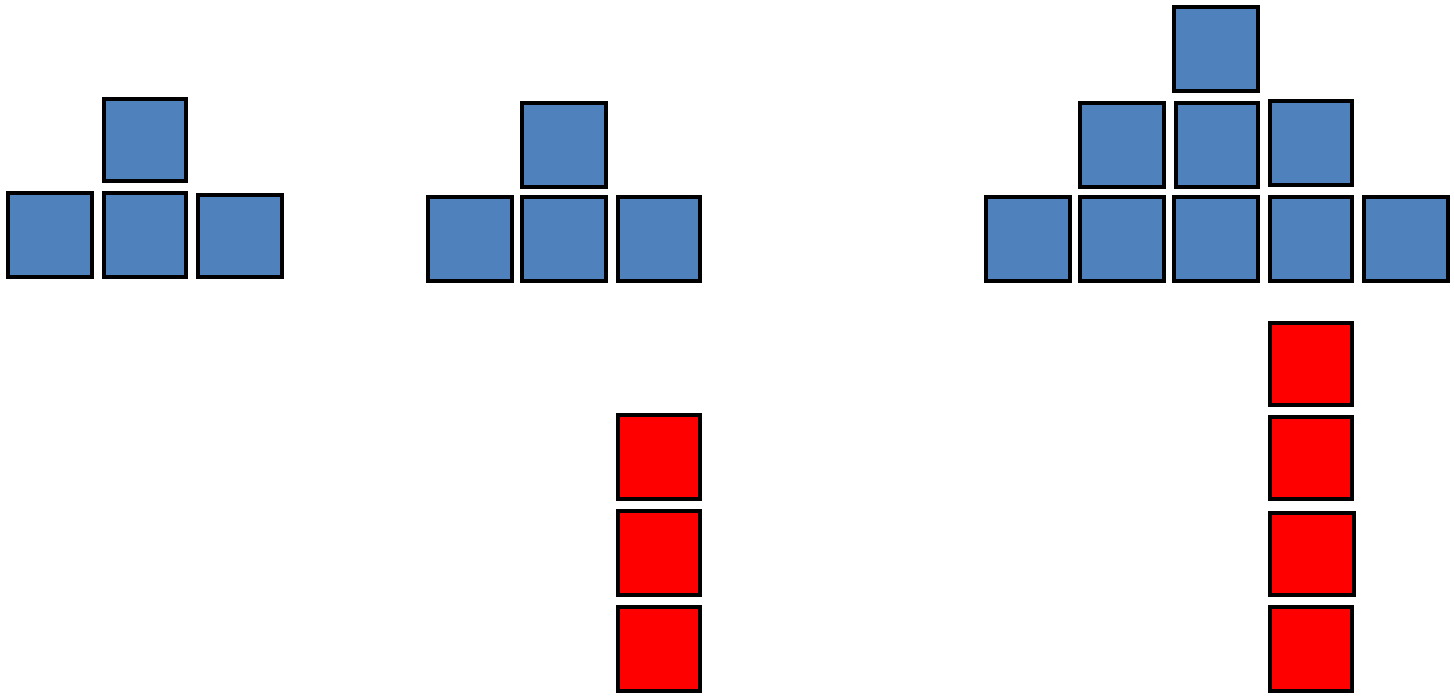
The Bowling Alley Method



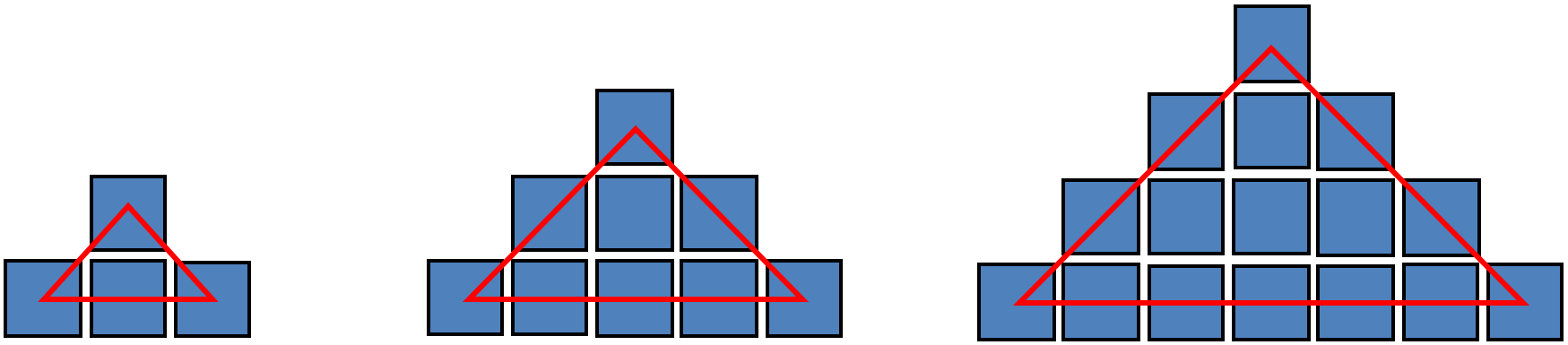
The Volcano Method



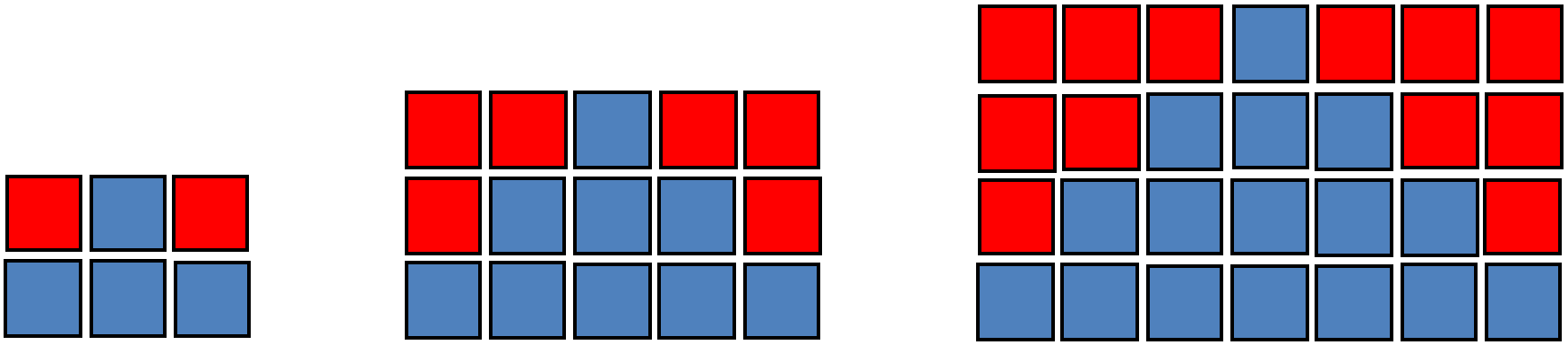
The Rocket Launch Method



Similar Triangles Method

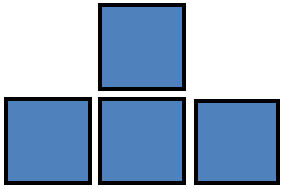


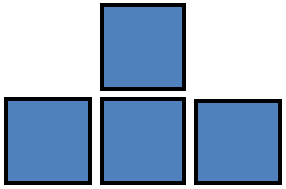
The Empty Triangles Method

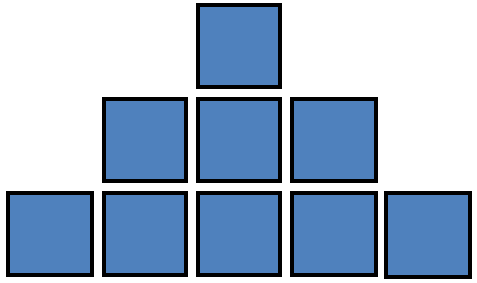


The Snow Method

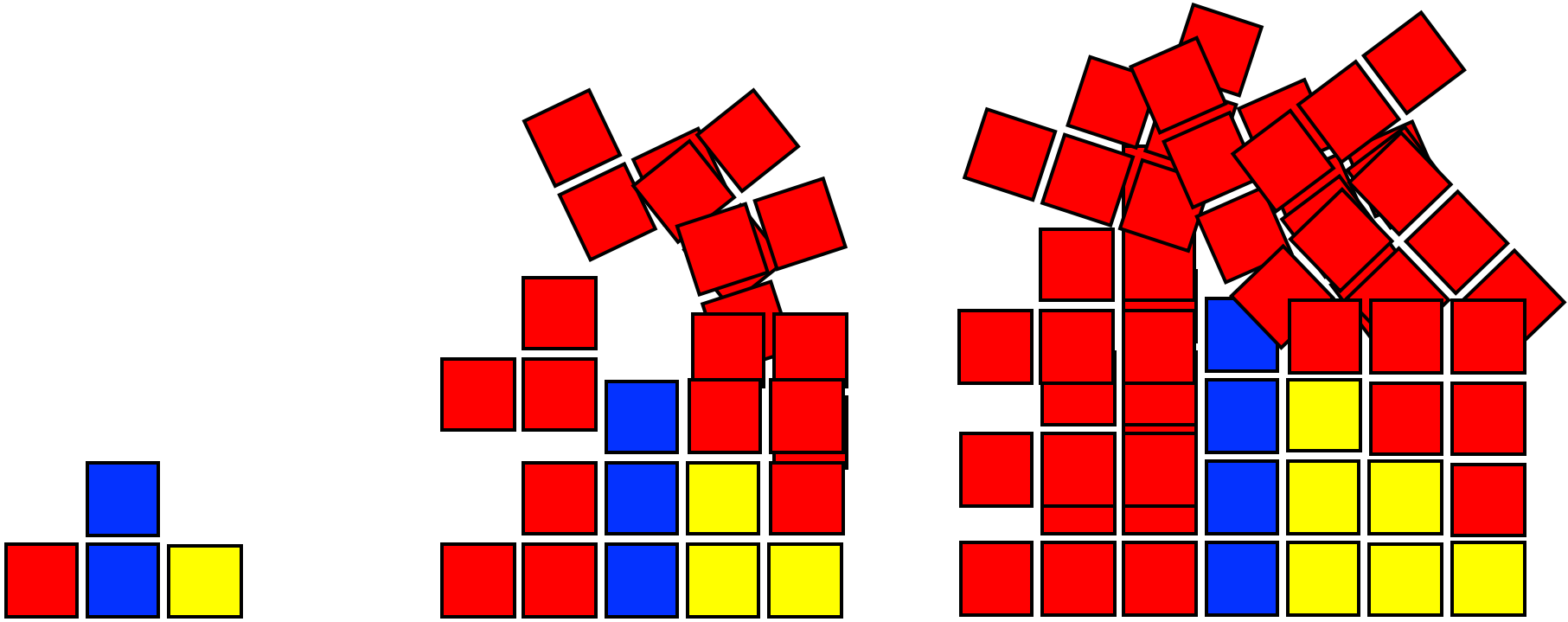
The Minecraft Tree Method

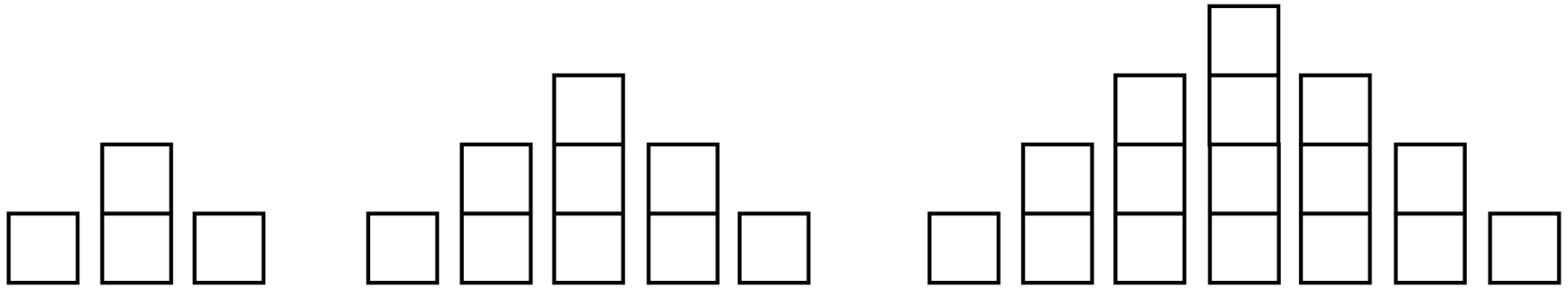






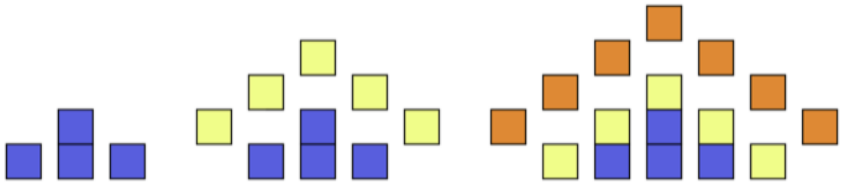
Square Method



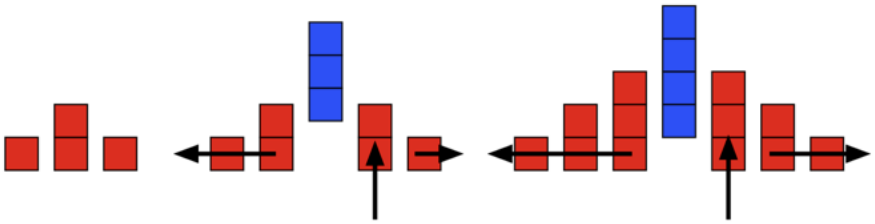


Case	# Blocks
1	4
2	9
3	16
n	$(n+1)^2$

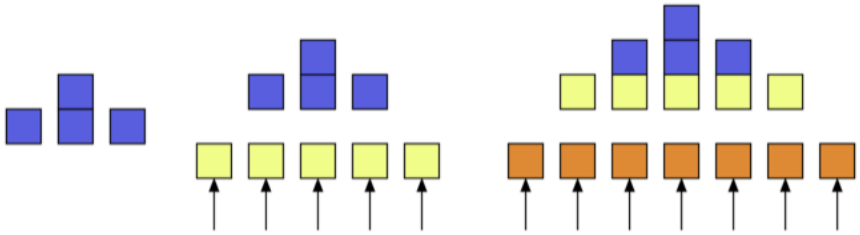
The raindrop method



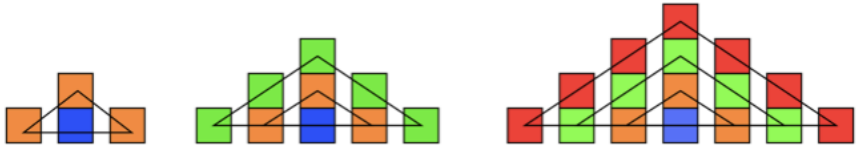
The rocket launch



