maths explained

Dyscalculia and Teaching the Foundations.

Steve Chinn

2024

copyright.Steve Chinn.2024

What is Dyscalculia?

An overview and some history

The ultimate source: Prof Mahesh Sharma. Math for all

copyright.Steve Chinn.2024

What is Dyscalculia?

In the words of Ben Goldacre,

“ I think you’ll find it’s a bit more complicated than that.”

copyright.Steve Chinn.2024

Constructivist

Focus on deeper level of understanding.

Involvement of a variety of materials and activities.

Teacher encourages discussions.

Students interact with materials to develop conceptual learning.

Students assimilate new concepts into prior knowledge.

copyright.Steve Chinn.2024

What is Dyscalculia? (Slightly) more historical

1933, Singer and Lowe. 1965,Bakwin and Bakwin

“A disability in counting.”

1979, Greenes

“Characterised by severe retardation in arithmetic achievement, ie, two or three years behind grade level”

copyright.Steve Chinn.2024

This webinar is based on 4 decades of teaching, teacher training and researching maths learning difficulties and dyscalculia: Of working with the ‘outliers’.

How I think we could be more efficacious in teaching maths to this population, and to many other learners.

Knowing learners.

What do I mean by that?

Knowing the maths

What do I mean by that?

The importance of effective communication

What do I mean by that?

Are we ‘tellers’ or ‘teachers’?

Telling implies a personal knowledge of your subject (maths)

Teaching requires a knowledge of our learners as well.

copyright.Steve Chinn.2024

Definition of dyscalculia. Historical, again (and maybe not too practical for the classroom?)

Kosc (1974) defined developmental dyscalculia as

‘a structural disorder of mathematical abilities which has its origin in a genetic or congenital disorder in those parts of the brain that are the anatomical-physiological substrate of the maturation of the mathematical abilities adequate to age, without a simultaneous disorder of general mental functions’. OK?

copyright.Steve Chinn.2024

Sally L Smith. 1978. **'No Easy Answers'** (10 'behaviours')

Counts on fingers

Cannot commit multiplication facts to memory.

Doesn't understand place value

Can't remember the sequence of steps to multiply and divide, has trouble switching from one process to another.

Subtracts smaller number in a column from larger number. In the problem 25 – 7 subtracts the 5 from the 7, and arrives at answer 22

25
- 7
22

copyright.Steve Chinn.2024

Awareness over the past three decades. (Ashcroft and Chinn book)

‘Mathematics for Dyslexics. A Teaching Handbook.’ (1993)

‘Mathematics for Dyslexics.
Including Dyscalculia.’ 3rd edition. (2007)

‘Mathematics for Dyslexics and Dyscalculics.
A Teaching Handbook.’ 4th edition. (2017)

copyright.Steve Chinn.2024

Dyscalculia definition. (UK). 2019 BDA /SASC

(It's getting longer)

Dyscalculia is a specific and persistent difficulty in understanding numbers which can lead to a diverse range of difficulties with mathematics.

It will be unexpected in relation to **age**, level of education and experience and occurs across all ages and abilities.

Mathematics difficulties are best thought of as a continuum, not a distinct category, and they have many causal factors.

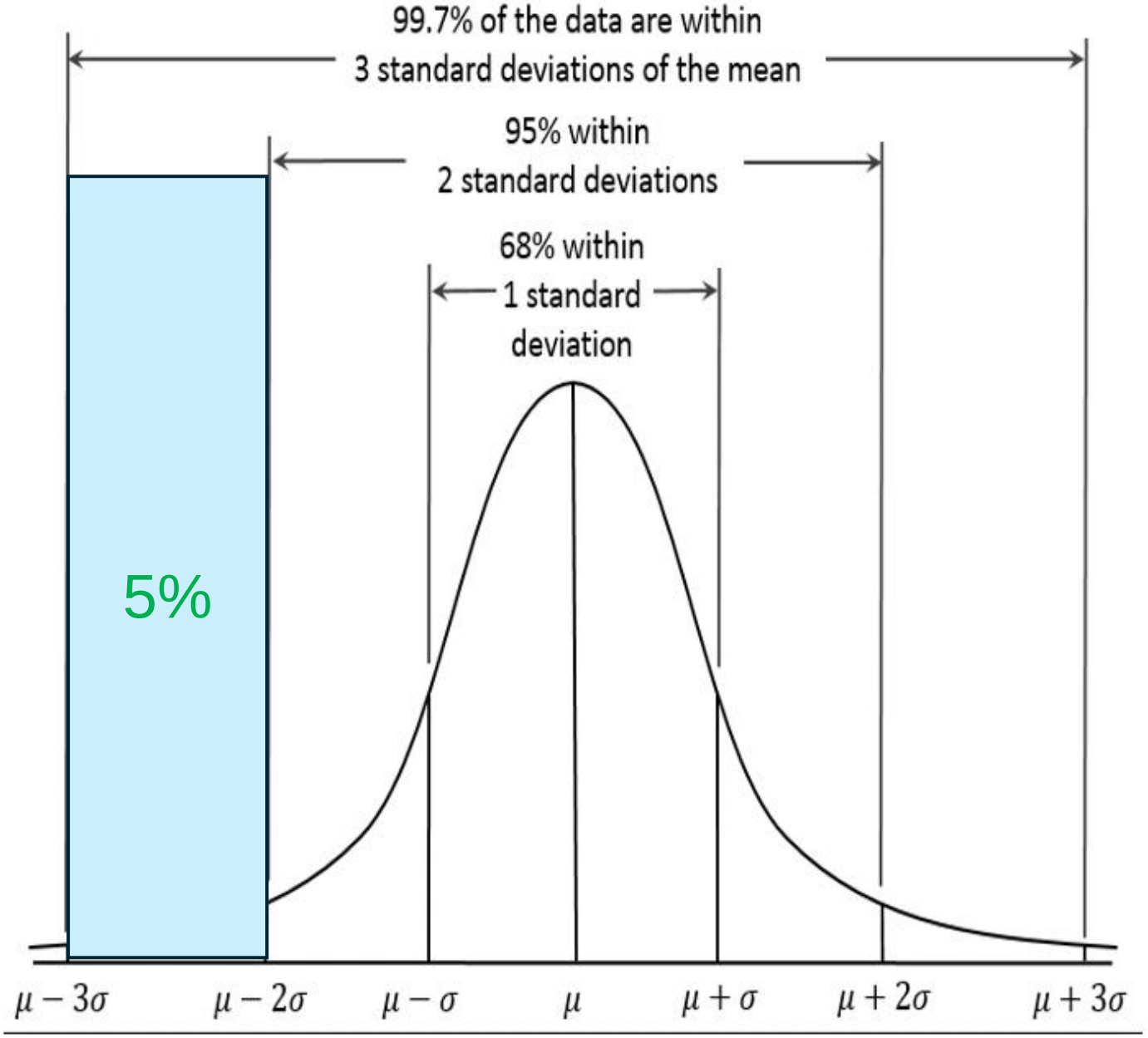
Dyscalculia falls at one end of the spectrum and will be distinguishable from other maths issues due to the severity of difficulties with number sense, including subitising, symbolic and non-symbolic magnitude comparison, and ordering.

It can occur singly but often co-occurs with other specific learning difficulties, mathematics anxiety and medical conditions.

copyright.Steve Chinn.2024

The normal distribution.

Are you measuring what you think you are measuring?



Definition (USA)

DSM-V: Dyscalculia 2013

Difficulties in production or comprehension of quantities, numerical symbols, or basic arithmetic operations that are not consistent with the person's chronological age, educational opportunities, or intellectual abilities.

copyright.Steve Chinn.2024

Can we quantify?

DSM-V: Dyscalculia

‘… not consistent with the person's chronological age, educational opportunities, or intellectual abilities.’

Basic maths/arithmetic

What do we quantify? The heterogeneity of the population.

copyright.Steve Chinn.2024

“ I think you’ll find it’s a bit more complicated than that.” Ben Goldacre

Mabbott and Bisanz (2008) noted that, ‘Children who experience difficulties in mathematics are a heterogeneous group’ (Beware definitions!)

and from Zhou and Cheng (2015): ‘mathematical competence is a constellation of abilities.’ (Can we narrow this down? Early onset?)

‘What mood are you in today? Can you work with me?’

copyright.Steve Chinn.2024

DSM-V Specific Learning Disability. Behaviours.

Difficulties remembering number facts.
Inaccurate or slow (?) arithmetic calculation. (stm. WM)
Ineffective or inaccurate mathematical reasoning.
Avoidance of activities requiring arithmetic (Affective)

The first three create the fourth – thanks to us adults.

In fact, maybe we create many of the 'dyscalculics'

The influence of maths culture and beliefs

copyright.Steve Chinn.2024

Definitions and testing: Positives and negatives: Cautions

Over literal interpretation of data.

Interactions.

Communication: Affective and cognitive.

Location.

Length and endurance.

Non-formulaic. Respond to the individual.

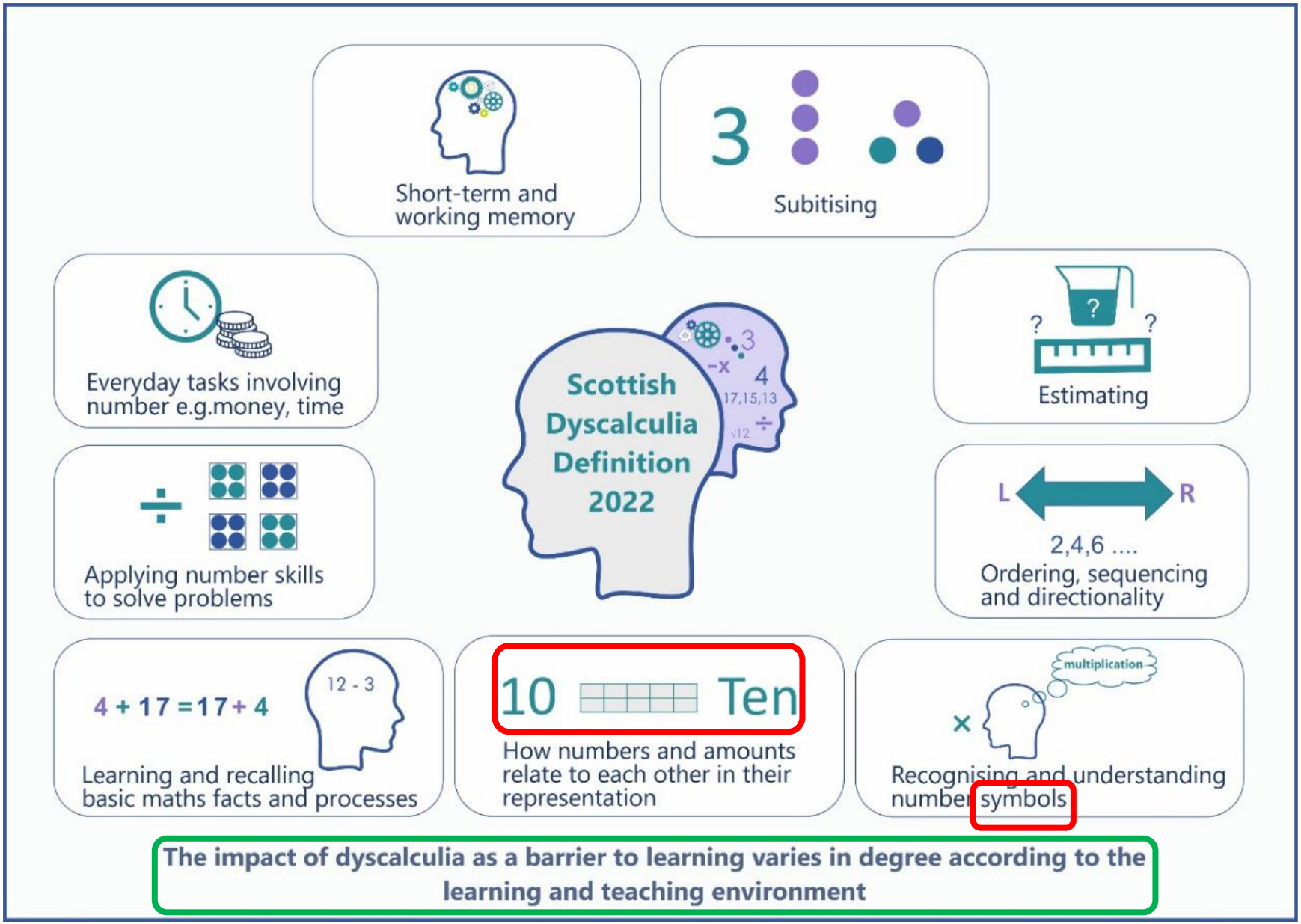
Being over-pedantic and/or literal in interpretations.

