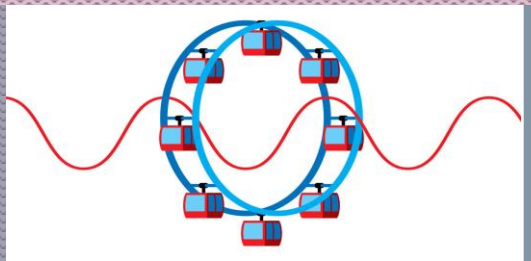
How I teach LC Higher Level....

Trigonometric Graphs

Meighan Duffy



Useful Prior Knowledge

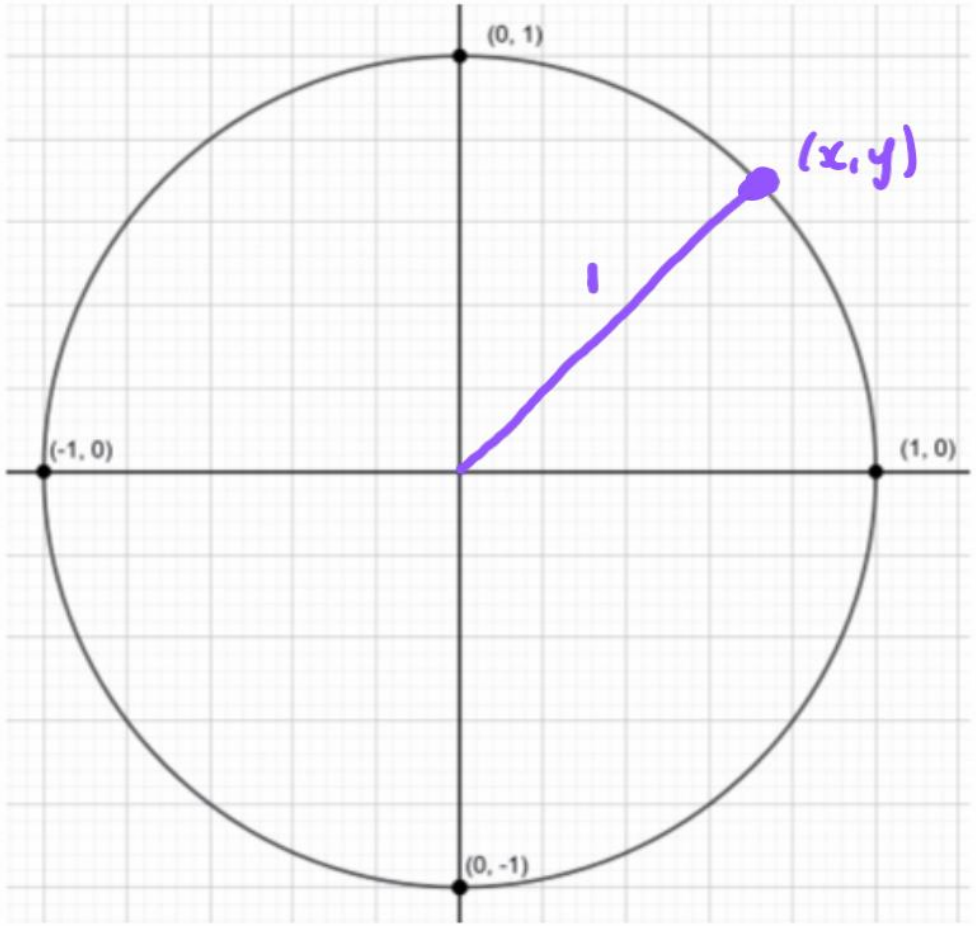
Functions

- Function notation
- Continuous vs discontinuous functions
- Asymptotes
- Domain and Range
- Graphing linear, quadratic, cubic, exponential & logarithmic functions
- Transformations of functions
- Using graphical methods to approximate values of $f(x) = 0, f(x) = k$

Trigonometry

- Trigonometric ratios and their relationship
- Special angles
- Pythagoras' theorem
- Radian measure

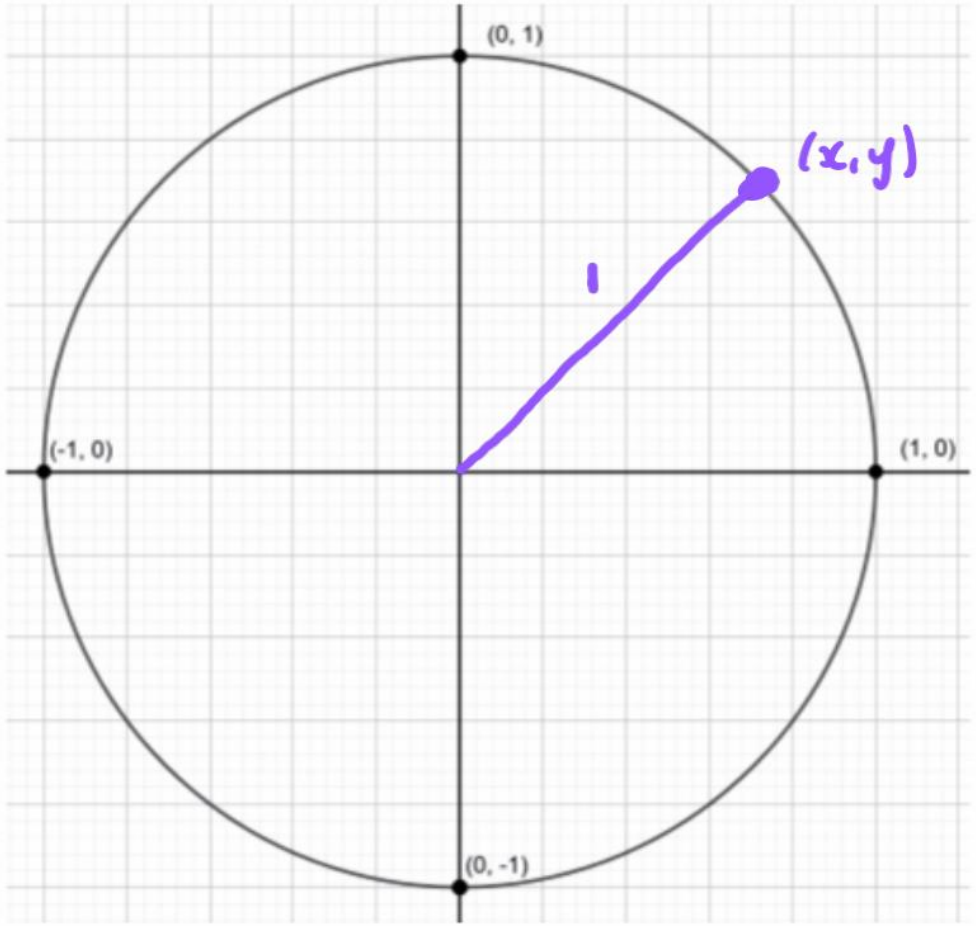
The Unit Circle



The Unit Circle

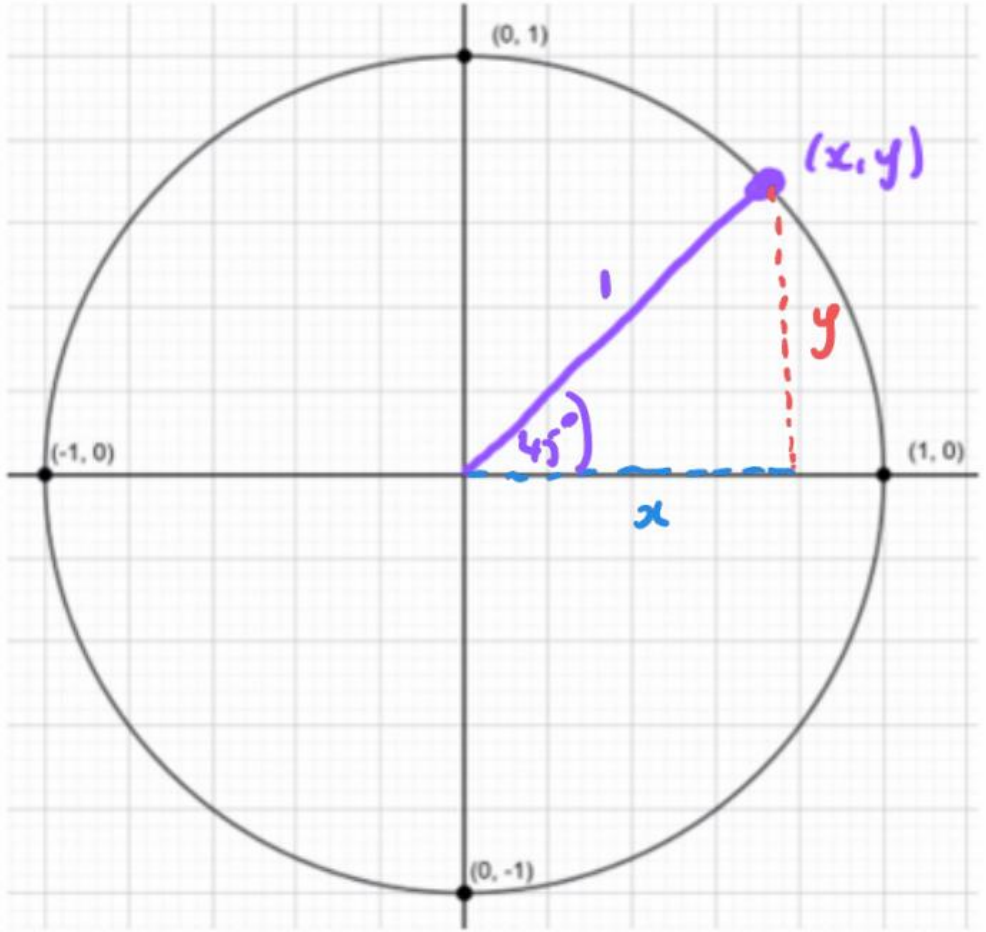
Group task:

› How could we estimate the values of the coordinates of this point?



