# Functions and Graphs
## - Real World (with Calculus)

| Checklist | Completed | Revisit |
|---|---|---|
| ▶ 2024 LCHL Paper 1 – Question 7 (c) | ☐ | ☐ |
| ▶ 2024 LCHL Paper 1 – Question 8 | ☐ | ☐ |
| ▶ 2024 LCHL Paper 1 – Question 10 | ☐ | ☐ |
| ▶ 2023 LCHL Paper 1 – Question 7 | ☐ | ☐ |
| ▶ 2022 LCHL Paper 1 – Question 7 | ☐ | ☐ |
| ▶ 2022 LCHL Paper 1 – Question 10 | ☐ | ☐ |
| ▶ 2021 LCHL Paper 1 – Question 8 | ☐ | ☐ |
| ▶ 2021 LCHL Paper 1 – Question 9 | ☐ | ☐ |
| ▶ 2021 LCHL Paper 1 – Question 10 | ☐ | ☐ |
| ▶ 2020 LCHL Paper 1 – Question 9 | ☐ | ☐ |
| ▶ 2019 LCHL Paper 1 – Question 8 | ☐ | ☐ |
| ▶ 2018 LCHL Paper 1 – Question 7 | ☐ | ☐ |
| ▶ 2017 LCHL Paper 1 – Question 7 | ☐ | ☐ |
| ▶ 2016 LCHL Paper 1 – Question 8 | ☐ | ☐ |
| ▶ 2015 LCHL Paper 1 – Question 7 | ☐ | ☐ |
| ▶ 2014 LCHL Paper 1 – Question 9 | ☐ | ☐ |

## ▶ 2024 LCHL Paper 1 – Question 7 (c)

Fiadh puts money in a savings account, and leaves it there for a number of years.

The following expression gives $F(t)$, the amount of money in the account in euro after $t$ years, where $t \in \mathbb{R}$, $t \geq 0$:

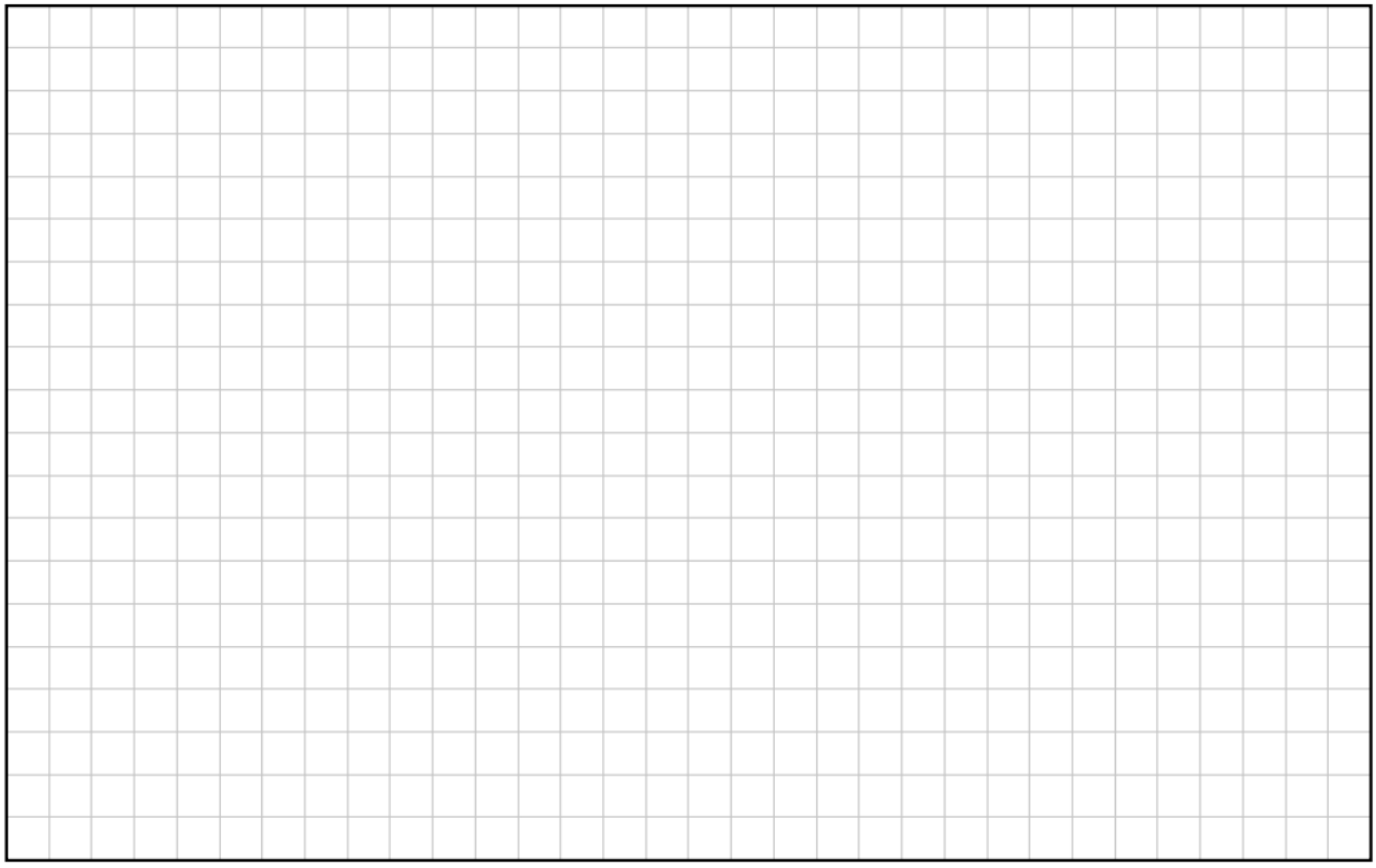
$$F(t) = 5000e^{0.04t}$$

Use this expression to answer parts **(c)(i)**, **(c)(ii)**, and **(c)(iii)**.

**(i)** Use differentiation to find the rate at which the amount of money in the account is increasing after 3·5 years. Give your answer correct to the nearest euro per year.

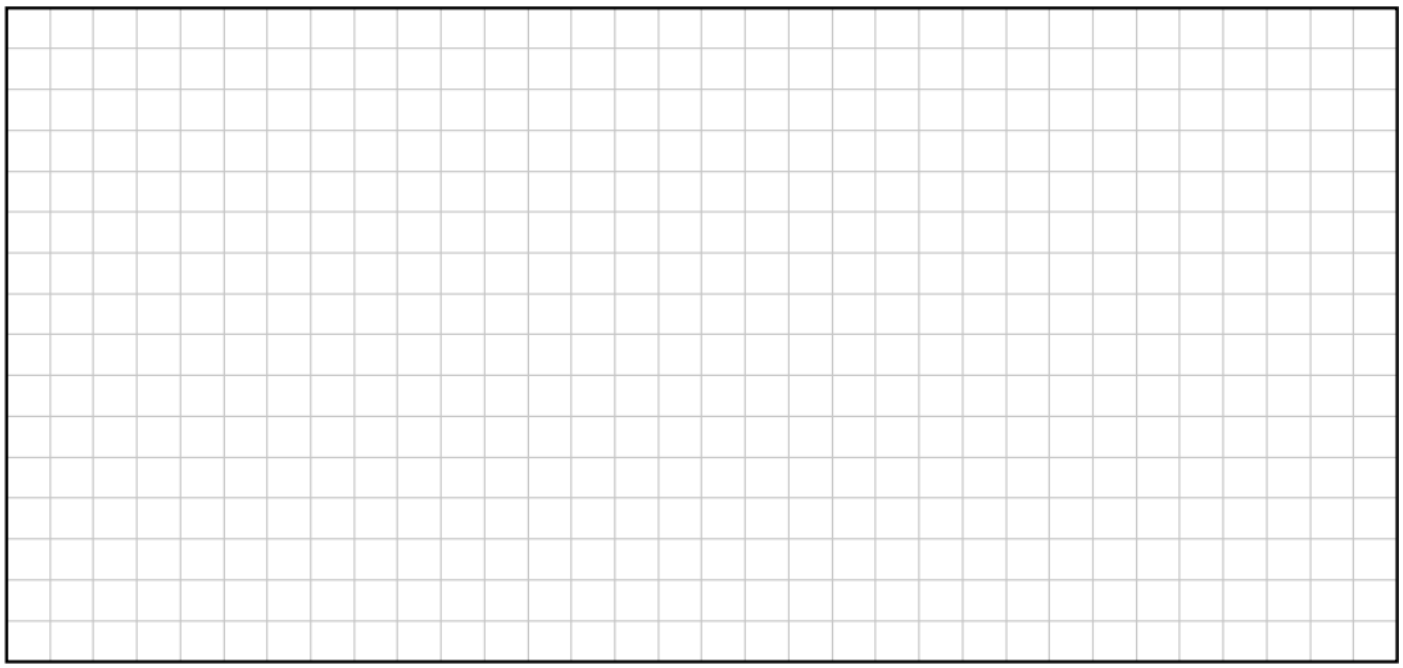
**(ii)** Use integration to find the average amount of money in the account over the first 5 years. Give your answer correct to the nearest euro.
Remember that the amount of money in the account after $t$ years is:

$$F(t) = 5000e^{0.04t}$$



**(iii)** Work out the annual rate of interest (AER) for this account.

That is, find the percentage increase in the amount of money in the account over the course of one year. Give your answer as a percentage, correct to 2 decimal places.