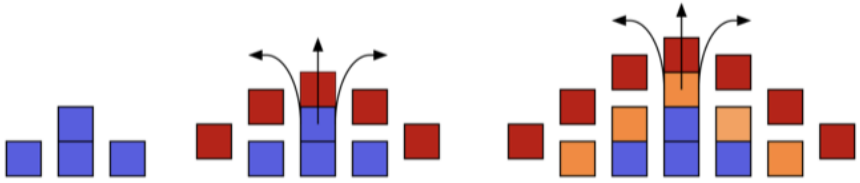
The bowling alley method



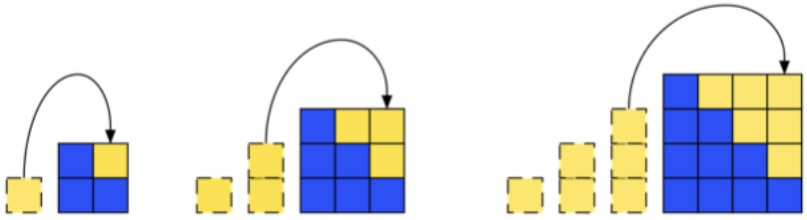
Triangular growth



The volcano method

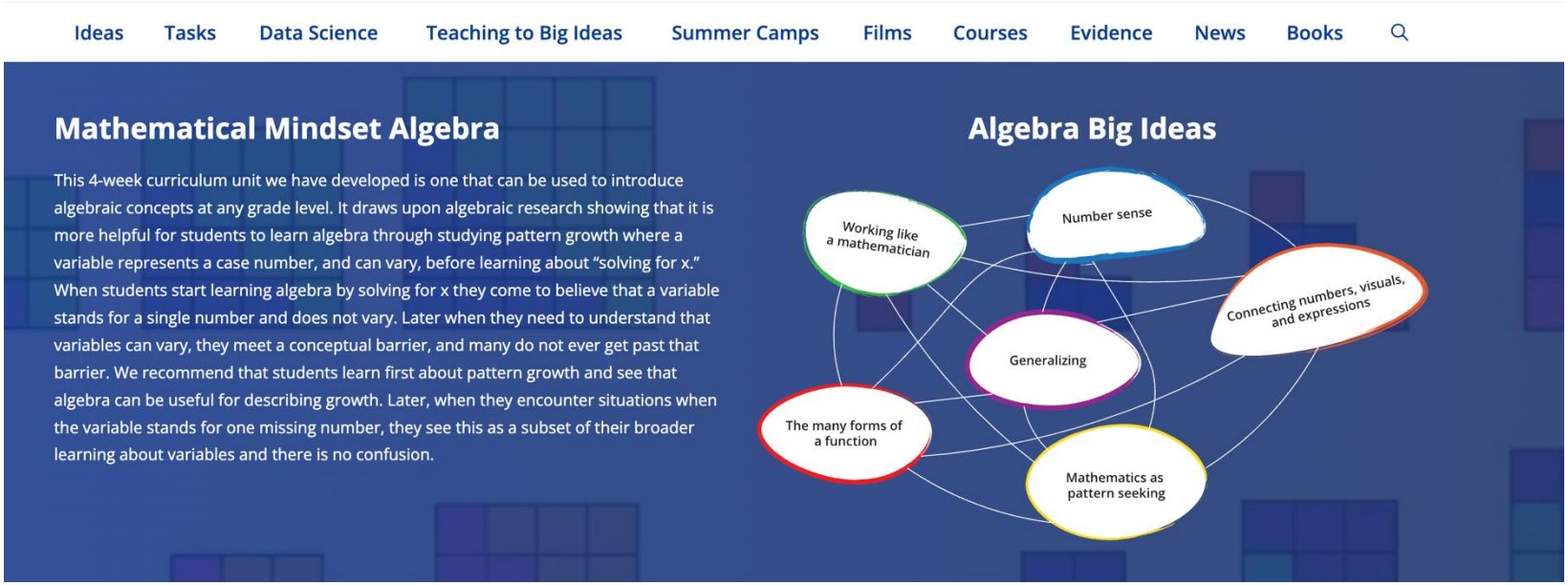


The square method



$(n+1)^2$

4 weeks + of Algebra Lessons



youcubed at Stanford University

Our latest unit

New Unit on youcubed – Basketball, Maths & Data

In collaboration with Stanford Women's Basketball Team

What Data is Collected on Players in Basketball?

Basketball Data

Height

Games played

Minutes/game

Points/game

Field goal %

2-point %

3-point %

Assists

Offensive rebounds

Defensive rebounds

Rebounds

Steals

Blocks

Turnovers

Fouls

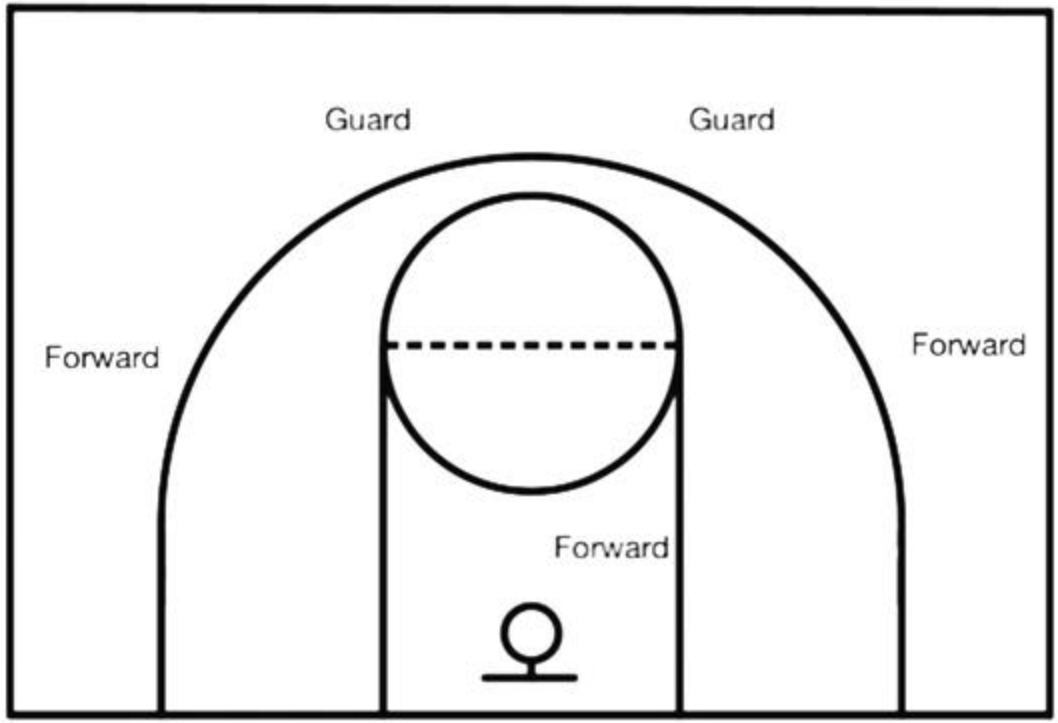
Fouls drawn


A Continuum of Engagement
OUR MATHEMATICIANS

A 3-pointer!!

