Function Deconstruction

Deconstruct the Function

The following functions are presented WITHOUT questions. Your job is to deconstruct the function and rigorously analyse it to extrapolate as much information as possible.

1. **Set the Scene**

   An initial exploration into the story that is being presented. Understand the notation. Use inputs to find outputs. Try and establish a reasonable domain given the real-world context of the question.

2. **Visualise the Function**

   Find points, sketch the graph of the function.

3. **Using Outputs to find Inputs!**

   Read values from the graph. Given outputs solve equations to find corresponding inputs. Find the roots of the equation.

4. **What Does Differentiation Do?**

   - Rate of Change
   - Slope
   - Increasing / Decreasing
   - Maximum and Minimum
   - Point of Inflection
   - Distance → Speed → Acceleration

5. **What Does Integration Do?**

   - Area under curve
   - Average value of the function
   - Integration as the reverse of differentiation
   - Acceleration → Speed → Distance

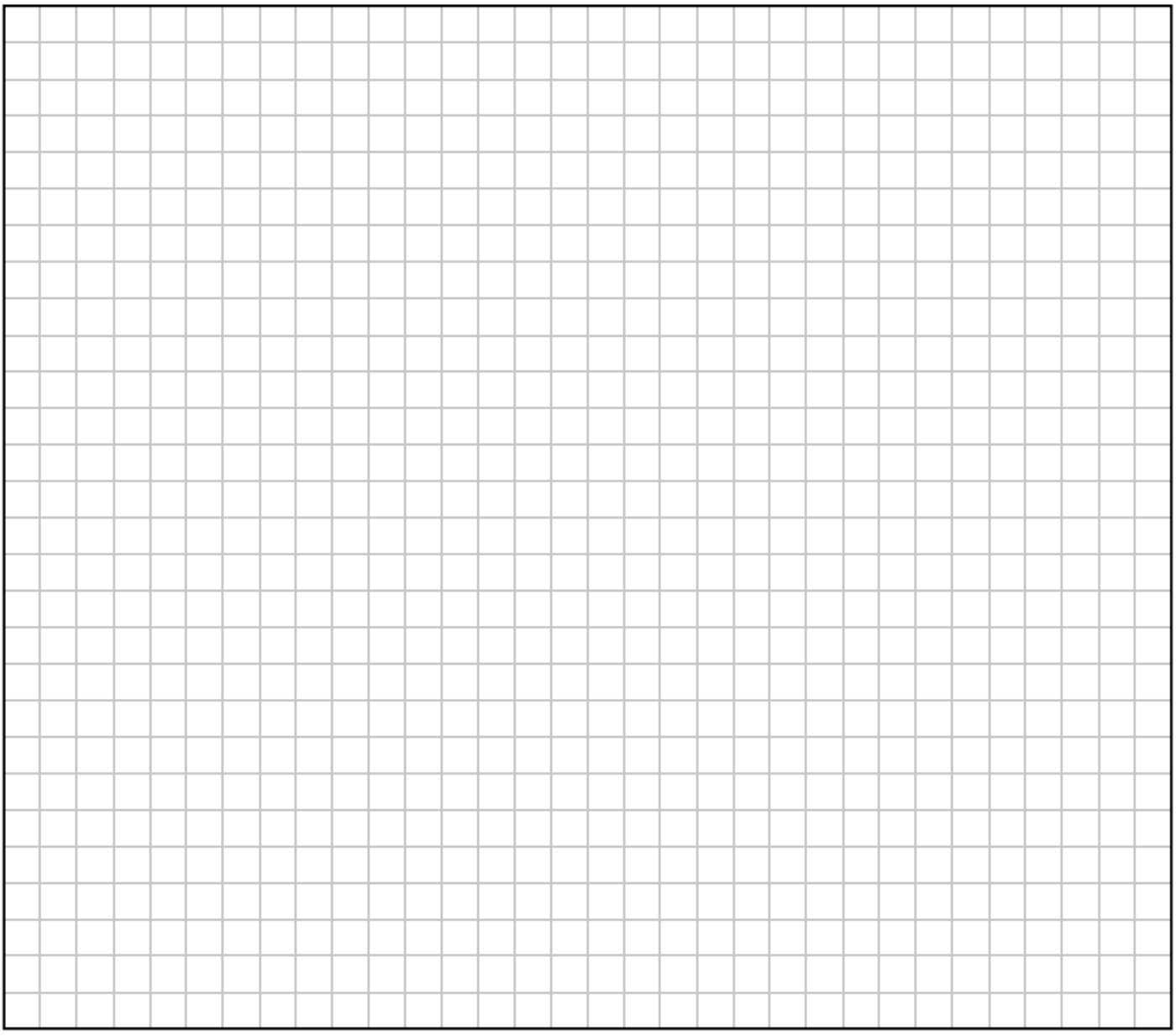
Click on the link for the IMTA presentation "How I Teach Functions and Calculus".