The Unit Circle

Class feedback & discussion:



$\cos 45° = \frac{x}{1} \Rightarrow \cos 45 = x$

$\frac{1}{\sqrt{2}} = x$

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

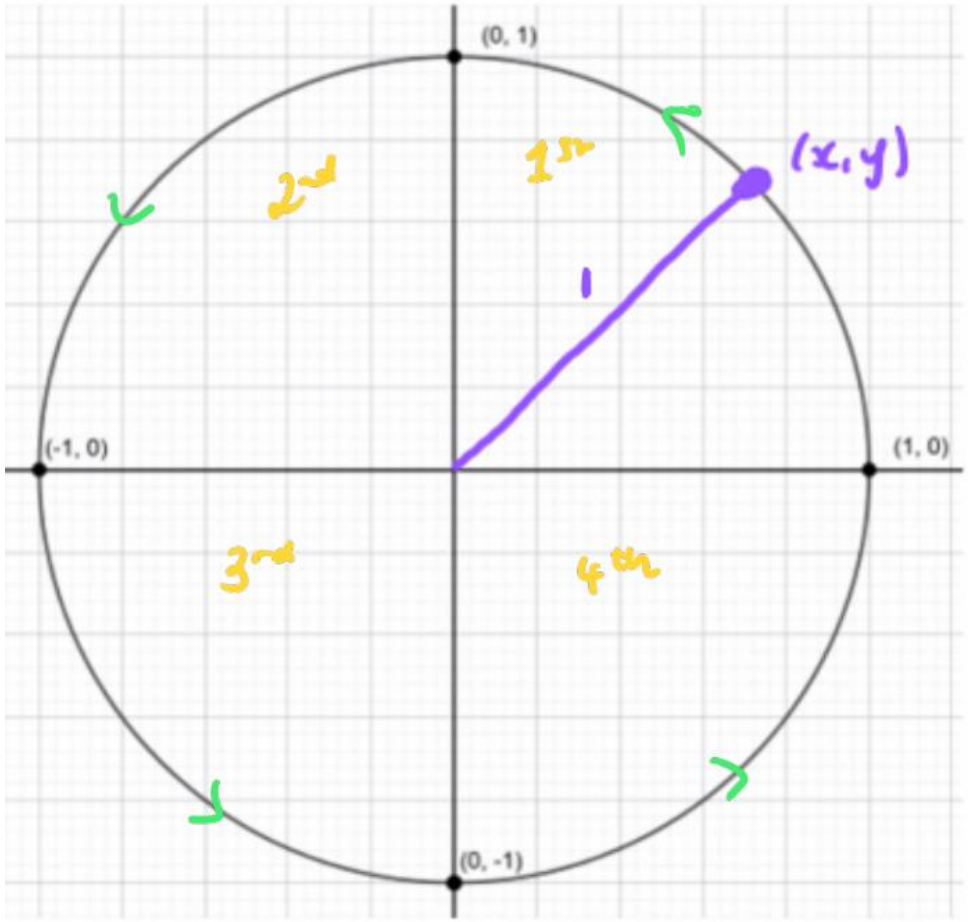
$\sin 45° = \frac{y}{1} \Rightarrow \sin 45° = y$

$\frac{1}{\sqrt{2}} = y$

The Unit Circle

Group task extension:

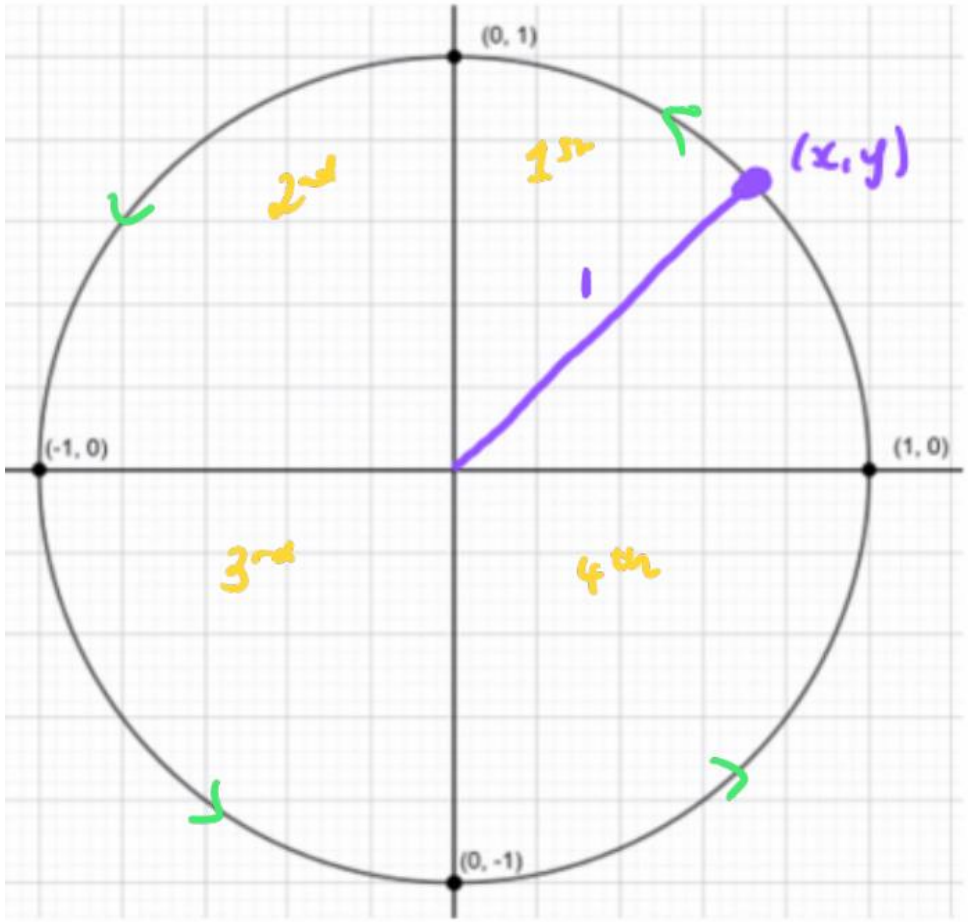
- Can we find more coordinates, using the same technique?



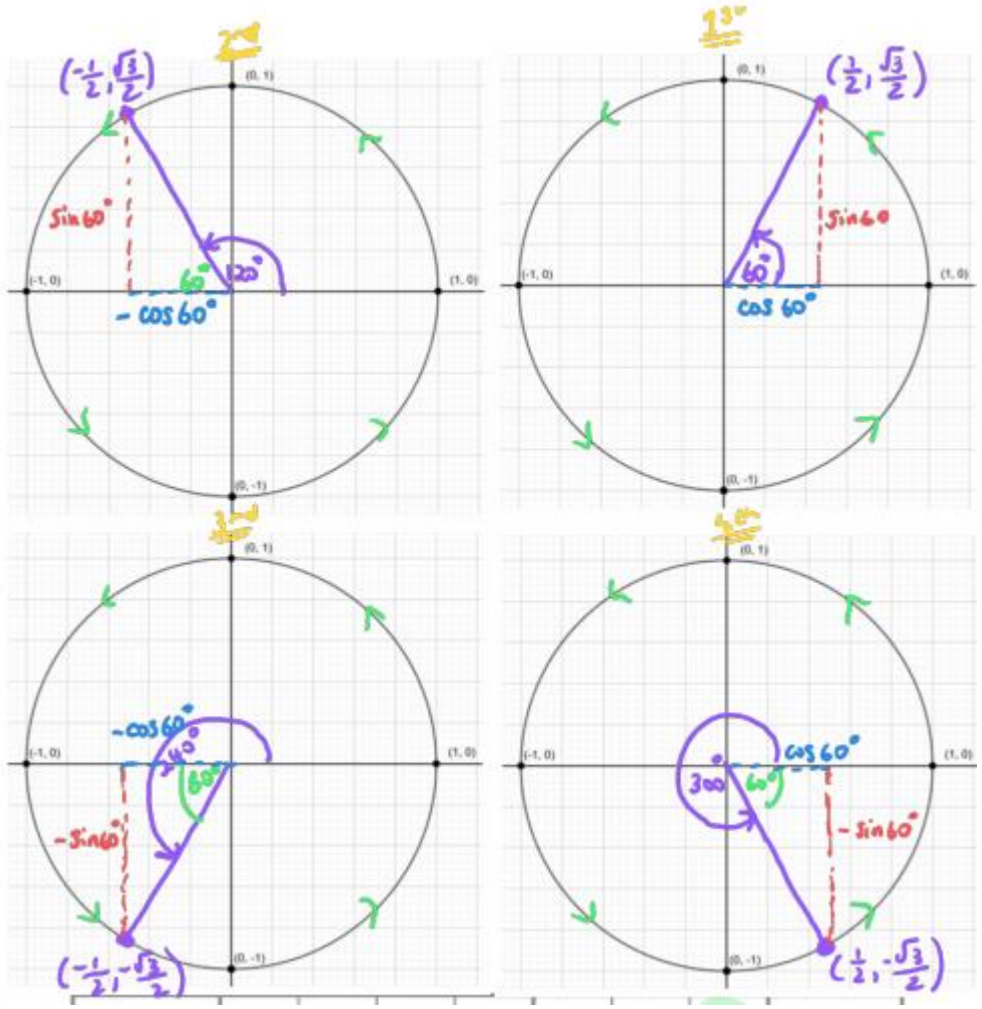
The Unit Circle

Group task extension:

- Can we find more coordinates, using the same technique?
- Can each group evaluate at least one coordinate in each quadrant?



The Unit Circle

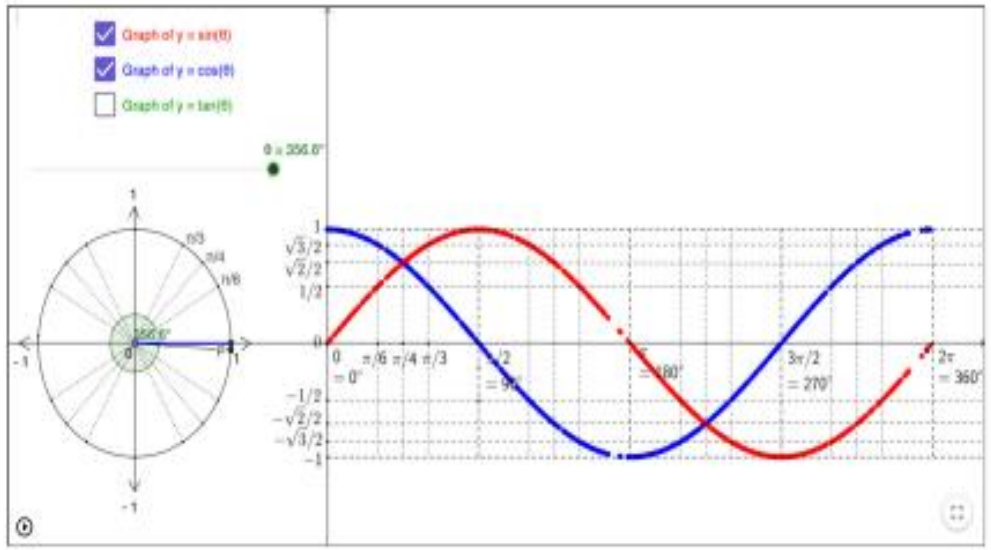


A (céimeanna)	0°	90°	180°	270°	30°	45°	60°	A (degrees)
A (raidiain)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	A (radians)
$\cos A$	1	0	−1	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\cos A$
$\sin A$	0	1	0	−1	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\sin A$
$\tan A$	0	-	0	-	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\tan A$

Using the values you've found for $\sin x$ and $\cos x$ in the domain $0 \leq x \leq 360$, graph the two functions.