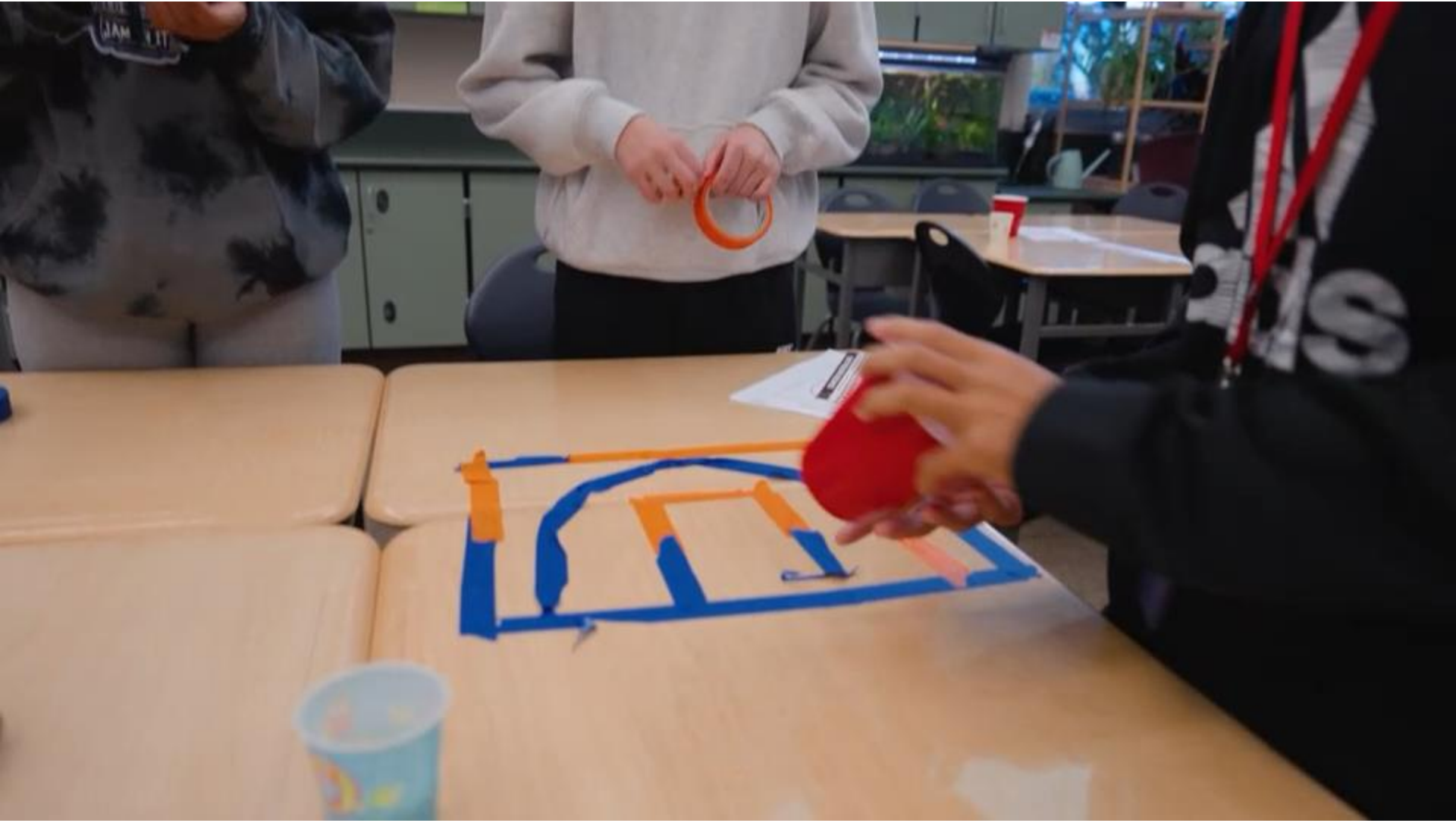
Mary Ashley Stevenson & Nunu Agara

Build your basketball court



Students try each move, and review what happens when the move happens in the game - do they get points? Does the ball move to the other team?

1. Offensive Rebound
2. Defensive Rebound
3. Field Goal Attempted- 2 point
   Field Goal Made- 2 point
4. Field Goal Attempted- 3 point
   Field Goal Made- 3 point
5. Assist
6. Block



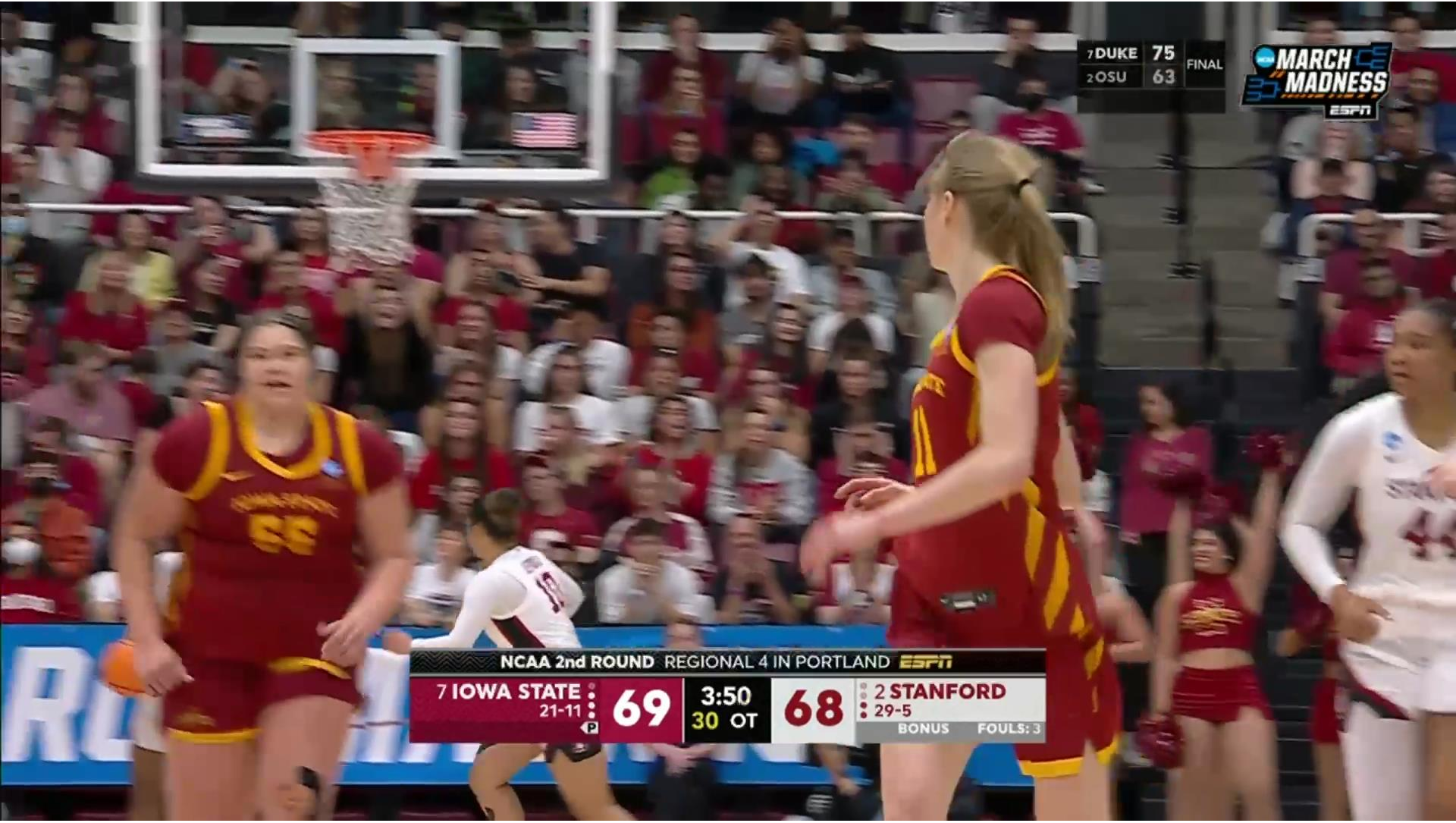
STANFORD BASKETBALL DATA

2 POINTERS (CIRCLE IF SCORED, CROSS OUT IF MISSED)

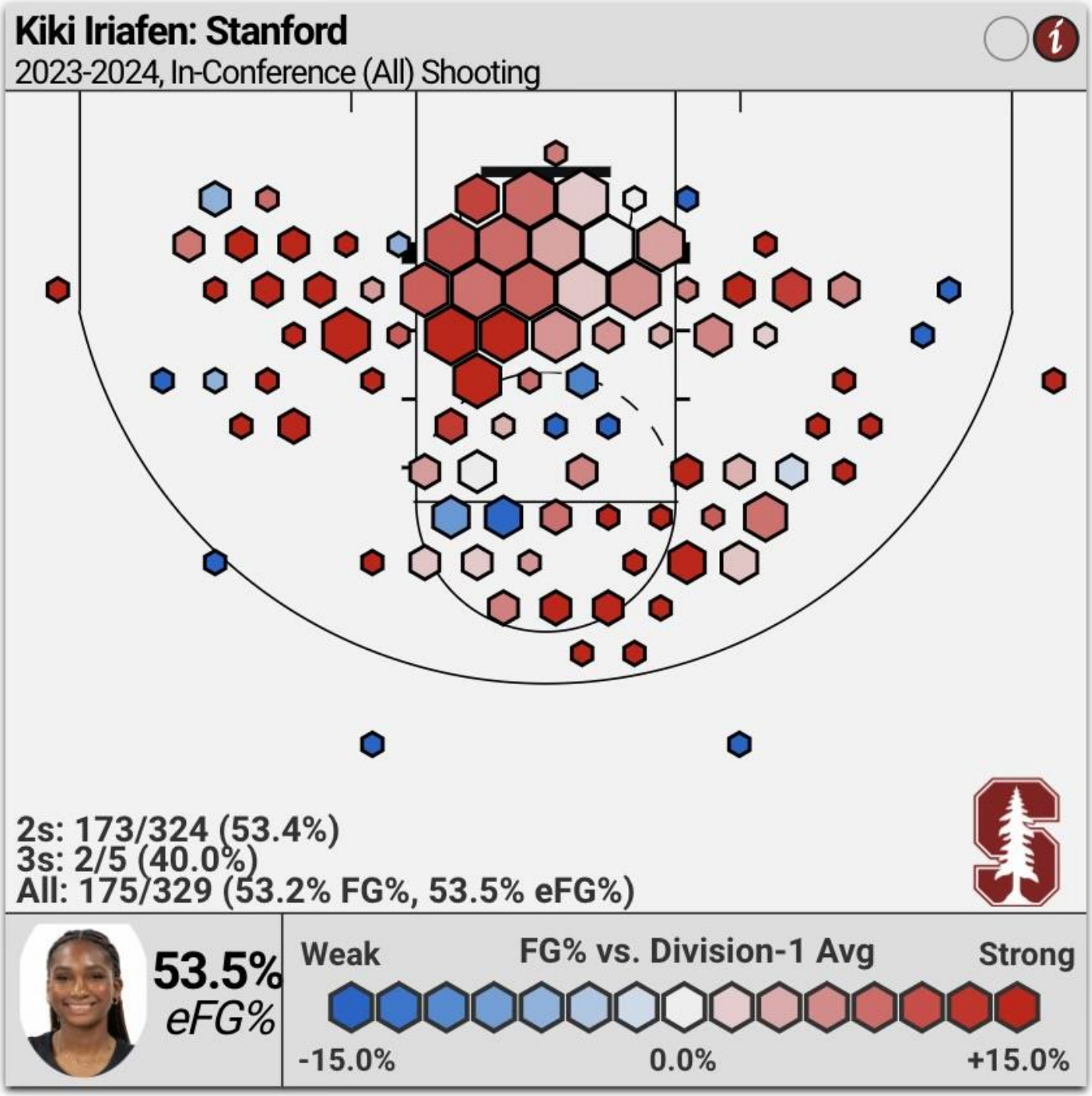
3 POINTERS (CIRCLE IF SCORED, CROSS OUT IF MISSED)

REBOUNDS - OFFENSIVE / DEFENSIVE (CIRCLE)

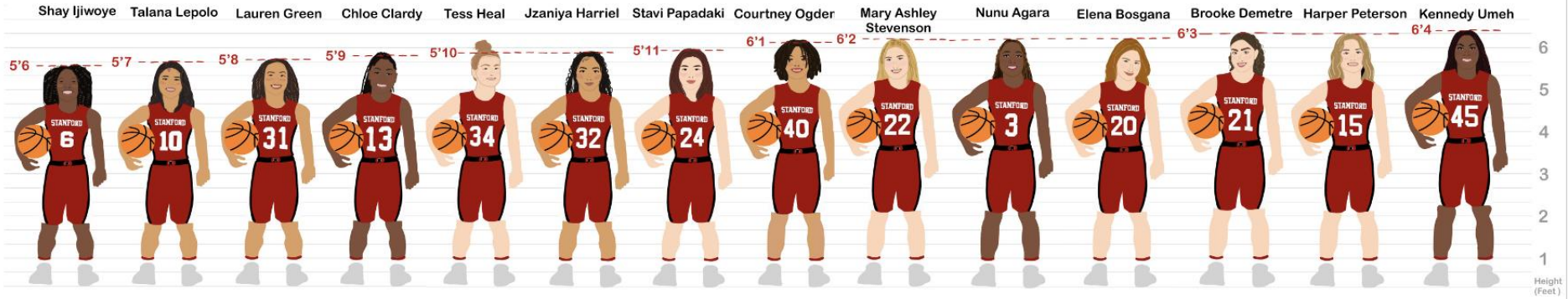
BLOCKS AND ASSISTS (CIRCLE)


7 DUKE 75
2 OSU 63
FINAL
MARCH MADNESS
ESPN
NCAA 2nd ROUND REGIONAL 4 IN PORTLAND ESPN
7 IOWA STATE
21-11
P
69
3:50
30 OT
68
2 STANFORD
29-5
BONUS
FOULS: 3

What do you notice,
What do you wonder?