Appropriate probing questions.

The power of listening.

copyright.Steve Chinn.2024

Scottish Working Definition of Dyscalculia. 2022



A journey through the early curriculum

copyright.Steve Chinn.2024

A journey through the early curriculum

This is not a prescription, nor is it the only route to follow.

I want to show how maths builds, or should do, from the start, the very basics. My focus is on the maths structure.

(In Numicon's 'Big Ideas', along with my co-authors, I show presentations/lesson plans in detail. *Philosophy*)

I want to show 'What else are you teaching?' at each stage of the curriculum. Maths is developmental…. Implications?

copyright.Steve Chinn.2024

The early curriculum

Regarding intervention ….. At any age ….

Go back to the basics,

in an age-appropriate way

Let’s start with a look at two extremes of teaching/curriculum styles. Which is more likely to work with your students?

Take a critical look. Maybe a bit of each?

copyright.Steve Chinn.2024

Constructivist or behaviourist?

Metacognition.

Cognitive style: Inchworms and grasshoppers

(You, or the learner?)

copyright.Steve Chinn.2024

Behaviouristic

Precision teaching (Nigel?)

A focus on developing skills.

Identification of a single algorithm/procedure.

Teacher provides sequence of steps.

Students memorise and follow a set procedure.

Mastery of skills prior to their application.

Individualised drill and rehearsal activities for mastery.

copyright.Steve Chinn.2024

Constructivist

Focus on deeper level of understanding.

Involvement of a variety of materials and activities.

Teacher encourages discussions.

Students interact with materials to develop conceptual learning.

Students assimilate new concepts into prior knowledge.

copyright.Steve Chinn.2024

Key Thinking Skills for problem solving in primary and lower secondary. From '**The Singapore Model Method for Learning Mathematics'**

Classifying

When do these start?

Comparing

Sequencing

Analysing parts and whole

Identifying patterns and relationships

Induction and deduction (Deduction moves from idea to observation. Induction moves from observation to idea).

Generalising

Verifying

Spatial visualisation

Reversing a sequence

copyright.Steve Chinn.2024

Two key skills for organising and processing knowledge.
(Desoete, 2015 Piaget, 1941 Krutetskii (1976)

Seriation and classification (as logical thinking skills) and procedural and conceptual counting knowledge are significant predictive indicators for mathematical learning disabilities in kindergarten. *(Beware 'reversing')*

Organising knowledge.

copyright.Steve Chinn.2024

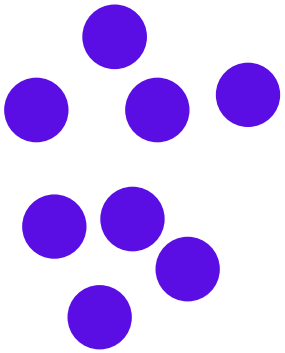
Subitising

The beginning

(Prof Brian Butterworth)

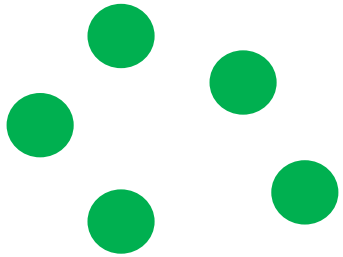
copyright.Steve Chinn.2024

Subitising. Quantity recognition



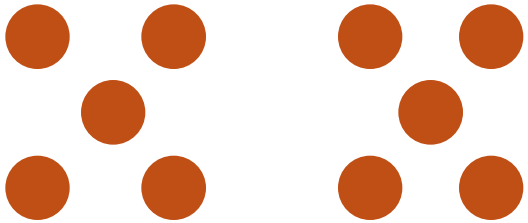
Perceptual subitising

Bigger or smaller?



subitising

Perceptual or conceptual?



chunking/partitioning

copyright.Steve Chinn.2024

Reading and numberness. (1) Sharma (2015 and 2022)

Fluent reading and fluency in numberness are analogous.

Research (Williams, 1995) in reading shows that phonemic awareness — the insight that words are composed of sounds and the ability to connect fluently grapheme to phoneme and phonological sensitivity — the ability to break words into meaningful "chunks" and then "blend" them fluently — are predictors of early reading performance ….

(better than IQ tests, readiness scores, or socioeconomic level) and essential for reading acquisition.

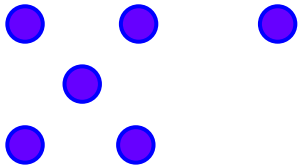
copyright.Steve Chinn.2024

Reading and numberness. (2) Sharma (2015 and 2022)

The sequencing, visual clustering, and decomposition/re-composition, representation of number orally and

The ability to associate a number to a cluster is like phonemic awareness and the ability to instantly recognize that a number is made of smaller numbers (decomposition/recomposition) is equivalent to chunking and blending.

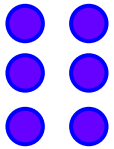
Numberness is a predictor for future proficiency and fluency in arithmetic.



Which?

Or both?

Why?



copyright.Steve Chinn.2024

Number sense. (Berch 2005) 30 components

A selection: What does each of these tell us?

2. Elementary abilities or intuitions about numbers *(subitising)* and arithmetic

4. Ability to make numerical magnitude comparisons. *(Is it bigger or smaller? More or less?)*

5. Ability to decompose numbers naturally. *(Using key/core/benchmark numbers)*

7. Ability to use the relationships among arithmetic operations to understand the base-10 number system.

copyright.Steve Chinn.2024

11. Possessing knowledge of the effects of operations on numbers.

12. Possessing fluency and flexibility with numbers. *(Estimation)*

13. Can understand number meanings.

14. Can understand multiple relationships among numbers.

15. Can recognize benchmark numbers and number patterns.

16. Can recognize gross numerical errors.

20. Can invent procedures for conducting numerical operations.

copyright.Steve Chinn.2024

21. Can represent the same number in multiple ways depending on the context and purpose of the representation. *(170 as 160 + 10)*

22. Can think or talk in a sensible way about the general properties of a numerical problem or expression—without doing any precise computation. *(Overviewing the problem – what is it about?)*

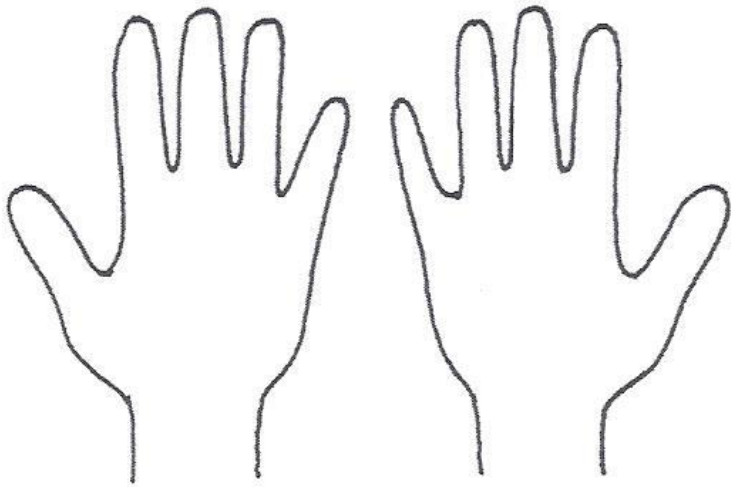
24. A non-algorithmic feel for numbers. *(Not procedure–dominant)*

26. A conceptual structure that relies on many links among mathematical relationships, mathematical principles, and mathematical procedures. *(Relationships between the four operations)*

30. A process that develops and matures with experience and knowledge *(Linking and inter-relating. The developmental nature of maths*).

copyright.Steve Chinn.2024

The role of 1 2 5 10



copyright.Steve Chinn.2024

Counting

copyright.Steve Chinn.2024

Putting the numbers 1 to 10 in order: with images to help?

10

4

3

6

1

9

8

5

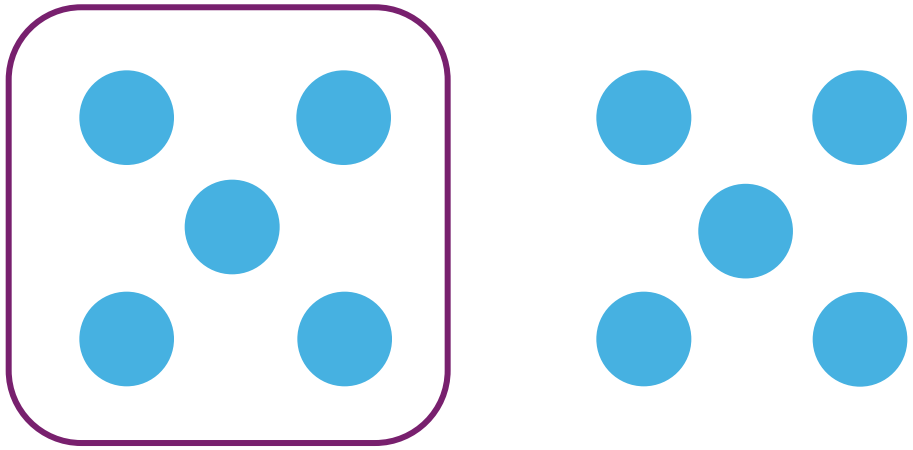
0

2

7

copyright.Steve Chinn.2024

Counting forwards. Adding 1 each time

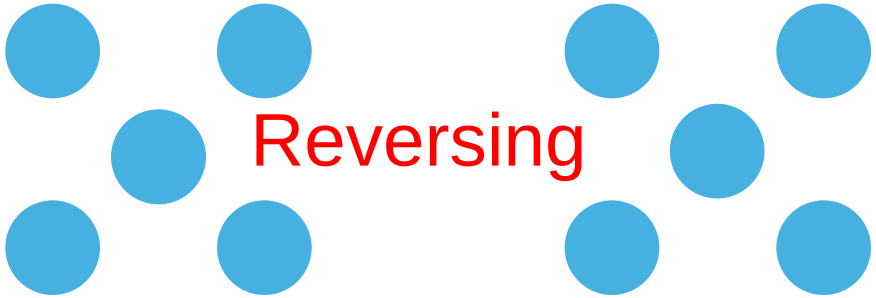


What else are you teaching? (always)

0 1 2 3 4 5 6 7 8 9 10

copyright.Steve Chinn.2024

Counting back. Subtracting 1 each time



10 9 8 7 6 5 4 3 2 1 **0**

copyright.Steve Chinn.2024

Chunky counters. Age appropriate?