The Unit Circle

$\cos 0$	$\cos 0$	1	$(0,1)$
$\cos 60$	$\cos \frac{\pi}{3}$	$\frac{1}{2}$	$(\frac{\pi}{3}, \frac{1}{2})$
$\cos 90$	$\cos \frac{\pi}{2}$	0	$(\frac{\pi}{2}, 0)$
$\cos 120$	$\cos \frac{2\pi}{3}$	$\frac{-1}{2}$	$(\frac{2\pi}{3}, \frac{-1}{2})$
$\cos 180$	$\cos \pi$	-1	$(\pi, -1)$
$\cos 240$	$\cos \frac{4\pi}{3}$	$-\frac{1}{2}$	$(\frac{4\pi}{3}, -\frac{1}{2})$
$\cos 270$	$\cos \frac{3\pi}{2}$	0	$(\frac{3\pi}{2}, 0)$
$\cos 300$	$\cos \frac{5\pi}{3}$	$\frac{1}{2}$	$(\frac{5\pi}{3}, \frac{1}{2})$
$\cos 360$	$\cos 2\pi$	1	$(2\pi, 1)$



GeoGebra

$\sin 0$	$\sin 0$	0	$(0,0)$
$\sin 60$	$\sin \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$
$\sin 90$	$\sin \frac{\pi}{2}$	1	$(\frac{\pi}{2}, 1)$
$\sin 120$	$\sin \frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$(\frac{2\pi}{3}, \frac{\sqrt{3}}{2})$
$\sin 180$	$\sin \pi$	0	$(\pi, 0)$
$\sin 240$	$\sin \frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$(\frac{4\pi}{3}, -\frac{\sqrt{3}}{2})$
$\sin 270$	$\sin \frac{3\pi}{2}$	-1	$(\frac{3\pi}{2}, -1)$
$\sin 300$	$\sin \frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$(\frac{5\pi}{3}, -\frac{\sqrt{3}}{2})$
$\sin 360$	$\sin 2\pi$	0	$(2\pi, 0)$

The Unit Circle

$\cos 0$	$\cos 0$	1	$(0,1)$
$\cos 60$	$\cos \frac{\pi}{3}$	$\frac{1}{2}$	$(\frac{\pi}{3}, \frac{1}{2})$
$\cos 90$	$\cos \frac{\pi}{2}$	0	$(\frac{\pi}{2}, 0)$
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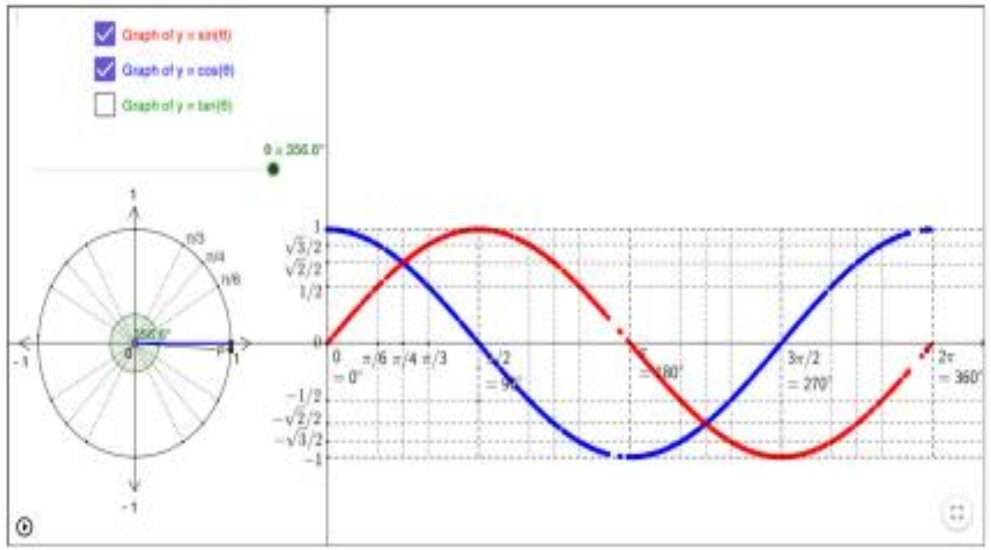
$\cos 180$	$\cos \pi$	-1	$(\pi, -1)$
$\cos 240$	$\cos \frac{4\pi}{3}$	$-\frac{1}{2}$	$(\frac{4\pi}{3}, -\frac{1}{2})$
$\cos 270$	$\cos \frac{3\pi}{2}$	0	$(\frac{3\pi}{2}, 0)$
$\cos 300$	$\cos \frac{5\pi}{3}$	$\frac{1}{2}$	$(\frac{5\pi}{3}, \frac{1}{2})$
$\cos 360$	$\cos 2\pi$	1	$(2\pi, 1)$



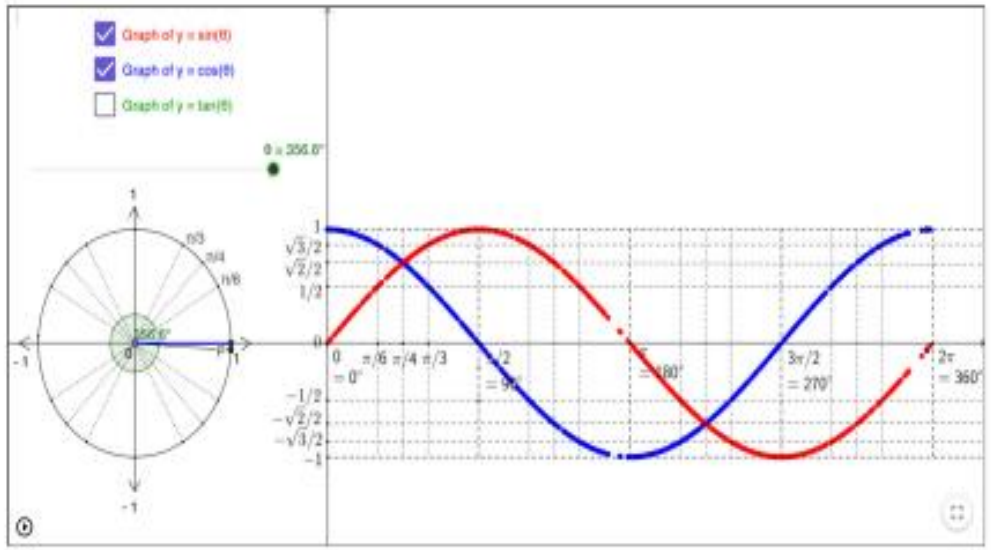
$\sin 0$	$\sin 0$	0	$(0,0)$
$\sin 60$	$\sin \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$
$\sin 90$	$\sin \frac{\pi}{2}$	1	$(\frac{\pi}{2}, 1)$
$\sin 120$	$\sin \frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$(\frac{2\pi}{3}, \frac{\sqrt{3}}{2})$
$\sin 180$	$\sin \pi$	0	$(\pi, 0)$
$\sin 240$	$\sin \frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$(\frac{4\pi}{3}, -\frac{\sqrt{3}}{2})$
$\sin 270$	$\sin \frac{3\pi}{2}$	-1	$(\frac{3\pi}{2}, -1)$
$\sin 300$	$\sin \frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$(\frac{5\pi}{3}, -\frac{\sqrt{3}}{2})$
$\sin 360$	$\sin 2\pi$	0	$(2\pi, 0)$

Extension Questions:

What similarities do you notice between the two functions?

The Unit Circle

$\cos 0$	$\cos 0$	1	$(0,1)$
$\cos 60$	$\cos \frac{\pi}{3}$	$\frac{1}{2}$	$(\frac{\pi}{3}, \frac{1}{2})$
$\cos 90$	$\cos \frac{\pi}{2}$	0	$(\frac{\pi}{2}, 0)$
$\cos 120$	$\cos \frac{2\pi}{3}$	$\frac{-1}{2}$	$(\frac{2\pi}{3}, \frac{-1}{2})$
$\cos 180$	$\cos \pi$	-1	$(\pi, -1)$
$\cos 240$	$\cos \frac{4\pi}{3}$	$-\frac{1}{2}$	$(\frac{4\pi}{3}, -\frac{1}{2})$
$\cos 270$	$\cos \frac{3\pi}{2}$	0	$(\frac{3\pi}{2}, 0)$
$\cos 300$	$\cos \frac{5\pi}{3}$	$\frac{1}{2}$	$(\frac{5\pi}{3}, \frac{1}{2})$
$\cos 360$	$\cos 2\pi$	1	$(2\pi, 1)$



$\sin 0$	$\sin 0$	0	$(0,0)$
$\sin 60$	$\sin \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$
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$\sin 180$	$\sin \pi$	0	$(\pi, 0)$
$\sin 240$	$\sin \frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$(\frac{4\pi}{3}, -\frac{\sqrt{3}}{2})$
$\sin 270$	$\sin \frac{3\pi}{2}$	-1	$(\frac{3\pi}{2}, -1)$
$\sin 300$	$\sin \frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$(\frac{5\pi}{3}, -\frac{\sqrt{3}}{2})$
$\sin 360$	$\sin 2\pi$	0	$(2\pi, 0)$

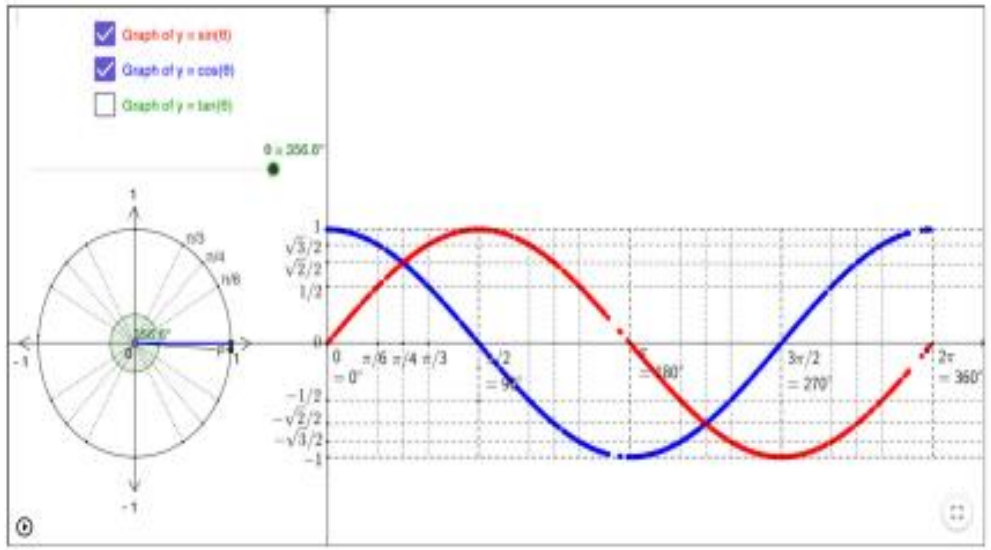
Extension Questions:

What similarities do you notice between the two functions?

What differences do you notice between the two functions?