## ▶ 2024 LCHL Paper 1 – Question 8

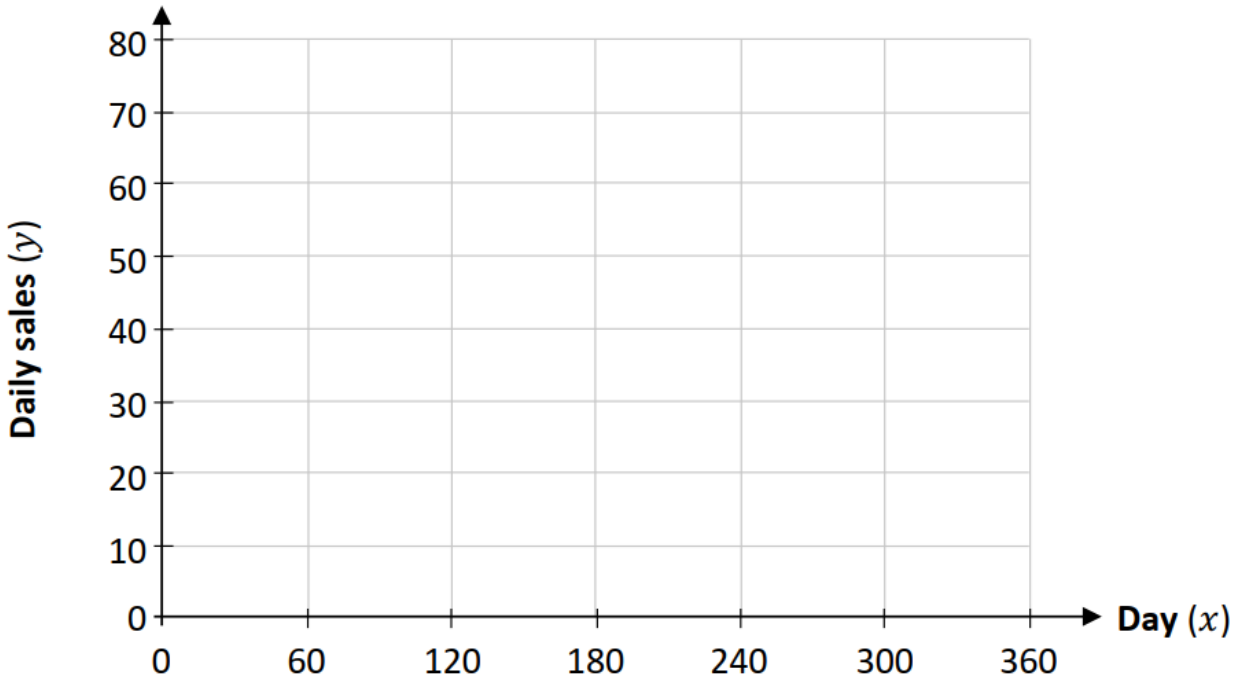
**(a)** A clothing company sells t-shirts. The daily sales of t-shirts over a 360-day period can be modelled using the following function, $T(x)$, where $0 \leq x \leq 360,\ x \in \mathbb{R}$:

$$T(x) = \left(\frac{x - 240}{60}\right)^3 + 70$$

**(i)** Fill in the table below, showing the value of $T(x)$ for each of the given values of $x$.
Give each value correct to the nearest whole number.
Two values have already been filled in.

| $x$ | 0 | 60 | 120 | 180 | 240 | 300 | 360 |
|---|---|---|---|---|---|---|---|
| $T(x)$ | | 43 | | | | | 78 |

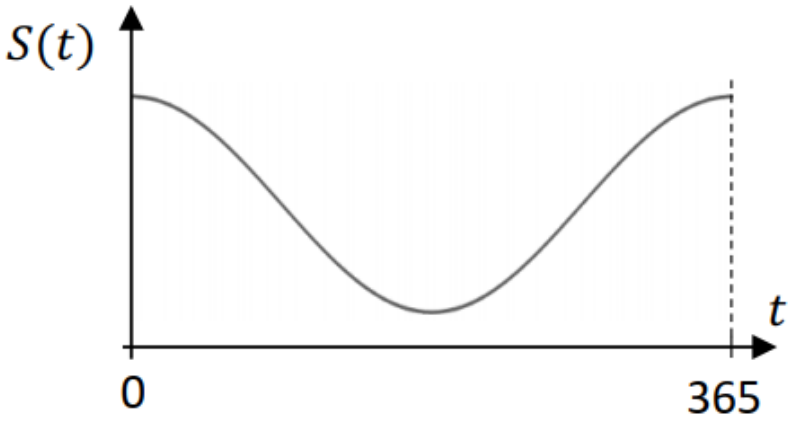
**(ii)** Draw the graph of $y = T(x)$ on the axes below, for $0 \leq x \leq 360,\ x \in \mathbb{R}$.



The clothing company also sells scarves. The daily sales of scarves can be modelled using the following function, $S(t)$, which is a periodic function with a period of 365 days:

$$S(t) = 21 + 19\cos\left(\frac{2\pi t}{365}\right)$$

Here, $t \in \mathbb{R}$ is the time, in days, from 1 January 2024, $t \geq 0$, and $\frac{2\pi t}{365}$ is in radians.




**(b)** Work out the maximum **and** minimum daily sales of scarves, according to $S(t)$.
You do **not** need to use differentiation.

Maximum = _____ Minimum = _____

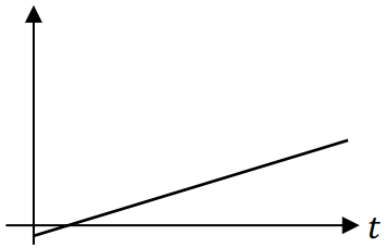
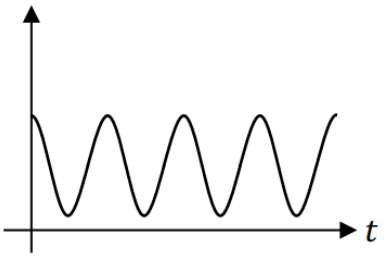
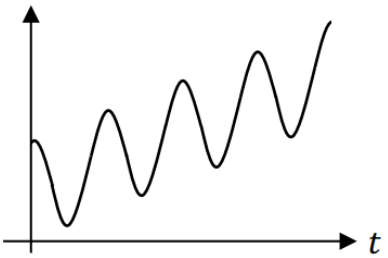
A different function, $C(t)$, can also be used to model the daily sales of scarves, where:

$$C(t) = S(t) - 2.4 + 0.03t$$

Again, $t \in \mathbb{R}$ is the time, in days, from 1 January 2024, $t \geq 0$, and $\frac{2\pi t}{365}$ is in radians.

**(c)** Find the value of $t$ for which $S(t)$ and $C(t)$ give the same number of daily sales.

**(d)** The graphs, **J**, **K**, and **L**, of three functions are shown below (not to scale).

| Graph J | Graph K | Graph L |
|---|---|---|
|  |  |  |



One of these graphs shows the function $C(t)$ over a number of years, where $C(t) = S(t) - 2.4 + 0.03t$. Write down which graph this is. Justify your answer.

Answer (**J**, **K**, or **L**):

Justification:

**(e)** The derivative of $C(t)$ is given by $C'(t) = 0.03 - \frac{38\pi}{365}\sin\left(\frac{2\pi t}{365}\right)$.

Find the value of $t$ that gives the first local maximum of $C$, that is, the first time that $C'(t) = 0$. Give your answer correct to the nearest day.

## ▶ 2024 LCHL Paper 1 – Question 10

A company grows and sells plants.

**(a)** The function $W(x)$ is defined below. It can be used to model the height, in mm, of a water spinach plant, for the first 35 days after it starts to grow.

$$W(x) = 0.667x + 1.5x^2 - 0.025x^3$$

Here, $x$ is the number of days after the plant starts to grow, where $0 \leq x \leq 35,\ x \in \mathbb{R}$.

**(i)** Use $W(x)$ to estimate the height of a water spinach plant after 15 days.

Give your answer correct to the nearest mm.

**(ii)** Write down $W'(x)$, the derivative of $W(x)$.

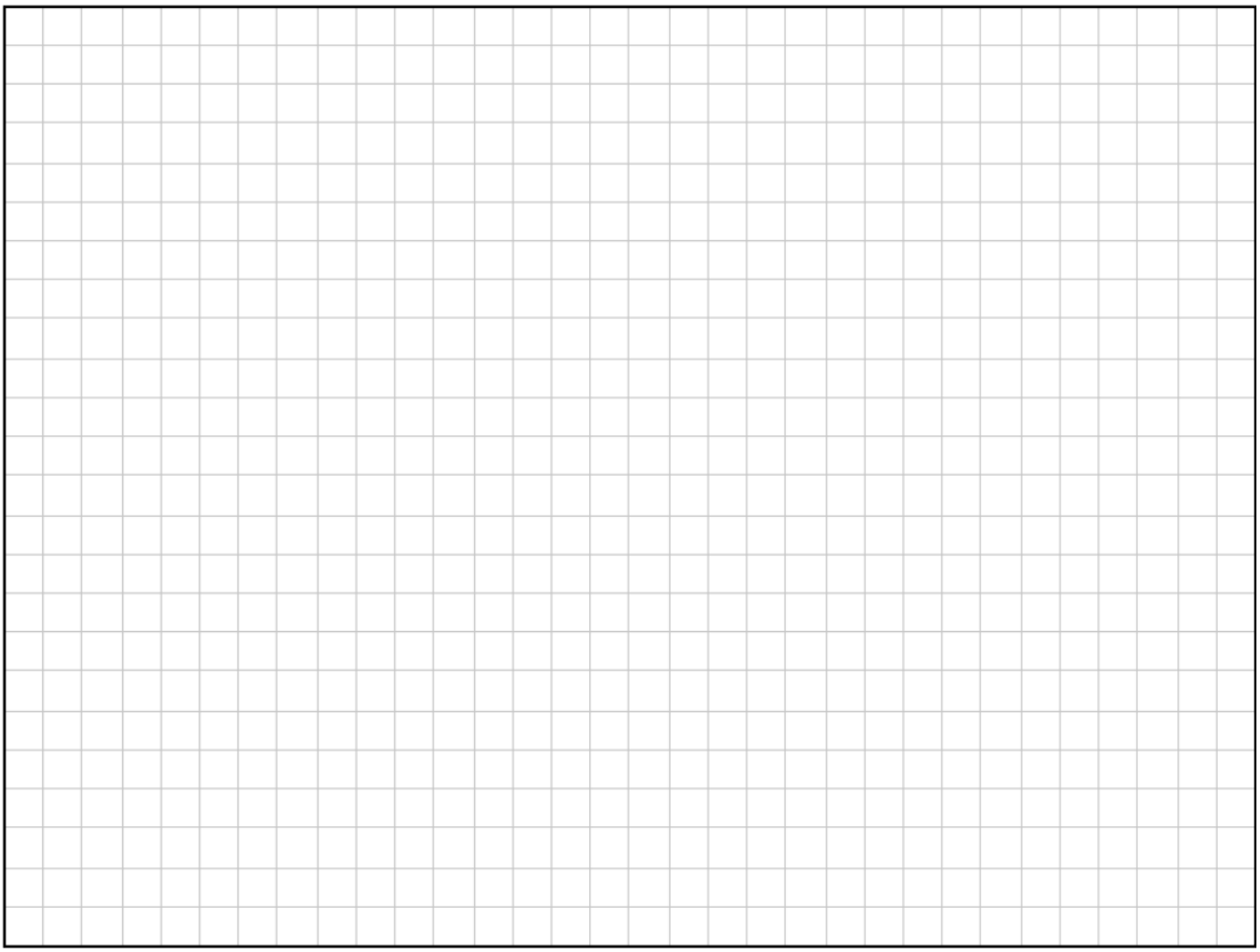
**(b)** The height of a different plant can be modelled by the function $P(x)$, where $x$ is again the number of days after the plant starts to grow.

