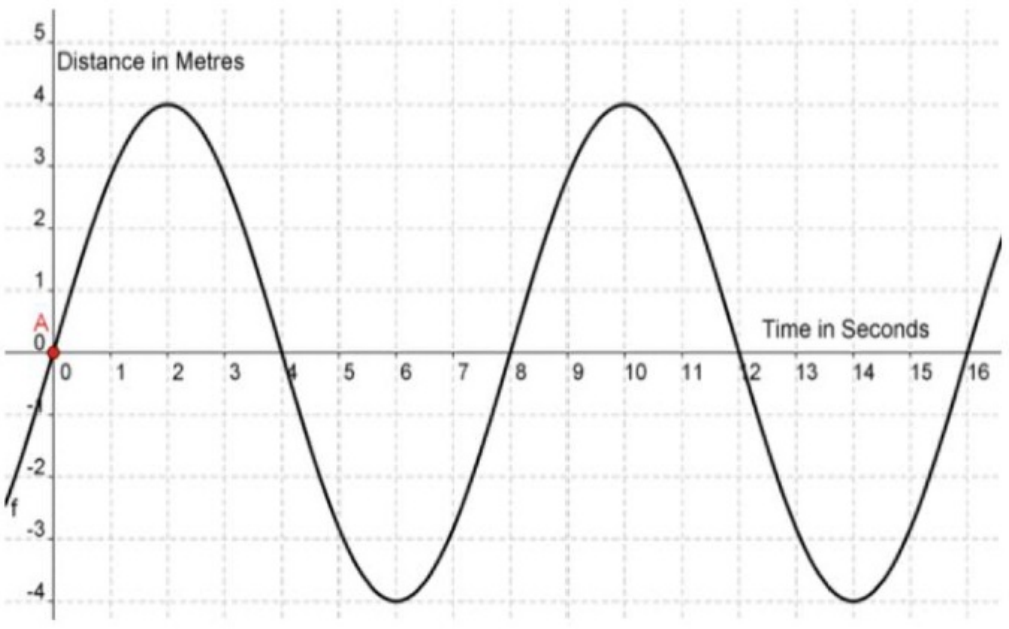
Modelling problems using trigonometric graphs

Question 1

A wrecking ball attached to a crane swings back and forth. The distance that the ball moves to the left and to the right of its resting position with respect to time is represented by the following graph.



i. What is the period of the crane's motion? Explain your answer.

ii. What is the equation of the horizontal midway line of the curve and what does it represent?

iii. What is the amplitude of the crane's motion? What does the amplitude represent in terms of the motion of the ball?

iv. How many complete swings will the wrecking ball make in four minutes?

v. What does point A represent?

vi. Could the motion of the wrecking ball be modelled using the cosine function? Explain your answer.

Question 2:

As a dolphin swims along by the side of a boat. It jumps to a height of four metres above the water surface and dives to a depth of four meters below the surface. It does this in a regular motion (simple harmonic motion). A passenger using a stopwatch and starting it just as the dolphin is at the surface determines that the dolphin completes one cycle every 8 seconds.

i. Using a sinusoid curve, sketch the graph of the dolphin's displacement relative to the water surface.

ii. After three seconds how high above the surface is the dolphin?

iii. Is the dolphin above or below the surface of the water after 37 seconds?

Question 3

A buoy in the ocean is bobbing up and down in harmonic motion.

At t = 0 seconds, the buoy is at its high point and returns to that high point every 8 seconds. The buoy moves a distance of 1.44 meters from its highest point to its lowest point.

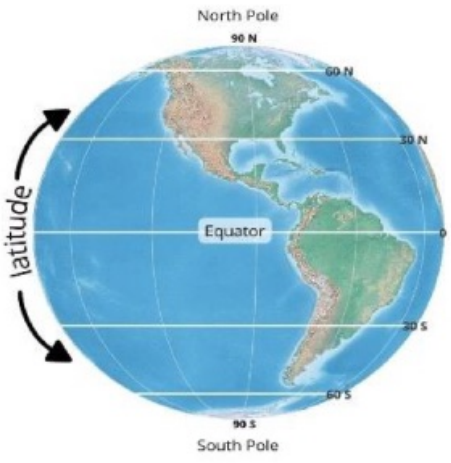
i. Using a sinusoidal function create a mathematical model to represent the depth of water under the buoy and sketch the curve.

ii. How high is the buoy at time = 29 seconds?

iii. Is it rising or falling at that time?

iv. How many times in 2 minutes will the buoy be at sea level

Question 4



At a certain latitude, the number (d) of hours of daylight in each day is given by,

$d = A + B \sin kt°$ where A and B are positive constants and t is the time in days after the spring equinox. Assuming the number of hours of daylight follows an annual cycle of 365 days;

i. Find the value of k correct to three decimal places.

