Leaving Certificate Higher Level Paper 1 Financial Maths

2024 Q7

2017 Q8

2023 Q8

2015 Q6

2020 Q5

2013 Q4

Paper Two
2018 Q8

2011 Q6

SEC sample
2014 Q8

- Review JC material
- Compound interest
- Taxation
- Margin/markup

- Evaluate compound interest $F = P(1 + i)^t$
- Evaluate depreciation (reducing balance) $F = P(1 - i)^t$
- Calculate saving regular amounts using geometric series

$$F = P(1 + i)^1 + P(1 + i)^2 + \ldots + P(1 + i)^n$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{P(1 + i)[1 - (1 + i)^n]}{1 - (1 + i)}$$

- Amortise (make equal repayments) loans and mortgages

$$P = \frac{A}{(1 + i)^1} + \frac{A}{(1 + i)^2} + \ldots + \frac{A}{(1 + i)^t}$$

$$A = P\frac{i(1 + i)^t}{(1 + i)^t - 1}$$

- Convert annual rates (APR or AER), r to monthly rates, i

$$(1 + i)^{12} = (1 + r)$$

$$i = \sqrt[12]{1 + r} - 1$$

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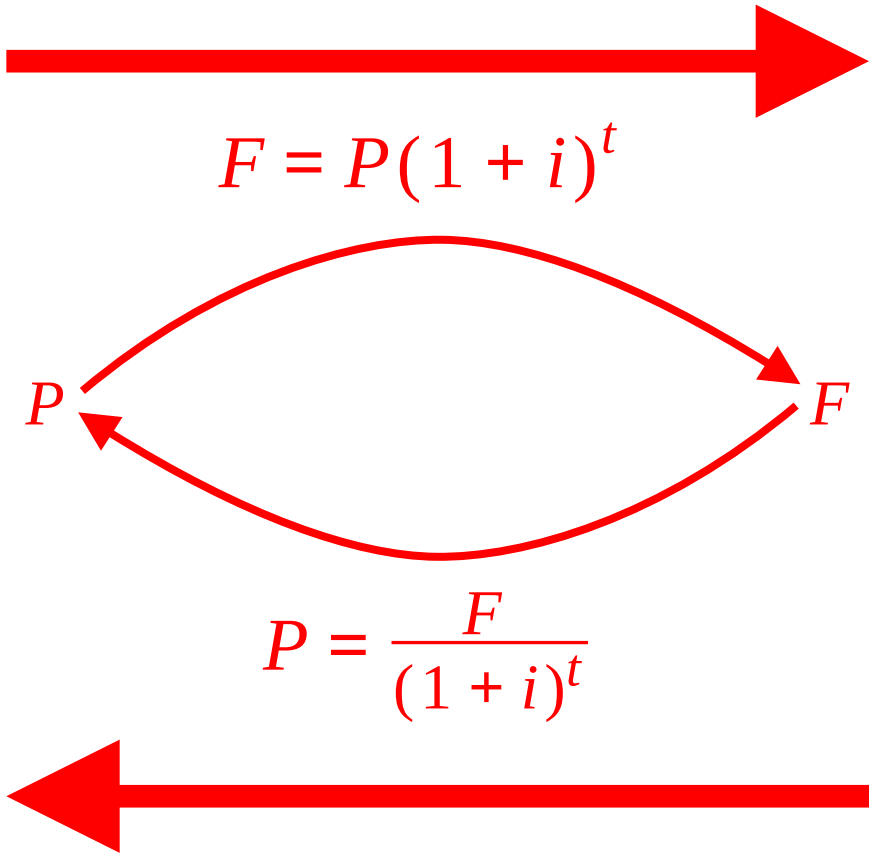
Forward compounded



Reverse discounted

Paper 1 Question 7
2024

50 marks

(a) Fiadh has a gross annual salary of €54 000.
She pays income tax at a rate of 20% on the first €40 000 of her salary and at a rate of 40% on the remainder.
She has an annual tax credit of €1775.
Work out her net annual pay, assuming that there are no other deductions.

$$40000 \times 0{\cdot}2 = 8000$$
$$14000 \times 0{\cdot}4 = 5600 \quad +$$
$$\text{Gross tax} = 13600$$
$$\text{Tax credit} = 1775 \quad -$$
$$\text{Tax payable} = 11825$$

$$54000 - 40000 = 14000$$

$$\text{Net annual pay} = 54000 - 11825$$
$$= €42175$$

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(b) Fiadh and her partner take out a 25-year mortgage with a monthly interest rate of 0·279%. They make equal monthly repayments of €1647·75 at the end of each month. They make the first repayment exactly one month after they take out the mortgage.

(i) Write down the present value of each of their first three monthly repayments, at the time when they take out the mortgage. Give each value as a fraction. Do not multiply out any powers.

$$\text{First month} = \frac{1647{\cdot}75}{1{\cdot}00279}$$

$$\text{Second month} = \frac{1647{\cdot}75}{(1{\cdot}00279)^2}$$

$$\text{Third month} = \frac{1647{\cdot}75}{(1{\cdot}00279)^3}$$

(b) Fiadh and her partner take out a 25-year mortgage with a monthly interest rate of 0·279%. They make equal monthly repayments of €1647·75 at the end of each month.
They make the first repayment exactly one month after they take out the mortgage.

(ii) Work out the amount of money that Fiadh and her partner borrowed for their mortgage.
Give your answer correct to the nearest euro.

$$n = 25\times12 = 300 \text{ repayments}$$

$$\frac{1647\cdot75}{1\cdot00279} + \frac{1647\cdot75}{1\cdot00279^2} + \frac{1647\cdot75}{1\cdot00279^3} + \ldots + \frac{1647\cdot75}{1\cdot00279^{300}}$$

$$S_n = \frac{a(1-r^n)}{1-r} \qquad a = \frac{1647\cdot75}{1\cdot00279} \qquad r = \frac{1}{1\cdot00279}$$

$$S_{300} = \frac{\frac{1647\cdot75}{1\cdot00279}\left(1-\frac{1}{1\cdot00279^{300}}\right)}{1-\frac{1}{1\cdot00279}} = €334562\cdot$$

15

(b) Fiadh and her partner take out a 25-year mortgage with a monthly interest rate of 0·279%. They make equal monthly repayments of €1647·75 at the end of each month.
They make the first repayment exactly one month after they take out the mortgage.