The derivative of this function is:

$$P'(x) = 1.1 + 2.73x - 0.078x^2$$

Find the range of values of $x$ for which $P'(x) > 24$.

In your answer, give each value correct to the nearest whole number.

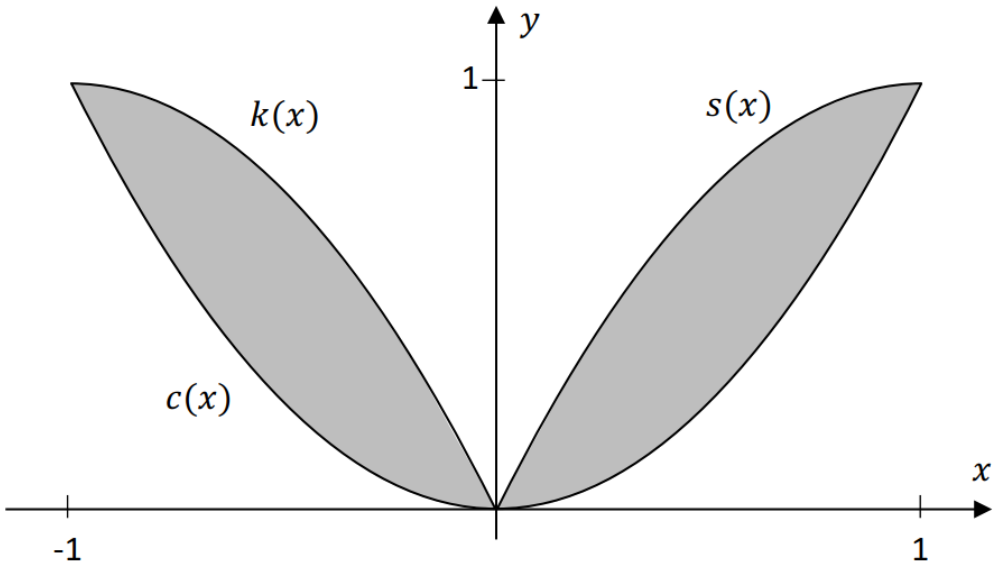


**(c)** The logo for the company is shown on the co-ordinate diagram below. The logo is the region enclosed by three curves, defined by the following functions $c$, $s$, and $k$:

$$c(x) = x^2, \qquad \text{for } -1 \leq x \leq 1, \qquad x \in \mathbb{R}$$

$$s(x) = 2x - x^2, \qquad \text{for } 0 \leq x \leq 1, \qquad x \in \mathbb{R}$$

$k(x)$ is the image of $s(x)$ under axial symmetry in the $y$-axis.




**(i)** Use integration to work out the area of the logo.

*Hint*: find the area of the logo in the first quadrant, and then double it.

**(ii)** The function $k$ can be written in the form $k(x) = -x^2 + bx + c$, where $b, c \in \mathbb{R}$ are constants. Find the value of $b$ and the value of $c$.

Remember that $k$ is the image of $s$ under axial symmetry in the $y$-axis.

$b =$ _____ $c =$ _____

**(d)** In this part, $p$ and $r$ are constants, with $p,\ r \in \mathbb{R}$ and $0 < r < 0.9p$.

The company sells bags of plant food.
The usual price of one bag is € $p$.

In a sale, the customer can choose to pay using either Option 1 or Option 2, as follows:

- **Option 1**: the usual price reduced by 10%, and then reduced by a further € $r$
- **Option 2**: the usual price reduced by € $r$, and then this new price reduced by 10%.

Which option (1 or 2), if either, is cheaper?
Write the price for each option (1 and 2) in terms of $p$ and $r$, to support your answer.

| **Cheaper option**: (Tick (✓) **one** box only) | Option 1 ☐ | Option 2 ☐ | It depends on the value of $p$ and of $r$ ☐ |
|---|---|---|---|

Price for Option 1, in terms of $p$ and $r$:

Price for Option 2, in terms of $p$ and $r$:

## ▶ 2023 LCHL Paper 1 – Question 7

Fiona is driving on a motorway. She passes a point **A** on the motorway. Her speed is given by:

$$v(t) = \frac{2}{3}t^3 - 6t^2 + 13t + 109$$

where $v$ is her speed in km/hour $t$ minutes after passing the point **A**, for $0 \leq t \leq 5$ and $t \in \mathbb{R}$.

**(a)** Work out Fiona's speed when she passes the point **A**.

**(b)** Work out Fiona's acceleration (that is, the rate at which her speed is increasing) 5 minutes after she passes the point **A**. Give your answer in km/hour per minute.

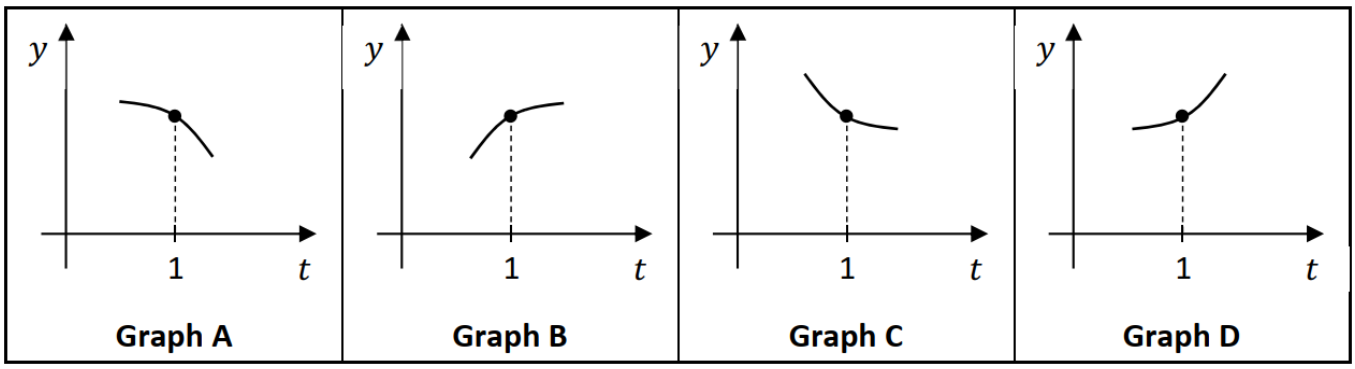
**(c)** Find the time (value of $t$) at which Fiona reaches her maximum speed, during the first 4 minutes after she passes the point **A**. Give your answer correct to 2 decimal places.

**(d)** Use integration to work out Fiona's average speed over the 5 minutes after she passes the point **A**. Give your answer correct to 2 decimal places.

**(e)** Taking $v'(t)$ to be the derivative of $v$, and $v''(t)$ to be the second derivative of $v$:

$$v'(1) > 0 \text{ and } v''(1) < 0.$$

Four graphs, **A**, **B**, **C**, and **D**, are shown below.




Close to where $t = 1$, the graph of $y = v(t)$ must look like one of the four graphs given above. Write down which graph this is. Justify your answer, using both $v'(1)$ **and** $v''(1)$.

Answer (**A**, **B**, **C**, or **D**):

Using $v'(1) > 0$:

Using $v''(1) < 0$:

*This question continues on the next page.*

There is an **Average Speed Zone** on the motorway, starting at the point **A** and ending at point **B**.
The distance from **A** to **B** along the motorway is 10 km.
Cameras record the time taken for each car to travel from the point **A** to the point **B**.
Each car's average speed from **A** to **B** is then calculated.

**(f)** Work out the **minimum** time, in minutes, that a driver could get from **A** to **B**, while not driving above 100 km/hour.

**(g)** Rohan drives from **A** to **B**.
He passes the point **A** driving at a constant speed of 120 km/hour. After 2 minutes driving at this speed, he starts to decelerate (reduce his speed) at a constant rate, until he reaches the point **B**.
Overall, his average speed in driving from **A** to **B** is 100 km/hour.

Work out Rohan's deceleration. Give your answer in km/hour per minute.

## ▶ 2022 LCHL Paper 1 – Question 7

Hannah is doing a training session. During this session, her heart-rate, $h(x)$, is measured in beats per minute (BPM), where $x$ is the time in minutes from the start of the session, $x \in \mathbb{R}$.

For the first 8 minutes of the session, Hannah does a number of exercises.

As she does these exercises, her heart-rate changes. In this time, $h(x)$ is given by:

$$h(x) = 2x^3 - 28.5x^2 + 105x + 70$$

**(a)** Work out Hannah's heart-rate 4 minutes after the start of the session.

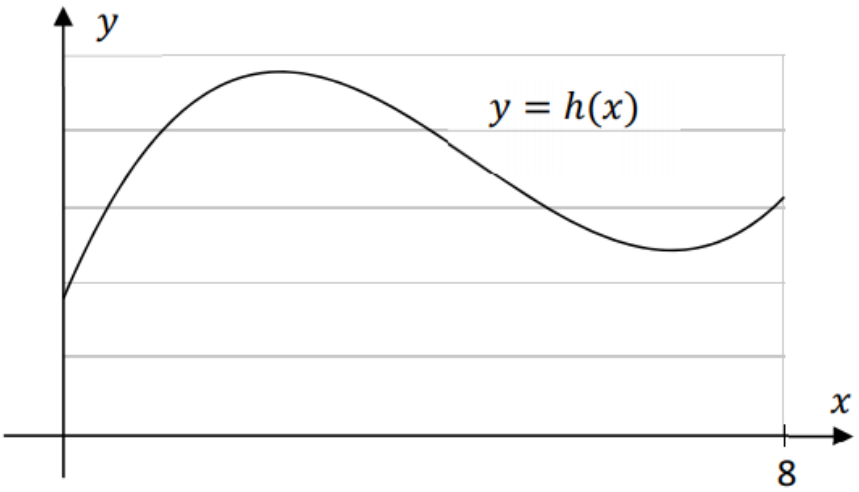
**(b)** Find $h'(x)$.

**(c)** Find $h'(2)$, **and** explain what this value means in the context of Hannah's heart-rate.

$h'(2)$ :

Explanation:

The graph below shows $y = h(x)$, where $0 \leq x \leq 8, x \in \mathbb{R}$.




**(d)** Find the least value and the greatest value of $h(x)$, for $0 \leq x \leq 8, x \in \mathbb{R}$.

Use calculus in your solution. You may also use information from the graph above, which is to scale.