Stanford Women Basketball 2024-2025 Heights
What do you notice? And what do you wonder?



youcubed®

Choose Your Dream team!

Your job is to choose your dream team of five players containing 2 guards and 3 forwards. Justify your choice of each player using graph tools, statistics and explanations. If there is time you may want to choose bench players too.

- Go to this link to find the data in CODAP. Here is a key to the data set.
- Find your group slides in the class slides and make a "poster".
  - Describe your findings with words and graphs. Include the mean and median for your data.

Be prepared to present your team and give arguments for why your team would win.

A Data Investigation!

An eg from Jo's students (Ana & Nick): The Kei'd and Sneakers

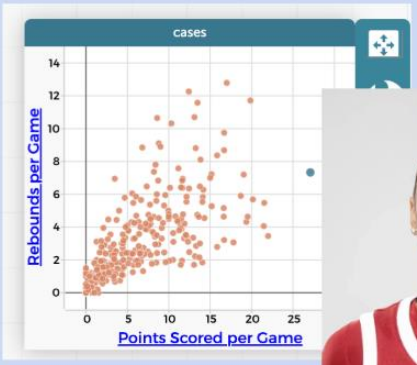
Potential Players

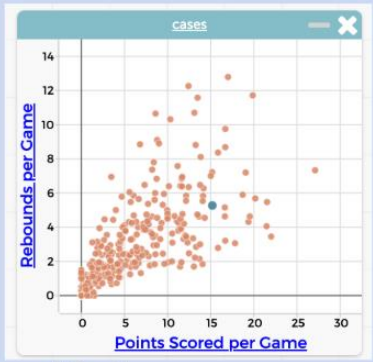
Cases (7 cases)

Index	Name	Height	PPG	RPG	APG	Pos	Place
1	Phillipina Kyei	80	12.42	12.42	2	F	Bench
2	Delanie Crawford	71	20.16	5.68	3	G	Bench
3	Juju Watkins	74	27.1	7.33	2.95	G	Start
4	Kiki Iriafen	75	19.86	11.71	2.76	F	Start
5	Cameron Brink	76	17	12.79	3.37	F	Start
6	Temira Poindexter	73	21.53	5.47	1.68	F	Start
7	Kailyn Gilbert	68	15.18	5.27	2.09	G	Start

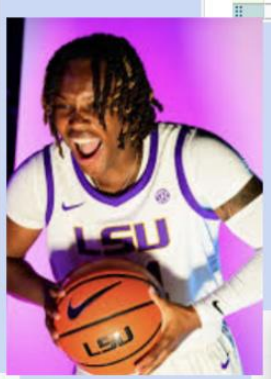
Guards:

Juju

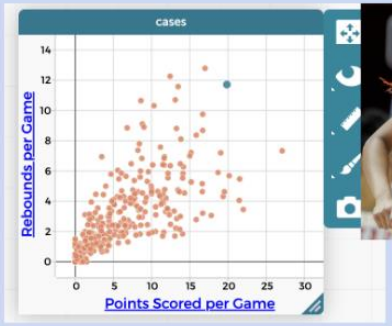


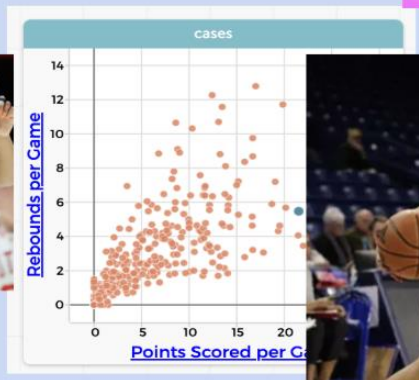


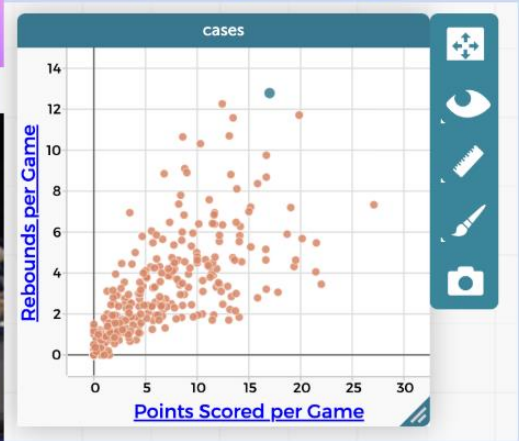
Kailyn



Forwards:







Kiki

Temira

Cameron

A Sabbatical Quarter!

A new book about maths ☺

Tahoe – Sierra Nevada Mountain Range, California.

Dougal

Chapters

1. A New Mathematical Relationship
2. Learning to learn
3. Valuing Struggle
4. Mathematics in the World
5. Mathematics as a Visual Experience
6. The Beauty of Mathematical Concepts and Connections
7. Diversity in Practice and Feedback
8. A New Mathematical Future

Harper Collins

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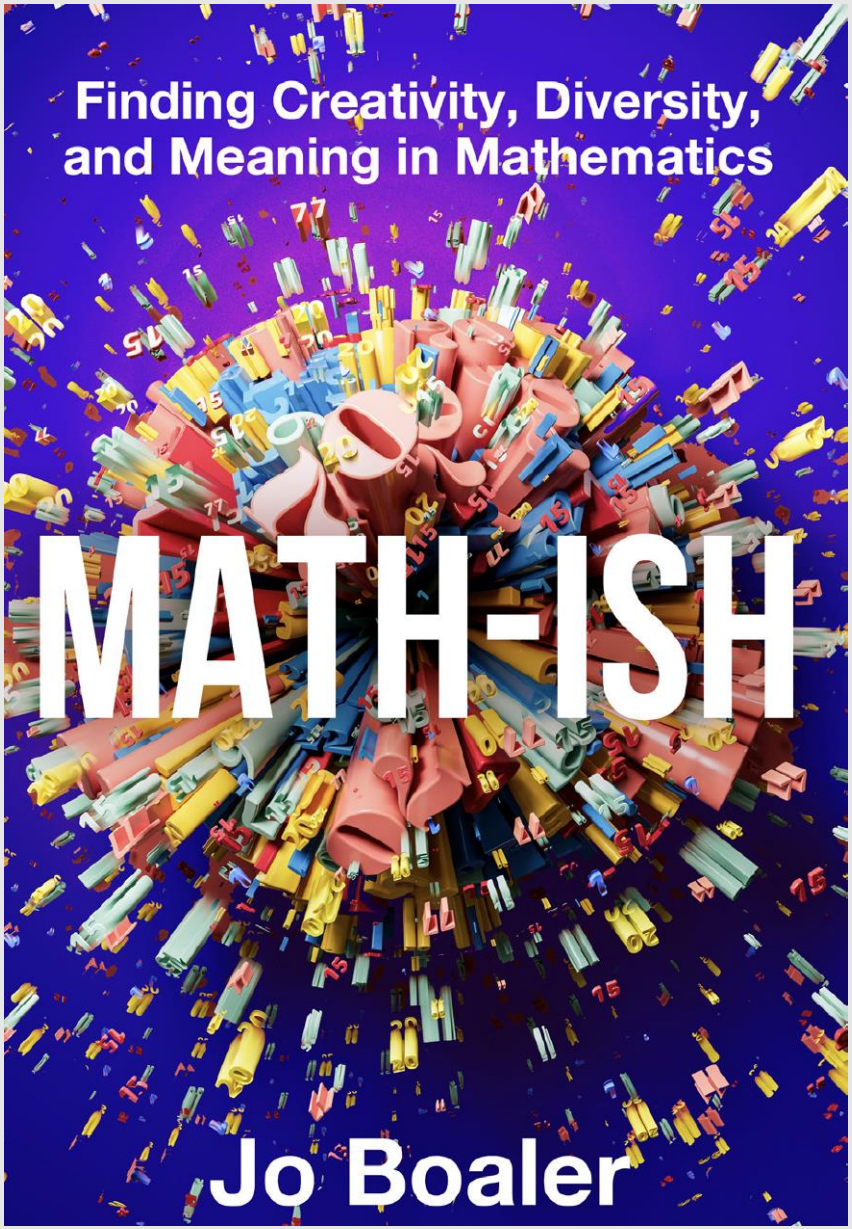
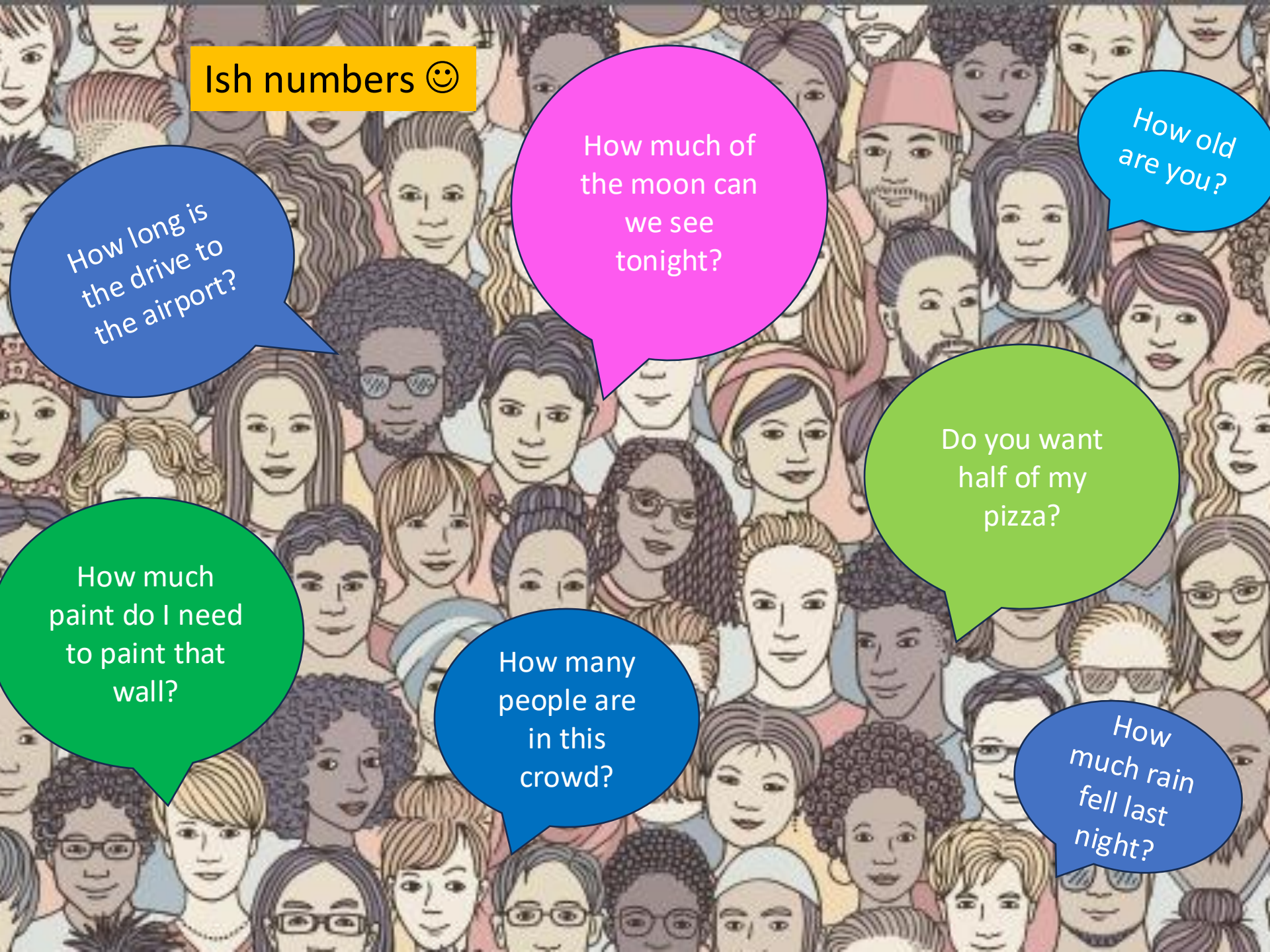
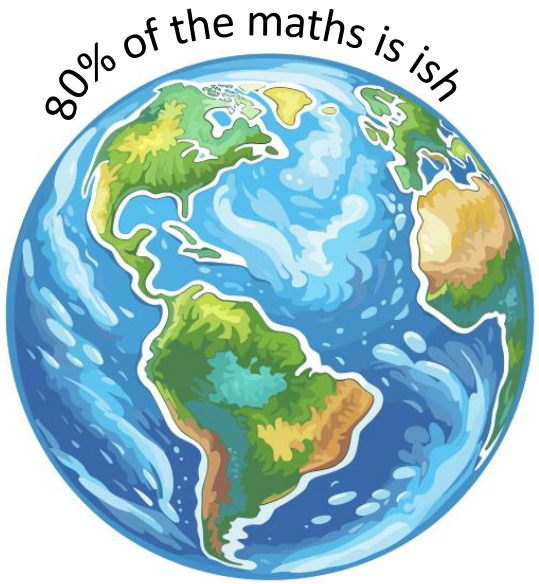