The Unit Circle

$\cos 0$	$\cos 0$	1	$(0,1)$
$\cos 60$	$\cos \frac{\pi}{3}$	$\frac{1}{2}$	$(\frac{\pi}{3}, \frac{1}{2})$
$\cos 90$	$\cos \frac{\pi}{2}$	0	$(\frac{\pi}{2}, 0)$
$\cos 120$	$\cos \frac{2\pi}{3}$	$\frac{-1}{2}$	$(\frac{2\pi}{3}, \frac{-1}{2})$
$\cos 180$	$\cos \pi$	-1	$(\pi, -1)$
$\cos 240$	$\cos \frac{4\pi}{3}$	$-\frac{1}{2}$	$(\frac{4\pi}{3}, -\frac{1}{2})$
$\cos 270$	$\cos \frac{3\pi}{2}$	0	$(\frac{3\pi}{2}, 0)$
$\cos 300$	$\cos \frac{5\pi}{3}$	$\frac{1}{2}$	$(\frac{5\pi}{3}, \frac{1}{2})$
$\cos 360$	$\cos 2\pi$	1	$(2\pi, 1)$



$\sin 0$	$\sin 0$	0	$(0,0)$
$\sin 60$	$\sin \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$
$\sin 90$	$\sin \frac{\pi}{2}$	1	$(\frac{\pi}{2}, 1)$
$\sin 120$	$\sin \frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$(\frac{2\pi}{3}, \frac{\sqrt{3}}{2})$
$\sin 180$	$\sin \pi$	0	$(\pi, 0)$
$\sin 240$	$\sin \frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$(\frac{4\pi}{3}, -\frac{\sqrt{3}}{2})$
$\sin 270$	$\sin \frac{3\pi}{2}$	-1	$(\frac{3\pi}{2}, -1)$
$\sin 300$	$\sin \frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$(\frac{5\pi}{3}, -\frac{\sqrt{3}}{2})$
$\sin 360$	$\sin 2\pi$	0	$(2\pi, 0)$

Extension Questions:

What similarities do you notice between the two functions?

What differences do you notice between the two functions?

What happens if you extend the domain?

THE SINE & COSINE GRAPHS

Key Points:

Both:

- ✓ Sinusoidal periodic functions
- ✓ Domain = R / (-∞,∞)
- ✓ The time to complete one full cycle (period) = 2π/360°
- ✓ Oscillates between values [-1,1] (Range)
- ✓ The $x\ -axis$ is the midway line

Sine Function:

- ✓ Starts at the midway line

Cosine Function:

- ✓ Starts at the max point

$y = \sin(x)$

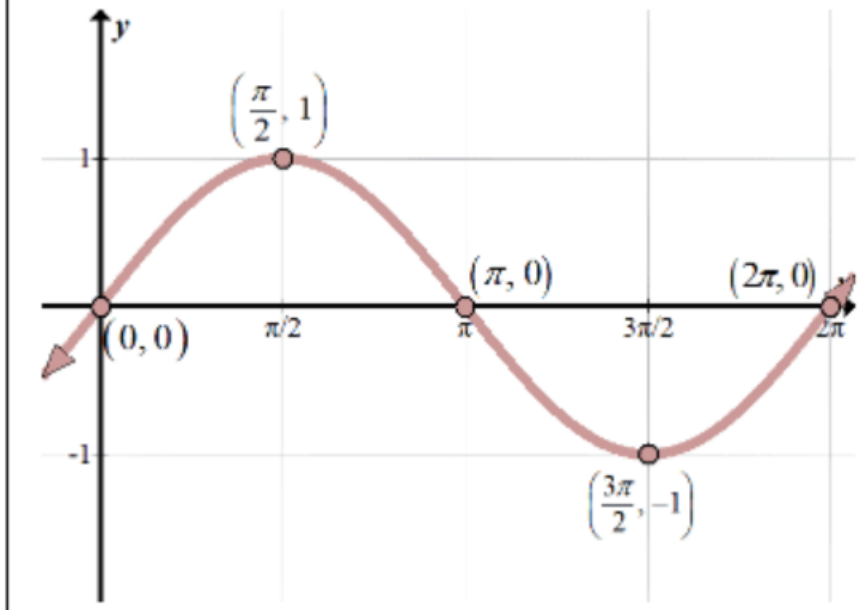
Odd

Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

Period: 2π

Zeros: $(\pi k, 0)$



$y = \cos(x)$

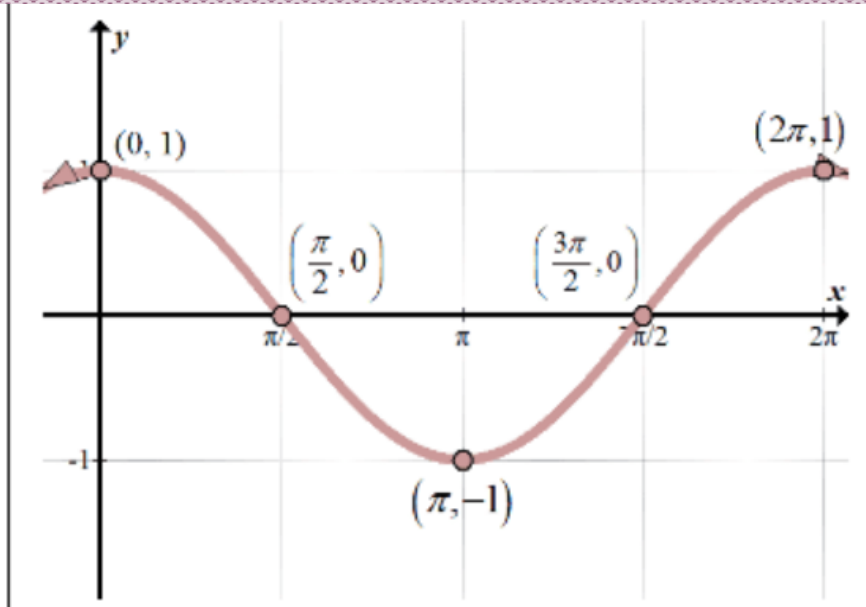
Even

Domain: $(-\infty, \infty)$

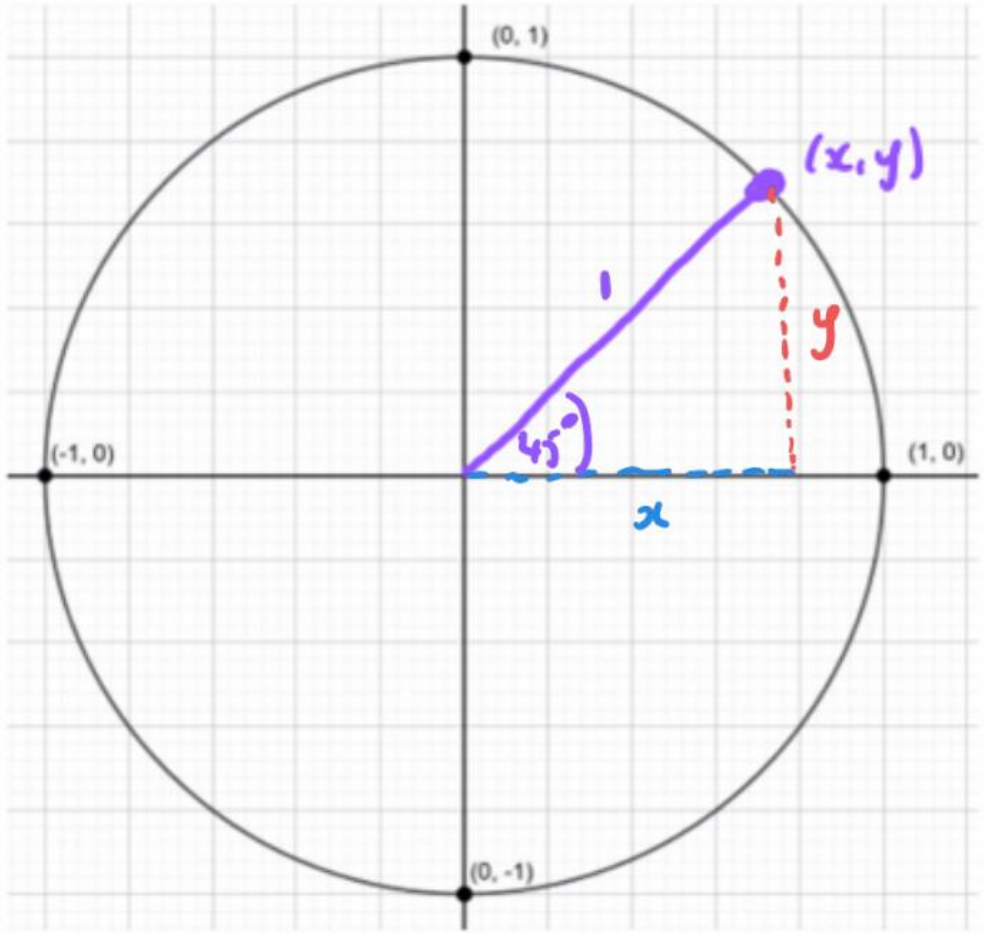
Range: $[-1, 1]$

Period: 2π

Zeros: $\left(\frac{\pi}{2} + \pi k, 0\right)$



What about the $\tan x$ function?



$\cos 45° = \frac{x}{1} \Rightarrow \cos 45 = x$

$\frac{1}{\sqrt{2}} = x$

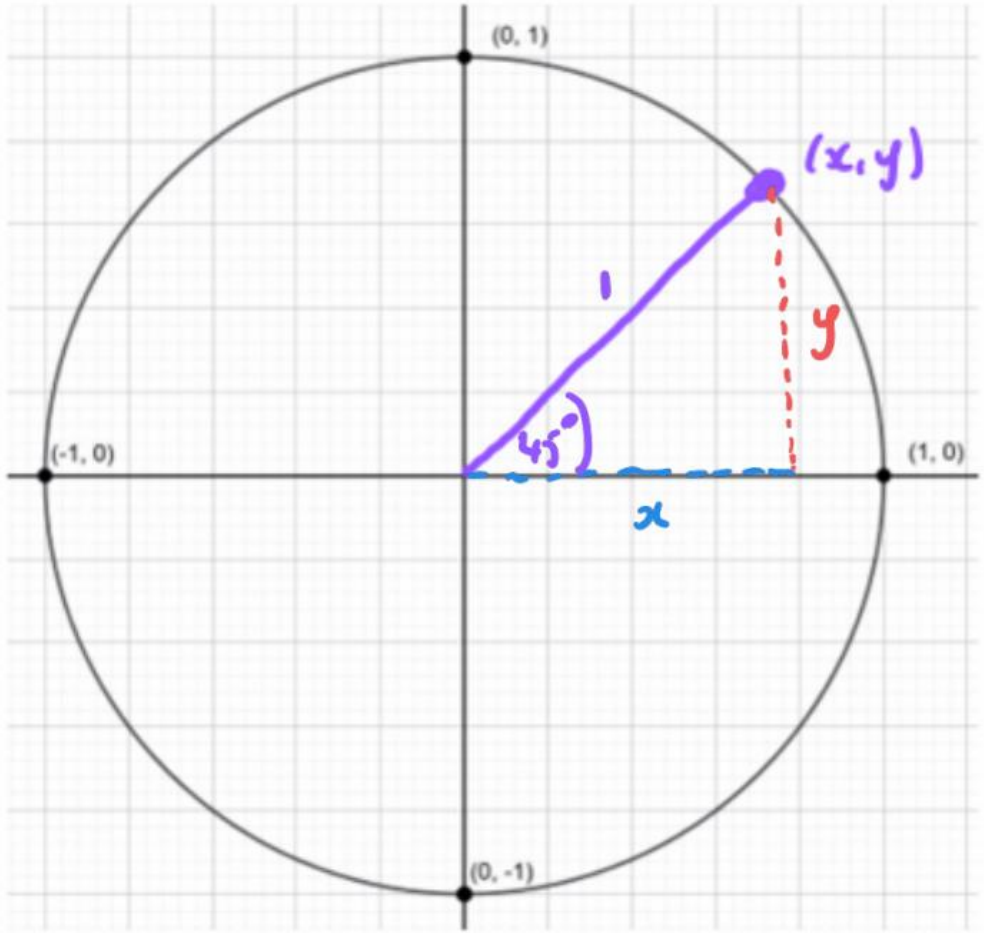
$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\sin 45° = \frac{y}{1} \Rightarrow \sin 45° = y$