(ii) Work out the amount of money that Fiadh and her partner borrowed for their mortgage.
Give your answer correct to the nearest euro.

$A = 1647{\cdot}75,\ i = 0{\cdot}00279,\ t = 25 \times 12 = 300$ repayments

Alternatively using amortisation formula:

$$A = P\,\frac{i(1+i)^t}{(1+i)^t - 1}$$

$$1647{\cdot}75 = \frac{P(0{\cdot}00279)(1{\cdot}00279)^{300}}{1{\cdot}00279^{300} - 1}$$

$$P = \frac{(1647{\cdot}75)(1{\cdot}00279^{300} - 1)}{(0{\cdot}00279)(1{\cdot}00279)^{300}} = €334563{\cdot}$$

(c) Fiadh puts money in a savings account, and leaves it there for a number of years. The following expression gives $F(t)$, the amount of money in the account in euro after t years, where $t \in \mathbb{R}, t \geq 0$:

$$F(t) = 5000e^{0{\cdot}04t}$$

Use this expression to answer parts **(c)(i)**, **(c)(ii)**, and **(c)(iii)**.

(i) Use differentiation to find the rate at which the amount of money in the account is increasing after $3{\cdot}5$ years. Give your answer correct to the nearest euro per year.

$$F'(t) = 5000(0{\cdot}04)e^{0{\cdot}04t}$$

$$F'(3{\cdot}5) = 5000(0{\cdot}04)e^{0{\cdot}04(3{\cdot}5)}$$

$$= €230 \text{ per year}$$

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(c) Fiadh puts money in a savings account, and leaves it there for a number of years. The following expression gives $F(t)$, the amount of money in the account in euro after t years, where $t \in \mathbb{R}, t \geq 0$:

$$F(t) = 5000e^{0{\cdot}04t}$$

Use this expression to answer parts **(c)(i)**, **(c)(ii)**, and **(c)(iii)**.

(ii) Use integration to find the average amount of money in the account over the first 5 years.
Give your answer correct to the nearest euro.
Remember that the amount of money in the account after t years is: $F(t) = 5000e^{0{\cdot}04t}$

$$\text{Average} = \frac{1}{b-a}\int_a^b f(x)dx = \frac{1}{5-0}\int_0^5 5000e^{0{\cdot}04t}dt$$

$$= \frac{1}{5}\left[\frac{5000e^{0{\cdot}04t}}{0{\cdot}04}\right]_0^5 = 25000\left(e^{0{\cdot}2} - 1\right)$$

$$= €5535.954$$

(c) Fiadh puts money in a savings account, and leaves it there for a number of years. The following expression gives $F(t)$, the amount of money in the account in euro after t years, where $t \in \mathbb{R}, t \geq 0$:

$$F(t) = 5000e^{0{\cdot}04t}$$

Use this expression to answer parts **(c)(i)**, **(c)(ii)**, and **(c)(iii)**.

(iii) Work out the annual rate of interest (AER) for this account. That is, find the percentage increase in the amount of money in the account over the course of one year. Give your answer as a percentage, correct to 2 decimal places.

$$\text{Interest in first year} = F(1) - F(0) = 5000e^{0{\cdot}04} - 5000e^{0}$$

$$= 204{\cdot}053871$$

$$\text{Interest rate} = \frac{204{\cdot}053871}{5000} \times 100$$

$$= 4{\cdot}0810\,\boxed{5}\,9$$

Paper 1 Question 8
2023

50 marks

Olga, Chen, Fiona, and Rohan all have bank accounts.

(a) Olga puts €3000 in a savings account. Interest is added annually at a rate of 2·4% per year.

Work out the amount in Olga's account after 5 years, correct to the nearest cent.

$$F = P(1+i)^t$$

$$= 3000(1{\cdot}024)^5$$

$$= €3377{\cdot}7099$$

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(b) **(i)** Explain what is meant by the "present value" of a payment of €1000 in 1 year's time, at a particular interest rate.

Present value is the amount of money that must be invested today to achieve €1000 in one year's time with interest

(ii) Chen puts a different amount in a savings account with the same interest rate (2·4% per year). After 6 years, Chen has €4000 in the account.

Work out how much money Chen put in the account initially, correct to the nearest cent.

$$P = \frac{F}{(1+i)^t}$$

$$= \frac{4000}{(1{\cdot}024)^6}$$

$$= €3469{\cdot}45$$

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(c) Fiona is taking out a loan at the same annual interest rate (2·4% per year). Fiona makes payments quarterly (that is, 4 times per year).

Work out the quarterly interest rate that would be equivalent to an APR of 2·4%. Give your answer as a **percentage**, correct to 2 decimal places.

$$(1+i)^4 = 1{\cdot}024$$

$$1+i = \sqrt[4]{1{\cdot}024}$$

$$i = \sqrt[4]{1{\cdot}024} - 1$$

$$= 0{\cdot}005946744$$

$$= 0{\cdot}59\%$$

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(d) Rohan wants to put the same amount of money in a savings account at the start of each month for 36 months so that, at the end of 3 years, he will have a total of €$12\,000$ in the account. Interest is calculated at a rate of $0{\cdot}11\%$ per month.

(i) Taking €A to be the amount Rohan puts in his account at the start of each month, write down a geometric series in €A to show the total amount of money in the account at the end of the 3 years. Include the first two and the last two terms.

$$A(1{\cdot}0011) + A(1{\cdot}0011)^2 + \cdots + A(1{\cdot}0011)^{35} + A(1{\cdot}0011)^{36} = 12000$$

(d) Rohan wants to put the same amount of money in a savings account at the start of each month for 36 months so that, at the end of 3 years, he will have a total of €12 000 in the account. Interest is calculated at a rate of 0·11% per month.

(ii) Hence, find the value of €A that will give a total of €12 000 in the account after 3 years.
Give your answer correct to the nearest cent.