Ish numbers ☺
How much of the moon can we see tonight?
How old are you?
How long is the drive to the airport?
Do you want half of my pizza?
How much paint do I need to paint that wall?
How many people are in this crowd?
How much rain fell last night?

Maths in the world and in classrooms



Our focus is out of balance

2 modes of thinking – detailed & big picture - We need to learn to move fluidly between the 2

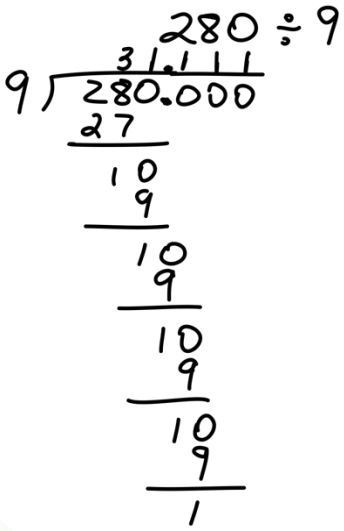
Dixon, M. L., De La Vega, A., Mills, C., Andrews-Hanna, J., Spreng, R. N., Cole, M. W., & Christoff, K. (2018). Heterogeneity within the frontoparietal control network and its relationship to the default and dorsal attention networks. *Proceedings of the National Academy of Sciences*, *115*(7), E1598-E1607.

When we move between ish and precise thinking

It activates different brain regions

The frontoparietal control network – A and B

Calculate it



For example:
280 divided by 9

Ish it ☺:

To ish you need a sense of the numbers
You need number sense!

Teacher reflection

I am starting to feel sad about young and older students that when asked to divide 272 by 8 mentally, they will try and do long division, without a real feel for whether the answer makes sense or not, inside their heads. Same for adding or subtracting mixed numbers. When asked to add 19 ¾ + 27 1/3 many students change these into improper fractions with a common denominator. These students will lose most sense of the numbers on the way. Is it that we don't take enough time for teaching true understanding? Is there not enough time for instilling joy of number sense and logical thinking?

Suzanne Dawson in our youcubed facebook group

Ask students to work out the ish number first

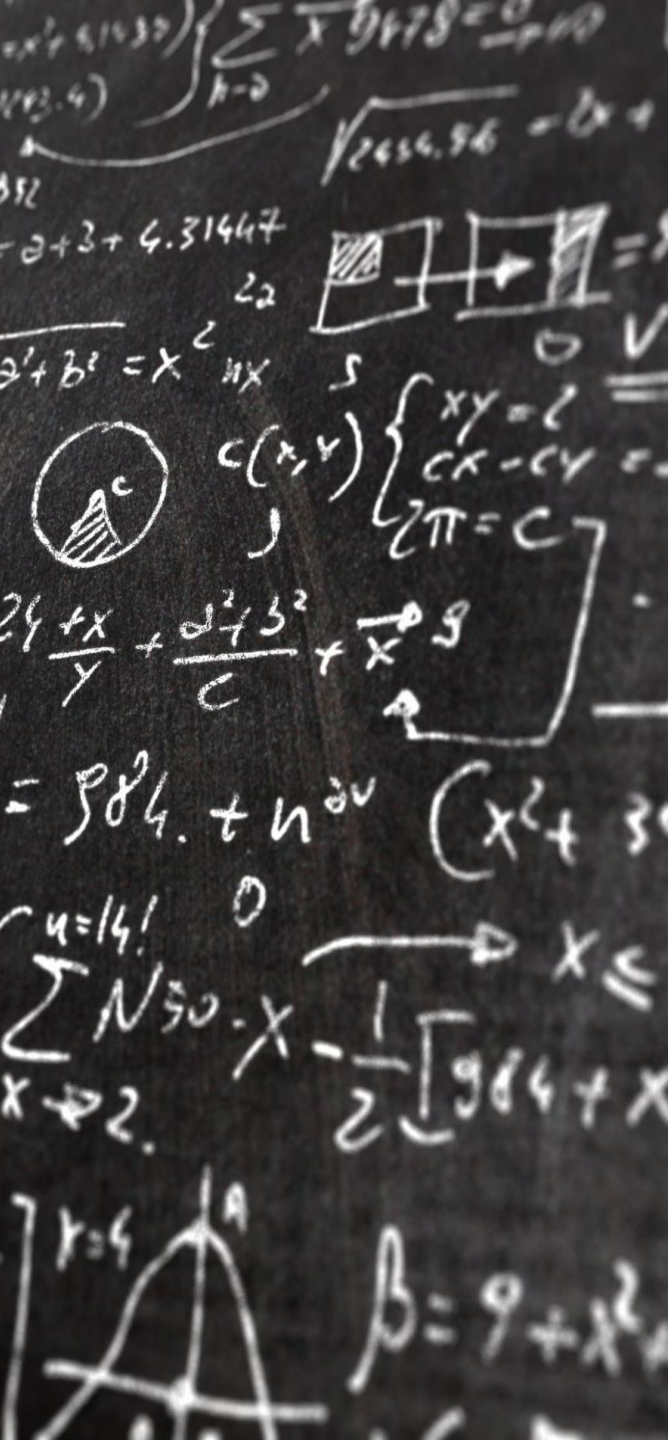
It will help them think conceptually and develop number sense

It is not just numbers that we can ish:

- Plotting Plot f (x+3)?
- What do you think it will look like?
- What do you think will happen?
- Small but transformative questions, that cause a pull back in thinking
- Gives access to deeper meaning

Ishing or estimating?

- Ishing is estimating but different
- When we ask students to estimate they think it is yet another method
- When we ask students to come up with ish numbers they think more freely, they lean in & engage



Ish-ing helps with standardized tests

$$\frac{12}{13} + \frac{7}{8}$$

1
2
19
21

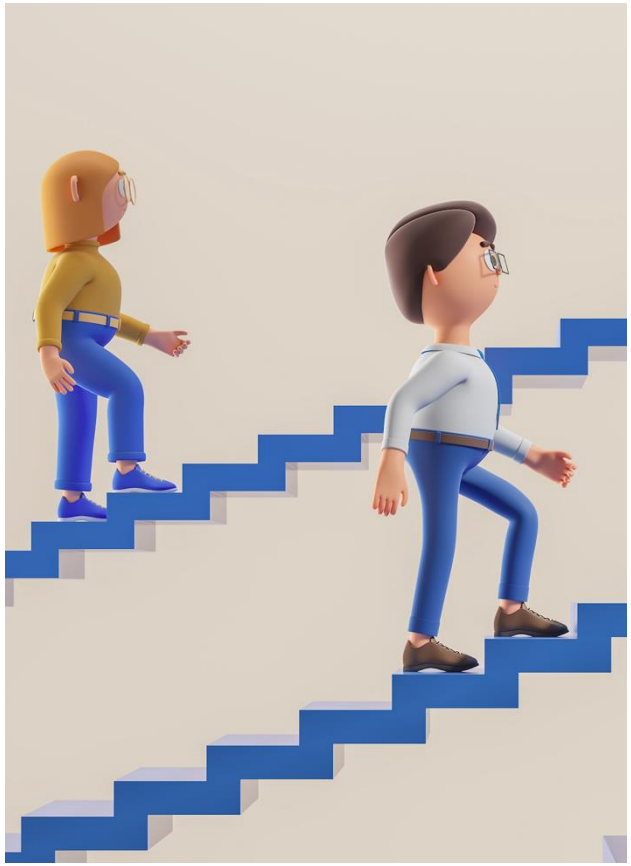
Most common answer: 19
Followed by 21

NAEP: Only 24% of 13-year-olds and 37% percent of 17-year-olds answered the question correctly.

Ishing helps with maths achievement!

Elementary students' ability to estimate, particularly magnitude, predicts formal mathematics achievement across the grades (Schneider et al., 2018).

Siegler et al. (2011) found that students' ability to estimate magnitudes on number lines, foundational for students' understanding of arithmetic and fractions, was an independent predictor of mathematics achievement-test scores all the way up to age 14.



Schneider, M., Merz, S., Stricker, J., De Smedt, B., Torbeyns, J., Verschaffel, L., & Luwel, K. (2018). Associations of number line estimation with mathematical competence: A meta-analysis. *Child Development*, *89*(5), 1467-1484.

Siegler, R., Thompson, C., & Schneider, M (2011). An Integrated Theory of Whole Number and Fractions Development. Cognitive Psychology, 62, 273-296.