▶ 2024 LCHL Paper 1 – Question 7 (c)

Fiadh puts money in a savings account, and leaves it there for a number of years.
The following expression gives $F(t)$, the amount of money in the account in euro after t years, where $t \in \mathbb{R}$, $t \geq 0$:

$$F(t) = 5000e^{0.04t}$$

1. Initial Exploration (Set the Scene)



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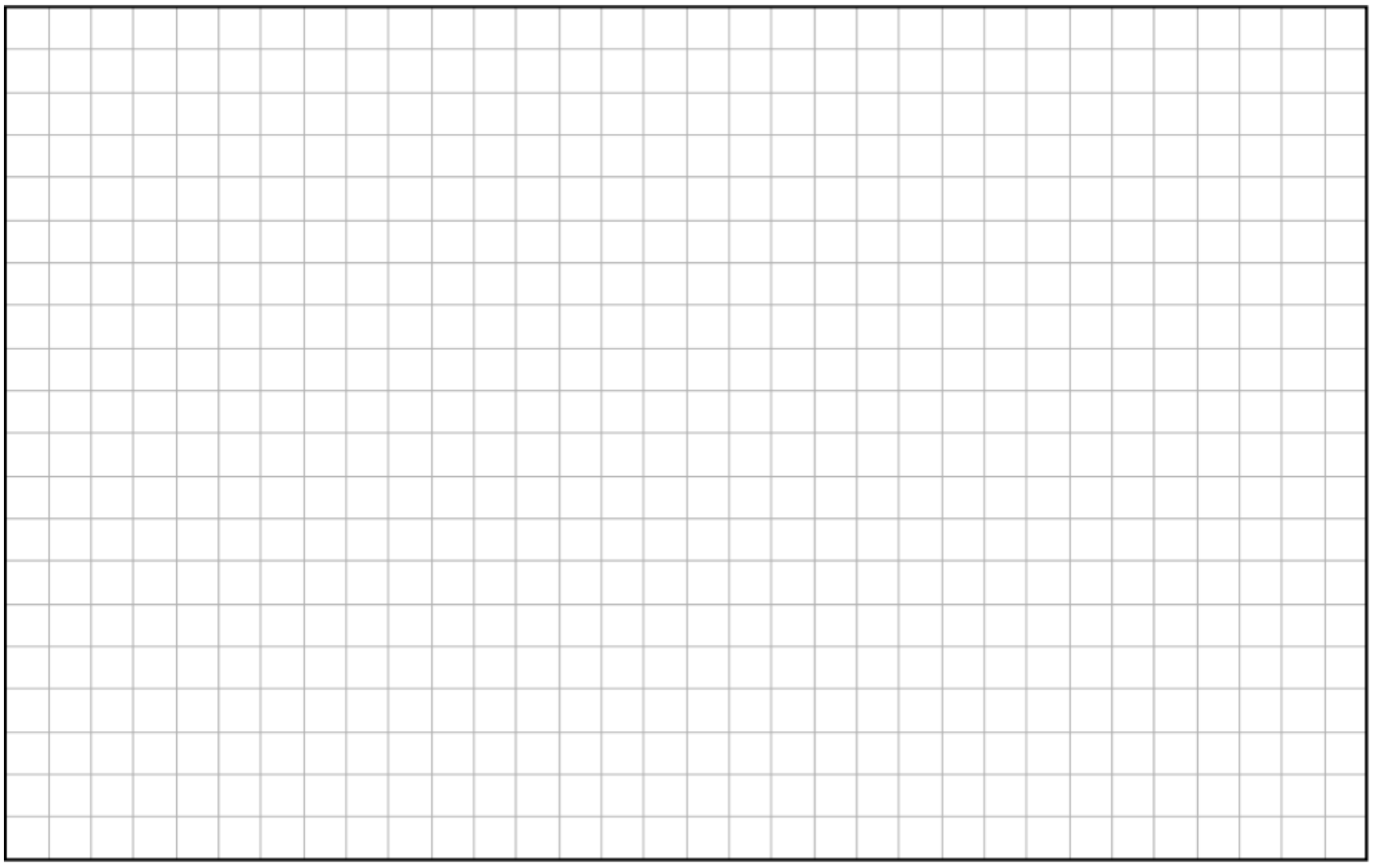
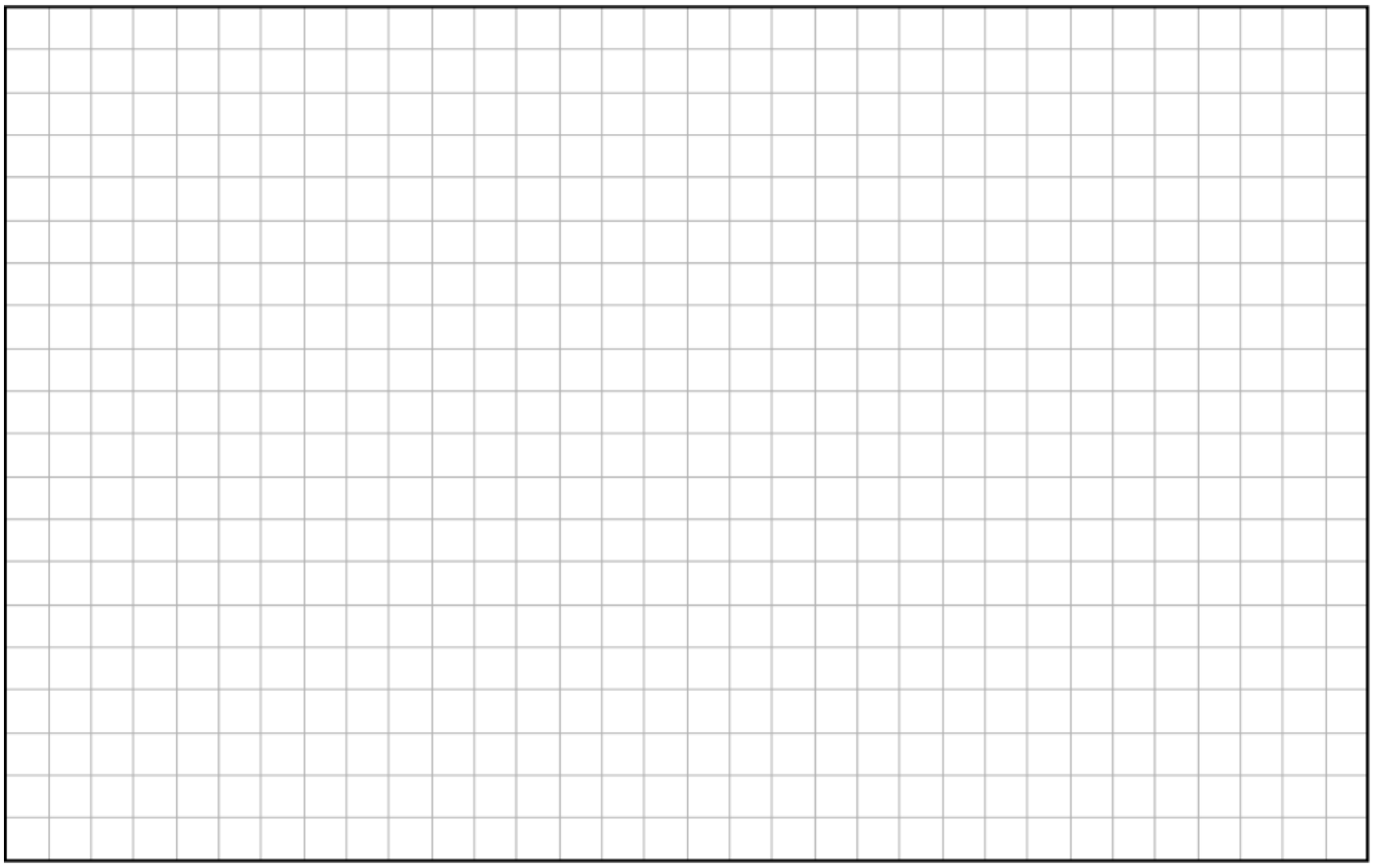