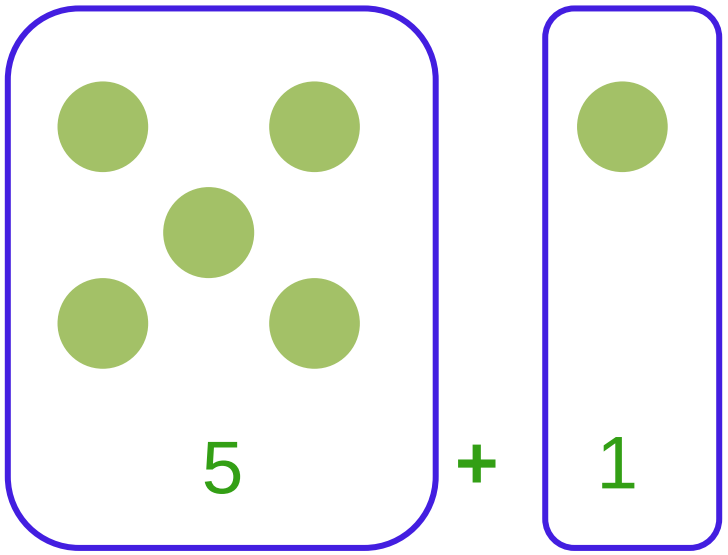
Two colour counters.

The implications of ‘going back’

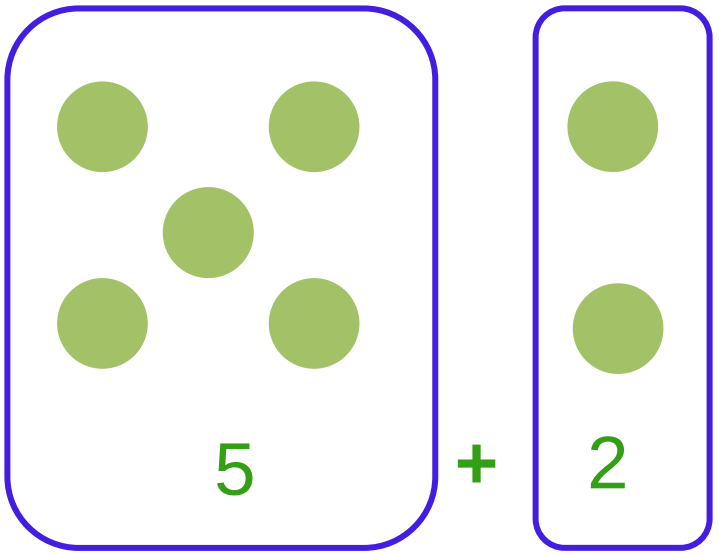
copyright.Steve Chinn.2024

What else are you teaching with 6 and 7? Just facts?
Long term mathematical memory.**

6

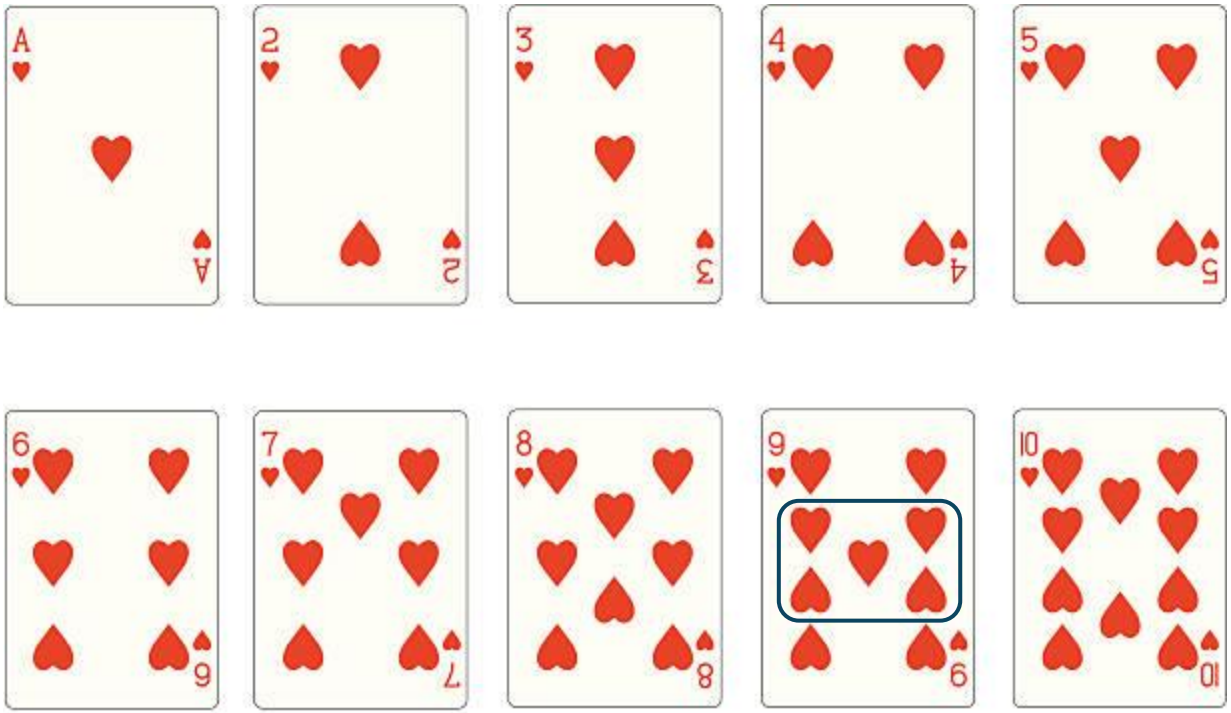


7



Chunking. Partitioning.
Decomposing and recomposing.
Using the key numbers to develop number sense.

** *Howard Gardner's multiple intelligences (memories)*

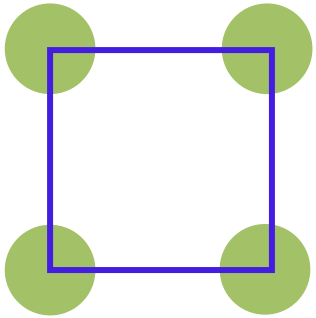


What patterns, relationships and inconsistencies, do you see?

Around 700 years.

copyright.Steve Chinn.2024

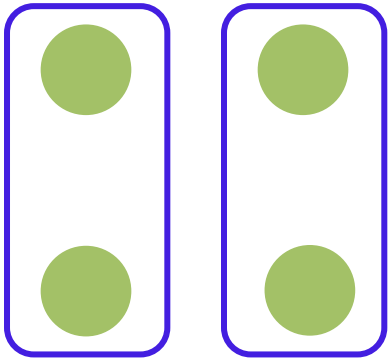
An example: What else are you teaching/could you teach with 4?



A square

Symbols

$4 = 2^2$



$4 = 2 + 2$

$4 = 2 \times 2$

Linking + and x

copyright.Steve Chinn.2024

‘How People Learn.’ **Key Finding 2**. NRC (2000)

To develop competence in an area of enquiry, students must:

(a) have a deep foundation of factual knowledge,

(b) understand facts and ideas in the context of a conceptual framework, and

(c) organise knowledge in ways that facilitate retrieval and application.

0 1 2 5 10 20 50 100 ……

Use patterns and generalisations

What else are you teaching?

copyright.Steve Chinn.2024

Number bonds for 10

Number combinations for 10

The key facts of all key facts?

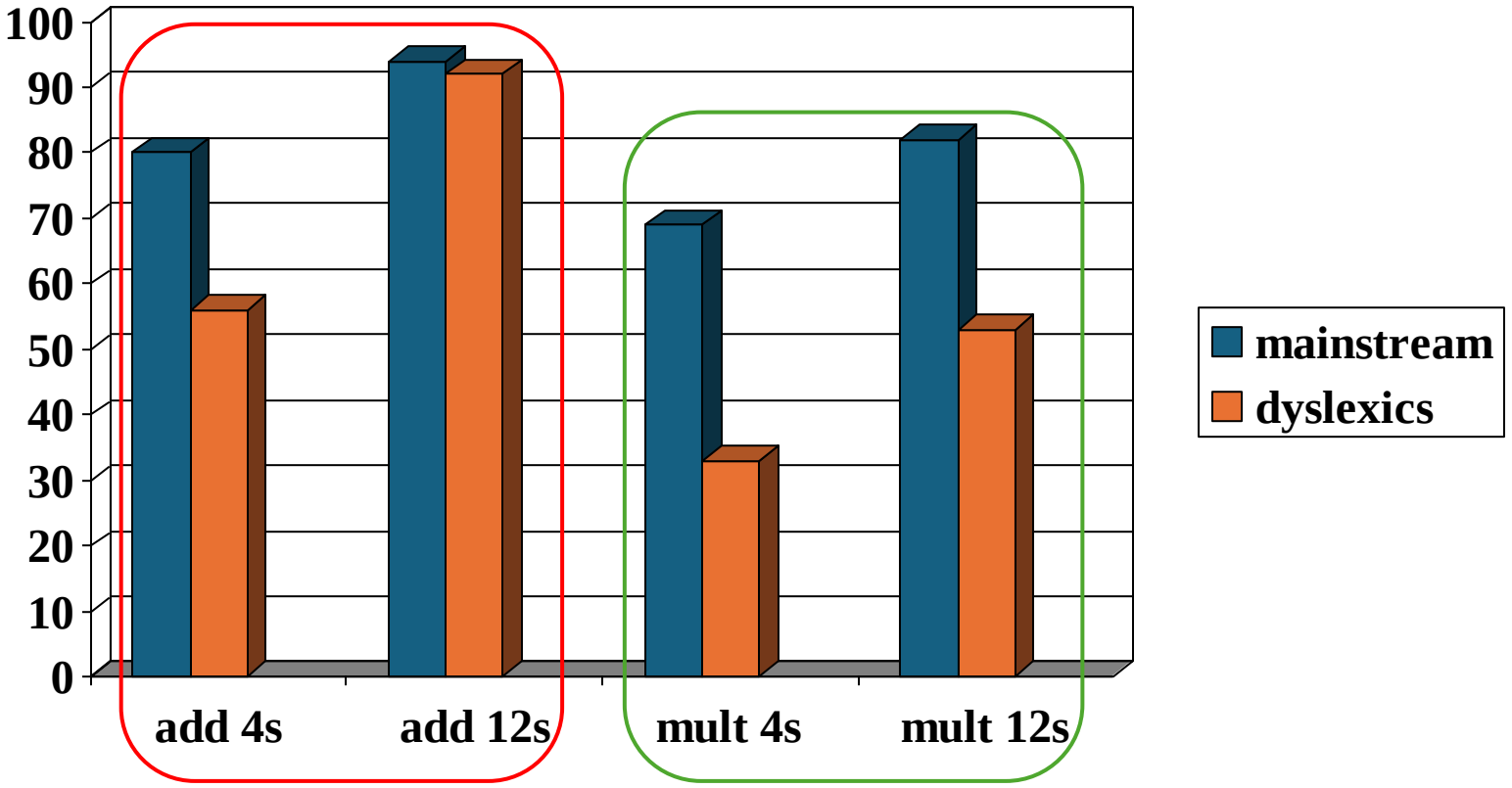
copyright.Steve Chinn.2024

Classroom research. 4s and 12 s retrieval. Chinn (1994)