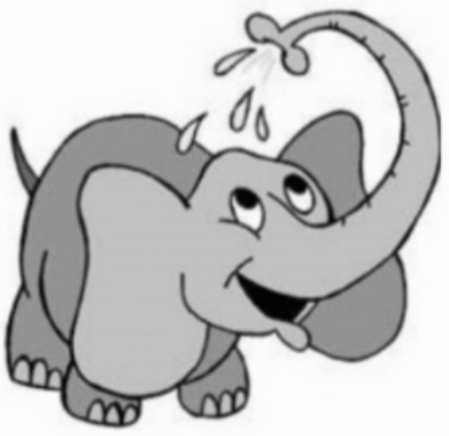
ii. If the shortest and longest days have 6 and 18 hours of daylight respectively state the values of A and B.

iii. Find in hours and minutes the amount of daylight on New Year's day which is 80 days before the spring equinox.

iv. A town at this latitude holds a fair twice a year on days that have exactly 10 hours of daylight. Find, in relation to the spring equinox, which two days these are

Question 5

A veterinary team are performing an operation on a large mammal in a zoo. They note that under anaesthetic the animal's temperature varies sinusoidally ie. it can be modelled using a sine wave, over time.
A maximum temperature of 48°C is reached 15 minutes after they begin the procedure. The minimum temperature of 24°C occurs 45 minutes into the procedure.



They wish to be able to predict the variance in the temperature of the animal by using a suitable graph.

(a) Using the information above:

(i) Write down the temperature of the animal at the start of the procedure.

(ii) Hence, or, otherwise, write down the equation of the midway line for the sinusoidal curve.

(iii) Write down the maximum height of the graph above the midway line.

(iv) Write down the period of the graph.

(b) The graph of the animal's temperature over time can be represented as the function:

$$f(t)=a+b\sin(ct).$$

(i) Using the information in part **(a)** Write down the function $f(t)$.

(ii) Draw the function $f(t)$ over the period of the two hour operation.

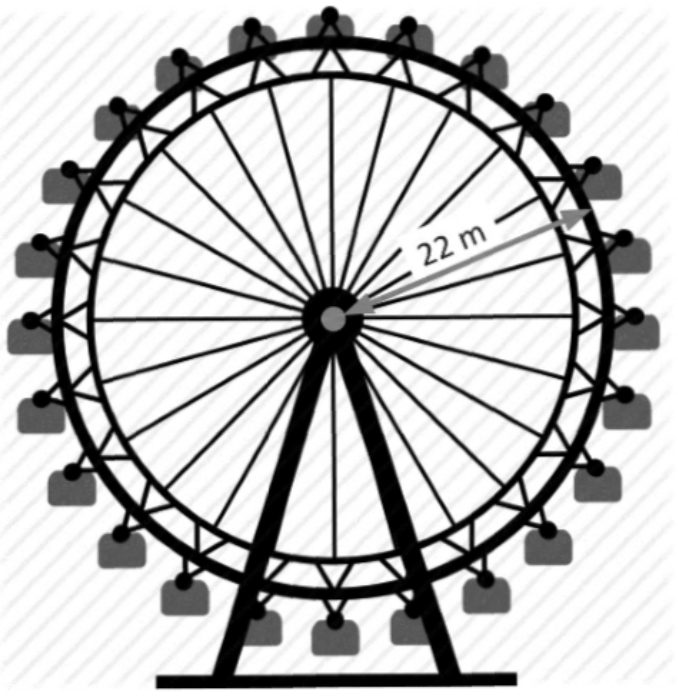
Question 6

A Ferris wheel has a radius of 22 m and is boarded from a platform that is 2 metres above the ground.

The 'six o'clock' position of the Ferris wheel is level with the loading platform for the viewing cars.

A viewing car reaches the highest point of the revolution after 3 minutes. The movement of the wheel can be represented by a sinusoidal function.

When this function is graphed, the horizontal axis represents time and the vertical axis represents the height of the first viewing car, as it continually moves around.



(a) In relation to the graph of the movement of the wheel, write down the equation of the horizontal midway line.

(b) Write down the range of heights a person will experience during one full revolution of the wheel.

(c) Write down the period of the graph representing the movement of the wheel.

(d) During Mathematics class, two students are deciding on the type of graph that should be used to represent the movement of the Ferris wheel. Tom states that he thinks the graph is a sine wave, while Joan believes it is a cosine wave.

Do you agree with Tom or Joan? Explain your answer fully.

(e) One of the functions below can be used to represent the motion of the Ferris wheel. Using your answer from parts **(a) – (d)**, choose the correct function and fill in the values for a, b, and c.

(i) $f(t) = a + bsin(ct)$

(ii) $f(t) = a - bsin(ct)$

(iii) $f(t) = a + bcos(ct)$

(iv) $f(t) = a - bcos(ct)$

(f) Draw a suitable graph to represent the motion of the Ferris wheel for one full revolution starting from the loading platform.