Least value of $h(x)$: _____ Greatest value of $h(x)$: _____

**(e)** How long after the start of the session is Hannah's heart-rate decreasing most quickly, within the first 8 minutes? Give your answer in minutes and seconds.

Remember that $h(x) = 2x^3 - 28.5x^2 + 105x + 70$

**(f)** Bruno, Karen and Martha start a training session at the same time as Hannah.

All of their heart-rates are measured in BPM.

**(i)** For the first 8 minutes of the session, Bruno's heart-rate, $b(x)$ is always 15 BPM more than Hannah's heart-rate.

Use this information to write $b'(x)$ **in terms of** $\boldsymbol{h'(x)}$, where $0 \leq x \leq 8, x \in \mathbb{R}$.

**(ii)** For the first 8 minutes of the session, Karen's heart-rate, $k(x)$, is always 10% less than Hannah's heart-rate.

Use this information to write $k'(x)$ in terms of $h'(x)$, where $0 \leq x \leq 8, x \in \mathbb{R}$.

**(g)** Martha does each exercise for a longer time than Hannah.

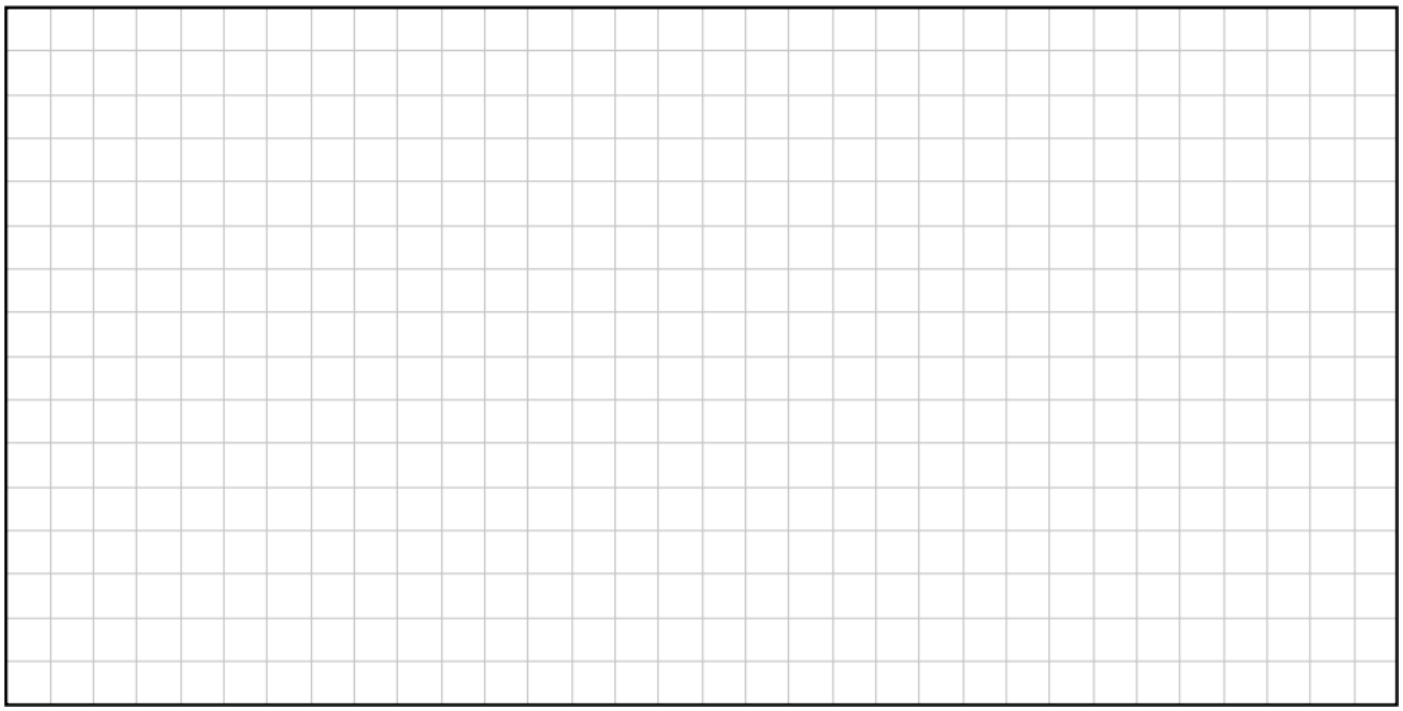
For $0 \leq x \leq 10$, Martha's heart-rate, $m(x)$, is:

$$m(x) = h(0.8x)$$

Use $h(x) = 2x^3 - 28.5x^2 + 105x + 70$ to write $m(x)$ in the form:

$$m(x) = ax^3 + bx^2 + cx + d$$

where $a, b, c, d \in \mathbb{R}$, for $0 \leq x \leq 10$.



## ▶ 2022 LCHL Paper 1 – Question 10

A student is asked to memorise a long list of digits, and then write down the list some time later. The proportion, $P$, of the digits recalled correctly after $t$ hours can be modelled by the function:

$$P(t) = 0.82 - 0.12\ln(t+1)$$

for $0 \leq t \leq 12$ and $t \in \mathbb{R}$.

**(a)** Find the proportion of the digits recalled correctly after 3 hours, according to this model. Give your answer correct to 2 decimal places.

**(b)** After how many hours would exactly 55% of the digits be recalled correctly, according to this model? Give your answer correct to 2 decimal places.

**(c)**

**(i)** Find the value of $P'(1)$.

**(ii)** $P'(t)$ is always negative for $0 \leq t \leq 12$, $t \in \mathbb{R}$. What does this tell you about the proportion of digits recalled correctly after $t$ hours, according to this model?

**(d)** Use calculus to show that the graph of $y = P(t)$ has **no** points of inflection, for $0 \leq t \leq 12$, $t \in \mathbb{R}$.

**(e)** If we learn a skill, and then don't practise it, how well we can do it usually decreases over time. For example, if you learn to play the guitar, and then don't play the guitar for a number of months, you will probably not be as good at playing the guitar the first time you try it again. This effect can be modelled by the following equation:

$$A = B(t+1)^c$$

where $A$ is a measure of how well the skill can be done at a certain time ($t = 0$);
$B$ is a measure of how well the skill can be done $t$ months later, without practising;
$c$ is a constant; and $A, B, c, t \in \mathbb{R}$.

**(i)** Write $c$ in terms of $\log_{10} A$, $\log_{10} B$ and $\log_{10}(t+1)$.

**(ii)** A student got 80% on a guitar exam.
After two **years** of not playing the guitar, the student got 47% on the same exam.

Use this to find the value of $c$ in the model above, correct to 3 decimal places.

## ▶ 2021 LCHL Paper 1 – Question 8

**(a)** The table in **Part (a)(ii)** below shows some of the values of the function:

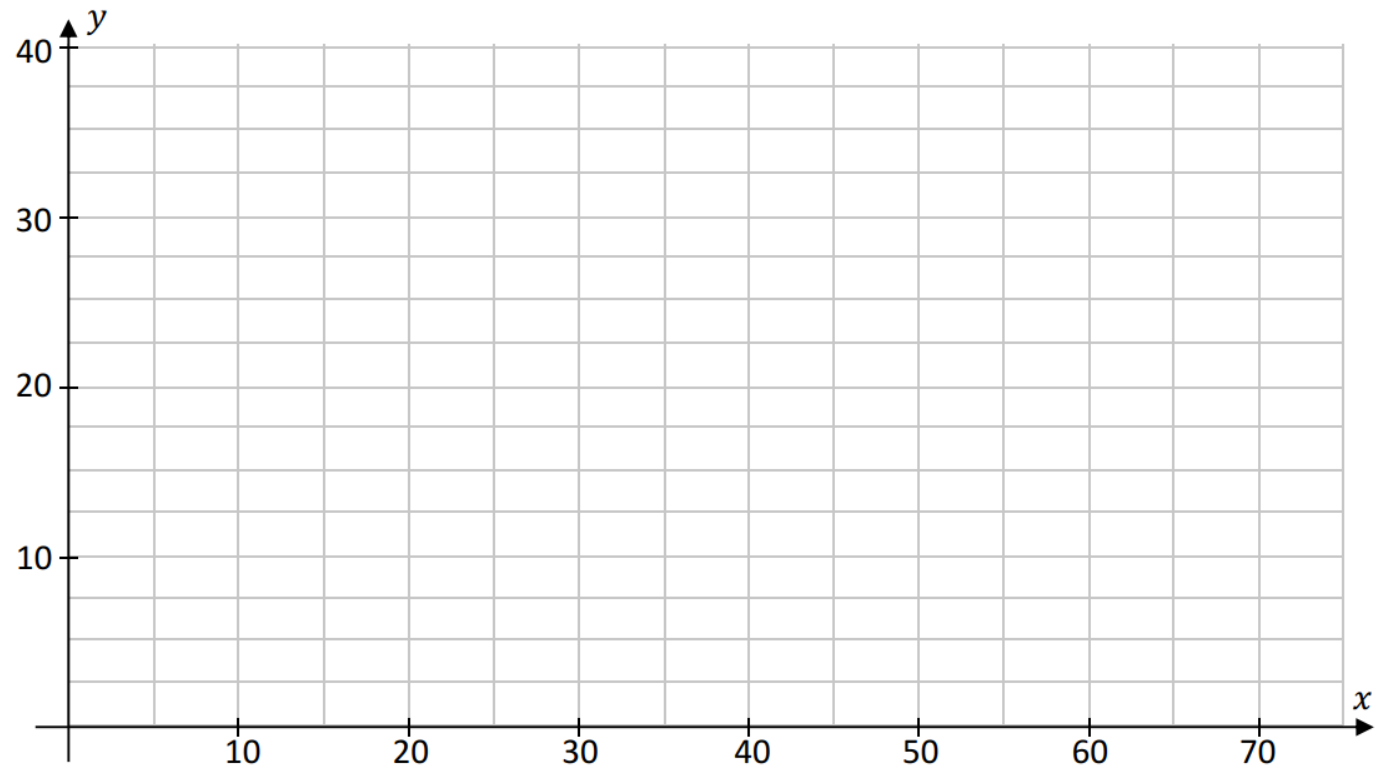
$h(x) = 0.001x^3 - 0.12x^2 + px + 5, x \in \mathbb{R}$,

in the domain $0 \leq x \leq 75$.