This is a geometric series $a = A(1{\cdot}0011), r = 1{\cdot}0011, n = 36$

$$A(1{\cdot}0011) + A(1{\cdot}0011)^2 + \cdots + A(1{\cdot}0011)^{35} + A(1{\cdot}0011)^{36} = 12000$$

$$S_n = \frac{a(1-r^n)}{1-r} \Rightarrow 12000 = \frac{A(1{\cdot}0011)(1 - 1{\cdot}0011^{36})}{1 - 1{\cdot}0011}$$

$$A = \frac{12000(1-1{\cdot}0011)}{(1{\cdot}0011)(1-1{\cdot}0011^{36})}$$

$$= €326{\cdot}600907$$

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(e) A park sells three types of ticket: child, student, and adult. The table below gives information on the price of each ticket and the percentage of tickets sold.

For example, 15% of all tickets sold are student tickets.

Type of ticket	Child	Student	Adult
Price of ticket	€11	€5 less than an adult ticket	€ x
Percentage	52%	15%	33%

The expected value of the price of a ticket is €13·85.

Work out the value of x, the price of an adult ticket.

$$11(0{\cdot}52) + (x - 5)0{\cdot}15 + 0{\cdot}33x = 13{\cdot}85$$

$$5{\cdot}72 + 0.15x - 5(0.15) + 0{\cdot}33x = 13{\cdot}85$$

$$x = \frac{13{\cdot}85 - 5{\cdot}72 + 0{\cdot}75}{0{\cdot}33 + 0{\cdot}15} = \frac{8{\cdot}88}{0{\cdot}48} = €18{\cdot}50$$

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(f) When an item is being sold:

- the **mark up** is the profit as a percentage of the cost price, and
- the **margin** is the profit as a percentage of the selling price.

A shop sells an item with a **margin** of 18%. Work out the **mark up** for this item. Give your answer as a percentage, correct to the nearest percent.

$$0{\cdot}18 = \frac{\text{Profit}}{\text{Selling price}} = \frac{\text{Profit}}{\text{Cost price} + \text{Profit}}$$

$$\text{Profit} = 0{\cdot}18(\text{Cost price} + \text{Profit})$$

$$\text{Profit} = 0{\cdot}18\text{Cost price} + 0{\cdot}18\text{Profit}$$

$$0{\cdot}18\text{Cost price} = 0{\cdot}82\text{Profit}$$

$$\text{Cost price} = \frac{0{\cdot}82}{0{\cdot}18}\text{Profit}$$

$$\text{Mark} - \text{up} = \text{Profit} \div \frac{0{\cdot}82}{0{\cdot}18}\text{Profit}$$

$$= \frac{0{\cdot}18}{0{\cdot}82} = 0{\cdot}21951 = 22\%$$

Paper 1 Question 9
2022

50 marks

Alex gets injections of a medicinal drug. Each injection has 15 mg of the drug. Each day, the amount of the drug left in Alex's body from an injection **decreases** by 40%. So, the amount of the drug (in mg) left in Alex's body t days after a single injection is given by:

$$15(0{\cdot}6)^t$$

where $t \in \mathbb{R}$.

(a) Find the amount of the drug left in Alex's body $2{\cdot}5$ days after a single 15 mg injection.
Give your answer in mg, correct to 2 decimal places.

$15(0{\cdot}6)^{2{\cdot}5} = 4{\cdot}18$ mg

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Alex gets injections of a medicinal drug. Each injection has 15 mg of the drug. Each day, the amount of the drug left in Alex's body from an injection **decreases** by 40%. So, the amount of the drug (in mg) left in Alex's body t days after a single injection is given by:

$$15(0{\cdot}6)^t$$

where $t \in \mathbb{R}$.

(b) How long after a single 15 mg injection will there be exactly 1 mg of the drug left in Alex's body? Give your answer in days, correct to 1 decimal place.

$$15(0{\cdot}6)^t = 1 \Rightarrow (0{\cdot}6)^t = \frac{1}{15}$$ Take the ln of both sides

$$t \ln 0{\cdot}6 = \ln\left(\frac{1}{15}\right)$$

$$t = \frac{\ln\left(\frac{1}{15}\right)}{\ln 0{\cdot}6} = 5{\cdot}30132\ldots$$

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Alex gets injections of a medicinal drug. Each injection has 15 mg of the drug. Each day, the amount of the drug left in Alex's body from an injection **decreases** by 40%. So, the amount of the drug (in mg) left in Alex's body t days after a single injection is given by:

$$15(0{\cdot}6)^t$$

where $t \in \mathbb{R}$.

Alex is given a 15 mg injection of the drug at the same time **every** day for a long period of time.

(c) Explain why the total amount of the drug, in mg, in Alex's body immediately after the 4th injection is given by:

$$15 + 15(0{\cdot}6) + 15(0{\cdot}6)^2 + 15(0{\cdot}6)^3$$

15 is the amount from injection just given

$15(0{\cdot}6)$: amount from injection 1 day ago

$15(0{\cdot}6)^2$: amount from injection 2 days ago

$15(0{\cdot}6)^3$: amount from injection 3 days ago

Alex gets injections of a medicinal drug. Each injection has 15 mg of the drug. Each day, the amount of the drug left in Alex's body from an injection **decreases** by 40%. So, the amount of the drug (in mg) left in Alex's body t days after a single injection is given by:

$$15(0{\cdot}6)^t$$

where $t \in \mathbb{R}$.

Alex is given a 15 mg injection of the drug at the same time **every** day for a long period of time.

(d) Find the total amount of the drug in Alex's body immediately after the 10th injection.

Give your answer in mg, correct to 2 decimal places.

$$15 + 15(0{\cdot}6) + 15(0{\cdot}6)^2 + 15(0{\cdot}6)^3 + \ldots + 15(0{\cdot}6)^9$$

This is a geometric series with $a = 15$, $r = 0{\cdot}6$, $n = 10$

$$S_n = \frac{a(1 - r^n)}{1 - r} \Rightarrow S_{10} = \frac{15(1 - 0{\cdot}6^{10})}{1 - 0{\cdot}6} = 37{\cdot}27325125$$

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Alex gets injections of a medicinal drug. Each injection has 15 mg of the drug. Each day, the amount of the drug left in Alex's body from an injection **decreases** by 40%. So, the amount of the drug (in mg) left in Alex's body t days after a single injection is given by:

$$15(0{\cdot}6)^t$$

where $t \in \mathbb{R}$.