3. Using outputs to find inputs!



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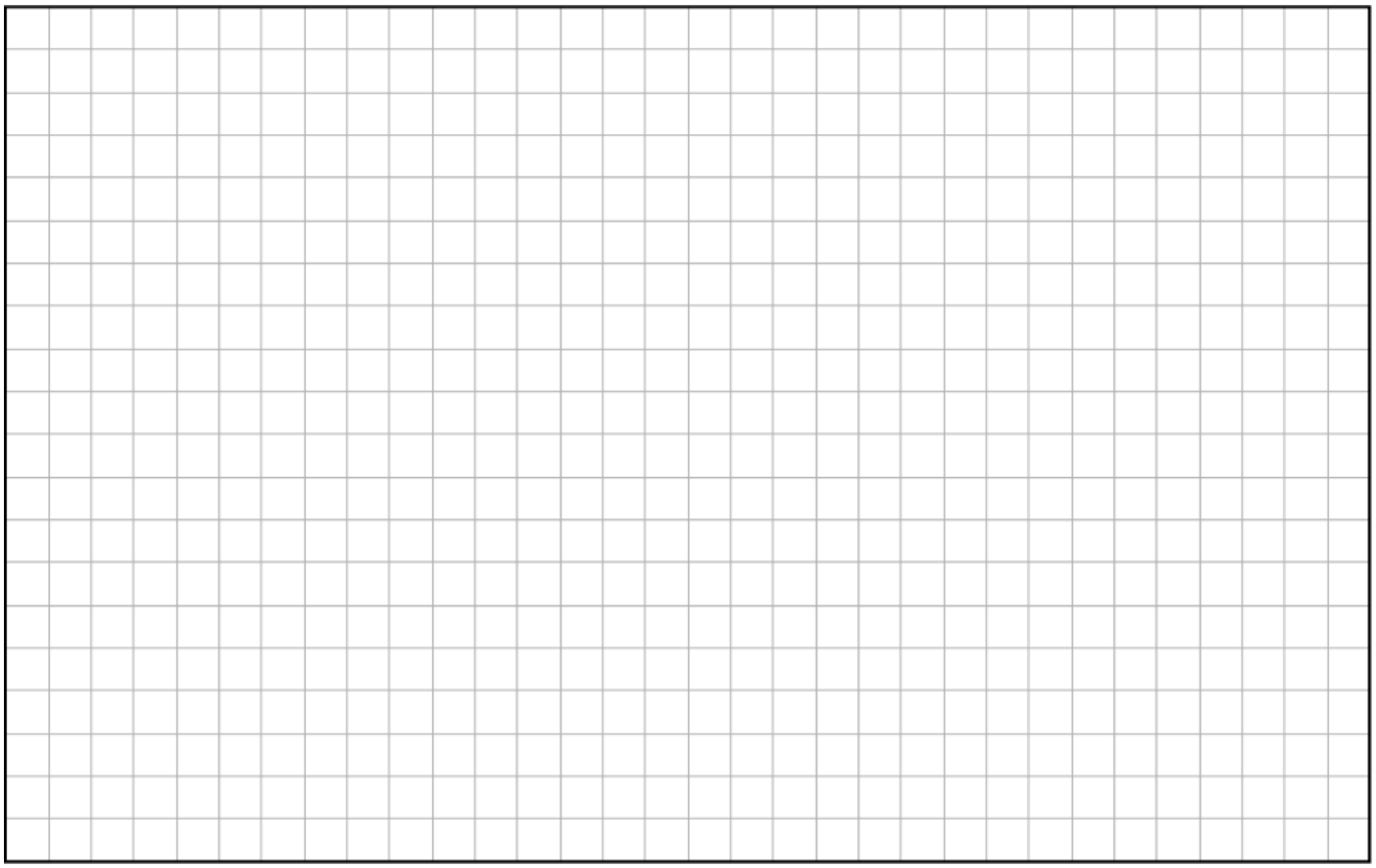