**(i)** Use $h(10) = 30$ to show that $p = 3.6$.

**(ii)** Complete the table below and hence draw the graph of $h(x)$ in the domain $0 \leq x \leq 75$ on the grid below.

| $x$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 75 |
|---|---|---|---|---|---|---|---|---|---|
| $h(x)$ | | 30 | | | 21 | | 5 | | 21·875 |



**(b)** The function $h(x)$ can be used to model the height above level ground (in metres) of a section of the path followed by a rollercoaster track, where $x$ is the horizontal distance from a fixed point.

**(i)** Find $h'(x)$, the derivative of $h(x)$.

**(ii)** Show that this section of the track reaches its maximum height above level ground when $x = 20$.

**(iii)** Find, using calculus, the height above ground, in metres, at the instant the track passes through an inflection point. The function $h(x) = 0.001x^3 - 0.12x^2 + 3.6x + 5$.

## ▶ 2021 LCHL Paper 1 – Question 9

**(a)** A cup of coffee is freshly brewed to 95° C.

The temperature, $T$, in degrees centigrade, of the coffee as it cools is given by the formula

$$T(t) = Ae^{-0.081t} + 20$$

where $A$ is constant and $t$ is time measured in minutes from when the coffee was brewed.

**(i)** Show that $A = 75$.

**(ii)** Explain what the value 20 in the formula represents in the context of the coffee cooling.

**(iii)** Find the **decrease** in the temperature of the coffee 10 minutes after brewing.
Give your answer correct to the nearest whole number.

**(b)** $T(t) = 75e^{-0.081t} + 20$ gives the temperature of the coffee at time $t$.

If the ideal temperature to drink coffee is 82°C, find the time, to the nearest second, that it takes for the coffee to reach this temperature.

**(c)** Find, to the nearest °C, the temperature the coffee has reached when $T'(t) = -4.05$, where $T'(t)$ is the rate at which the coffee is cooling, in °C per minute.

## ▶ 2021 LCHL Paper 1 – Question 9 (d)

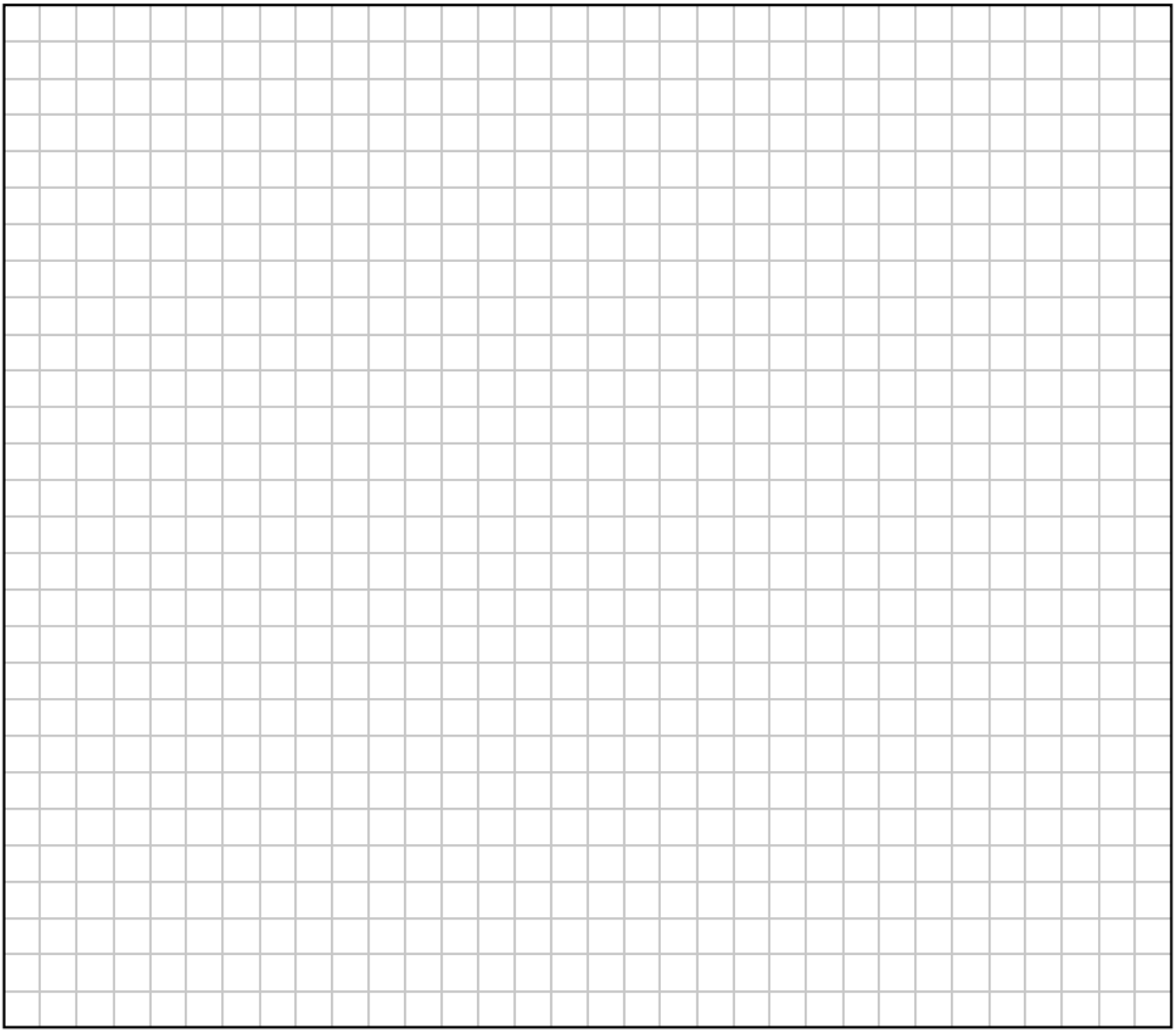
A sugar cube is put into the coffee.

The sugar keeps its cube shape as it dissolves.

As the sugar cube dissolves, its volume decreases at the constant rate of $\frac{1}{20}$ cm$^3$/sec.

Let $x(t)$ be the side length of the cube at time $t$.

Find the rate of change of $x(t)$ when the volume of the cube reaches $\frac{1}{64}$ cm$^3$.



## ▶ 2021 LCHL Paper 1 – Question 10 (a)

Water is flowing into and being removed from a tank.

The volume of water in the tank is measured on a daily basis, starting at day 0.

The volume of water, in litres, in the tank can be modelled by the formula

$$V(t) = 60 + 41t - 3t^2, \text{ while } V(t) \geq 0,$$

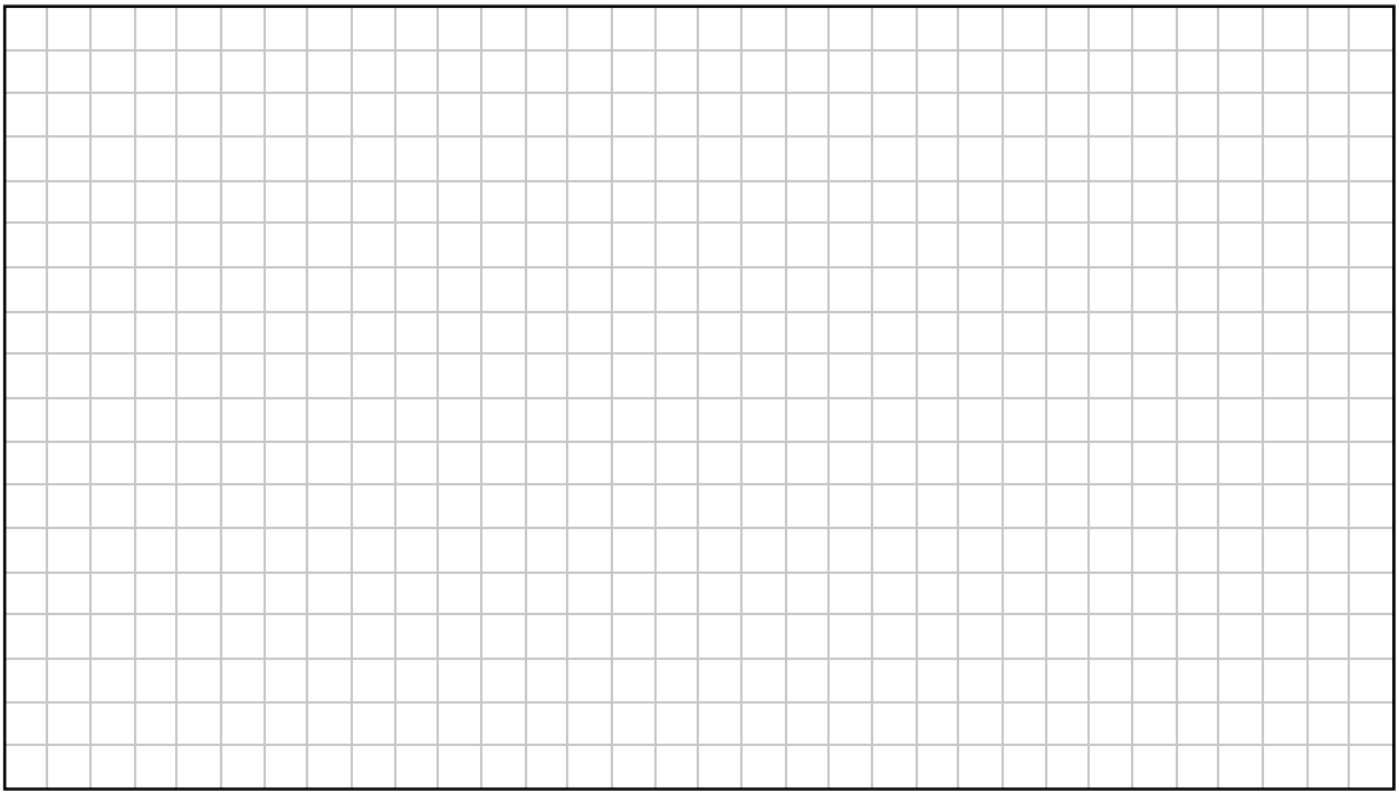
where $t \in \mathbb{R}$ is the time in days, starting at day 0.

Use the function $V(t)$ to answer the following 4 questions.