4 + 5	7 + 9	8 + 6	7 x 2	6 x 4
7 + 9	9 + 8	5 + 6	5 x 8	7 x 9
6 + 7	8 + 9	8 + 7	6 x 8	7 x 5
5 + 5	7 + 8	8 + 5	7 x 7	6 x 6
5 + 5	9 + 6		3 x 8	4 x 9

copyright.Steve Chinn.2024

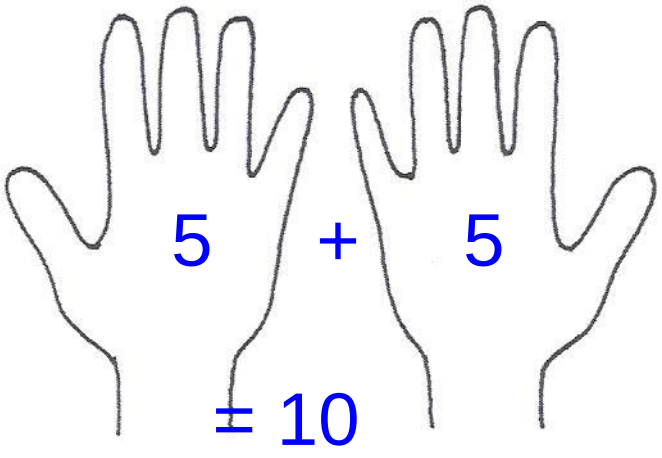
Retrieval of + and x facts (and the role of time)



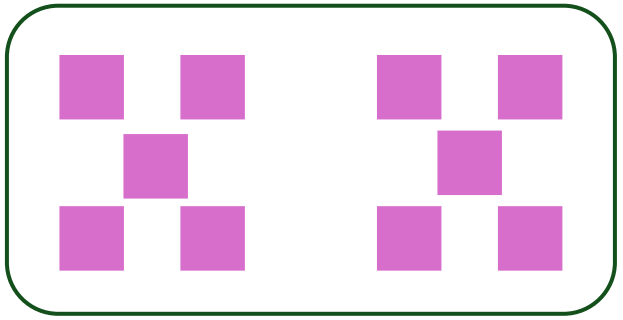
copyright.Steve Chinn.2024

Number combinations for 10. Core facts.

(Putting place value for 10 on hold for a moment)



What can you see in the pattern?



5 + 5

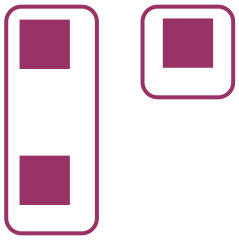
= 10

copyright.Steve Chinn.2024

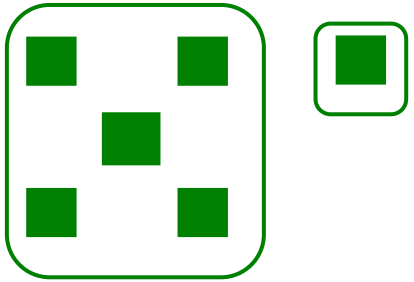
Numbers within numbers. Numbersense. Decomposing

The core/key numbers (1, 2, 5, 10) in 3, 6, 7 and 9

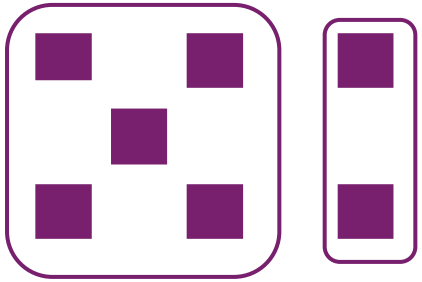
3 = 2 + 1

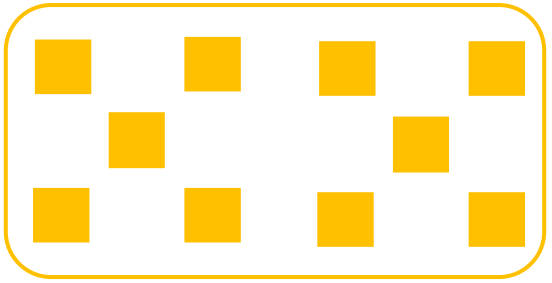


6 = 5 + 1



7 = 5 + 2





9 = 10 – 1 (estimation?)

copyright.Steve Chinn.2024

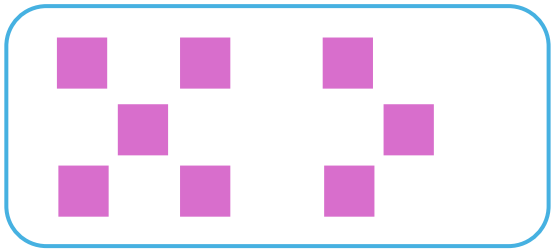
Number combinations for 10 and the commutative property

7 + 3 = 10

3 + 7 = 10

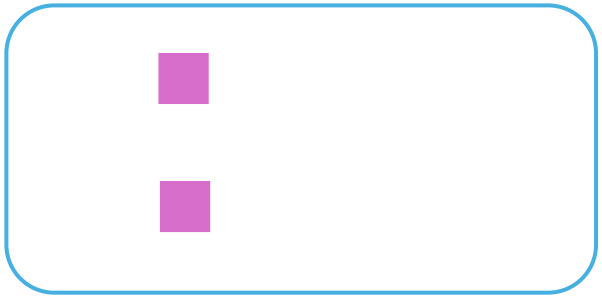
copyright.Steve Chinn.2024

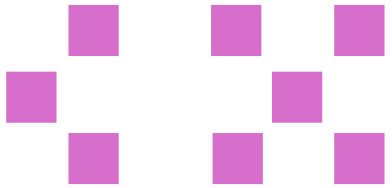
Number combinations for 10: Visuals



8 + 2

vocabulary

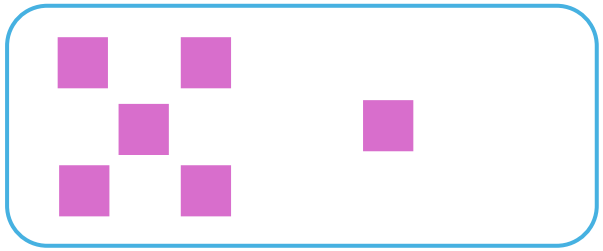




2 + 8

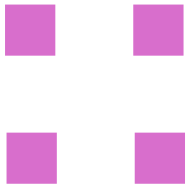
copyright.Steve Chinn.2024

Number combinations for 10

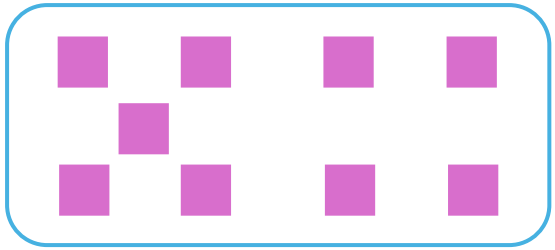


6

+



4



9

+

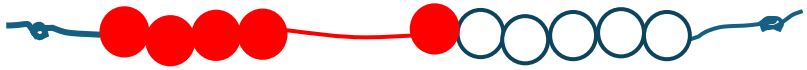
1

copyright.Steve Chinn.2024

Bead strings and the commutative property

5 + 5

6 + 4



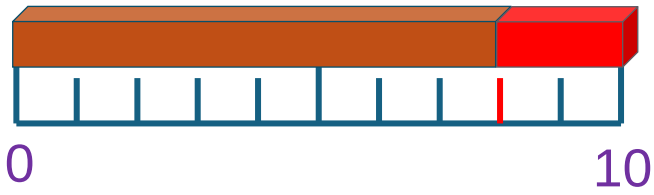
4 + 6

The 10 is conserved.

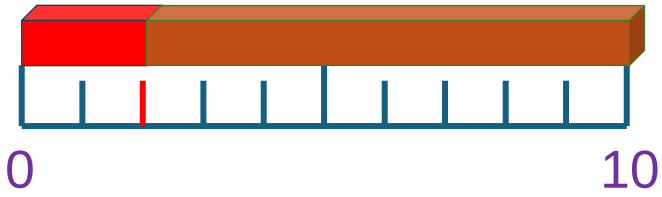
copyright.Steve Chinn.2024

Understanding addition: The commutative property. Visualisations. (Adding on?)