I am not saying:

- That precision is not important
- I am saying we need both – they need to be in balance

Math-ish

NAEP Grade 4 Question

Subtract.

$$\begin{array}{r} 3.48 \\ -\ 1.46 \\ \hline \end{array}$$

a) 1.02
b) 2.02
c) 4.94
d) 5.94

https://www.nationsreportcard.gov/mathematics/sample-questions/?grade=4

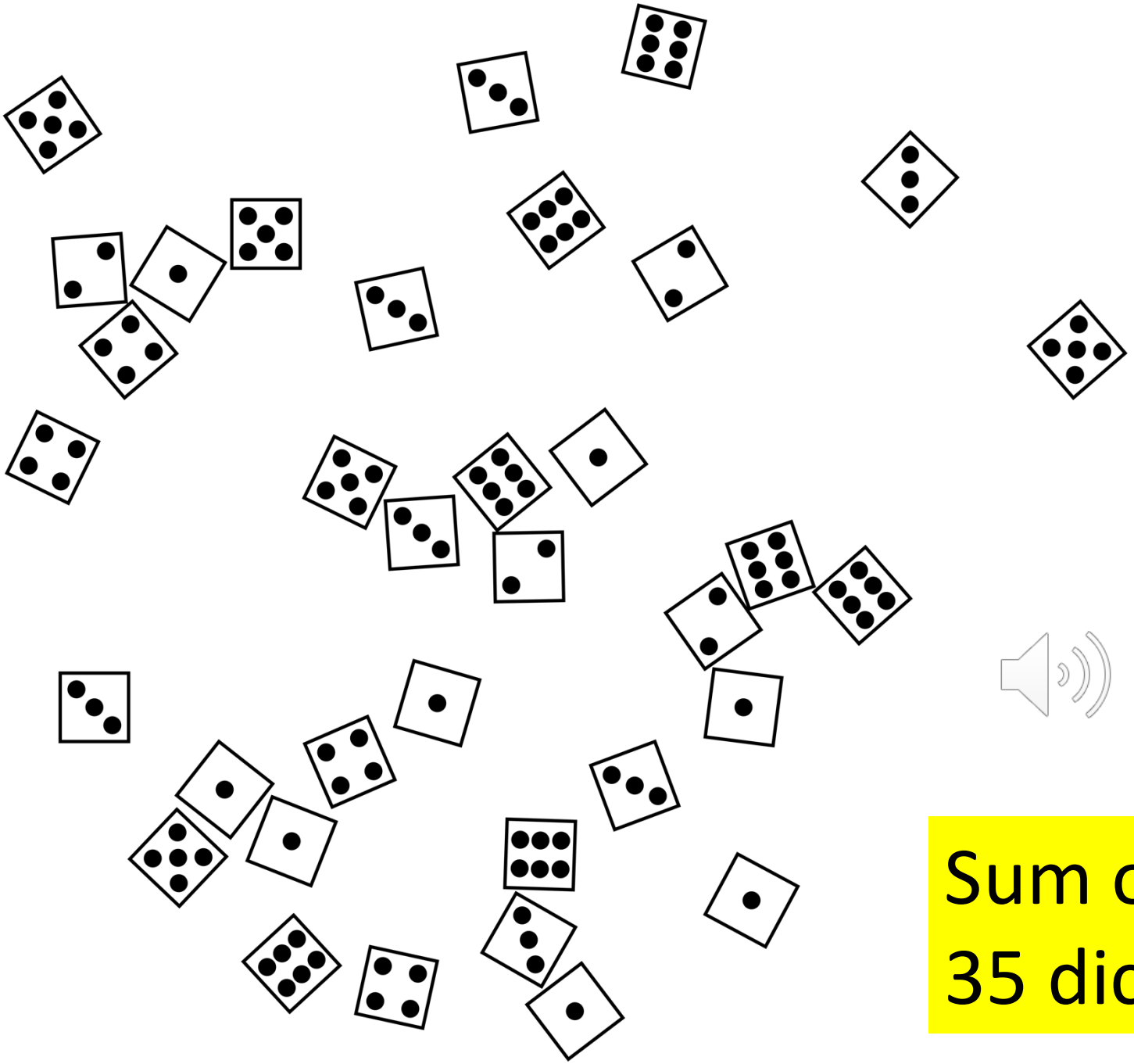
A rectangle has an area of 377 square feet, and one side length of 13 feet. What is the other side length?

a) 20 feet

b) 29 feet

c) 32 feet

d) 85 feet



a) 60
b) 90
c)120
d)150

Sum of the 35 dice?

From a conference attendee

I was at the CMC North Conference in Asilomar this past weekend and I sat in on your Math –ish presentation. I was able to teach my students about -ish numbers and we used it yesterday in class. I teach 6th grade and we are working on finding the unit price.

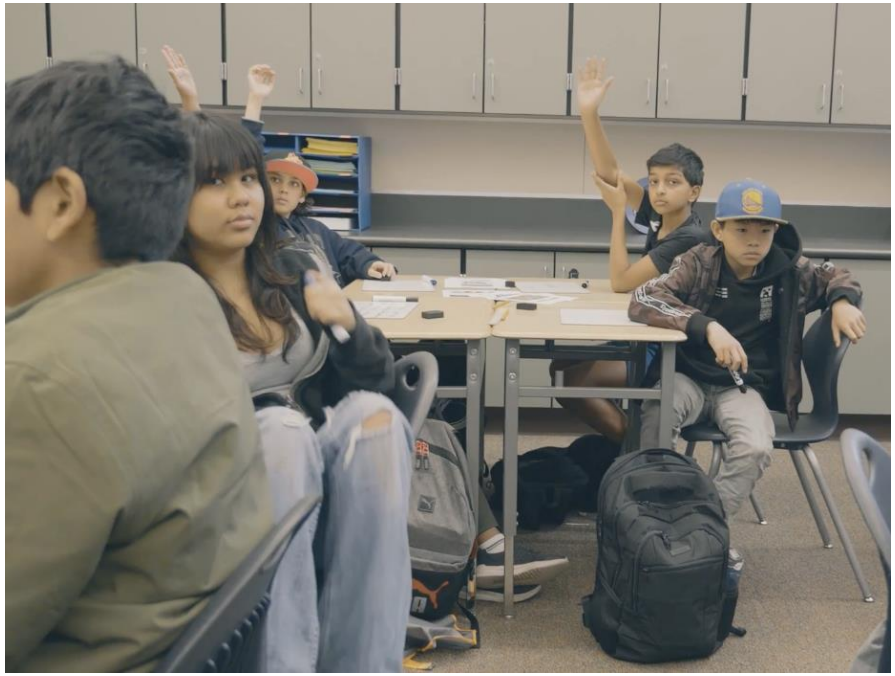
I had the students find an -ish number for the unit price before I gave them a calculator to do the calculations.

Even my less confident students were getting involved in the conversation and when we did calculate it the students could see their -ish answer was close and if it was not they talked about why it was not close. I was amazed at how much conversation I got out of the students!

Today while they were working I heard students helping each other and to start they were asking "what is your -ish number?".

Shelby Craig

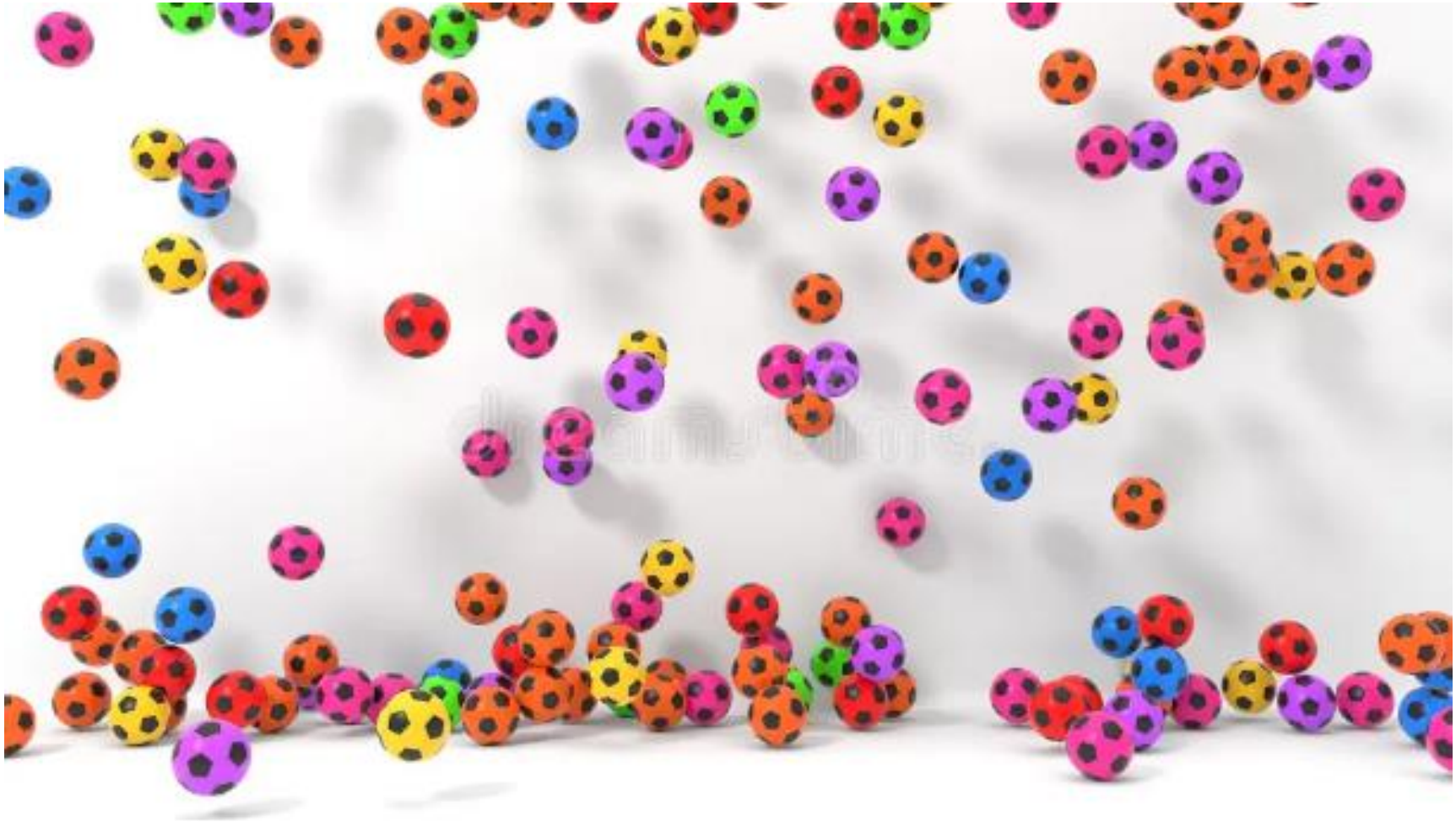
The students who used to sit there quietly in my class & keep to themselves are now raising their hands



Shelby Craig, Dublin, CA


How many soccer balls -ish?

What is the ish-ratio of purple to green soccer balls?



What is the
Ish-ratio of
Cats to dogs


ISH
NUMBERS

What do you think?

How would ishing be useful to you?

What topics could I ish with?

Addition
Subtraction
Multiplication
Division
Measurement
Time
Money
Fractions
Adding Fractions
Multiplying Fractions
Dividing Fractions
Fraction Equivalence
Volume
Area
Surface area
Perimeter
Mean, mode and median
Data collection
Dot plots
Histograms
Box Plots
Measures of variability
Ratio
Percentages

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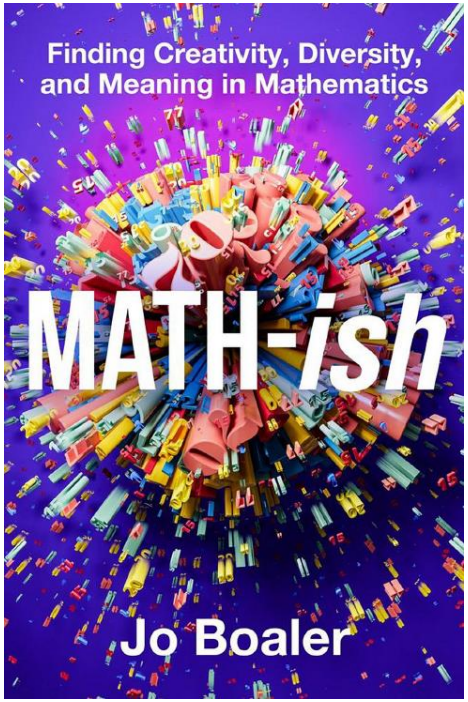