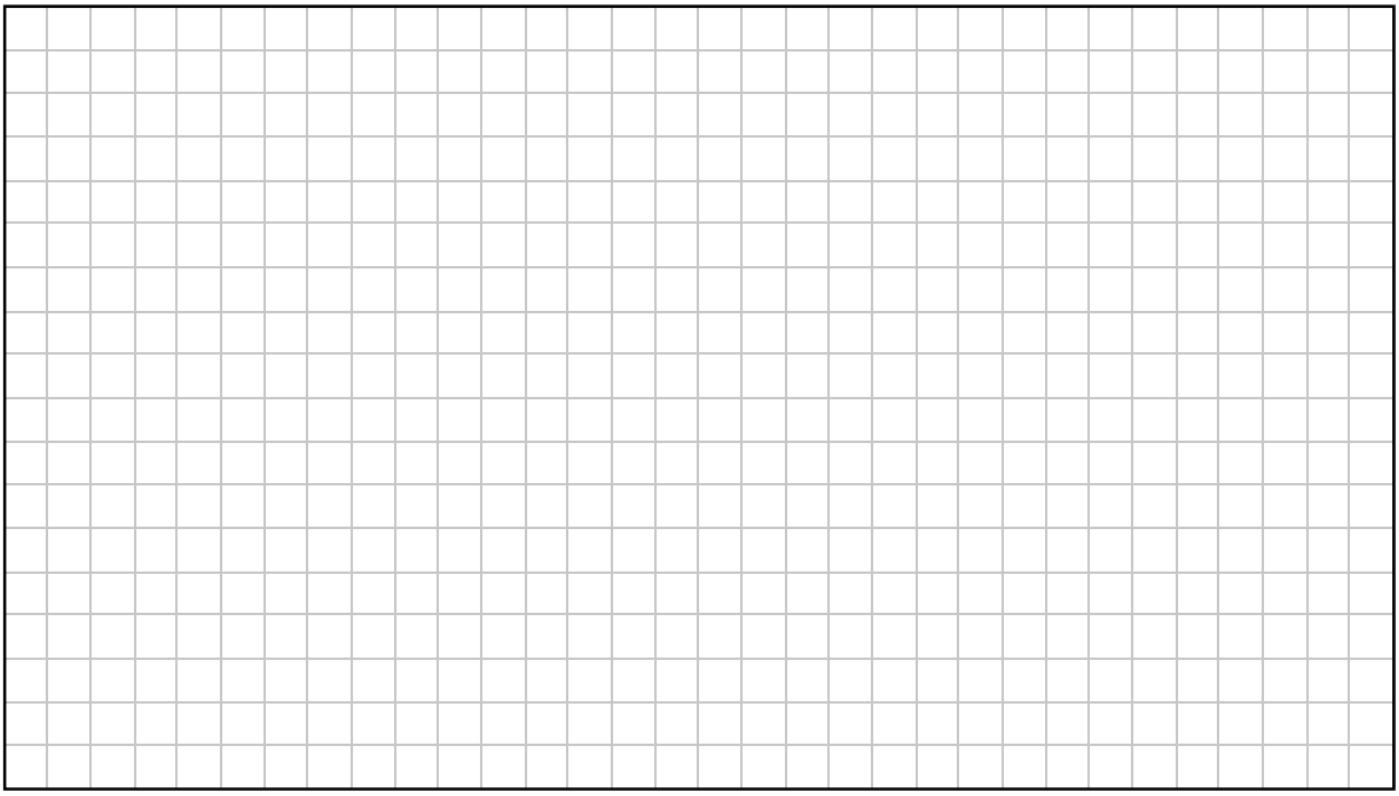
**(i)** Find the value of $t$ when the tank empties.

**(ii)** Find the rate at which the volume of water in the tank is changing when $t = 5$.

**(iii)** Find the value of $t$ when the volume of water in the tank is a maximum.



**(iv)** Find the maximum volume of the water in the tank, correct to the nearest litre.



## ▶ 2020 LCHL Paper 1 – Question 9

**(a)** The number of bacteria in the early stages of a growing colony of bacteria can be approximated using the function:

$$N(t) = 450e^{0.065t}$$

where $t$ is the time, measured in hours, since the colony started to grow, and $N(t)$ is the number of bacteria in the colony at time $t$.

**(i)** Find the number of bacteria in the colony after 4·5 hours.

Give your answer correct to the nearest whole number.

**(ii)** Find the time, in **hours**, that it takes the colony to grow to 790 bacteria.

Give your answer correct to 1 decimal place.

**(b)** Using the function $N(t) = 450e^{0.065t}$, find the average number of bacteria in the colony during the period from $t = 3$ to $t = 12$.

Give your answer correct to the nearest whole number.

**(c)** Find the rate at which $N(t) = 450e^{0.065t}$ is changing when $t = 12$.

Give your answer correct to one decimal place.

Interpret this value in the context of the question.

Rate:

Interpretation:

**(d)** After $k$ hours, the rate of increase of $N(t)$ is greater than 90 bacteria per hour.

Find the least value of $k$, where $k \in \mathbb{N}$.

**(e)** The number of bacteria in the early stages of a **different** colony of bacteria can be approximated using the function:

$$P(t) = 220e^{0.17t}$$

where $P(t)$ is the number of bacteria and $t$ is measured in hours.

Assume that both colonies start growing at the same time.

Find the time, to the nearest hour, at which the number of bacteria in both colonies will be equal.

## ▶ 2019 LCHL Paper 1 – Question 8

The weekly revenue produced by a company manufacturing air conditioning units is seasonal.

The revenue (in euro) can be approximated by the function:

$$r(t) = 22\,500\cos\left(\frac{\pi}{26}t\right) + 37\,500,\ \ t \geq 0$$

where $t$ is the number of weeks measured from the beginning of July and $\left(\frac{\pi}{26}t\right)$ is in radians.

**(a)** Find the approximate revenue produced 20 weeks after the beginning of July.

Give your answer correct to the nearest euro.

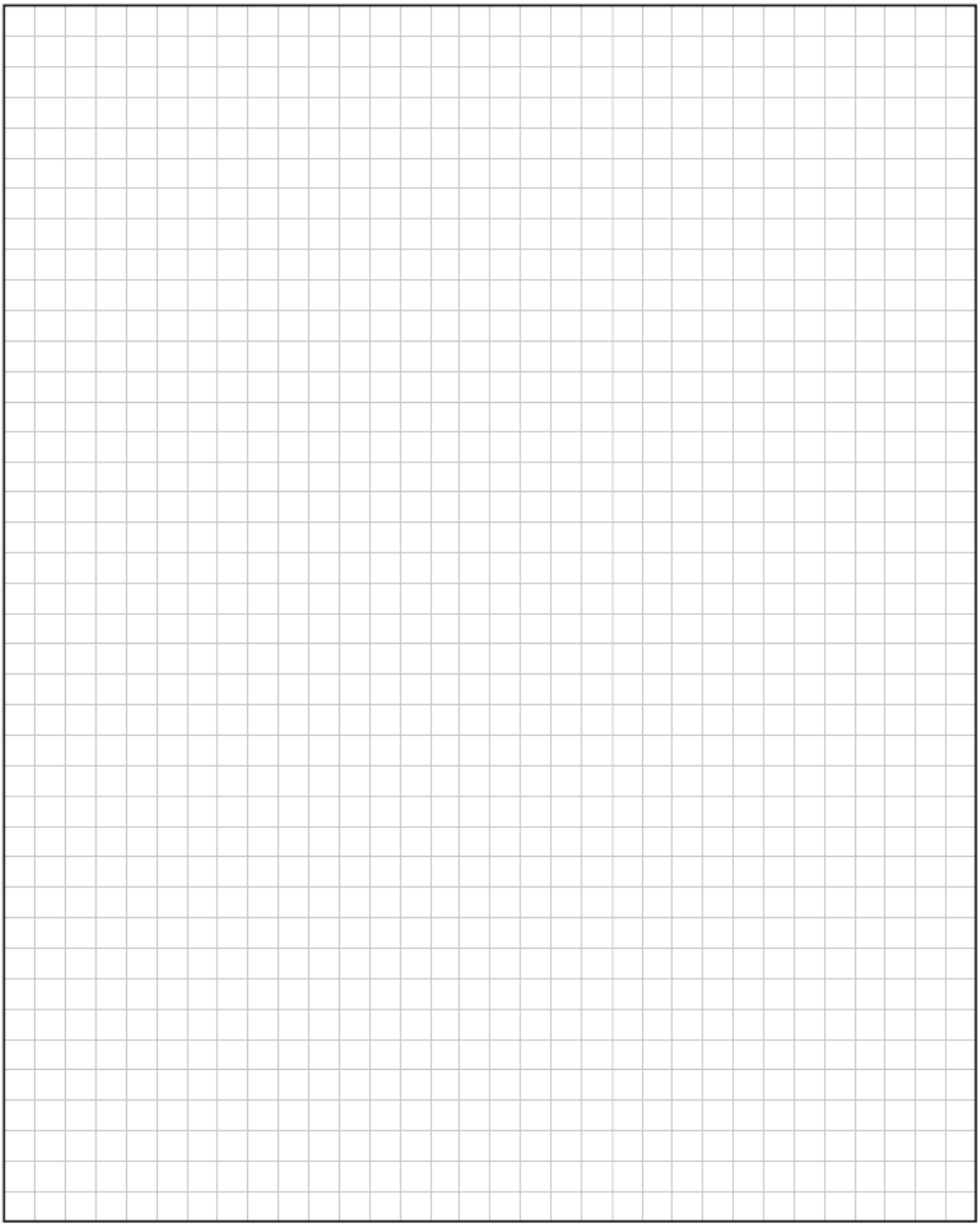
**(b)** Find the two values of the time $t$, within the first 52 weeks, when the revenue is approximately €26 250.

**(c)** Find $r'(t)$, the derivative of $r(t) = 22\,500\cos\left(\frac{\pi}{26}t\right) + 37\,500$.

**(d)** Use calculus to show that the revenue is increasing 30 weeks after the beginning of July.

**(e)** Find a value for the time $t$, within the first 52 weeks, when the revenue is at a minimum.

Use $r''(t)$, to verify your answer.



## ▶ 2018 LCHL Paper 1 – Question 7

The time, in days of practice, it takes Jack to learn to type $x$ words per minute (wpm) can be modelled by the function:

$t(x) = k\left[\ln\left(1 - \frac{x}{80}\right)\right]$, where $0 \leq x \leq 70$, $x \in \mathbb{R}$, and $k$ is a constant.