Alex is given a 15 mg injection of the drug at the same time **every** day for a long period of time.

(e) Use the formula for the sum to infinity of a geometric series to estimate the amount of the drug (in mg) in Alex's body, after a long period of time during which he gets daily injections.

$$a = 15,\ r = 0{\cdot}6,\ n = \infty$$

$$S_\infty = \frac{a}{1-r} = \frac{15}{1-0{\cdot}6} = 37{\cdot}5 \text{ mg}$$

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(f) Jessica also gets daily injections of a medicinal drug at the same time every day. She gets d mg of the drug in each injection, where $d \in \mathbb{R}$. Each day, the amount of the drug left in Jessica's body from an injection **decreases** by 15%.

(i) Use the sum of a geometric series to show that the total amount of the drug (in mg) in Jessica's body immediately after the nth injection, where $n \in \mathbb{N}$, is:

$$\frac{20d(1-0{\cdot}85^n)}{3}$$

$$d + d(0{\cdot}85) + d(0{\cdot}85)^2 + d(0{\cdot}85)^3 + \ldots + d(0{\cdot}85)^{n-1}$$

This is a geometric series with $a = d$, $r = 0{\cdot}85$

$$S_n = \frac{a(1-r^n)}{1-r} \Rightarrow S_n = \frac{d(1-0{\cdot}85^n)}{1-0{\cdot}85} = \frac{20d(1-0{\cdot}85^n)}{3}$$

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(f) Jessica also gets daily injections of a medicinal drug at the same time every day. She gets d mg of the drug in each injection, where $d \in \mathbb{R}$. Each day, the amount of the drug left in Jessica's body from an injection **decreases** by 15%.

(ii) Immediately after the 7th injection, there are 50 mg of the drug in Jessica's body. Find the amount of the drug in one of Jessica's daily injections.

Give your answer correct to the nearest mg.

$$S_n = \frac{20d(1 - 0{\cdot}85^n)}{3}$$

$$S_7 = \frac{20d(1 - 0{\cdot}85^7)}{3} = 50$$

$$20d(1 - 0{\cdot}85^7) = 150$$

$$d = \frac{150}{20(1 - 0{\cdot}85^7)}$$

$$= 11{\cdot}038 \ldots \text{ mg}$$

5

Paper 1 Question 5
2020

25 marks

(a) A couple agree to take out a €250 000 mortgage in order to purchase a new home. The loan is to be paid back monthly over 25 years with the repayments due at the end of each month. The bank charges an annual percentage rate (APR) which is equivalent to a monthly rate of 0·287%.

Using the amortisation formula, or otherwise, find the couples' monthly repayment on the mortgage.

Give your answer in euro correct to the nearest cent.

$P = 250{,}000$, $i = 0{\cdot}00287$, $t = 25 \times 12 = 300$ repayments

Amortisation formula: $A = P\dfrac{i(1+i)^t}{(1+i)^t - 1}$

$$A = \frac{250{,}000(0{\cdot}00287)(1{\cdot}00287)^{300}}{(1{\cdot}00287)^{300} - 1}$$

$$= €1244{\cdot}0630\ 15$$

(b) Another couple agree to take out a mortgage of €350 000, at a rate of 0·3% per month, in order to purchase a new home. This loan is also to be paid back monthly over 25 years with the repayments due at the end of each month. The amount of each repayment is €1771. After exactly 11 years of repayments, the couple receive a financial windfall. They decide to repay the remaining balance on the mortgage. Write down a **series** (including the first two and last two terms) which shows the total of the present values of all the remaining monthly repayments due over the remaining 14 years of the mortgage (after the last monthly repayment at the end of year 11). **Hence**, find how much the couple will need to repay in order to clear their mortgage entirely. Give your answer correct to the nearest cent.

$$n = 14 \times 12 = 168 \text{ repayments}$$

$$\text{Remaining} = \frac{1771}{(1{\cdot}003)^1} + \frac{1771}{(1{\cdot}003)^2} + \ldots + \frac{1771}{(1{\cdot}003)^{167}} + \frac{1771}{(1{\cdot}003)^{168}}$$

(b) Another couple agree to take out a mortgage of €350 000, at a rate of 0·3% per month, in order to purchase a new home. This loan is also to be paid back monthly over 25 years with the repayments due at the end of each month. The amount of each repayment is €1771.

$$S_n = \frac{a(1-r^n)}{1-r} \Rightarrow S_{168} = \frac{\frac{1771}{1{\cdot}003}\left(1-\frac{1}{1{\cdot}003^{168}}\right)}{1-\frac{1}{1{\cdot}003}}$$

$$= €233{,}438{\cdot}2549$$ (10)

Give your answer correct to the nearest cent.

$$n = 14 \times 12 = 168 \text{ repayments}$$

$$\text{Remaining} = \frac{1771}{(1{\cdot}003)^1} + \frac{1771}{(1{\cdot}003)^2} + \ldots + \frac{1771}{(1{\cdot}003)^{167}} + \frac{1771}{(1{\cdot}003)^{168}}$$

Paper 2 Question 8
2018

10 marks

(c) *Acme Confectionery* has an employee pension plan. For an employee who qualifies for the full pension, *Acme Confectionery* will pay a sum of €20 000 on the day of retirement. It will then pay a sum on the same date each subsequent year for the next 25 years. Each year the employee is paid a sum that is 1% more than the amount paid in the previous year.