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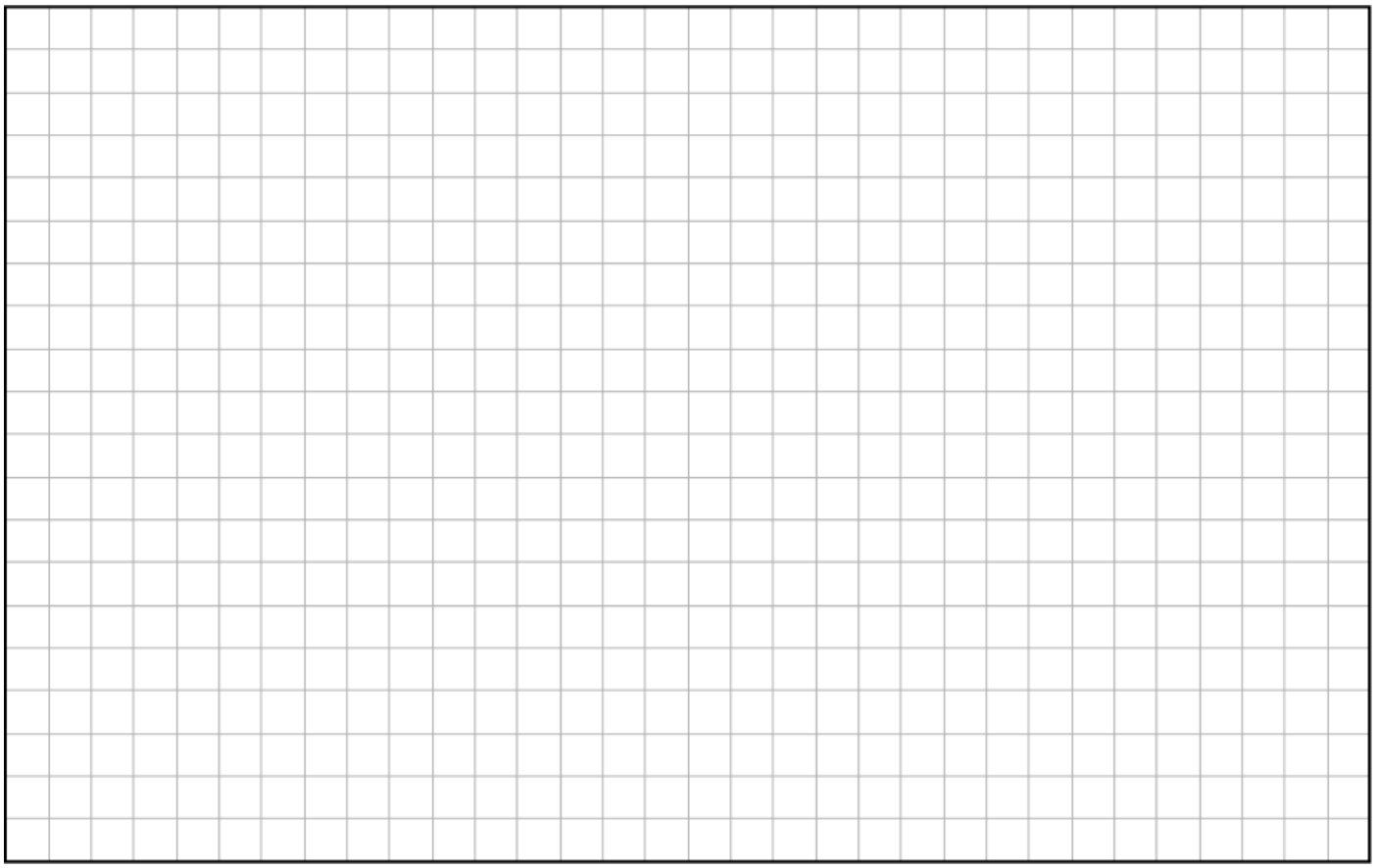
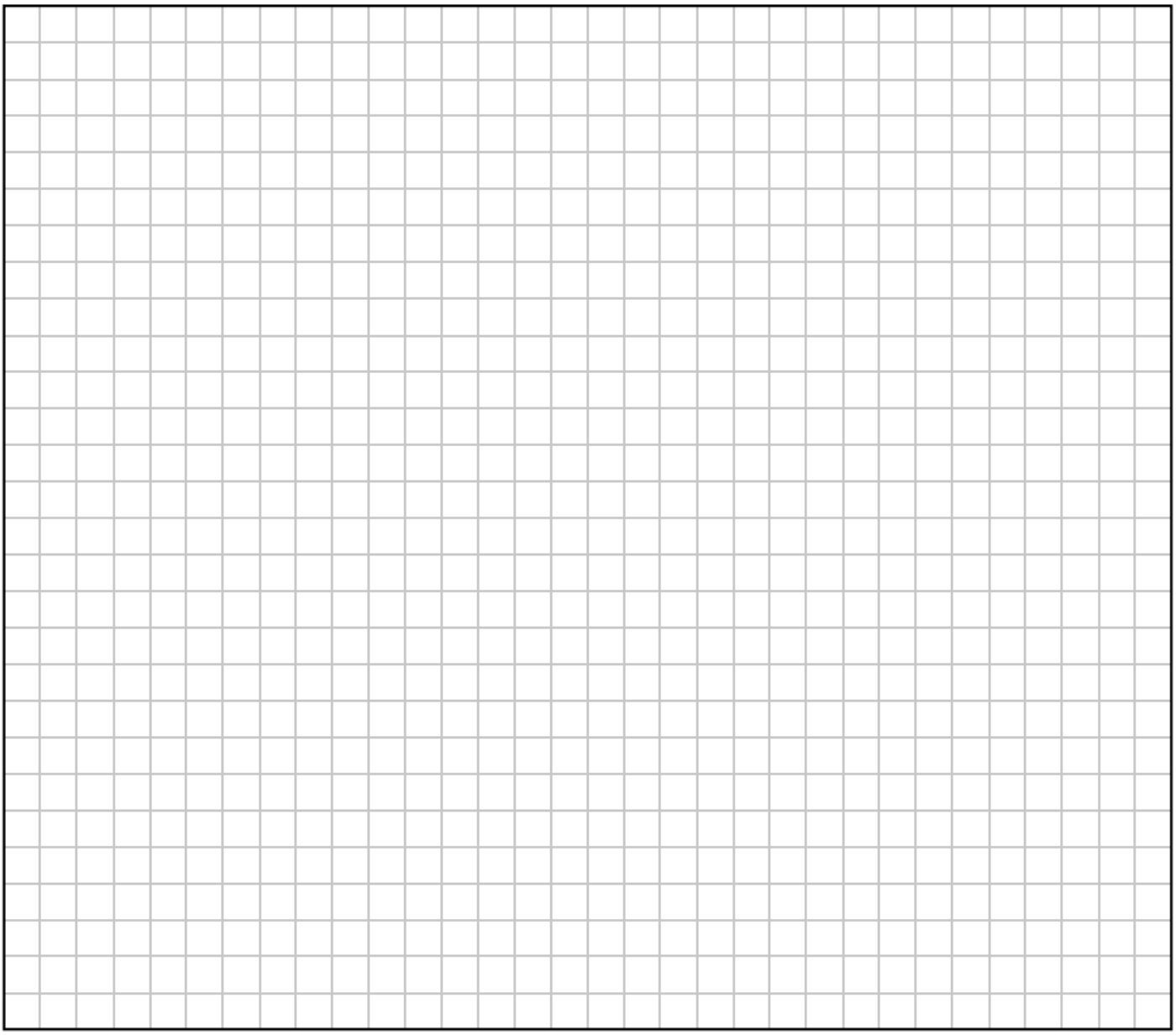