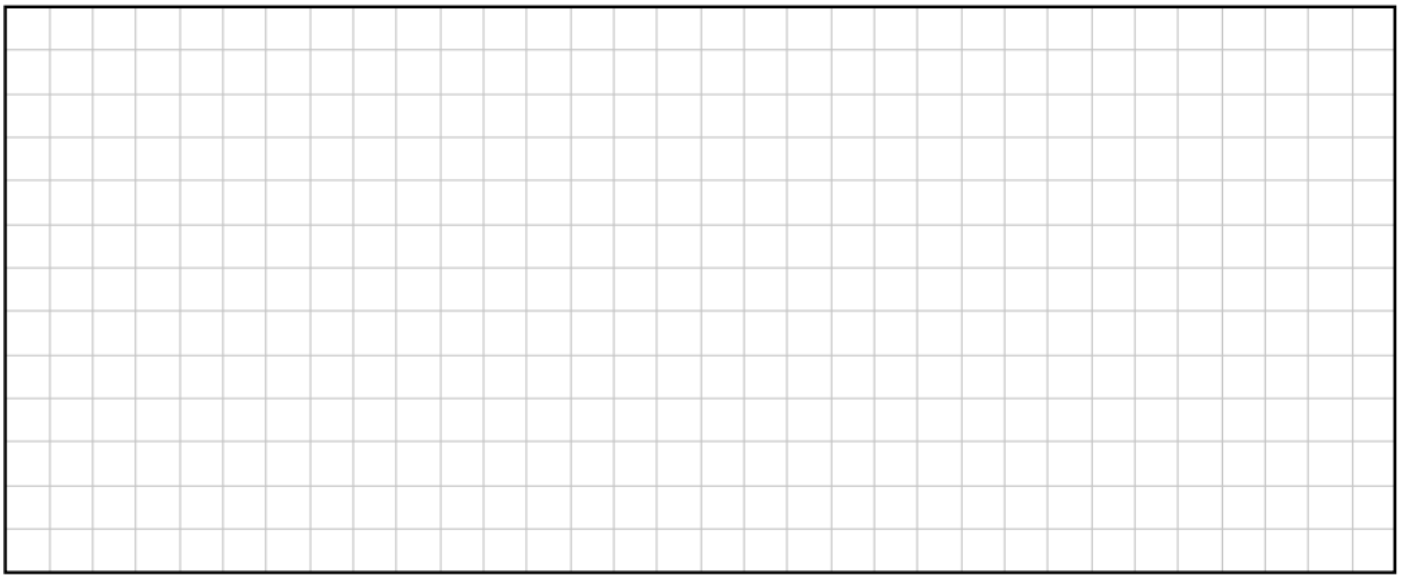
**(a)** Based on the function $t(x)$, Jack can learn to type 35 wpm in 35·96 days. Write the function above in terms of $k$ and **hence** show that $k = -62 \cdot 5$, correct to 1 decimal place.

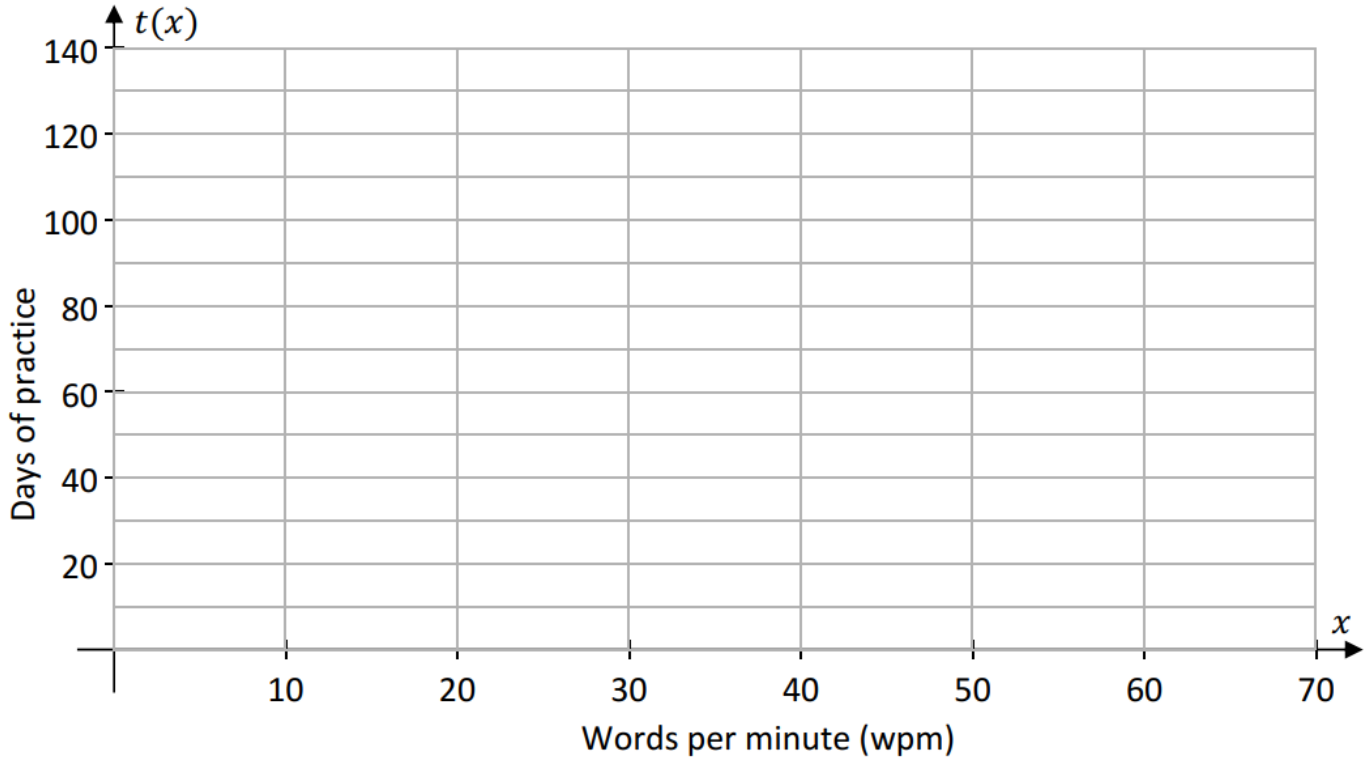
**(b)** Find the number of wpm that Jack can learn to type with 100 days of practice.

Give your answer correct to the nearest whole number.

**(c)** Complete the table below, correct to the nearest whole number **and hence** draw the graph of $t(x)$ for $0 \leq x \leq 70, x \in \mathbb{R}$.

| $x$ (wpm) | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 |
|---|---|---|---|---|---|---|---|---|
| $t(x)$ (days) | | | | | | | | |





**(d)** A simpler function that could also be used to model the number of days needed to attain $x$ wpm is $p(x) = 1 \cdot 5x$.

Draw, on the diagram above, the graph of $p(x)$ for $0 \leq x \leq 70, x \in \mathbb{R}$.

**(e)** Let $h(x) = p(x) - t(x)$

**(i)** Use your graphs above to estimate the solution to $h(x) = 0$ for $x > 0$.

**(ii)** Use calculus to find the maximum value of $h(x)$ for $0 \leq x \leq 70, x \in \mathbb{R}$.

Give your answer correct to the nearest whole number.

## ▶ 2017 LCHL Paper 1 – Question 7

Sometimes it is possible to predict the future population in a city using a function.

The population in Sapphire City, over time, can be predicted using the following function:

$$p(t) = Se^{0.1t} \times 10^6.$$

The population in Avalon, over time, can be predicted using the following function:

$$q(t) = 3.9e^{kt} \times 10^6.$$

In the functions above, $t$ is time, in years; $t = 0$ is the beginning of 2010; and both $S$ and $k$ are constants.

The population in Sapphire City at the beginning of 2010 is 1 100 000 people.

**(a)** Find the value of $S$.

**(b)** Find the predicted population in Sapphire City at the beginning of 2015.

**(c)** Find the predicted change in the population in Sapphire City during 2015.

The predicted population in Avalon at the beginning of 2011 is 3 709 795 people.

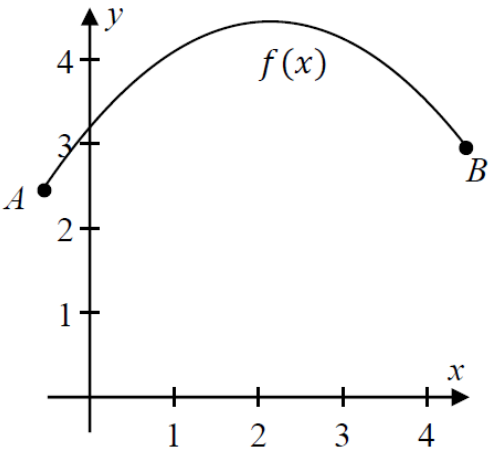
**(d)** Write down and solve an equation in $k$ to show that $k = -0.05$, correct to 2 decimal places.

**(e)** Find the year during which the populations in both cities will be equal.

**(f)** Find the predicted average population in Avalon from the beginning of 2010 to the beginning of 2025.

**(g)** Use the function $q(t) = 3.9e^{-0.05t} \times 10^6$ to find the predicted rate of change of the population in Avalon at the beginning of 2018.

## ▶ 2016 LCHL Paper 1 – Question 8 (a)




The diagram shows Sarah's first throw at the basket in a basketball game. The ball left her hands at $A$ and entered the basket at $B$. Using the co-ordinate plane with $A(-0.5,\ 2.565)$ and $B(4.5,\ 3.05)$, the equation of the path of the centre of the ball is

$f(x) = -0.274x^2 + 1.193x + 3.23,$

where both $x$ and $f(x)$ are measured in metres.

**(i)** Find the maximum height reached by the centre of the ball, correct to three decimal places.

**(ii)** Find the acute angle to the horizontal at which the ball entered the basket. Give your answer correct to the nearest degree.

Sarah took a second throw. This throw followed the path of the parabola $g(x)$ as shown.

The ball left Sarah's hands at the point $C(0,\ 2)$.

The graph $y = g(x)$ is the image of the graph $y = f(x)$ under the translation which maps $A$ onto $C$.

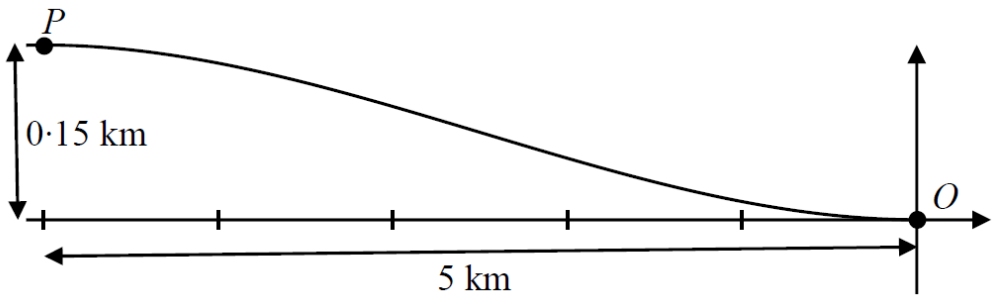
**(iii)** Using your result from part **a(i)**, show that the centre of this ball reached its maximum height at the point $(2.677,\ 3.964)$, correct to three decimal places.