8 + 2 = 10



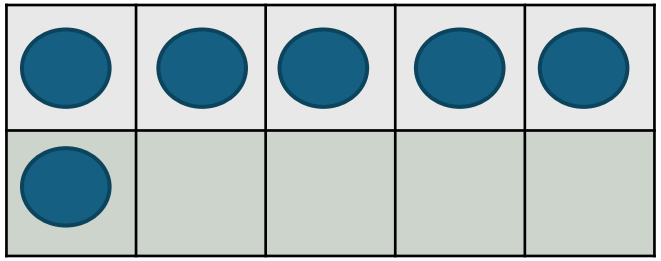
2 + 8 = 10

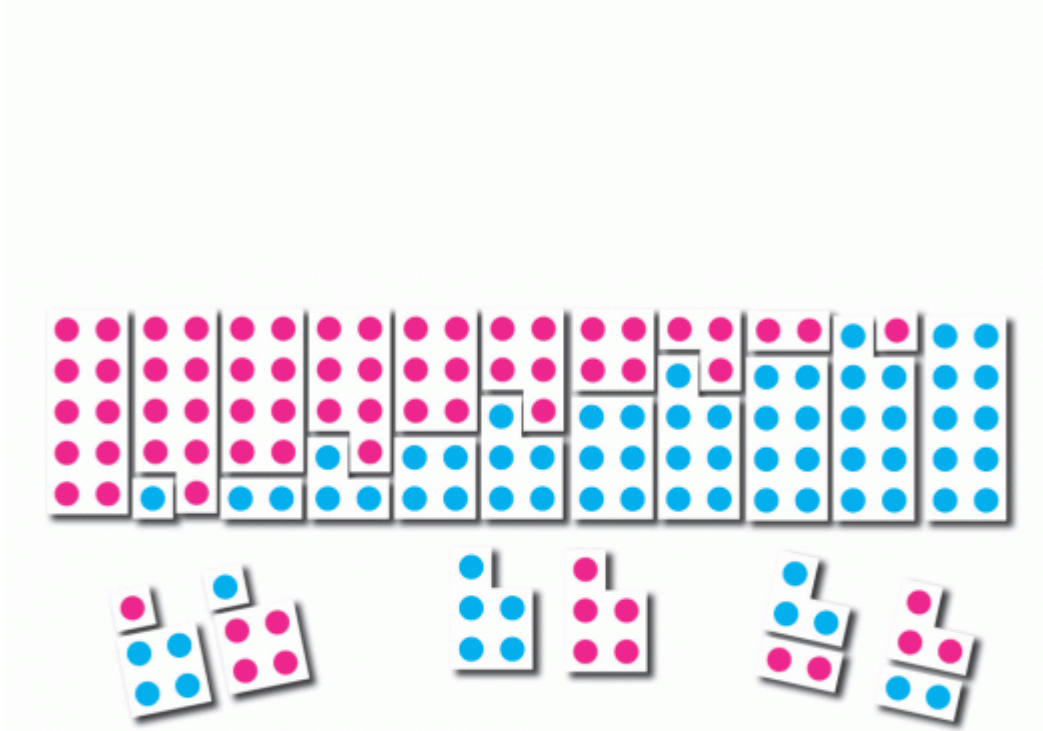


copyright.Steve Chinn.2024

Some more apparatus: *What are the characteristics?*

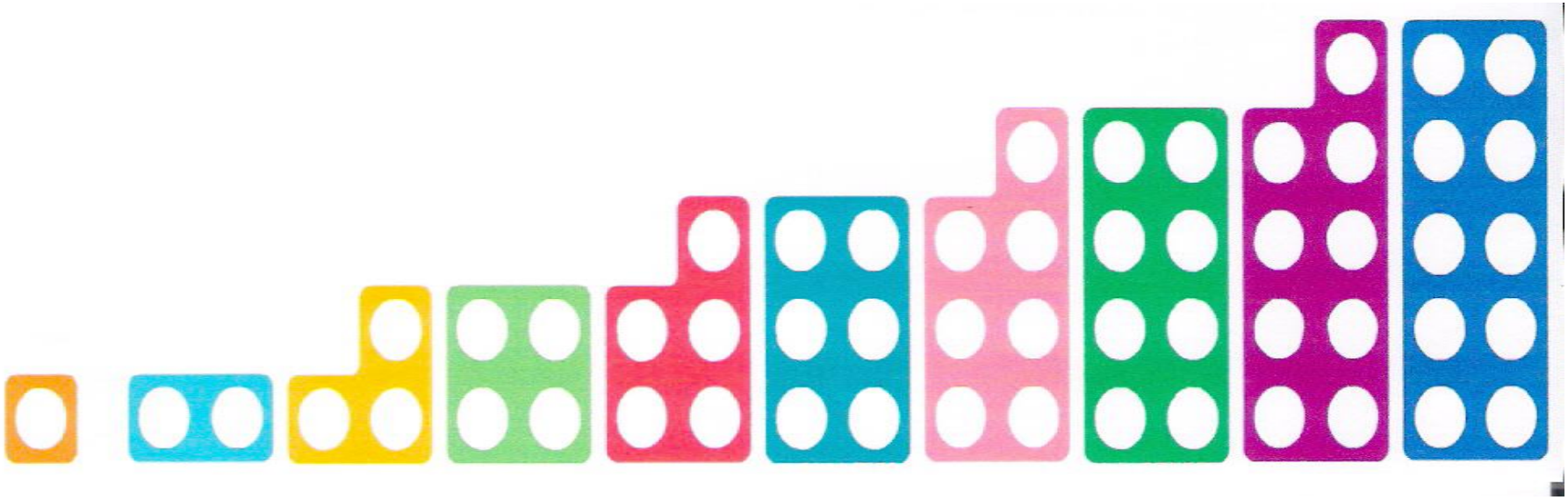
10-frames





copyright.Steve Chinn.2024

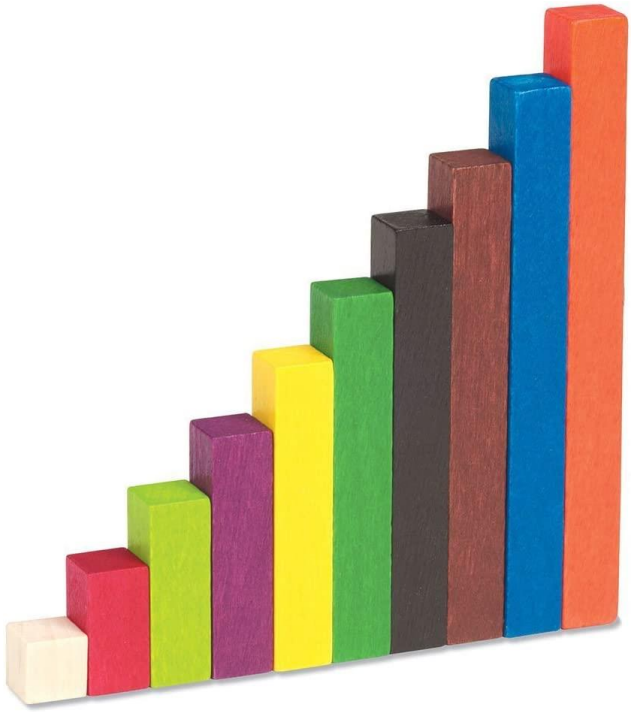
Stern/Numicon shapes



1	2	3	4	5	6	7	8	9	10
odd	even	odd	even	odd	even	odd	even	odd	even

copyright.Steve Chinn.2024

Discrete and continuous. Cuisenaire rods.



1 2 3 4 5 6 7 8 9 **10**

10 20 30 40 50 60 70 80 90 **100**

100 200 300 400 500 600 700 800 900 **1000**

.1 .2 .3 .4 .5 .6 .7 .8 .9 **1.0**

copyright.Steve Chinn.2024

Coinage and money

Be aware of inconsistencies

At the other extreme, in 2015, the Zimbabwean

100-trillion-dollar note was worth 40 US cents

copyright.Steve Chinn.2024

Good/not good characteristics?

Analysing and knowing what you are using and its effectiveness with your learners.

copyright.Steve Chinn.2024

6 year old pupil. Visualisation

32 - 19 = 27

‘Show me how you worked out your answer’

copyright.Steve Chinn.2024

Place value and numbers.

For Chinese children in Hong Kong, Ho et al (2015) found that *a persistently low level of early place-value understanding appeared to be associated with later low mathematical outcomes.*

A simple place value test given half-way through 1st grade identified 95% of the children who were low achievers in 2nd Grade.

copyright.Steve Chinn.2024

Maths Anxiety
The affective domain

Check out ‘The Maths Anxiety Trust’

copyright.Steve Chinn.2024

maths explained

Dyscalculia and Teaching the Foundations.

END

copyright.Steve Chinn.2024

maths explained

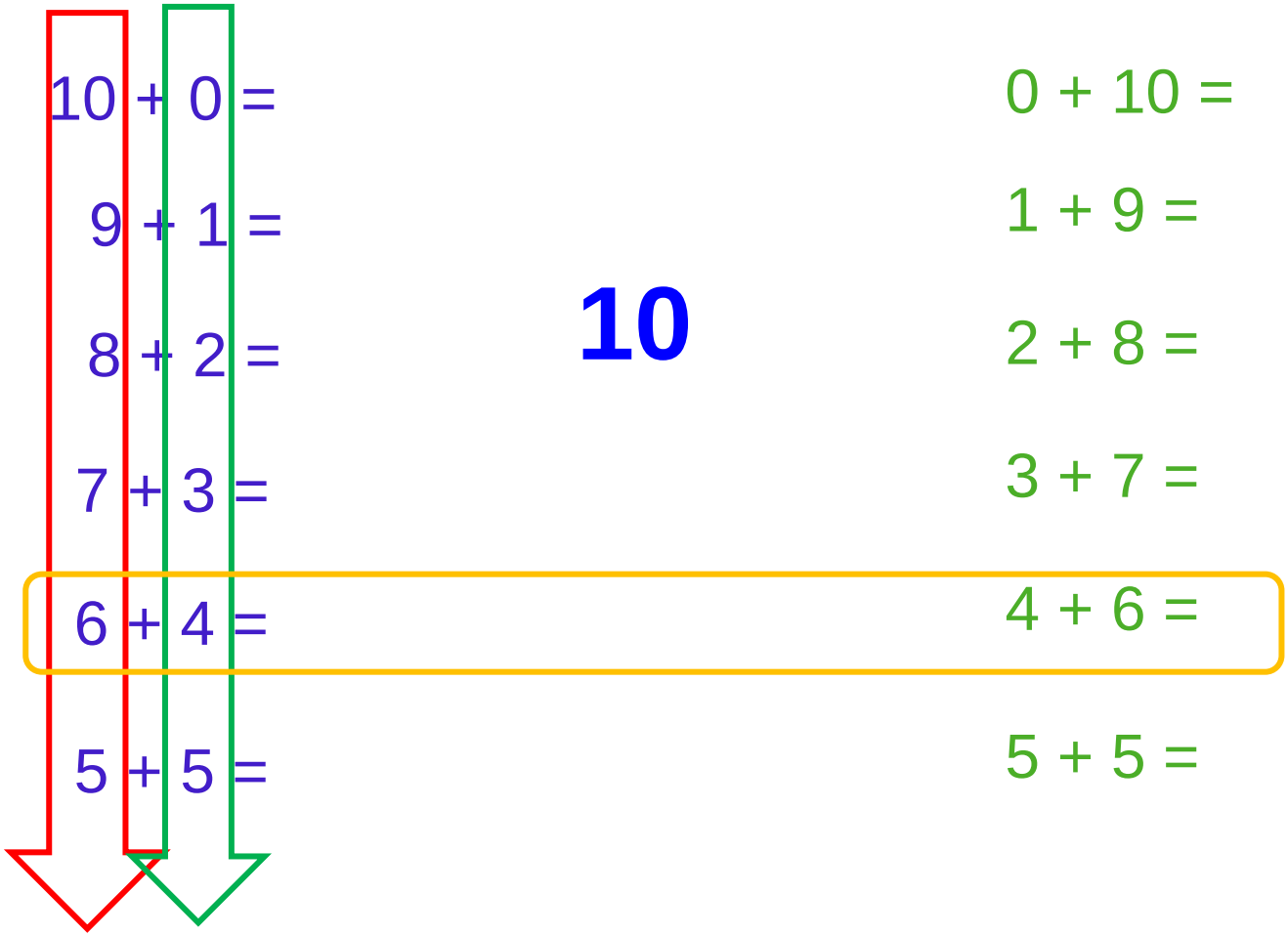
Dyscalculia and Maths Learning Difficulties

Steve Chinn

Oct 22nd 2024

copyright.Steve Chinn.2024

The number combinations for 10. Symbols only. Thoughts?



copyright.Steve Chinn.2024

Number combinations for 10. Symbols

1 2 3 4 5 6 7 8 9

9 8 7 6 5 4 3 2 1

copyright.Steve Chinn.2024

Adding by casting out tens. What are the benefits?