What sum of money must the company have set aside on the day of retirement in order to fund this pension? Assume an annual interest rate (AER) of 2·4%.

$$\text{Value of fund} = 20{,}000 + \frac{20000(1\cdot 01)^1}{(1\cdot 024)^1} + \ldots + \frac{20000(1\cdot 01)^{25}}{(1\cdot 024)^{25}}$$

$$S_n = \frac{a(1-r^n)}{1-r} \Rightarrow S_{26} = \frac{20{,}000\left(1-\left(\frac{1\cdot 01}{1\cdot 024}\right)^{26}\right)}{1-\frac{1\cdot 01}{1\cdot 024}}$$

$$= €440{,}132\cdot 4015$$

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Paper 1 Question 8
2017

55 marks

(a) When a loan of €P is repaid in equal repayments of amount €A, at the end of each of t equal periods of time, where i is the periodic compound interest rate (expressed as a decimal), the formula below can be used to find the amount of each repayment.

$$A = P\,\frac{i(1+i)^t}{((1+i)^t - 1)}$$

Show how this formula is derived. You may use the formula for the sum of a finite geometric series.

$S_n = \dfrac{a(1-r^n)}{1-r}$ ➡ $P = \dfrac{\dfrac{A}{(1+i)}\left(1 - \dfrac{1}{(1+i)^t}\right)}{1 - \dfrac{1}{(1+i)}}$

$P = \dfrac{A}{(1+i)^1} + \dfrac{A}{(1+i)^2} + \cdots + \dfrac{A}{(1+i)^t}$ Multiply by $(1+i)^t$

$P = \dfrac{\dfrac{A}{(1+i)}\left(\dfrac{(1+i)^t - 1}{(1+i)^t}\right)}{\dfrac{i}{(1+i)}}$ Multiply by i

$P = \dfrac{A(1+i)^t - A}{(1+i)^t} \cdot \dfrac{(1+i)}{i}$ Divide by $(1+i)^t - 1$

Geometric series ➡ $a = \dfrac{A}{(1+i)}$ and $r = \dfrac{1}{(1+i)}$

$P = \dfrac{A(1+i)^t - A}{i(1+i)^t}$

$P = \dfrac{A((1+i)^t - 1)}{i(1+i)^t}$

$A = P\,\dfrac{i(1+i)^t}{((1+i)^t - 1)}$

(b) Alex has a credit card debt of €5000. One method of clearing this debt is to make a fixed repayment at the end of each month. The amount of this repayment is 2·5% of the original debt.

(i) What is the fixed monthly repayment, €A, required to pay the debt of €5000?

$A = 5000(0{\cdot}025) = €125$ 10

(ii) The annual percentage rate (APR) charged on debt by the credit card company is 21·75%, fixed for the term of the debt. Find as a percentage, correct to 3 significant figures, the monthly interest rate that is equivalent to an APR of 21·75%.

$(1 + i)^{12} = 1{\cdot}2175$

$1 + i = \sqrt[12]{1{\cdot}2175}$

$i = \sqrt[12]{1{\cdot}2175} - 1 = 0{\cdot}01653518226$

$i = 1{\cdot}65\%$ 10

(iii) Assume Alex pays the fixed monthly repayment, €A, each month and does not have any further transactions on that card. Complete the table below to show how the balance of the debt of €5000 is reducing each month for the first three months, assuming an APR of 21·75%, charged and compounded monthly.

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Payment number	Fixed monthly payment, €A	€A		New balance of debt (€)
		Interest	Previous balance reduced by (€)	
0				5000
1	125	82·50	42·50	4957·50
2	125	81·80	43·20	4914·30
3	125	81·09	43·91	4870·39

$5000(0{\cdot}0165) = 82{\cdot}50$

$4957{\cdot}50(0{\cdot}0165) = 81{\cdot}79875$

$4914{\cdot}30(0{\cdot}0165) = 81{\cdot}08595$

(iv) Using the formula you derived on the previous page, or otherwise, find how long it would take to pay off a credit card debt of €5000, using the repayment method outlined at the beginning of **part (b)** above.
Give your answer in months, correct to the nearest month.

$$A = P\,\frac{i(1+i)^t}{(1+i)^t - 1}$$

$$125 = \frac{5000(0{\cdot}0165)(1{\cdot}0165)^t}{(1{\cdot}0165)^t - 1}$$

$$125((1{\cdot}0165)^t - 1) = 5{,}000(0{\cdot}0165)(1{\cdot}0165)^t$$

$$125(1{\cdot}0165)^t - 125 = 82{\cdot}5(1{\cdot}0165)^t$$

$$125(1{\cdot}0165)^t - 82{\cdot}5(1{\cdot}0165)^t = 125$$

$$42{\cdot}5(1{\cdot}0165)^t = 125$$

$$(1{\cdot}0165)^t = \frac{125}{42{\cdot}5}$$

$$t\ln 1{\cdot}0165 = \ln\frac{125}{42{\cdot}5}$$

$$t = \frac{\ln\frac{125}{42{\cdot}5}}{\ln 1{\cdot}0165}$$

$$= 65{\cdot}920373$$

$$= 66 \text{ months}$$

(v) Alex decides to borrow €5000 from the local Credit Union to pay off this credit card debt of €5000. The APR charge for the Credit Union loan is 8·5% fixed for the term of the loan. The loan is to be repaid in equal weekly repayments, at the end of each week, for 156 weeks. Find the amount of each weekly repayment.

$$(1 + i)^{52} = 1{\cdot}085$$

$$1 + i = \sqrt[52]{1{\cdot}085}$$

$$i = \sqrt[52]{1{\cdot}085} - 1$$

$$i = 0{\cdot}001570077$$

$$A = P\,\frac{i(1+i)^t}{(1+i)^t - 1}$$

$$A = \frac{5{,}000(\,0{\cdot}00157)(1{\cdot}00157)^{156}}{(1{\cdot}00157)^{156} - 1} = €36{\cdot}1613$$

(vi) How much will Alex save by paying off the credit card debt using the loan from the Credit Union instead of paying the fixed repayment from **part (b)(i)** each month to the credit card company?

Credit card payments = $125 \times 66 = €8250{\cdot}00$

Credit union payments = $36{\cdot}16 \times 156 = €5640{\cdot}96$

Saving = €2609·04 5

Paper 1 Question 6
2015

25 marks

(a) Donagh is arranging a loan and is examining two different repayment options.