▶ 2024 LCHL Paper 1 – Question 8 (a)

A clothing company sells t-shirts. The daily sales of t-shirts over a 360-day period can be modelled using the following function, $T(x)$, where $0 \leq x \leq 360,\ x \in \mathbb{R}$:

$$T(x) = \left(\frac{x - 240}{60}\right)^3 + 70$$

1. Initial Exploration (Set the Scene)



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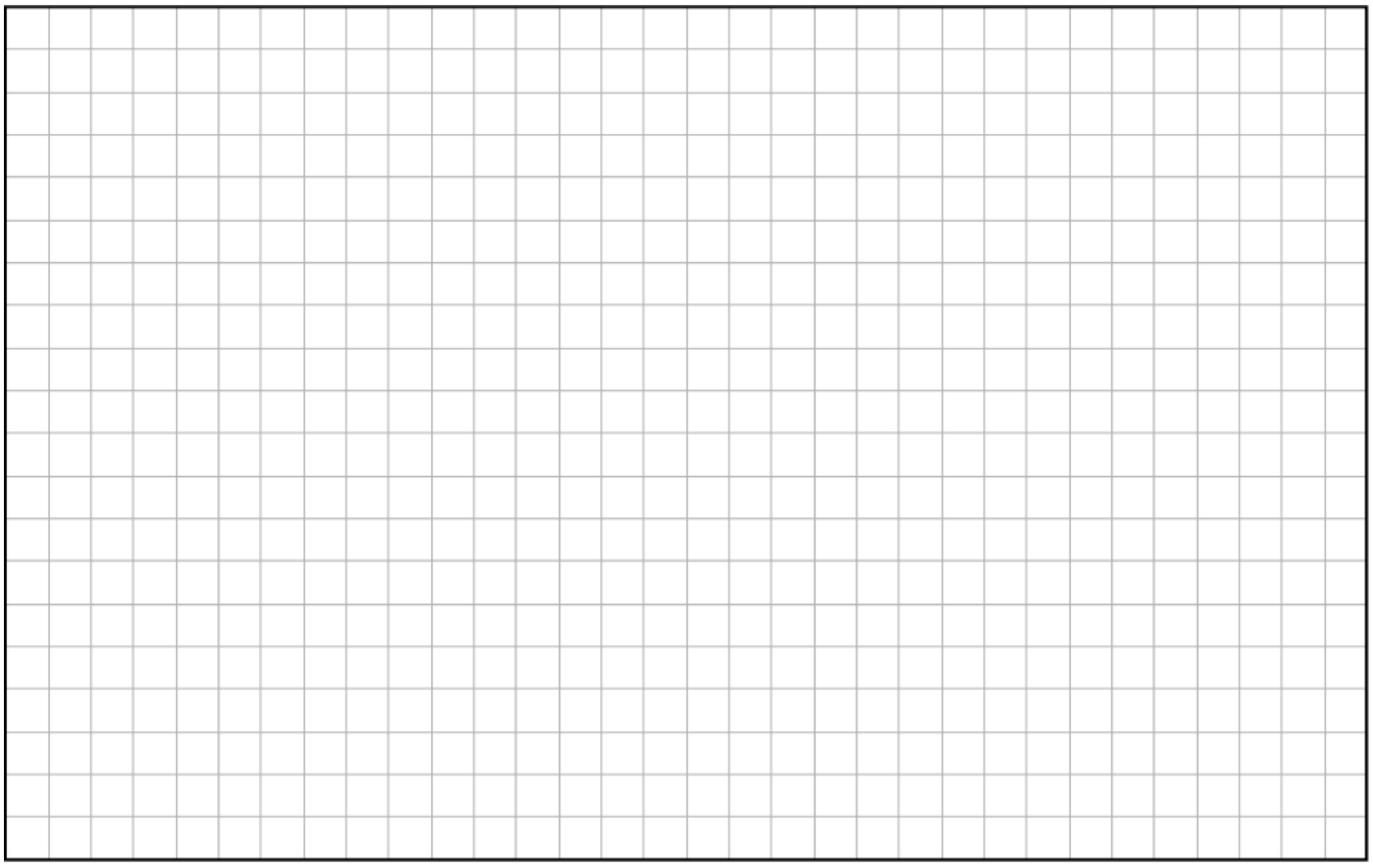