WEAYT

It's about overviewing and then connecting the numbers.

Relationships and connections.

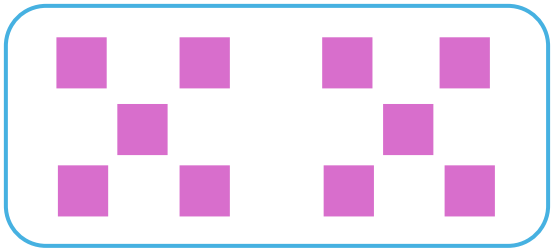
WM

Estimate?

copyright.Steve Chinn.2024

Subtraction with 10 combinations

vocabulary

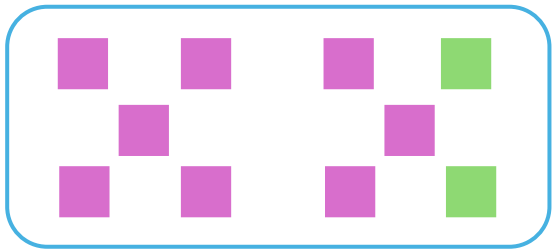


= 8

10 - 2

copyright.Steve Chinn.2024

Subtraction with counting on



8 9 10

Linking the operations of + and –

Reversing

copyright.Steve Chinn.2024

Place value.

A key concept

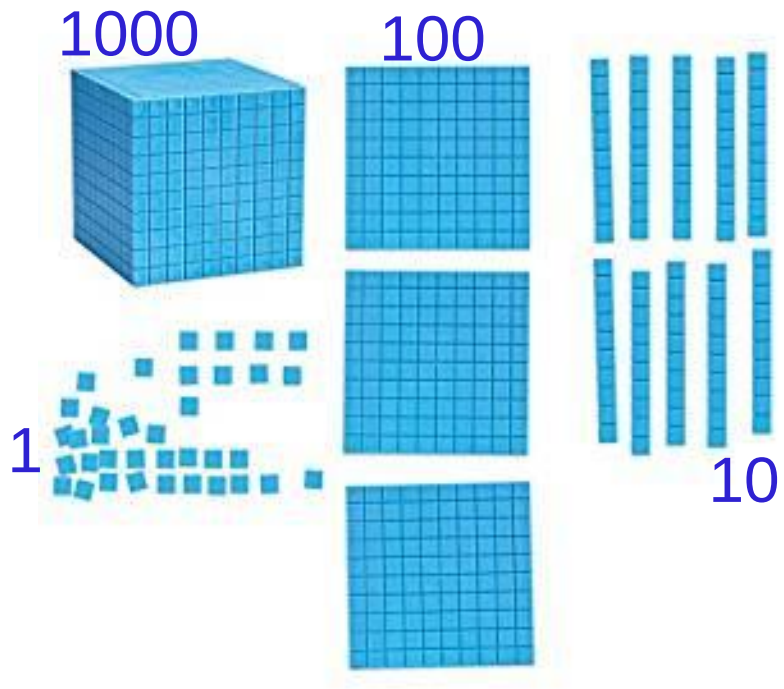
Piaget and
Hattie

Learner perceptions. Primary colours. Weight.
Ask your learners.

Scaffolding

copyright.Steve Chinn.2024

Base ten (or Dienes) blocks



Present the symbols alongside the materials.

Age appropriate?

Big numbers and patterns: eg million

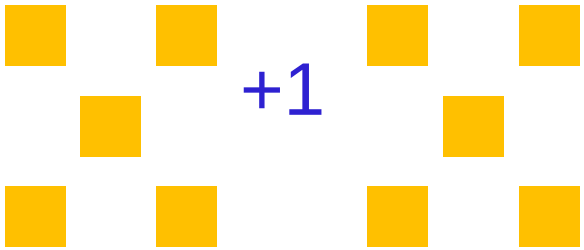
copyright.Steve Chinn.2024

Two key moments in the base 10 system

nine → ten 9 → 10

nine ← ten 9 ← 10

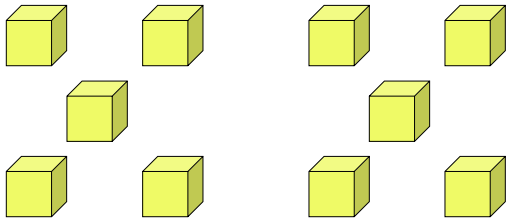
Interpret 'place value'

9  +1 10 - 1

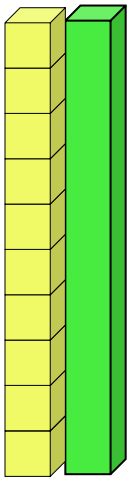
We use two (or more) digits to show numbers that are bigger than 9

copyright.Steve Chinn.2024

ten ones = one ten. Visualisation. (‘What do you see?’)



ten ones



1 ten

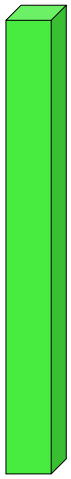
<u>Vocabulary:</u>
Renaming
Trading
Carrying ……

copyright.Steve Chinn.2024

The digits and where they are placed

place value
and
zero

one ten



1 0

zero ones

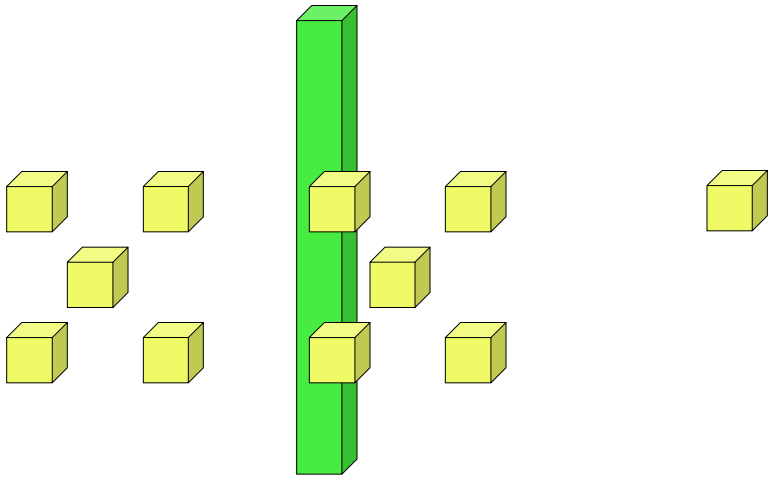
one ten

copyright.Steve Chinn.2024

The digits and where they are placed

place value

eleven



11

one ten

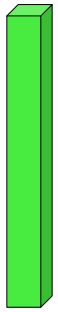
one (one)

copyright.Steve Chinn.2024

The ‘teen’ numbers

Place value columns

Scaffolding the concept



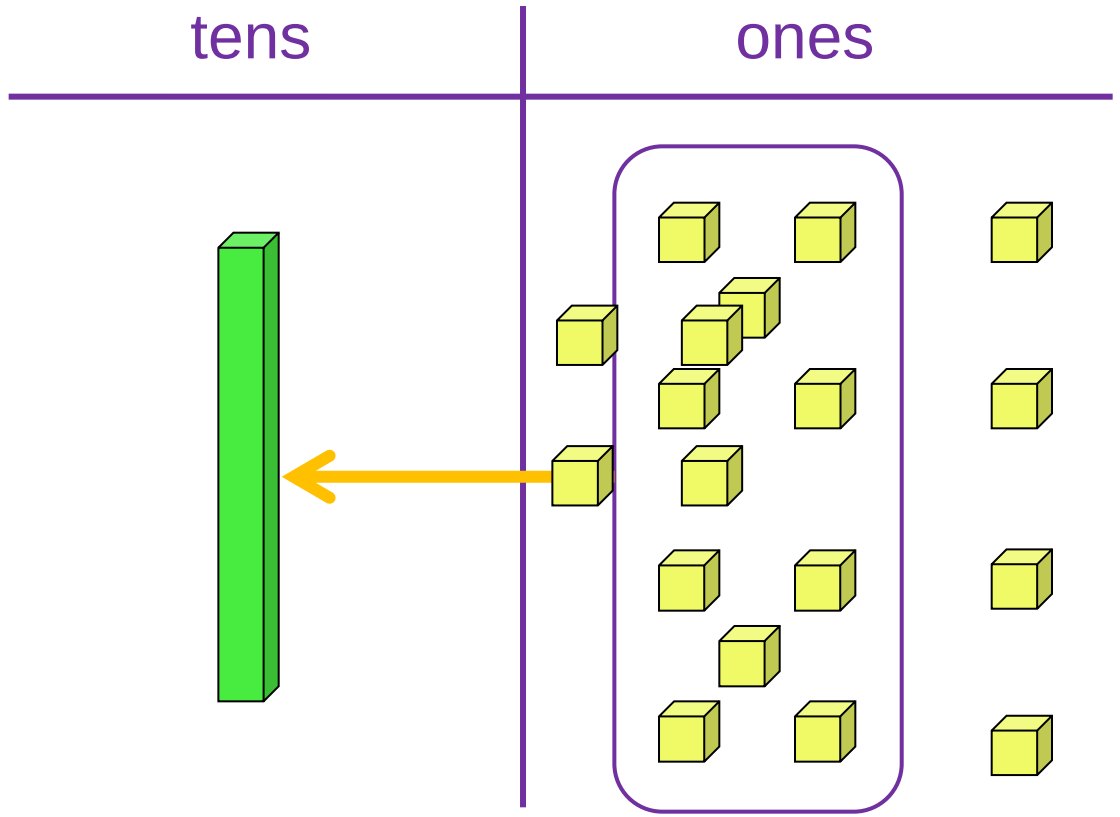
	tens	ones
thirteen	1	3
fourteen	1	4
fifteen	1	5
sixteen	1	6