(i) Bank A will charge him a monthly interest rate of 0·35%. Find, correct to three significant figures, the annual percentage rate (APR) that is equivalent to a monthly interest rate of 0·35%.

$(1 + 0{\cdot}0035)^{12} = 1{\cdot}042818007$

$\text{APR} = 0{\cdot}042818007$

$\text{APR} = 4{\cdot}28\%$

(a) Donagh is arranging a loan and is examining two different repayment options.

(ii) Bank B will charge him a rate that is equivalent to an APR of 4·5%. Find, correct to three significant figures, the monthly interest rate that is equivalent to an APR of 4·5%.

$$(1 + i)^{12} = 1{\cdot}045$$

$$1 + i = \sqrt[12]{1{\cdot}045}$$

$$i = \sqrt[12]{1{\cdot}045} - 1$$

$$i = 0{\cdot}003674809$$

$$i = 0{\cdot}367\%$$

10

(b) Donagh borrowed €80 000 at a monthly interest rate of 0·35%, fixed for the term of the loan, from Bank A. The loan is to be repaid in equal monthly repayments over ten years. The first repayment is due one month after the loan is issued. Calculate, correct to the nearest euro, the amount of each monthly repayment.

$P = 80{,}000$, $i = 0{\cdot}0035$, $t = 10 \times 12 = 120$ repayments

$$80{,}000 = \frac{A}{(1{\cdot}0035)^1} + \frac{A}{(1{\cdot}0035)^2} + \ldots + \frac{A}{(1{\cdot}0035)^{120}}$$

This is a geometric series $a = \frac{A}{1{\cdot}0035}$, $r = \frac{1}{1{\cdot}0035}$, $n = 120$

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad \Rightarrow \quad 80{,}000 = \frac{\frac{A}{1{\cdot}0035}\left(1 - \frac{1}{(1{\cdot}0035)^{120}}\right)}{\left(1 - \frac{1}{1{\cdot}0035}\right)}$$

(b) Donagh borrowed €80 000 at a monthly interest rate of 0·35%, fixed for the term of the loan, from Bank A. The loan is to be repaid in equal monthly repayments over ten years. The first repayment is due one month after the loan is issued. Calculate, correct to the nearest euro, the amount of each monthly repayment.

$P = 80{,}000$, $i = 0{\cdot}0035$, $t = 10 \times 12 = 120$ repayments

$$A = \frac{80{,}000\left(1 - \frac{1}{1{\cdot}0035}\right)}{\frac{1}{1{\cdot}0035}\left(1 - \frac{1}{(1{\cdot}0035)^{120}}\right)} = €818{\cdot}52$$

15

(b) Donagh borrowed €80 000 at a monthly interest rate of 0·35%, fixed for the term of the loan, from Bank A. The loan is to be repaid in equal monthly repayments over ten years. The first repayment is due one month after the loan is issued. Calculate, correct to the nearest euro, the amount of each monthly repayment.

$P = 80{,}000$, $i = 0{\cdot}0035$, $t = 10 \times 12 = 120$ repayments

Alternatively using amortisation formula:

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$

$$A = \frac{80{,}000(0{\cdot}0035)(1{\cdot}0035)^{120}}{(1{\cdot}0035)^{120} - 1}$$

$= €818{\cdot}52$

Paper 1 Question 8
SEC Sample 2014

50 marks

Pádraig is 25 years old and is planning for his pension. He intends to retire in forty years' time, when he is 65. First, he calculates how much he wants to have in his pension fund when he retires. Then, he calculates how much he needs to invest in order to achieve this. He assumes that, in the long run, money can be invested at an inflation-adjusted annual rate of 3%. Your answers throughout this question should therefore be based on a 3% annual growth rate.

(a) Write down the present value of a future payment of €20,000 in one year's time.

$F = 20{,}000$, $i = 0{\cdot}03$ and $t = 1$

$F = P(1 + i)^t$

$$\frac{20{,}000}{(1{\cdot}03)^1} = P$$

$P = €19{,}417{\cdot}48573$

Pádraig is 25 years old and is planning for his pension. He intends to retire in forty years' time, when he is 65. First, he calculates how much he wants to have in his pension fund when he retires. Then, he calculates how much he needs to invest in order to achieve this. He assumes that, in the long run, money can be invested at an inflation-adjusted annual rate of 3%. Your answers throughout this question should therefore be based on a 3% annual growth rate.

(b) Write down, in terms of t, the present value of a future payment of €20,000 in t years' time.

$F = 20{,}000,\ i = 0{\cdot}03$

$F = P(1 + i)^t$

$$\frac{20{,}000}{(1{\cdot}03)^t} = P$$

(c) Pádraig wants to have a fund that could, from the date of his retirement, give him a payment of €20,000 at the start of each year for 25 years. Show how to use the sum of a geometric series to calculate the value, on the date of retirement, of the fund required.

€20,000 is drawn down immediately.

The next €20,000 will not be drawn down for another year so its present value on the day of retirement is $\frac{20000}{(1{\cdot}03)^1}$.

The next €20,000 will not be drawn down for two years so its present value on the day of retirement is $\frac{20000}{(1{\cdot}03)^2}$.

➡ Value of fund = $20{,}000 + \frac{20000}{(1{\cdot}03)^1} + \frac{20000}{(1{\cdot}03)^2} + \ldots + \frac{20000}{(1{\cdot}03)^{24}}$

(c) Pádraig wants to have a fund that could, from the date of his retirement, give him a payment of €20,000 at the start of each year for 25 years. Show how to use the sum of a geometric series to calculate the value, on the date of retirement, of the fund required.

This is a geometric series with $a = 20{,}000$, $r = \frac{1}{1{\cdot}03}$, $n = 25$

$$S_n = \frac{a(1 - r^n)}{1 - r} \Rightarrow S_{25} = \frac{20{,}000\left(1 - \frac{1}{1{\cdot}03^{25}}\right)}{1 - \frac{1}{1{\cdot}03}}$$

$$= €358{,}710{\cdot}8424$$

$$\Rightarrow \text{Value of fund} = 20{,}000 + \frac{20000}{(1{\cdot}03)^1} + \frac{20000}{(1{\cdot}03)^2} + \ldots + \frac{20000}{(1{\cdot}03)^{24}}$$

(d) Pádraig plans to invest a fixed amount of money every month in order to generate the fund calculated in part **(c)**.
His retirement is $40 \times 12 = 480$ months away.