$\frac{1}{\sqrt{2}} = y$

How can we use what we know about the relationship between sine & cosine and the unit circle, to graph the tan function?

What about the $\tan x$ function?



$\cos 45° = \frac{x}{1} \Rightarrow \cos 45 = x$

$\frac{1}{\sqrt{2}} = x$

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$\sin 45° = \frac{y}{1} \Rightarrow \sin 45° = y$

$\frac{1}{\sqrt{2}} = y$

How can we use what we know about the relationship between sine & cosine and the unit circle, to graph the tan function?

Trigonometric Ratios

What about the $\tan x$ function?

$$\tan x = \frac{\sin x}{\cos x}$$

$\sin 0$	$\sin 0$	0	$(0,0)$
$\sin 60$	$\sin\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$(\frac{\pi}{3},\frac{\sqrt{3}}{2})$
$\sin 90$	$\sin\frac{\pi}{2}$	1	$(\frac{\pi}{2},1)$
$\sin 120$	$\sin\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$(\frac{2\pi}{3},\frac{\sqrt{3}}{2})$
$\sin 180$	$\sin\pi$	0	$(\pi,0)$
$\sin 240$	$\sin\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$(\frac{4\pi}{3},-\frac{\sqrt{3}}{2})$
$\sin 270$	$\sin\frac{3\pi}{2}$	-1	$(\frac{3\pi}{2},-1)$
$\sin 300$	$\sin\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$(\frac{5\pi}{3},-\frac{\sqrt{3}}{2})$
$\sin 360$	$\sin 2\pi$	0	$(2\pi,0)$

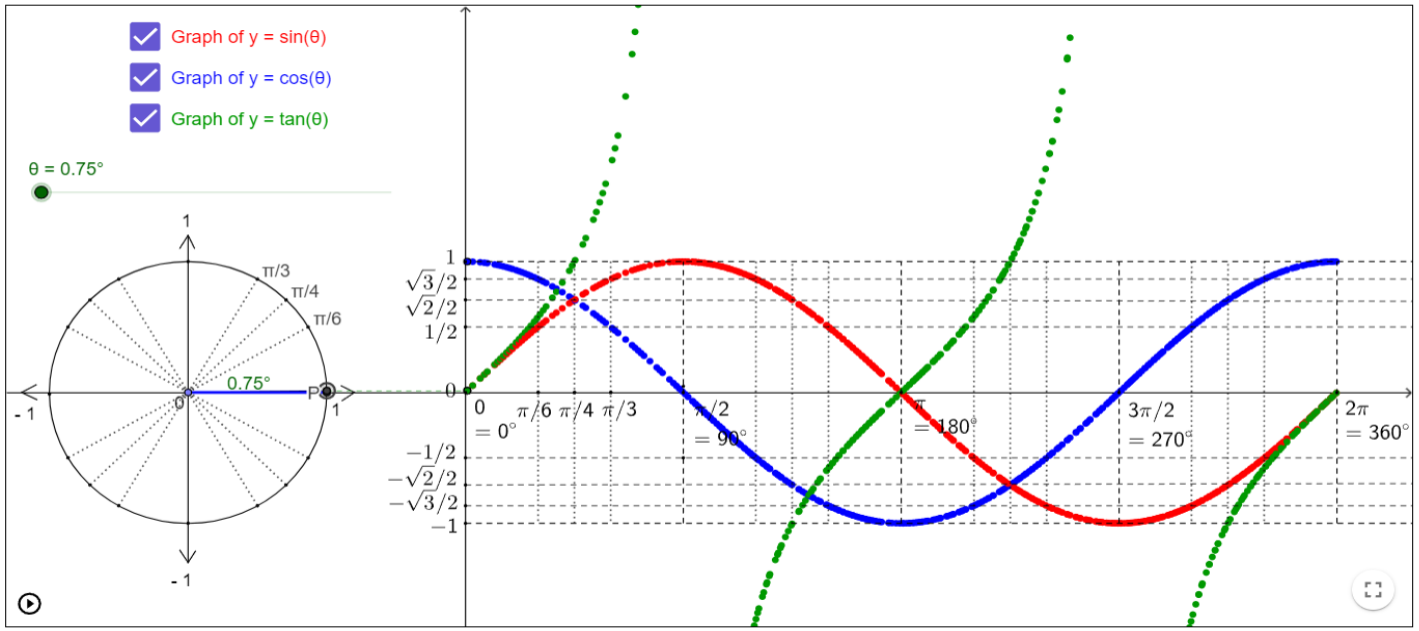
$\cos 0$	$\cos 0$	1	$(0,1)$
$\cos 60$	$\cos\frac{\pi}{3}$	$\frac{1}{2}$	$(\frac{\pi}{3},\frac{1}{2})$
$\cos 90$	$\cos\frac{\pi}{2}$	0	$(\frac{\pi}{2},0)$
$\cos 120$	$\cos\frac{2\pi}{3}$	$-\frac{1}{2}$	$(\frac{2\pi}{3},-\frac{1}{2})$
$\cos 180$	$\cos\pi$	-1	$(\pi,-1)$
$\cos 240$	$\cos\frac{4\pi}{3}$	$-\frac{1}{2}$	$(\frac{4\pi}{3},-\frac{1}{2})$
$\cos 270$	$\cos\frac{3\pi}{2}$	0	$(\frac{3\pi}{2},0)$
$\cos 300$	$\cos\frac{5\pi}{3}$	$\frac{1}{2}$	$(\frac{5\pi}{3},\frac{1}{2})$
$\cos 360$	$\cos 2\pi$	1	$(2\pi,1)$

$\tan 0$	$\tan 0$	$\frac{0}{1}$	0	$(0,0)$
$\tan 60$	$\tan\frac{\pi}{3}$	$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$	$\sqrt{3}$	$(\frac{\pi}{3},\sqrt{3})$
$\tan 90$	$\tan\frac{\pi}{2}$	$\frac{1}{0}$	$undefined$	-
$\tan 120$	$\tan\frac{2\pi}{3}$	$\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$	$-\sqrt{3}$	$(\frac{2\pi}{3},-\sqrt{3})$
$\tan 180$	$\tan\pi$	$\frac{0}{-1}$	0	$(\pi,0)$
$\tan 240$	$\tan\frac{4\pi}{3}$	$\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$	$\frac{1}{2}$	$(\frac{4\pi}{3},\frac{1}{2})$
$\tan 270$	$\tan\frac{3\pi}{2}$	$\frac{-1}{0}$	$undefined$	-
$\tan 300$	$\tan\frac{5\pi}{3}$	$\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$	$-\sqrt{3}$	$(\frac{5\pi}{3},-\sqrt{3})$
$\tan 360$	$\tan 2\pi$	$\frac{0}{1}$	0	$(2\pi,0)$

What about the $\tan x$ function?

$$\tan x = \frac{\sin x}{\cos x}$$

GeoGebra



tan 0	tan 0	$\frac{0}{1}$	0	(0,0)
tan 60	$\tan\frac{\pi}{3}$	$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$	$\sqrt{3}$	$(\frac{\pi}{3},\sqrt{3})$
tan 90	$\tan\frac{\pi}{2}$	$\frac{1}{0}$	$undefined$	-
tan 120	$\tan\frac{2\pi}{3}$	$\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$	$-\sqrt{3}$	$(\frac{2\pi}{3},-\sqrt{3})$
tan 180	$\tan\pi$	$\frac{0}{-1}$	0	$(\pi,0)$
tan 240	$\tan\frac{4\pi}{3}$	$\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$	$\frac{1}{2}$	$(\frac{4\pi}{3},\frac{1}{2})$
tan 270	$\tan\frac{3\pi}{2}$	$\frac{-1}{0}$	$undefined$	-
tan 300	$\tan\frac{5\pi}{3}$	$\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$	$-\sqrt{3}$	$(\frac{5\pi}{3},-\sqrt{3})$
tan 360	$\tan 2\pi$	$\frac{0}{1}$	0	$(2\pi,0)$

Phet - Trig Functions

THE TAN GRAPH

Key Points:

- ✓ Discontinuous function
- ✓ Domain = R \{±90°, ±270°,…} – asymptote every 180°/π radians
- ✓ The time to complete one full cycle (period) = π/180°
- ✓ Range = R or (-∞,∞)

$y = \tan(x)$

Odd

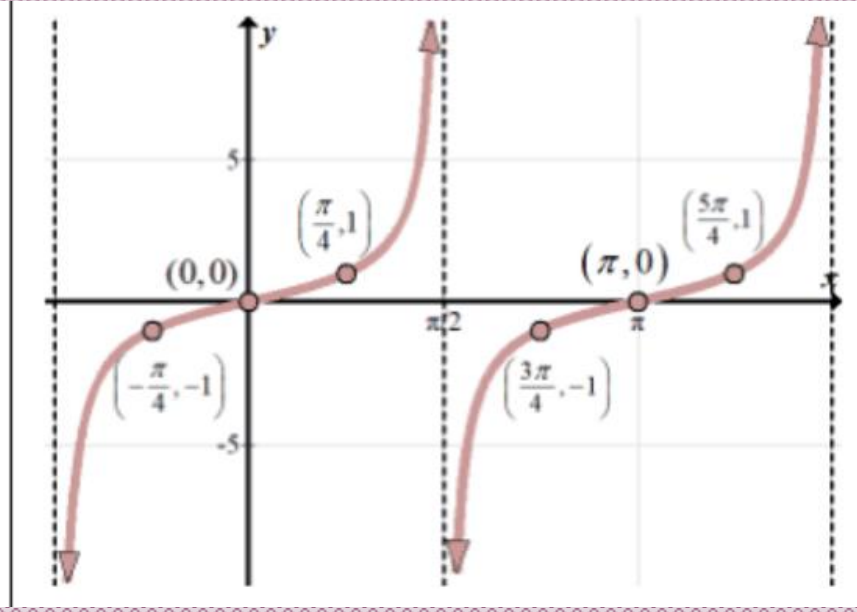
Domain: $x \neq \frac{\pi}{2} + \pi k$

Range: $(-\infty, \infty)$

Asymptotes: $x = \frac{\pi}{2} + \pi k$

Period: π

Zeros: $(\pi k, 0)$



Transformations of trigonometric graphs

Transformations of Trigonometric Functions

Using the following (https://www.geogebra.org/m/n7H6ucxM) or any graphing calculator, complete the following tables:

Function	Mid- line	Min Value	Max Value	Range	Period
$y = c\sin x$ and $y = c\cos x$					
$y = 2\sin x$					
$f(x) = \frac{1}{2}\cos x$					
$f(x) = -5\cos x$					
$y = \sin bx$ and $y = \cos bx$					
$y = \cos 2x$					
$f(x) = \sin\frac{3x}{2}$					
$y = \cos\frac{2x}{3}$					
$y = a + \sin x$ and $y = a + \cos x$					
$f(x) = -1 + \sin x$					
$f(x) = 4 + \cos x$					
$y = -2 + \cos x$					
$y = 3 + \sin x$					

Transformations of trigonometric graphs

Function	Mid- line	Min Value	Max Value	Range	Period
$y = b\sin x$ and $y = b\cos x$					
$y = 2\sin x$	$y=0$	-2	2	$[-2,2]$	2π rads/360°
$f(x) = \frac{1}{2}\cos x$	$y=0$	-0.5	0.5	$[-0.5,0.5]$	2π rads/360°
$f(x) = -5\cos x$	$y=0$	-5	5	$[-5,5]$	2π rads/360°
$y = \sin cx$ and $y = \cos cx$					
$y = \cos 2x$	$y=0$	-1	1	$[-1,1]$	π rads/180°
$f(x) = \sin\frac{3x}{2}$	$y=0$	-1	1	$[-1,1]$	$\frac{4}{3}\pi$ rads/240°
$y = \cos\frac{2x}{3}$	$y=0$	-1	1	$[-1,1]$	3π rads/540°
$y = a + \sin x$ and $y = a + \cos x$					
$f(x) = -1 + \sin x$	$y=-1$	-2	0	$[-2,0]$	2π rads/360°
$f(x) = 4 + \cos x$	$y=4$	3	5	$[3,5]$	2π rads/360°
$y = -2 + \cos x$	$y=-2$	-3	-1	$[-3,-1]$	2π rads/360°
$y = 3 + \sin x$	$y=3$	2	4	$[2,4]$	2π rads/360°

Transformations of trigonometric graphs

Transformation	Change observed, from basic functions: $y = \sin x$ and $y = \cos x$
$y = c \sin x$ and $y = c \cos x$	
$y = \sin bx$ and $y = \cos bx$	
$y = a + \sin x$ and $y = a + \cos x$	

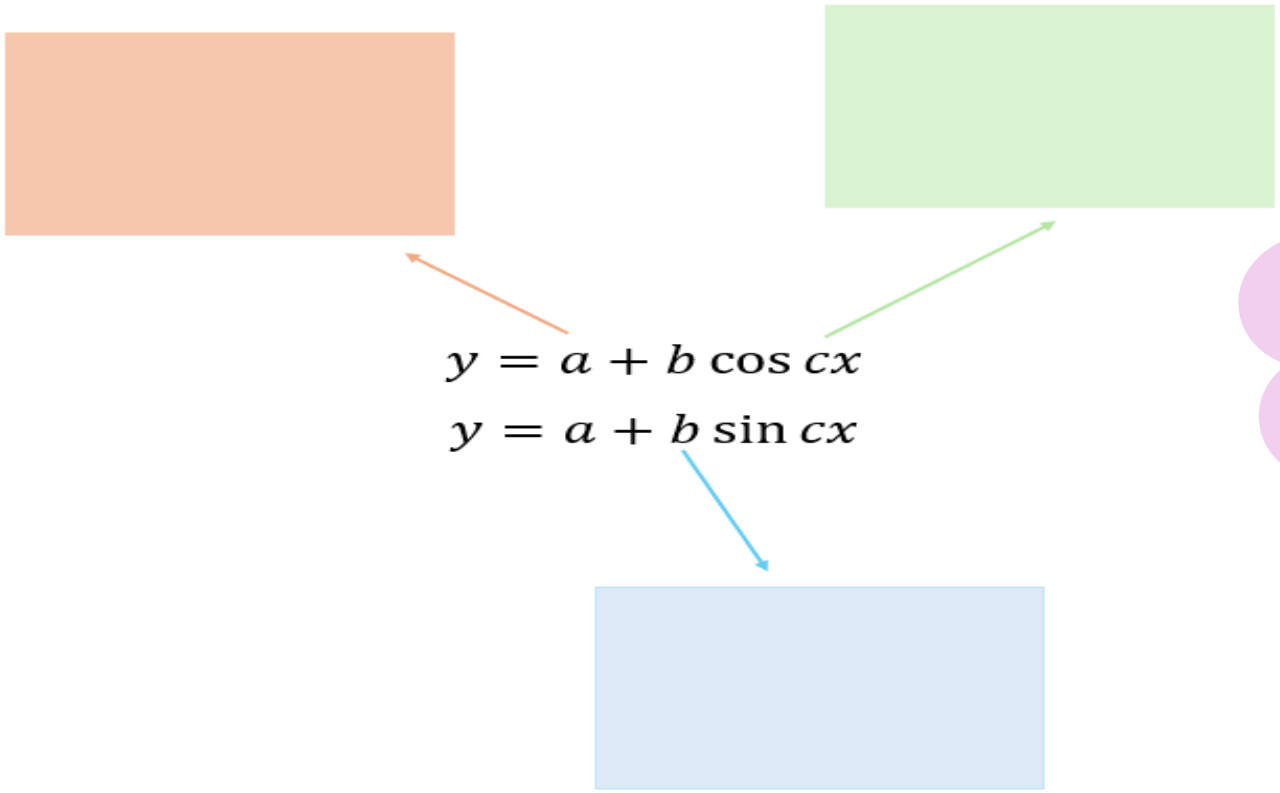
Transformations of trigonometric graphs

Transformation	Change observed, from basic functions: $y = \sin x$ and $y = \cos x$
$y = b\sin x$ and $y = b\cos x$	→ If $\|b\| > 1$ then the graph is stretched along the y-axis. → If $\|b\| < 1$ e.g $\frac{1}{2}$, the graph shrinks along the y-axis → the range changes but the period doesn't → If b is negative e.g $y = -\cos x$ then $y = \cos x$ is reflected in the x-axis
$y = \sin cx$ and $y = \cos cx$	→ c = no. of cycles in $2\pi/360°$ ⇒ If c = 2 then the period is just π rads / 180° ⇒ twice as quick → only the inputs/x-values and ∴ period is effected by this transformation.
$y = a + \sin x$ and $y = a + \cos x$	→ If $a > 1$ then there will be a translation of a units in the +ve direction of the y-axis → If $a < 1$ then there will be a translation of a units in the -ve direction of the y-axis

→ a = the midway line $a + |b|$ = max value

→ Range = $[a - |b|, a + |b|]$ $a - |b|$ = min value

Transformations of trigonometric graphs



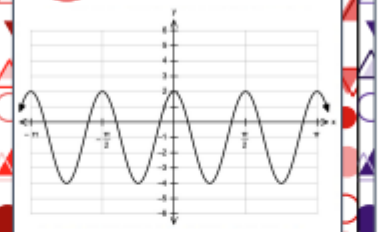
Students' Individualised summaries – they can adjust & add to this after doing exam paper questions too.

Matching Activity
$y = a \pm b\ sin\ cx$ & $y = a \pm b\ cos\ cx$

0 Which trigonometric function has the following characteristics

- A cosine function
- Amplitude of 3
- Period of $\pi/2$
- Midline $y = -1$
- Passes through (0, 2)

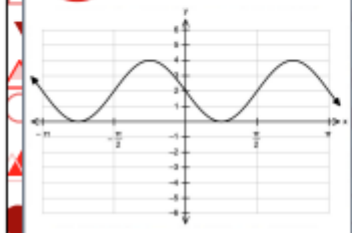
A



1 Which trigonometric function has the following characteristics

- A y-intercept of 2
- Amplitude of 2
- One full cycle from 0 to π
- Maximum at 4
- Minimum at 0

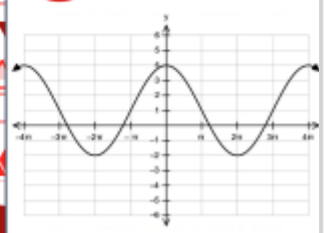
E



2 Which trigonometric function has the following characteristics

- A cosine function
- Passes through (0, 4)
- Maximum at 4
- Decreasing from 0 to 2π
- Amplitude of 3

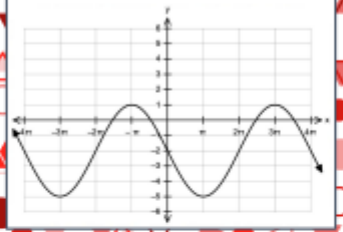
B



3 Which trigonometric function has the following characteristics

- Midline at y = -2
- Amplitude of 3
- One full cycle from 0 to 4π

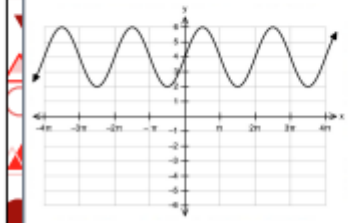
F



4

$y = 2\sin(x) + 4$

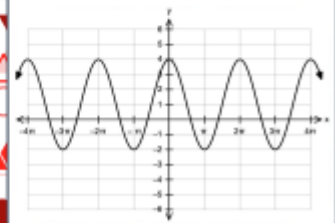
I



5

$y = 3\cos(x) + 1$

C



Full 20-piece activity included in additional materials

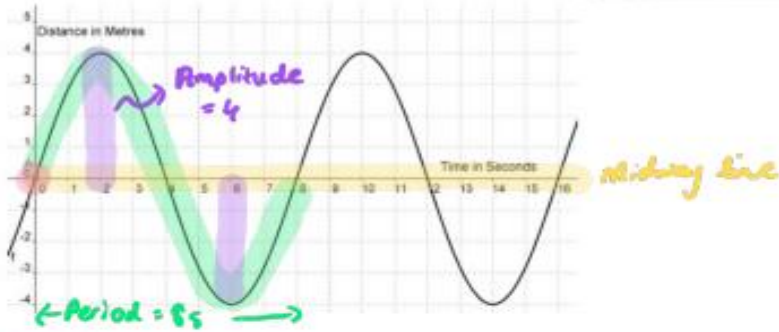
Real-life scenarios:
modelled using trigonometric graphs

Exam style questions!