consistency

copyright.Steve Chinn.2024

Doubles. 2x 2 + 2

Example: 7 + 7 = 14



5 + 2

5 + 2

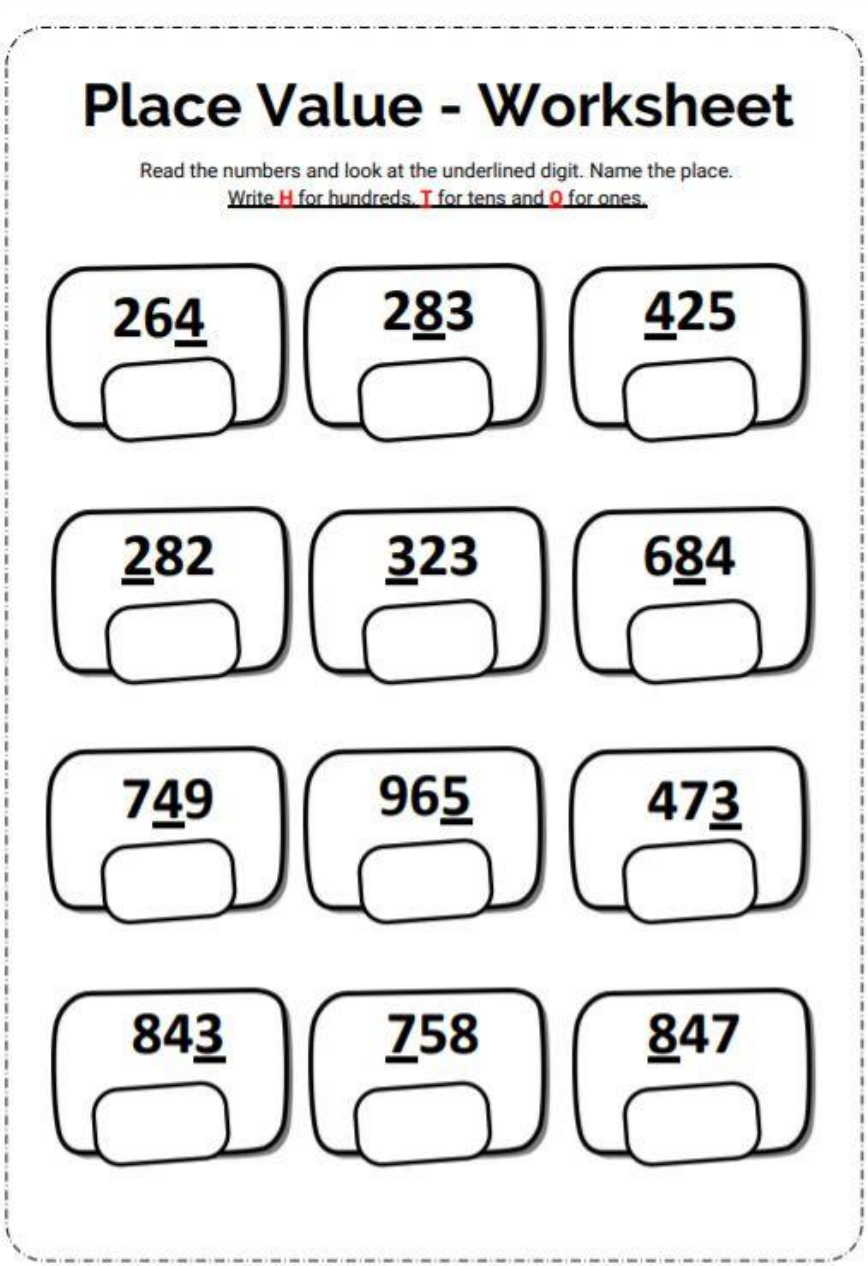
7

+

7

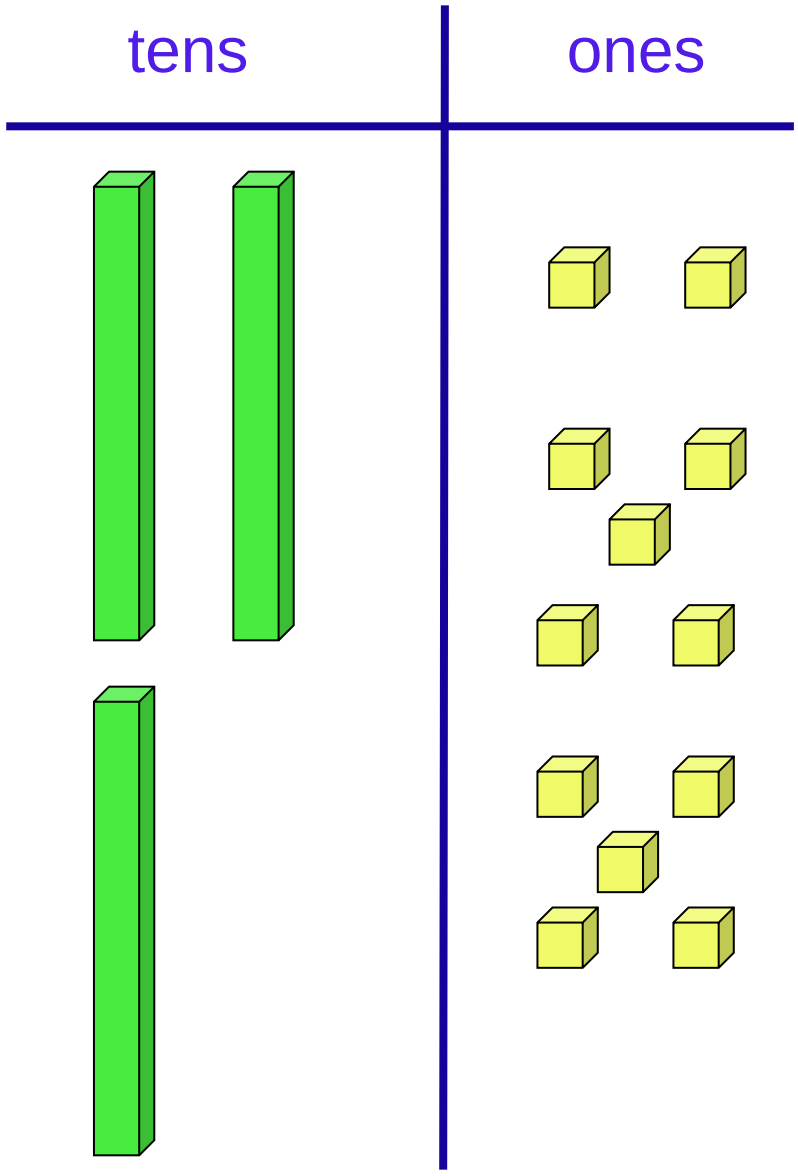
copyright.Steve Chinn.2024

From Pinterest



copyright.Steve Chinn.2024

Counting and renaming



37 thirty seven

38 thirty eight

39 thirty nine

30 thirty ten

10

30 thirty eleven

11

30 thirty twelve

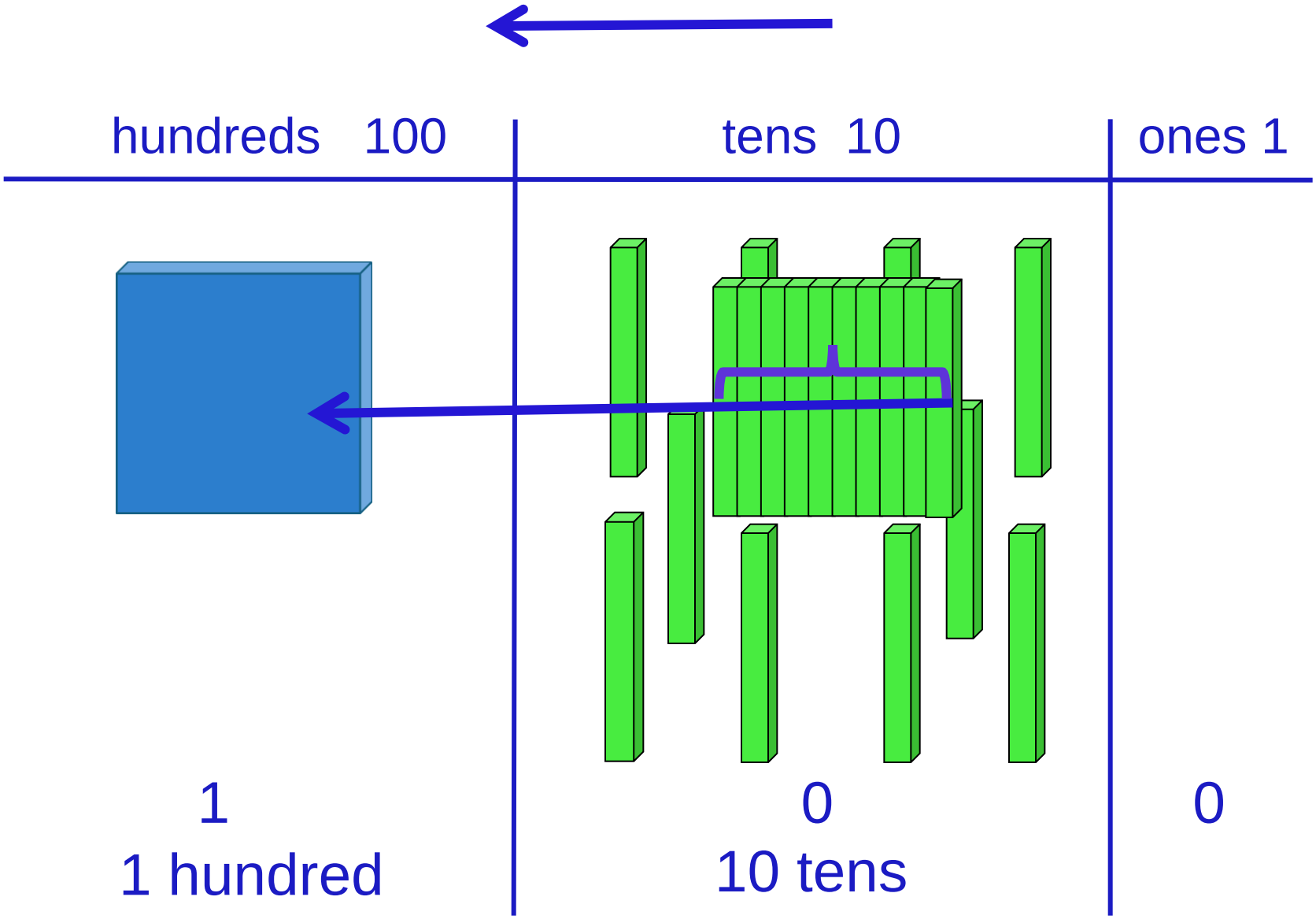
12

thirty twelve = forty two

42 = 30 + 12

copyright.Steve Chinn.2024

Trading: 10 tens for 1 hundred



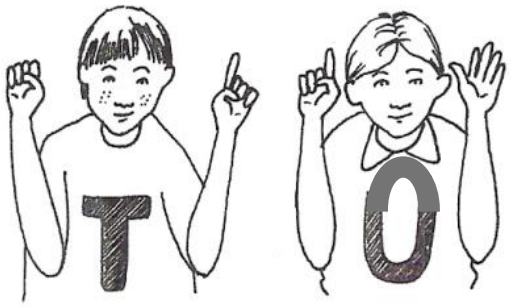
copyright.Steve Chinn.2024

The sheep.

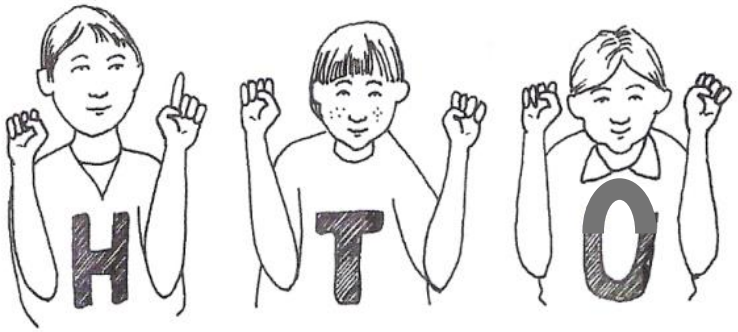
copyright.Steve Chinn.2024

The sheep counters.