(i) Find, correct to four significant figures, the rate of interest per month that would, if paid and compounded monthly, be equivalent to an effective annual rate of 3%.

$$(1 + i)^{12} = 1{\cdot}03$$

$$1 + i = \sqrt[12]{1{\cdot}03}$$

$$i = \sqrt[12]{1{\cdot}03} - 1$$

$$i = 0{\cdot}00246627$$

$$i = 0{\cdot}2466\%$$

(d) Pádraig plans to invest a fixed amount of money every month in order to generate the fund calculated in part **(c)**.
His retirement is $40 \times 12 = 480$ months away.

(ii) Write down, in terms of n and P, the value on the retirement date of a payment of €P made n months before the retirement date.

$F = P(1 + 0{\cdot}002466)^n$

$F = P(1{\cdot}002466)^n$

(d) Pádraig plans to invest a fixed amount of money every month in order to generate the fund calculated in part **(c)**.
His retirement is 40 × 12 = 480 months away.

(iii) If Pádraig makes 480 equal monthly payments of €P from now until his retirement, what value of P will give the fund he requires?

His first payment P will be compounded 480 times @ 0·2466%
His second payment P will be compounded 479 times…

$$P(1{\cdot}002466)^1 + P(1{\cdot}002466)^2 + \ldots + P(1{\cdot}002466)^{480} = €358{,}710{\cdot}84$$

This is a geometric series with

$a = P(1{\cdot}002466)$, $r = 1{\cdot}002466$, $n = 480$ and $S_{480} = 358{,}710{\cdot}84$

$$S_n = \frac{a(1-r^n)}{1-r} \Rightarrow 358{,}710{\cdot}84 = \frac{P(1{\cdot}002466)(1-1{\cdot}002466^{480})}{1-1{\cdot}002466}$$

$$P = \frac{358{,}710{\cdot}84(1-1{\cdot}002466)}{1{\cdot}002466(1-1{\cdot}002466^{480})}$$

$$= €390{\cdot}17655585$$

(e) If Pádraig waits for ten years before starting his pension investments, how much will he then have to pay each month in order to generate the same pension fund?

If he waits 10 years he will only have $30 \times 12 = 360$ payments

$$P(1{\cdot}002466)^1 + P(1{\cdot}002466)^2 + \ldots + P(1{\cdot}002466)^{360} = €358{,}710{\cdot}84$$

This is a geometric series with

$a = P(1{\cdot}002466)$, $r = 1{\cdot}002466$, $n = 360$ and $S_{360} = 358{,}710{\cdot}84$

$$S_n = \frac{a(1 - r^n)}{1 - r} \Rightarrow 358{,}710{\cdot}84 = \frac{P(1{\cdot}002466)(1 - 1{\cdot}002466^{360})}{1 - 1{\cdot}002466}$$

$$P = \frac{358{,}710{\cdot}84(1 - 1{\cdot}002466)}{1{\cdot}002466(1 - 1{\cdot}002466^{360})}$$

$$= €618{\cdot}3518013$$

Paper 1 Question 4
2013

25 marks

(a) Niamh has saved to buy a car. She saved an equal amount at the beginning of each month in an account that earned an annual equivalent rate (AER) of 4%.

(i) Show that the rate of interest, compounded monthly, which is equivalent to an AER of 4% is 0·327%, correct to 3 decimal places.

$$(1 + i)^{12} = 1{\cdot}04$$

$$1 + i = \sqrt[12]{1{\cdot}04}$$

$$i = \sqrt[12]{1{\cdot}04} - 1$$

$$i = 0{\cdot}00327374$$

$$i = 0{\cdot}327\%$$

5

(a) Niamh has saved to buy a car. She saved an equal amount at the beginning of each month in an account that earned an annual equivalent rate (AER) of 4%.

(ii) Niamh has €15000 in the account at the end of 36 months. How much has she saved each month, correct to the nearest euro?

Let A be the amount invested monthly

The first payment accrues an interest of $A(1{\cdot}00327)^{36}$

The second payment accrues an interest of $A(1{\cdot}00327)^{35}$ etc.

$\Rightarrow A(1{\cdot}00327)^{36} + A(1{\cdot}00327)^{35} + \ldots A(1{\cdot}00327)^{1} = €15000$

This is a geometric series with

$a = A(1{\cdot}00327)$, $r = 1{\cdot}00327$, $n = 36$ and $S_{36} = 15000$

$$S_n = \frac{a(1-r^n)}{1-r} \Rightarrow 15000 = \frac{1{\cdot}00327A(1-1{\cdot}00327^{36})}{1-1{\cdot}00327}$$

$$A = \frac{15000(1-1{\cdot}00327)}{1{\cdot}00327(1-1{\cdot}00327^{36})}$$

$A = €392{\cdot}02$

(b) Conall borrowed to buy a car. He borrowed €15 000 at a monthly interest rate of 0·866%. He made 36 equal monthly payments to repay the entire loan. How much, to the nearest euro, was each of his monthly payments?

$P = 15{,}000$, $i = 0{\cdot}00866$, $n = 36$ repayments

$$15{,}000 = \frac{A}{(1{\cdot}00866)^1} + \frac{A}{(1{\cdot}00866)^2} + \ldots + \frac{A}{(1{\cdot}00866)^{36}}$$

This is a geometric series $a = \frac{A}{1{\cdot}00866}$, $r = \frac{1}{1{\cdot}00866}$, $n = 36$

$$S_n = \frac{a(1-r^n)}{1-r} \qquad 15{,}000 = \frac{\frac{A}{1{\cdot}00866}\left(1 - \frac{1}{(1{\cdot}00866)^{36}}\right)}{\left(1 - \frac{1}{1{\cdot}00866}\right)}$$

$$A = \frac{15{,}000\left(1 - \frac{1}{1{\cdot}00866}\right)}{\frac{1}{1{\cdot}00866}\left(1 - \frac{1}{(1{\cdot}00866)^{36}}\right)} = €487{\cdot}7731293$$

(b) Conall borrowed to buy a car. He borrowed €15 000 at a monthly interest rate of 0·866%. He made 36 equal monthly payments to repay the entire loan. How much, to the nearest euro, was each of his monthly payments?