Modelling problems using trigonometric graphs

Question 1

A wrecking ball attached to a crane swings back and forth. The distance that the ball moves to the left and to the right of its resting position with respect to time is represented by the following graph.



i. What is the period of the crane's motion? Explain your answer.

The period is 8 seconds. It takes 8 seconds for the wrecking ball to complete 1 full swing.

ii. What is the equation of the horizontal midway line of the curve and what does it represent?

$y = 0 / d = 0$

The midway line represents when the wrecking ball is at the resting/middle positio in its swing

iii. What is the amplitude of the crane's motion? What does the amplitude represent in terms of the motion of the ball?

Amplitude = 4m

This represents the maximum distance the wrecking ball swings left and right from the resting/middle position.

iv. How many complete swings will the wrecking ball make in four minutes?

4mins → 4 × 60 = 240s

No. of swings = $\frac{240}{8}$ = 30 swings

v. What does point A represent?

Point A represents the starting position

vi. Could the motion of the wrecking ball be modelled using the cosine function? Explain your answer.

Yes if the wrecking ball started at one of its extreme points - the left/right position.

Real-life scenarios:
modelled using trigonometric graphs

Exam style questions!

Question 2

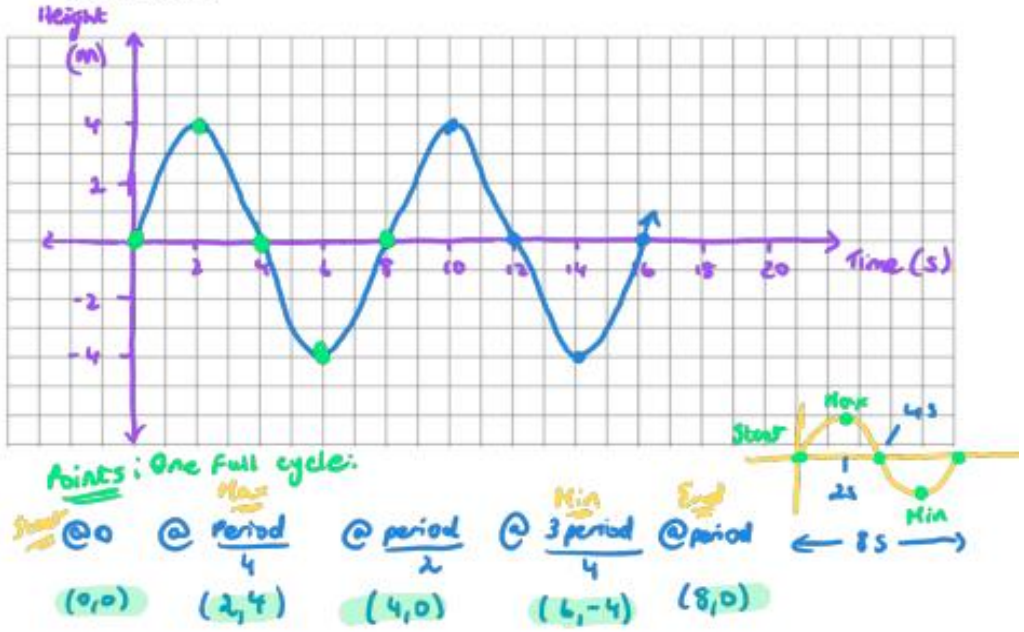
As a dolphin swims along by the side of a boat. It jumps to a height of four metres above the water surface and dives to a depth of four meters below the surface. It does this in a regular motion (simple harmonic motion). A passenger using a stopwatch and starting it just as the dolphin is at the surface determines that the dolphin completes one cycle every 8 seconds.

Amplitude = 4m

Sine function (+ve)

Period = 8s

i. Using a sinusoid curve, sketch the graph of the dolphin's displacement relative to the water surface.



Points: One full cycle:

@0	@ Period/4	@ period/2	@ 3 period/4	@ period
(0,0)	(2,4)	(4,0)	(6,-4)	(8,0)

ii. After three seconds how high above the surface is the dolphin?

$a \pm b\sin cx$

$\text{Period} = \frac{2\pi}{c} \Rightarrow c = \frac{\pi}{4}$

$8 = \frac{2\pi}{c}$

$\Rightarrow 0 + 4\sin\frac{\pi}{4}x$

$h = 4\sin\frac{\pi}{4}t$

@ $t = 3s$

$h = 4\sin\frac{\pi}{4}(3)$

$h = 2{\cdot}828m$

iii. Is the dolphin above or below the surface of the water after 37 seconds?

$h = 4\sin\frac{\pi}{4}(37)$

$h = -2{\cdot}828m$

∴ the dolphin is below the surface of the water after 37s

Real-life scenarios:
modelled using trigonometric graphs

Exam style questions!

Question 3

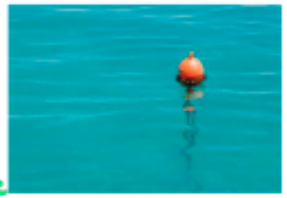
A buoy in the ocean is bobbing up and down in harmonic motion.

→ +ve cosine function

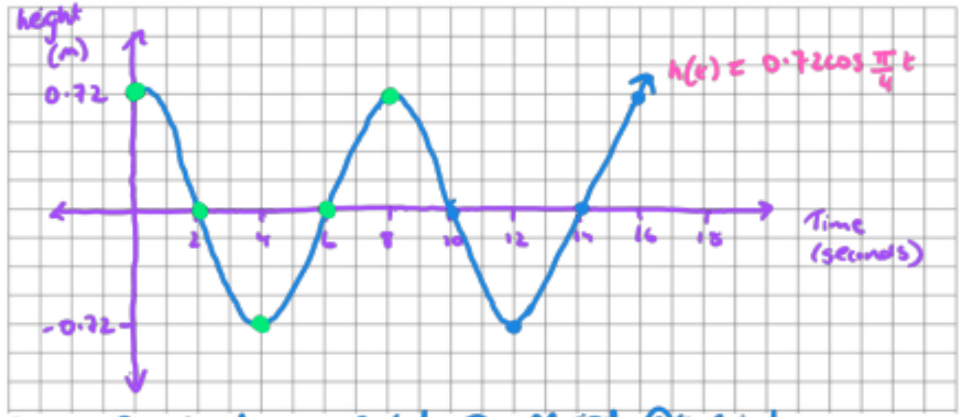
At t = 0 seconds, the buoy is at its high point and returns to that high point every 8 seconds. The buoy moves a distance of 1.44 meters from its highest point to its lowest point.

period = 8s

$\frac{1.44}{2} = 0.72m$ ↳ amplitude

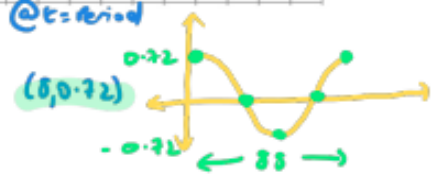


i. Using a sinusoidal function create a mathematical model to represent the depth of water under the buoy and sketch the curve.



@t=0 @t=period/4 @t=period/2 @t=3period/4 @t=period

(0, 0.72) (2, 0) (4, -0.72) (6, 0) (8, 0.72)



ii. How high is the buoy at time = 29 seconds?

$h(29) = 0.72\cos\left(\frac{\pi}{4}(29)\right) \simeq -0.5091...$

0.509m below the middle/normal depth

iii. Is it rising or falling at that time?

$\frac{29}{8} = 3.625$ ↳ over halfway into 4th cycle

∴ rising

iv. How many times in 2 minutes will the buoy be at sea level

2 × 60 = 120s

↳ twice in every cycle

$\frac{120}{8} = 15$ full cycle

∴ 30 times

$y = a \pm b\cos cx$

- Period = 8s

$\frac{2\pi}{c} = 8 \Rightarrow c = \frac{\pi}{4}$

- midway line @ y=0 ⇒ a = 0
- Amplitude = |b| = 0.72, +ve ⇒ b = 0.72

⇒ $h(t) = 0.72\cos\frac{\pi}{4}t$

Real-life scenarios:
modelled using trigonometric graphs

Exam style questions!

Question 4

At a certain latitude, the number (d) of hours of daylight in each day is given by,

measured in degrees

$d = A + B \sin kt°$ where A and B are positive constants and t is the time in days after the spring equinox. Assuming the number of hours of daylight follows an annual cycle of 365 days;

↳ Period = 365

i. Find the value of k correct to three decimal places.

$\text{Period} = \frac{360°}{k} = 365$

$\frac{360}{365} = k$

$0.986 = k$

ii. If the shortest and longest days have 6 and 18 hours of daylight respectively state the values of A and B.

min max

$\frac{6+18}{2} = 12$ → mid way line ⇒ $A = 12$

⇒ $d = 12 + 6\sin 0.986t°$

Amplitude $= 18 - 12 = 6$ ⇒ $B = 6$

iii. Find in hours and minutes the amount of daylight on New Year's day which is 80 days before the spring equinox.

↳ $t = -80$

$d = 12 + 6\sin 0.986(-80)$

$d = 6.11$ hrs

⇒ 6hrs 7mins

$0.11(60) = 6.6 \approx 7$ mins

Real-life scenarios:
modelled using trigonometric graphs

Exam style questions!

Question 5

A veterinary team are performing an operation on a large mammal in a zoo. They note that under anaesthetic the animal's temperature varies sinusoidally ie. it can be modelled using a sine wave, over time.
A maximum temperature of 48°C is reached 15 minutes after they begin the procedure. The minimum temperature of 24°C occurs 45 minutes into the procedure.

They wish to be able to predict the variance in the temperature of the animal by using a suitable graph.

(a) Using the information above:

(i) Write down the temperature of the animal at the start of the procedure.

max = 48°C
min = 24°C
$\Rightarrow \frac{24+48}{2} = 36°C$

(ii) Hence, or, otherwise, write down the equation of the midway line for the sinusoidal curve.

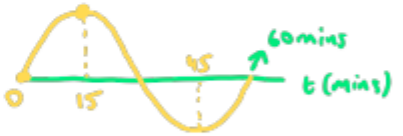
$y = 36$

(iii) Write down the maximum height of the graph above the midway line.

$48 - 36 = 12°C$

(iv) Write down the period of the graph.

60 mins



(b) The graph of the animal's temperature over time can be represented as the function:

$$f(t) = a + b\sin(ct).$$

(i) Using the information in part **(a)** Write down the function $f(t)$.