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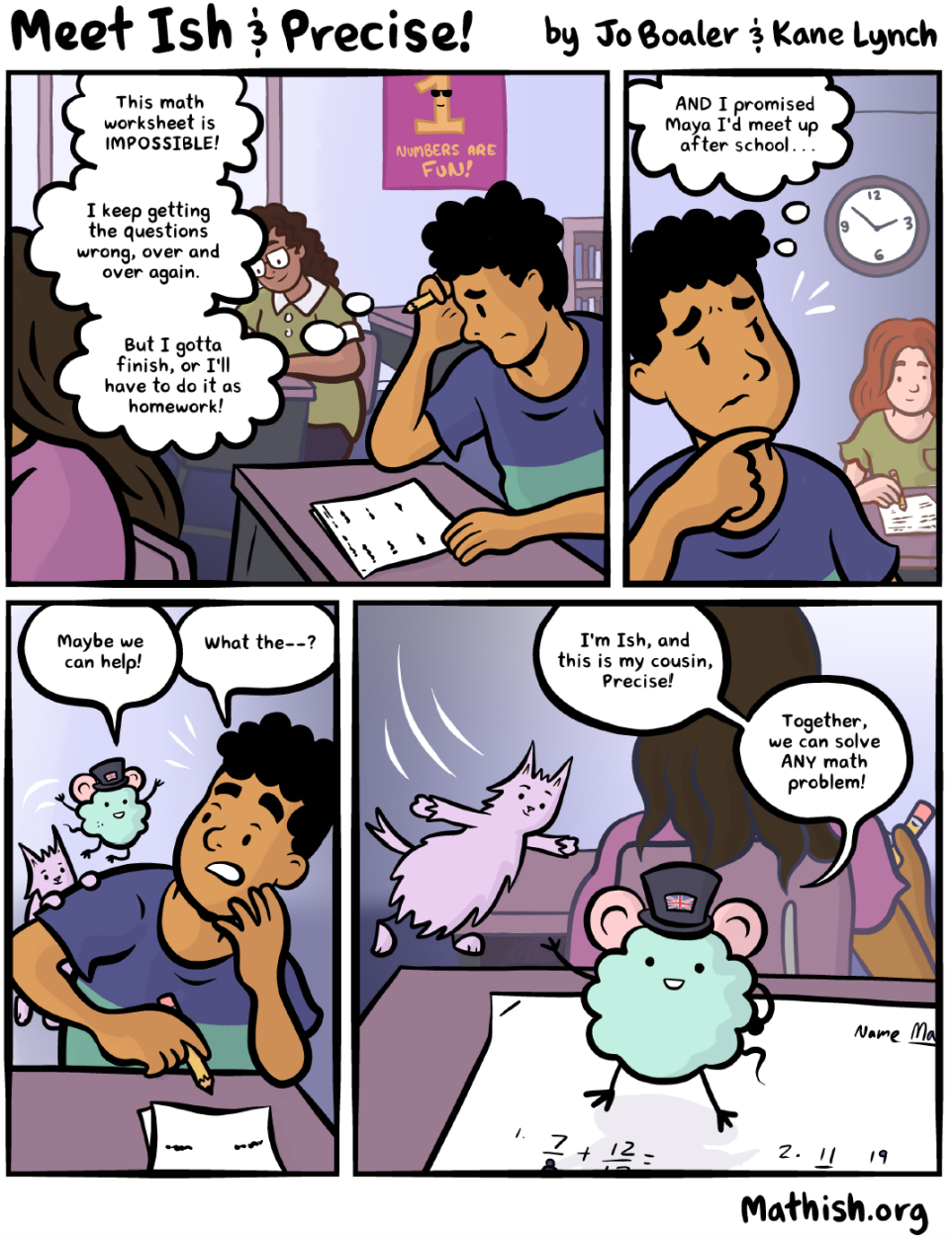
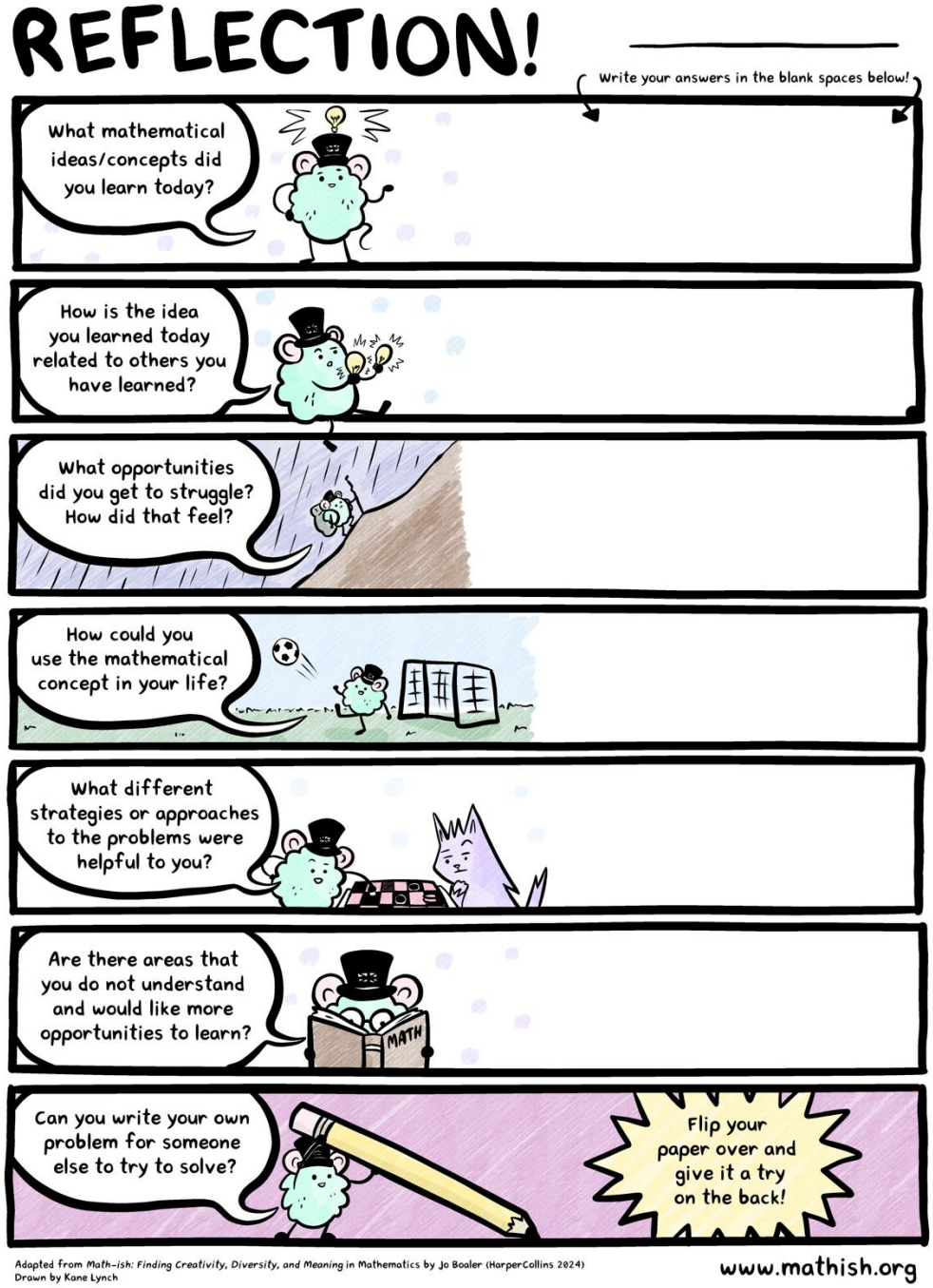
A comic introducing ish to your students

www.mathish.org





From chapter 2

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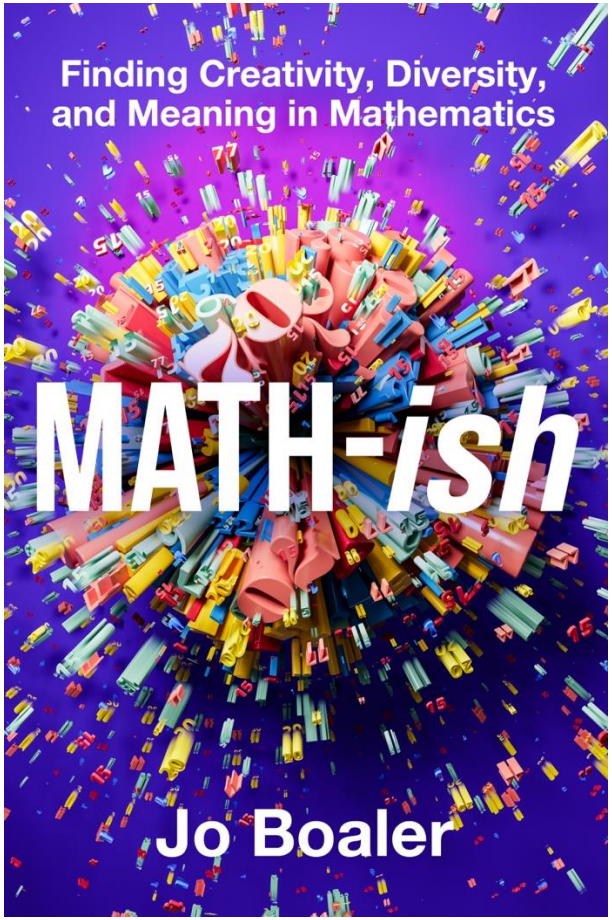
Available also in French and in Spanish

Please write a book review….



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How to use interactively:

Weight: 2.5oz.

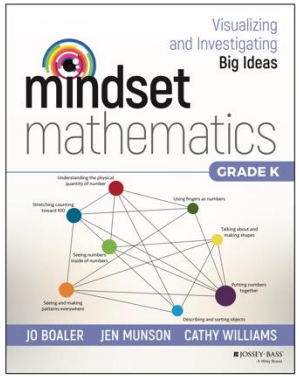
Made in China.

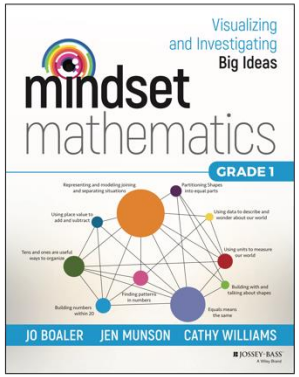
Quantity:

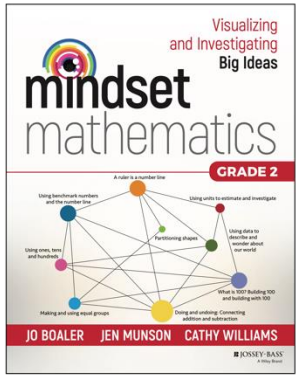
1

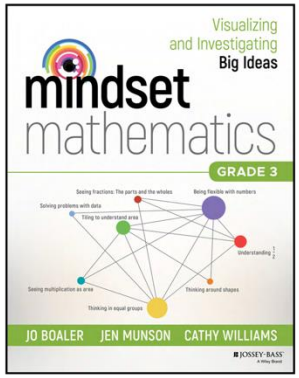
ADD TO CART

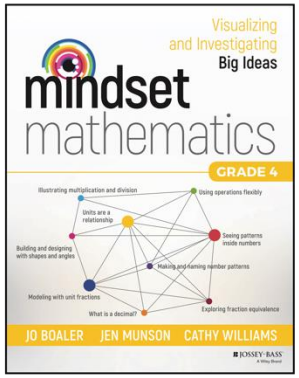
Open, Creative Maths Tasks from Jo Boaler, Jen Munson & Cathy Williams