Alternatively using amortisation formula:

$$A = P\,\frac{i(1+i)^t}{(1+i)^t - 1}$$

$$A = \frac{15{,}000(0{\cdot}00866)(1{\cdot}00866)^{36}}{(1{\cdot}00866)^{36} - 1}$$

$$= €486{\cdot}77310293$$

Paper 1 Question 6
2011

50 marks

Most lottery games in the USA allow winners of the jackpot prize to choose between two forms of the prize: an *annual-payments* option or a *cash-value* option. In the case of the *New York Lotto*, there are 26 annual payments in the *annual-payments* option, with the first payment immediately, and the last payment in 25 years' time. The payments increase by 4% each year. The amount advertised as the jackpot prize is the total amount of these 26 payments. The *cash-value* option pays a smaller amount than this.

(a) If the amount of the first annual payment is A, write down, in terms of A, the amount of the second, third, fourth and 26th payments.

1st payment(now): A

2nd payment: $A(1{\cdot}04)$

3rd payment: $A(1{\cdot}04)^2$

4th payment: $A(1{\cdot}04)^3$

26th payment: $A(1{\cdot}04)^{25}$

Most lottery games in the USA allow winners of the jackpot prize to choose between two forms of the prize: an *annual-payments* option or a *cash-value* option. In the case of the *New York Lotto*, there are 26 annual payments in the *annual-payments* option, with the first payment immediately, and the last payment in 25 years' time. The payments increase by 4% each year. The amount advertised as the jackpot prize is the total amount of these 26 payments. The *cash-value* option pays a smaller amount than this.

(b) The 26 payments form a geometric series. Use this fact to express the advertised jackpot prize in terms of A.

$$A + A(1{\cdot}04)^1 + A(1{\cdot}04)^2 + \ \ldots + A(1{\cdot}04)^{25}$$

$$a = A,\ r = 1{\cdot}04,\ n = 26$$

$$S_n = \frac{a(1 - r^n)}{1 - r} \Rightarrow S_{26} = \frac{A(1 - 1{\cdot}04^{26})}{1 - 1{\cdot}04}$$

$$= \frac{A(1 - 1{\cdot}04^{26})}{-0{\cdot}04}$$

Most lottery games in the USA allow winners of the jackpot prize to choose between two forms of the prize: an *annual-payments* option or a *cash-value* option. In the case of the *New York Lotto*, there are 26 annual payments in the *annual-payments* option, with the first payment immediately, and the last payment in 25 years' time. The payments increase by 4% each year. The amount advertised as the jackpot prize is the total amount of these 26 payments. The *cash-value* option pays a smaller amount than this.

(c) Find, correct to the nearest dollar, the value of A that corresponds to an advertised jackpot prize of \$21·5 million.

$$\frac{A(1 - 1{\cdot}04^{26})}{-0{\cdot}04} = 21{,}500{,}000$$

$$A = \frac{21{,}500{,}000(-0{\cdot}04)}{(1 - 1{\cdot}04^{26})}$$

$$A = \$485{,}198{\cdot}68$$

10

(d) A winner who chooses the *cash-value* option receives, immediately, the total of the present values of the 26 annual payments. The interest rate used for the present-value calculations is 4·78%. We want to find the cash value of the prize referred to in part **(c)**.

(i) Complete the table below to show the actual amount and the present value of each of the first three annual payments.

payment number	time to payment (years)	actual amount	present value
1	0	$485,199	$485,199
2	1	$504,606·96	$481,587·10
3	2	$524,791·24	$478,002·08

$\$485{,}199(1{\cdot}04) = \$504{,}606{\cdot}96 \quad \div 1{\cdot}0478$

$\$485{,}199(1{\cdot}04)^2 = \$524{,}791{\cdot}24 \quad \div 1{\cdot}0478^2$

The payments increase by 4% each year

(d) A winner who chooses the *cash-value* option receives, immediately, the total of the present values of the 26 annual payments. The interest rate used for the present-value calculations is 4·78%. We want to find the cash value of the prize referred to in part **(c)**.

(ii) Write down, in terms of n, an expression for the present value of the n^{th} annual payment.

payment number	time to payment (years)	actual amount	present value
1	0	\$485,199	\$485,199
2	1	\$504,606·96	\$481,587·10
3	2	\$524,791·24	\$478,002·08

Part **(d) (i)** and **(ii)** 5 $\dfrac{485{,}199(1{\cdot}04)^{n-1}}{(1{\cdot}0478)^{n-1}}$

(d) A winner who chooses the *cash-value* option receives, immediately, the total of the present values of the 26 annual payments. The interest rate used for the present-value calculations is 4·78%. We want to find the cash value of the prize referred to in part **(c)**.

(iii) Find the amount of prize money payable under the *cash-value* option. That is, find the total of the present values of the 26 annual payments. Give your answer in millions, correct to one decimal place.

$$a = 485{,}199,\ r = \frac{1{\cdot}04}{1{\cdot}0478},\ n = 26$$

$$S_n = \frac{a(1 - r^n)}{1 - r} \Rightarrow S_{26} = 485{,}199\left(\frac{1 - \left(\frac{1{\cdot}04}{1{\cdot}0478}\right)^{26}}{1 - \left(\frac{1{\cdot}04}{1{\cdot}0478}\right)}\right)$$

$$= \$11{\cdot}5083\text{million}$$

(e) The jackpot described in parts **(c)** and **(d)** above was won by an Irish woman earlier this year. She chose the *cash-value* option. After tax, she received \$7·9 million. What percentage of tax was charged on her winnings?

Tax: \$11·5 million – \$7·9 million = \$3·6 million

$$\text{Tax} = \frac{3{\cdot}6}{11{\cdot}5} \times 100 = 31{\cdot}3\%$$

Part **(d)(iii)** and **(e)** 5