- Midway line = 36 = a
- Amplitude = |b| = 12 +ve sine function $\therefore$ b = 12
- Period = 60

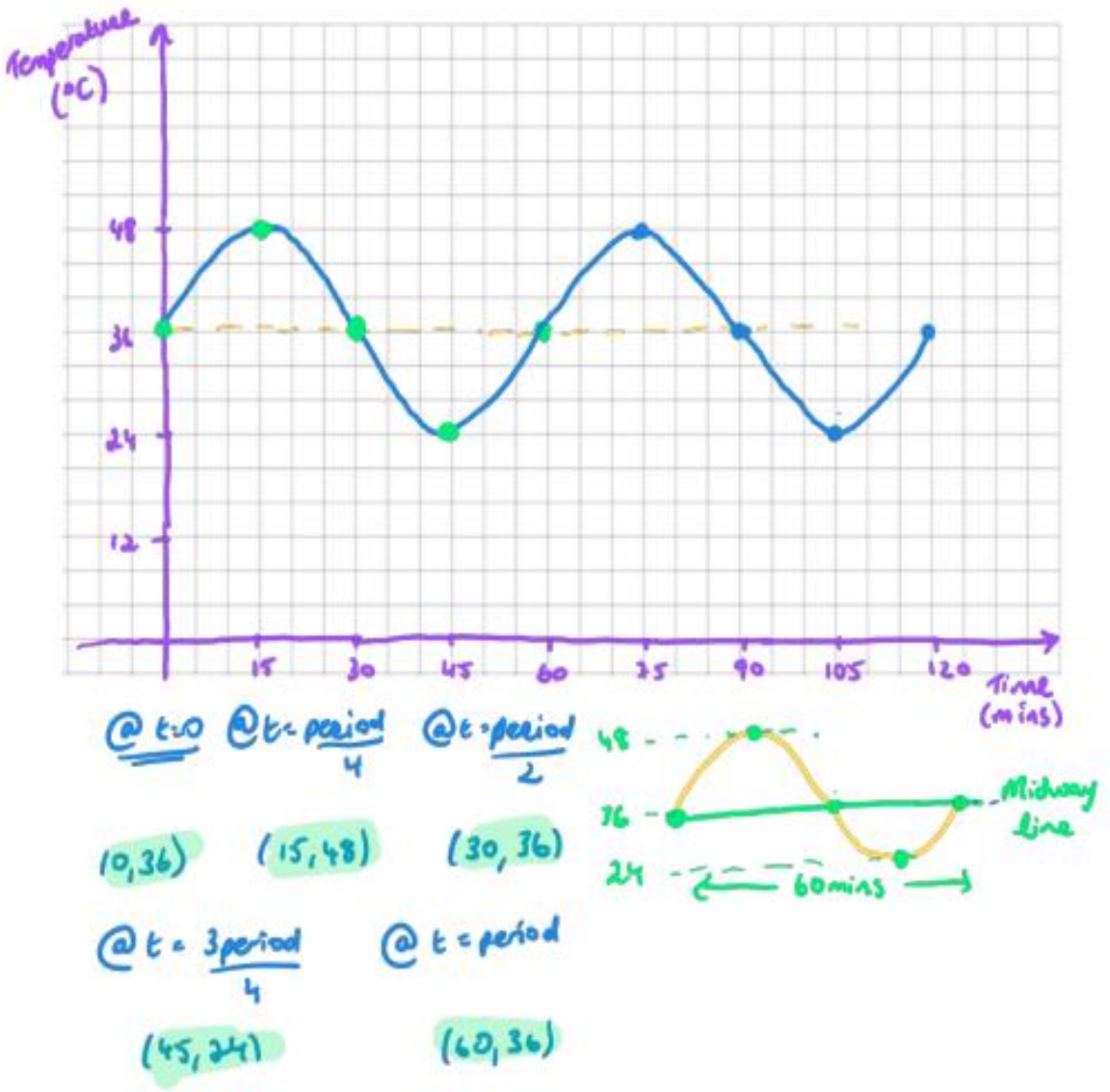
$\frac{2\pi}{c} = 60 \Rightarrow c = \frac{\pi}{30}$ or $\frac{360}{c} = 60$, $6 = c$

$\therefore f(t) = 36 + 12\sin(6t)$
or
$f(t) = 36 + 12\sin\left(\frac{\pi}{30}t\right)$

Real-life scenarios:
modelled using trigonometric graphs

Exam style questions!

(ii) Draw the function $f(t)$ over the period of the two hour operation.



Real-life scenarios:
modelled using trigonometric graphs

Exam style questions!

Question 6:

A Ferris wheel has a radius of 22 m and is boarded from a platform that is 2 metres above the ground.

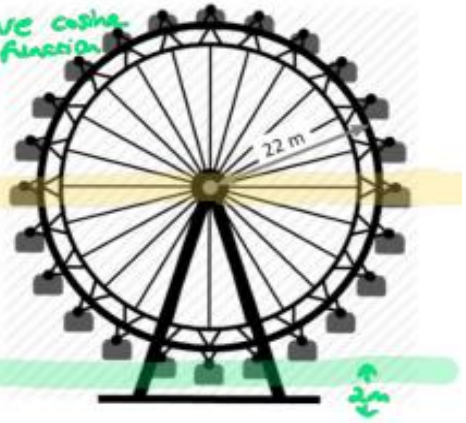
-ve cosine function

↳ starts @ lowest point

The 'six o'clock' position of the Ferris wheel is level with the loading platform for the viewing cars.

A viewing car reaches the highest point of the revolution after 3 minutes. The movement of the wheel can be represented by a sinusoidal function.

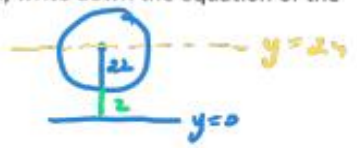
When this function is graphed, the horizontal axis represents time and the vertical axis represents the height of the first viewing car, as it continually moves around.



(a) In relation to the graph of the movement of the wheel, write down the equation of the horizontal midway line.

$22+2$

$\therefore y = 24$



(b) Write down the range of heights a person will experience during one full revolution of the wheel.

Max = 24+22 = 46m

Min = 24-22 = 2m

Range = [2m, 44m]

(c) Write down the period of the graph representing the movement of the wheel.

3mins to reach highest point

∴ Period = 6mins

(d) During Mathematics class, two students are deciding on the type of graph that should be used to represent the movement of the Ferris wheel. Tom states that he thinks the graph is a sine wave, while Joan believes it is a cosine wave.

Do you agree with Tom or Joan? Explain your answer fully.

Joan, the ferris wheel is boarded from 2m above ground - the lowest point in the motion. ∴ it is a negative cosine graph.

Real-life scenarios:
modelled using trigonometric graphs

Exam style questions!

(e) One of the functions below can be used to represent the motion of the Ferris wheel. Using your answer from parts **(a) – (d)**, choose the correct function and fill in the values for a, b, and c.

(i) $f(t) = a + b\sin(ct)$

(ii) $f(t) = a - b\sin(ct)$

(iii) $f(t) = a + b\cos(ct)$

(iv) $f(t) = a - b\cos(ct)$

Amplitude = |b| ∴ b = 22

midway line ⇒ a = 24

No. of cycles in 2π / 360°

Period = $\frac{2\pi}{c}$ / $\frac{360^\circ}{c}$

$\therefore f(t) = 24 - 22\cos\left(\frac{\pi}{3}t\right)$

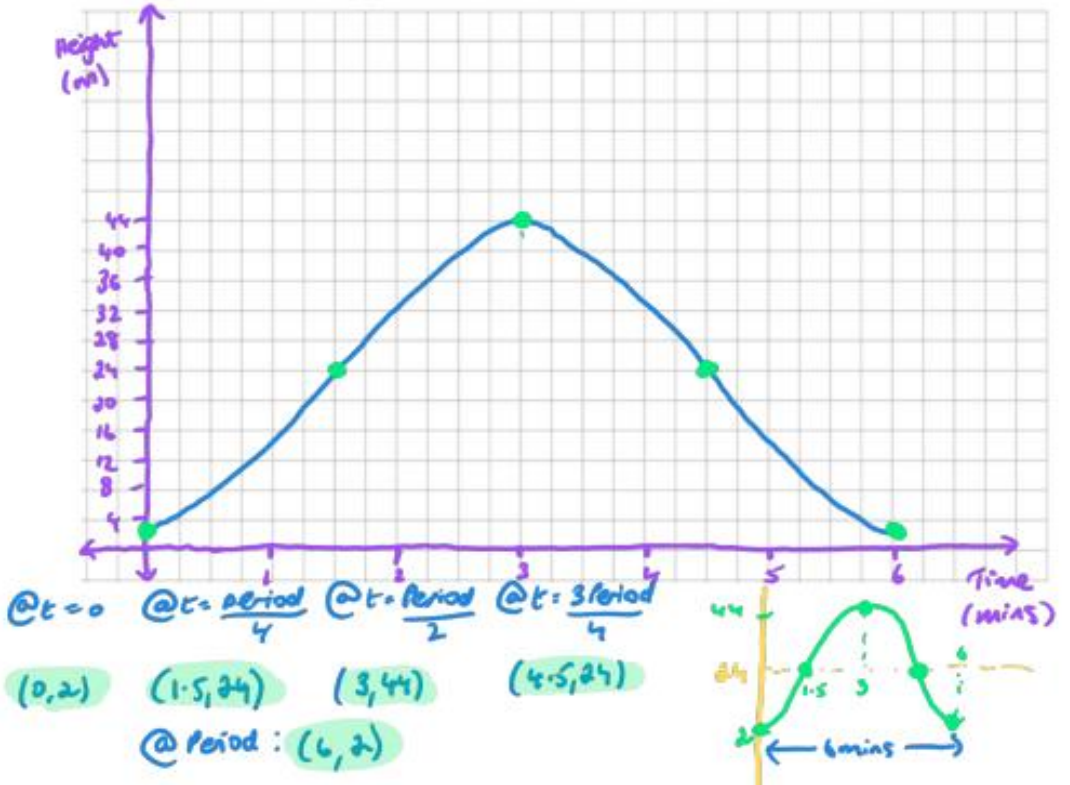
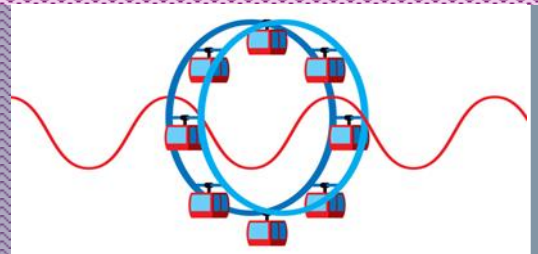
or

$f(t) = 24 - 22\cos(60t)$

$\frac{2\pi}{c} = 6 \Rightarrow c = \frac{\pi}{3}$ (rads)

$\frac{360}{c} = 6 \Rightarrow c = 60$ (degrees)

(f) Draw a suitable graph to represent the motion of the Ferris wheel for one full revolution starting from the loading platform.



THANK
YOU