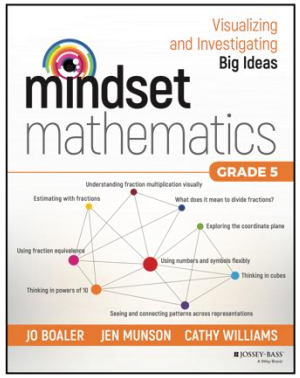


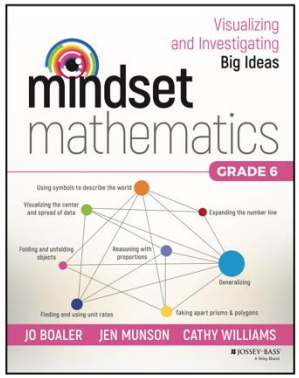


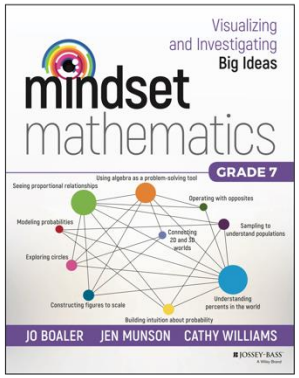


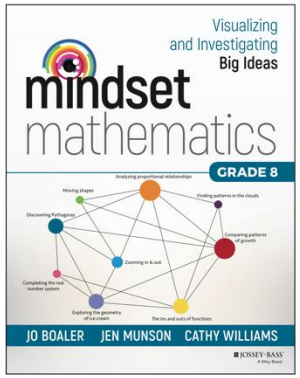












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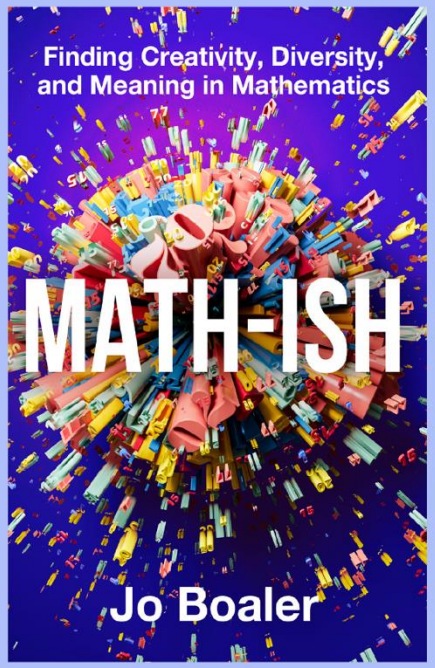
Math-ish workshop

Come and join Jo & the youcubed team for a maths party!
You will learn practical strategies to unlock your students' maths potential.



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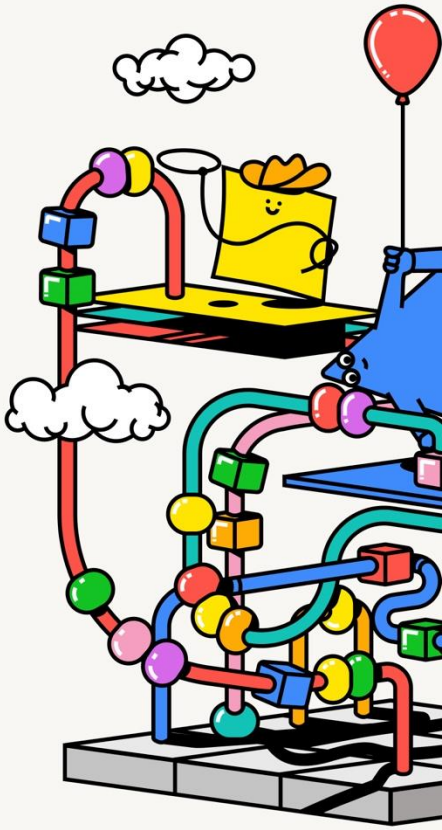
We also have online PD!

All attendees receive a free, limited edition plushie!

STRUGGLY



A platform for students and parents to understand the value of struggle, and to experience mathematics – as a creative, visual, connected subject – together.

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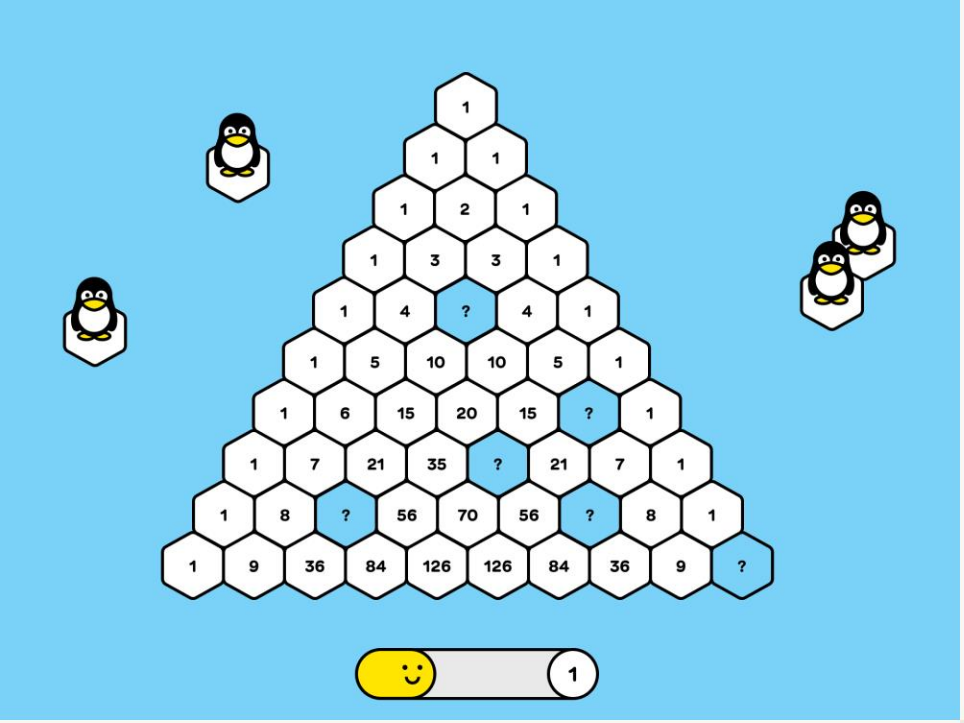
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1 2 1
1 3 3 1
1 4 ? 4 1
1 5 10 10 5 1
1 6 15 20 15 ? 1
1 7 21 35 ? 21 7 1
1 8 ? 56 70 56 ? 8 1
1 9 36 84 126 126 84 36 9 ?
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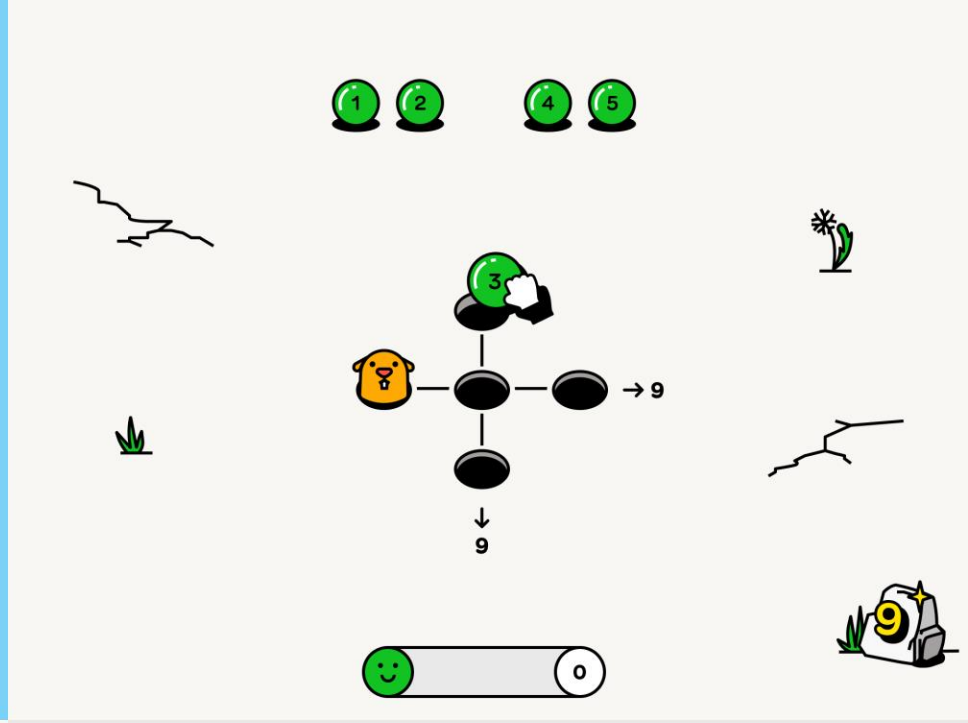
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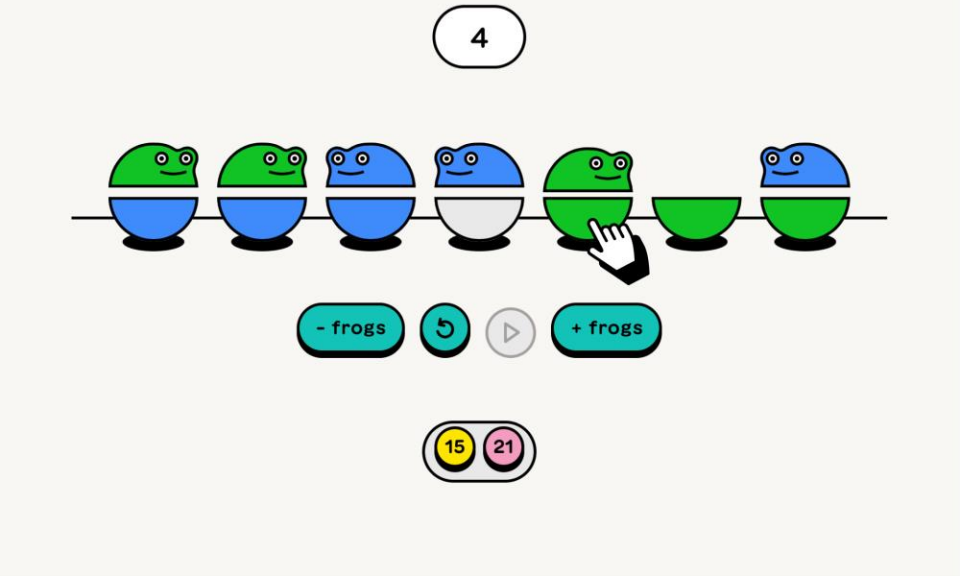
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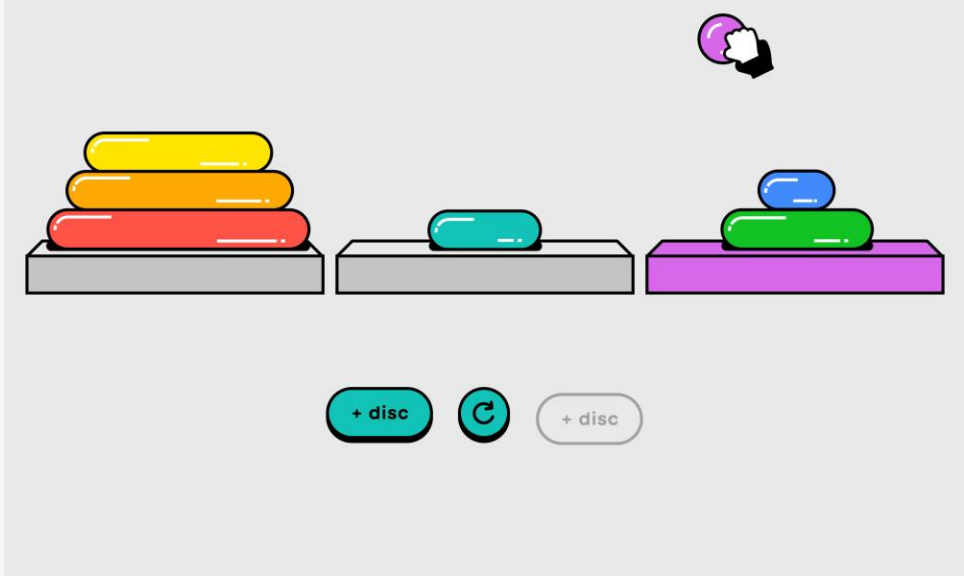
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STRUGGLY

Free
for
60
days

Brain growth

Struggle

Self-belief

Brain connections

Depth

youcubed
at Stanford University

Translating research into lessons, videos, news

Contact me: joboaler@Stanford.edu