**(iv)** Hence, or otherwise, find the equation of the parabola $g(x)$.

## ▶ 2015 LCHL Paper 1 – Question 7

A plane is flying horizontally at $P$ at a height of 150 m above level ground when it begins its descent.

$P$ is 5 km, horizontally, from the point of touchdown $O$. The plane lands horizontally at $O$.




Taking $O$ as the origin, $(x,\ f(x))$ approximately describes the path of the plane's descent where $f(x) = 0.0024x^3 + 0.018x^2 + cx + d, \quad -5 \leq x \leq 0$, and both $x$ and $f(x)$ are measured in km.

**(a)**

**(i)** Show that $d = 0$.

**(ii)** Using the fact that $P$ is the point $(-5,\ 0.15)$, or otherwise, show that $c = 0$.

**(b)**

**(i)** Find the value of $f'(x)$, the derivative of $f(x)$, when $x = -4$.

**(ii)** Use your answer to part (b) (i) above to find the angle at which the plane is descending when it is 4 km from touchdown. Give your answer correct to the nearest degree.

**(c)** Show that $(-2.5, 0.075)$ is the point of inflection of the curve $y = f(x)$.

**(d)**

**(i)** If $(x,\ y)$ is a point on the curve $y = f(x)$, verify that $(-x - 5, -y + 0.15)$ is also a point on $y = f(x)$.

**(ii)** Find the image of $(-x - 5, -y + 0.15)$ under symmetry in the point of inflection.

### ▶ 2014 LCHL Paper 1 – Question 9

Ciarán is preparing food for his baby and must use cooled boiled water. The equation $y = Ae^{kt}$ describes how the boiled water cools. In this equation:

- $t$ is the time, in minutes, from when the water boiled,
- $y$ is the *difference* between the water temperature and room temperature at time $t$, measured in degrees Celsius,
- $A$ and $k$ are constants.

The temperature of the water when it boils is $100°C$ and the room temperature is a constant $23°C$.

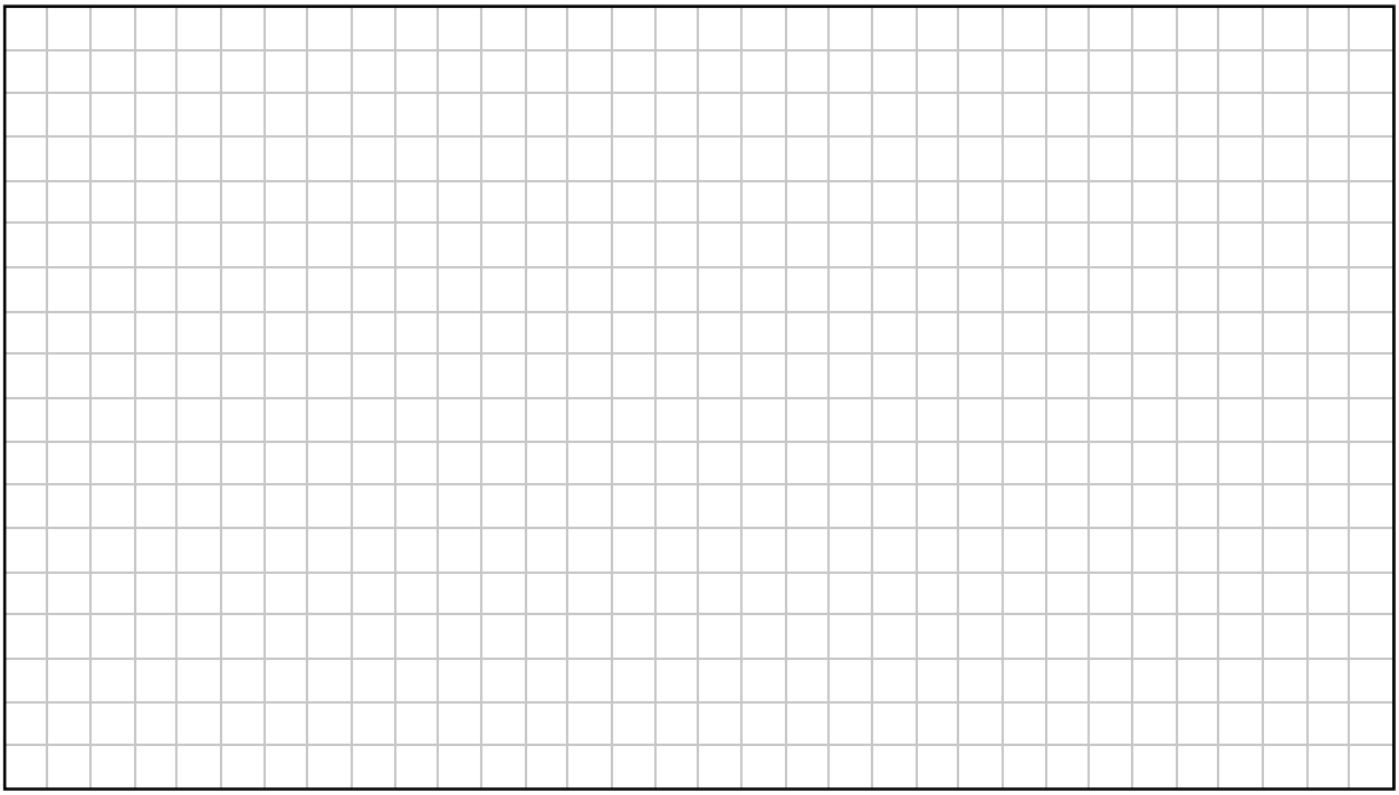
**(a)** Write down the value of the temperature difference, $y$, when the water boils, and find the value of $A$.

$y =$

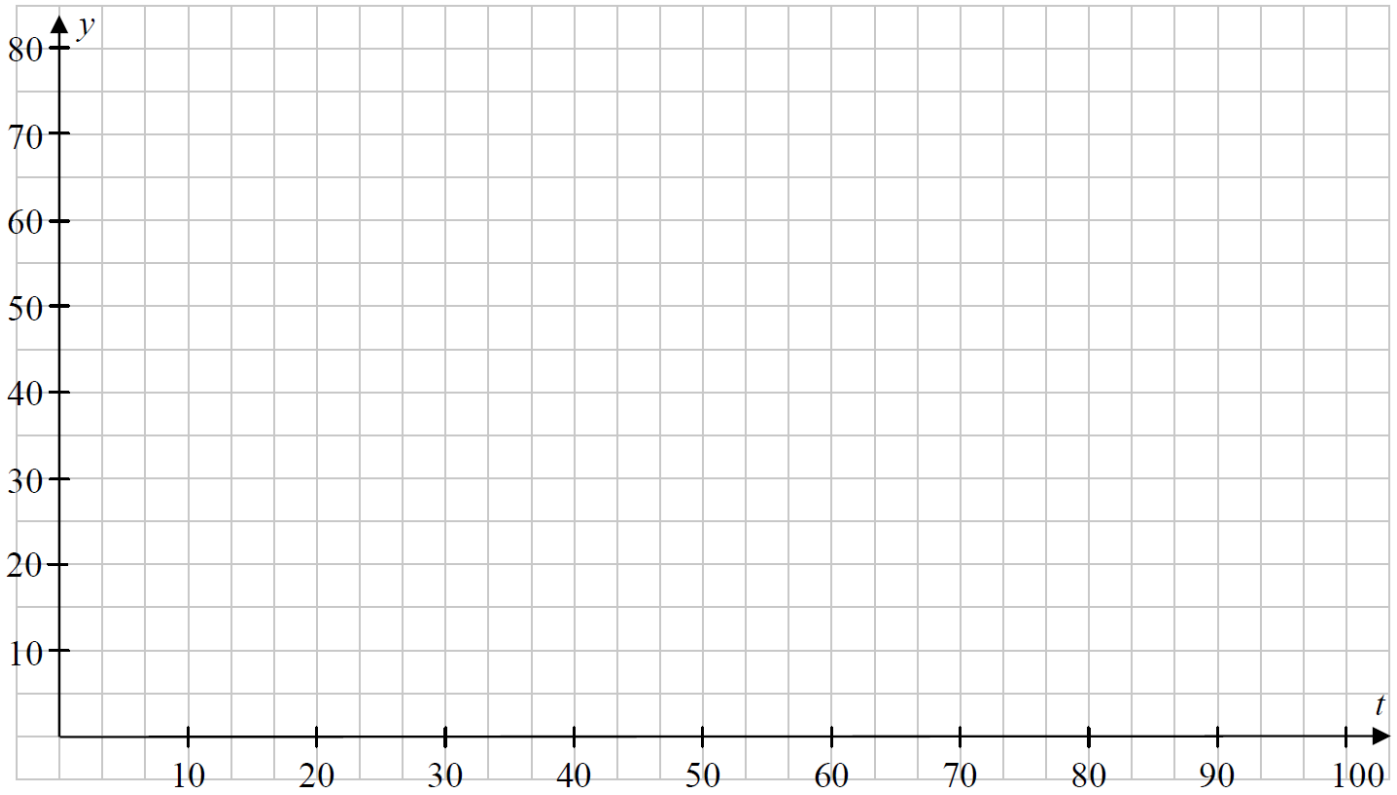
$A =$

**(b)** After five minutes, the temperature of the water is $88°C$.
Find the value of $k$, correct to three significant figures.

**(c)** Ciarán prepares the food for his baby when the water has cooled to $50°C$. How long does it take, correct to the nearest minute, for the water to cool to this temperature?



**(d)** Using your values for $A$ and $k$, sketch the curve $f(t) = Ae^{kt}$ for $0 \leq t \leq 100$, t $\in \mathbb{R}$.




**(e)**

**(i)** On the same diagram, sketch a curve $g(t) = Ae^{mt}$ showing the water cooling at a faster rate, where $A$ is the value from part (a), and $m$ is a constant. Label each graph clearly.

**(ii)** Suggest one possible value for $m$ for the sketch you have drawn and give a reason for your choice.

**(f)**

**(i)** Find the rates of change of the function $f(t)$ after 1 minute and after 10 minutes.

Give your answers correct to two decimal places.

**(ii)** Show that the rate of change of $f(t)$ will always increase over time.