16 sheep



100 sheep



copyright.Steve Chinn.2024

Note that early maths is neither easy nor consistent, especially vocabulary.

Vocabulary of numbers (inc. fractions) – inconsistent/illogical (Szucs and Morsani, 2014)

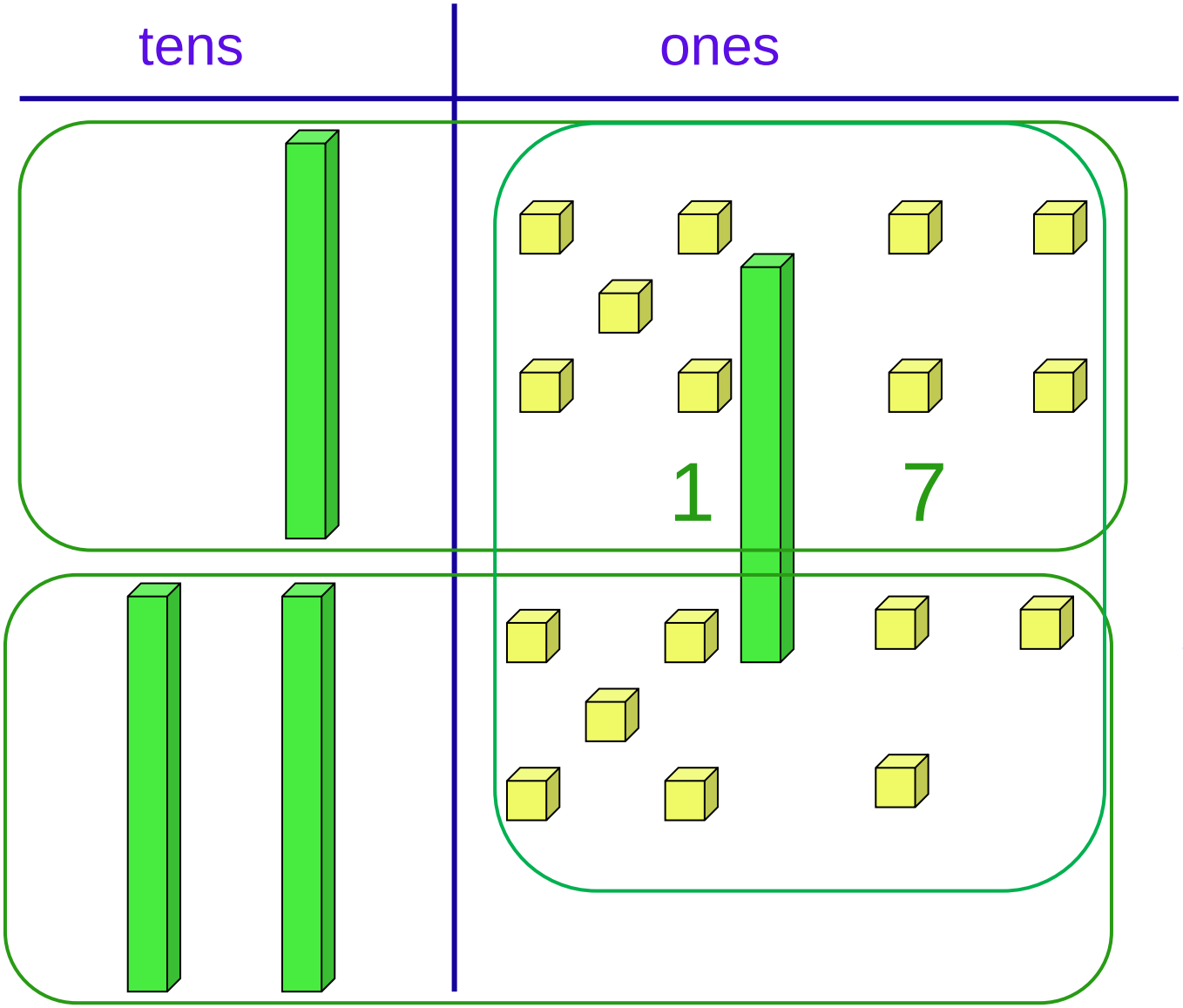
Vocabulary of operations … Is consistency possible?

Sequence of numbers - -

place value - - zero

copyright.Steve Chinn.2024

Adding a 2-digit number to a 2-digit number. Scaffolding.



1

1 9

+2 8

4 7

vocabulary

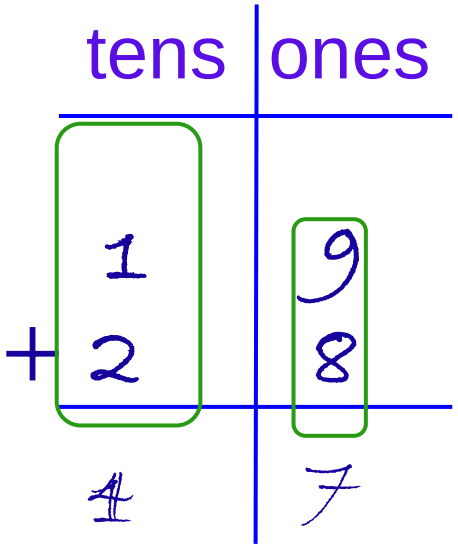
De- and re-composition

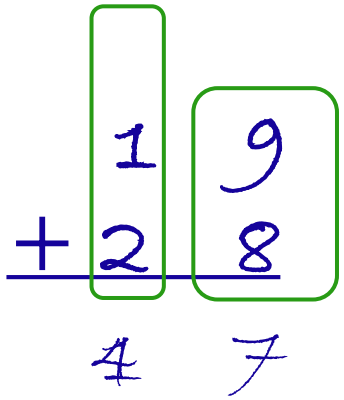
trade

copyright.Steve Chinn.2024

Adding a 2 digit number to a 2 digit number

Withdrawing the scaffolding





copyright.Steve Chinn.2024

School students' confidence when answering diagnostic questions online. Colin Foster, Simon Woodhead, Craig Barton & Alison Clark-Wilson Educational Studies in Mathematics. 2021

Have you ever been so certain about a fact that when you found out you were wrong the shock caused you to remember the right answer forever?

Work by Andrew Butler and his team at Duke University appears to show that the hypercorrection effect persists for about a week, but longer than this and the 'high-confidence error' (misconception) will persist.

Inhibition.

copyright.Steve Chinn.2024

School students' confidence when answering diagnostic questions online. Colin Foster, Simon Woodhead, Craig Barton & Alison Clark-Wilson Educational Studies in Mathematics. 2021

We found a positive correlation between student confidence and mean facility, higher confidence for boys than for girls and lower confidence for students classified as socioeconomically disadvantaged, even after accounting for facility.

We found that confidence was lower for older students, and this was particularly marked across the primary to secondary school transition.

Eedi.co.uk free access to over 50,000 multiple-choice **diagnostic** mathematics questions (+ confidence)

copyright.Steve Chinn.2024

A recent paper on maths anxiety from the Derby University team. (2021)

https://isres.org/interventions-to-address-mathematics-anxiety-an-overview-and-recommendations-244-s.html#.YcCDNHzP02w…

Tom Hunt

David Sheffield

Dominic Petronzi

Debilitative and facilitative. State and trait.

copyright.Steve Chinn.2024

Conclusion and Recommendations. 1

The existence of MA across a range of countries, and therefore, educational systems, is evidenced through the studies considered in this review.

Research has indicated that MA can develop in children as young as four years of age (Petronzi et al., 2017)

with attitudes formed because of classroom-based negativity (Fraser et al., 2013).

Therefore, it is important that suitable strategies are implemented in early schooling to minimise the development or exacerbation of MA.

Those strategies that appear to be most effective in addressing MA are those that facilitate cognitive control, with an emphasis on attentional processes and emotion regulation. Thus, in the main, these studies provide support for the debilitating anxiety model.

Next …….

copyright.Steve Chinn.2024

Conclusion and Recommendations. 2

We propose the need for targeted strategies that are time and cost effective, especially those that lend themselves to a range of educational contexts, such as the home and virtual learning environment.

Since MA is associated with avoidance (Chinn, 2012; Choe et al., 2019), as a starting point it is important that academic settings are inviting and encourage greater participation among maths learners (Hlalele, 2012).

Future work would benefit from testing a wider range of hypotheses pertaining to plausible targets associated with MA, including maths self-efficacy, social support, growth mindset and resilience.

copyright.Steve Chinn.2024

Let’s look at what is involved in this belief

Stm

WM

LTM

Speed of processing

Anxiety and the affective domain

Expectations

Inhibition

Maths

copyright.Steve Chinn.2024

Definition (USA)

DSM-V: Dyscalculia 2013

Difficulties in production or comprehension of quantities, numerical symbols, or basic arithmetic operations that are not consistent with the person's chronological age, educational opportunities, or intellectual abilities.

copyright.Steve Chinn.2024

DSM-V Specific Learning Disability

Difficulties remembering number facts.
Inaccurate or slow arithmetic calculation.
Ineffective or inaccurate mathematical reasoning.
Avoidance of activities requiring arithmetic.

The first three create the fourth – thanks to us adults.

In fact, maybe we create many of the 'dyscalculics'

The influence of maths culture and beliefs

copyright.Steve Chinn.2024

Can we quantify?

DSM-V: Dyscalculia

‘… not consistent with the person's chronological age, educational opportunities, or intellectual abilities.’

Basic maths/arithmetic

What do we quantify? The heterogeneity of the population.

copyright.Steve Chinn.2024

Dyscalculia definition. (UK). 2019 BDA /SASC

(It's getting longer)

Dyscalculia is a specific and persistent difficulty in understanding numbers which can lead to a diverse range of difficulties with mathematics.

It will be unexpected in relation to age, level of education and experience and occurs across all ages and abilities.

Mathematics difficulties are best thought of as a continuum, not a distinct category, and they have many causal factors.

Dyscalculia falls at one end of the spectrum and will be distinguishable from other maths issues due to the severity of difficulties with number sense, including subitising, symbolic and non-symbolic magnitude comparison, and ordering.

It can occur singly but often co-occurs with other specific learning difficulties, mathematics anxiety and medical conditions.

copyright.Steve Chinn.2024

Recent paper:

Basic Symbolic Number Skills, but Not Formal Mathematics Performance, Longitudinally Predict Mathematics Anxiety in the First Years of Primary School

Patrick A. O'Connor Kinga Morsanyi and Teresa McCormack

Published: 1 November 2023

Mathematical anxiety (MA) and mathematics performance typically correlate negatively in studies of adolescents and adults, but not always amongst young children, with some theorists questioning the relevance of MA to mathematics performance in this age group. Evidence is also limited in relation to the developmental origins of MA and whether MA in young children can be linked to their earlier mathematics performance.

copyright.Steve Chinn.2024