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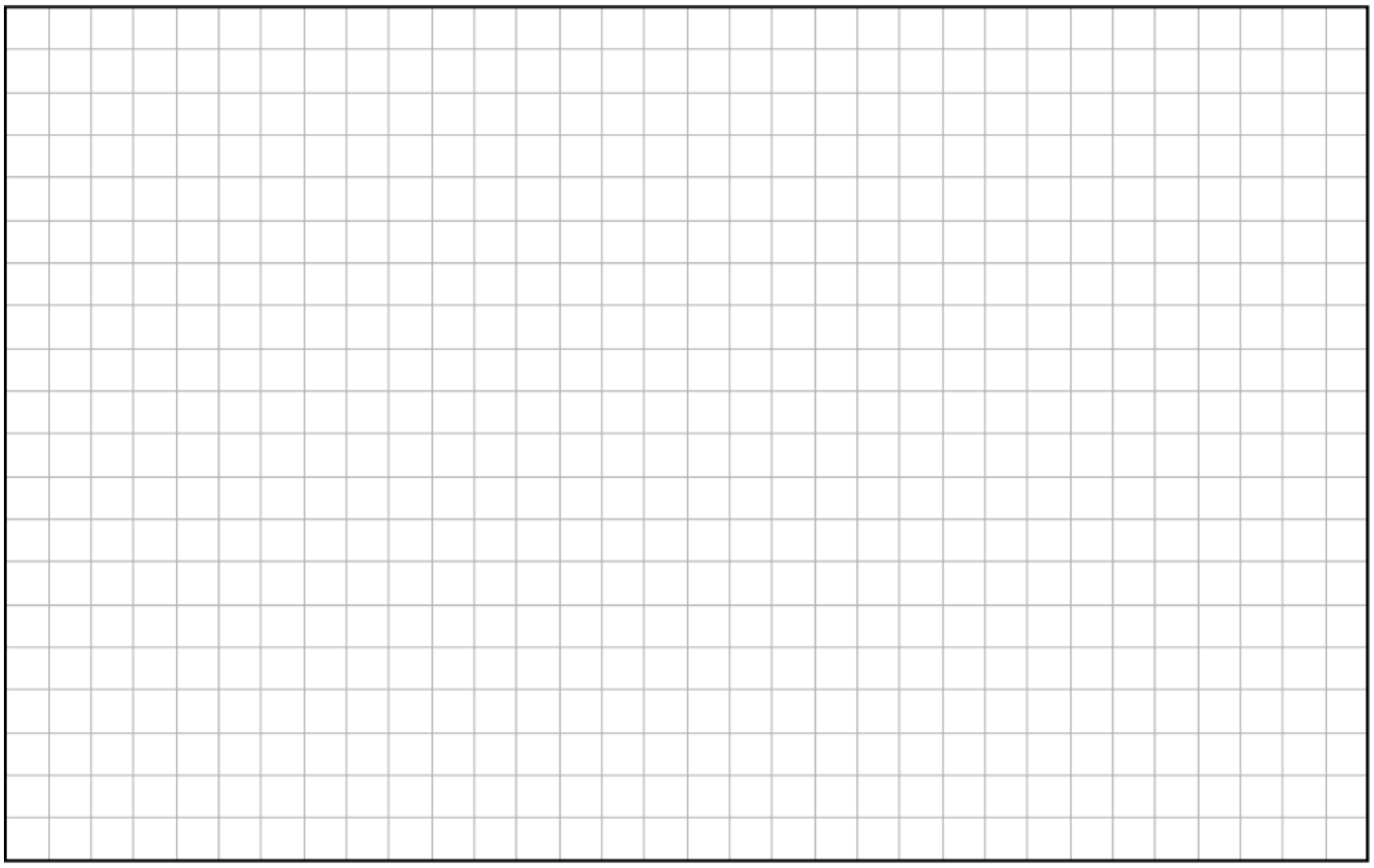

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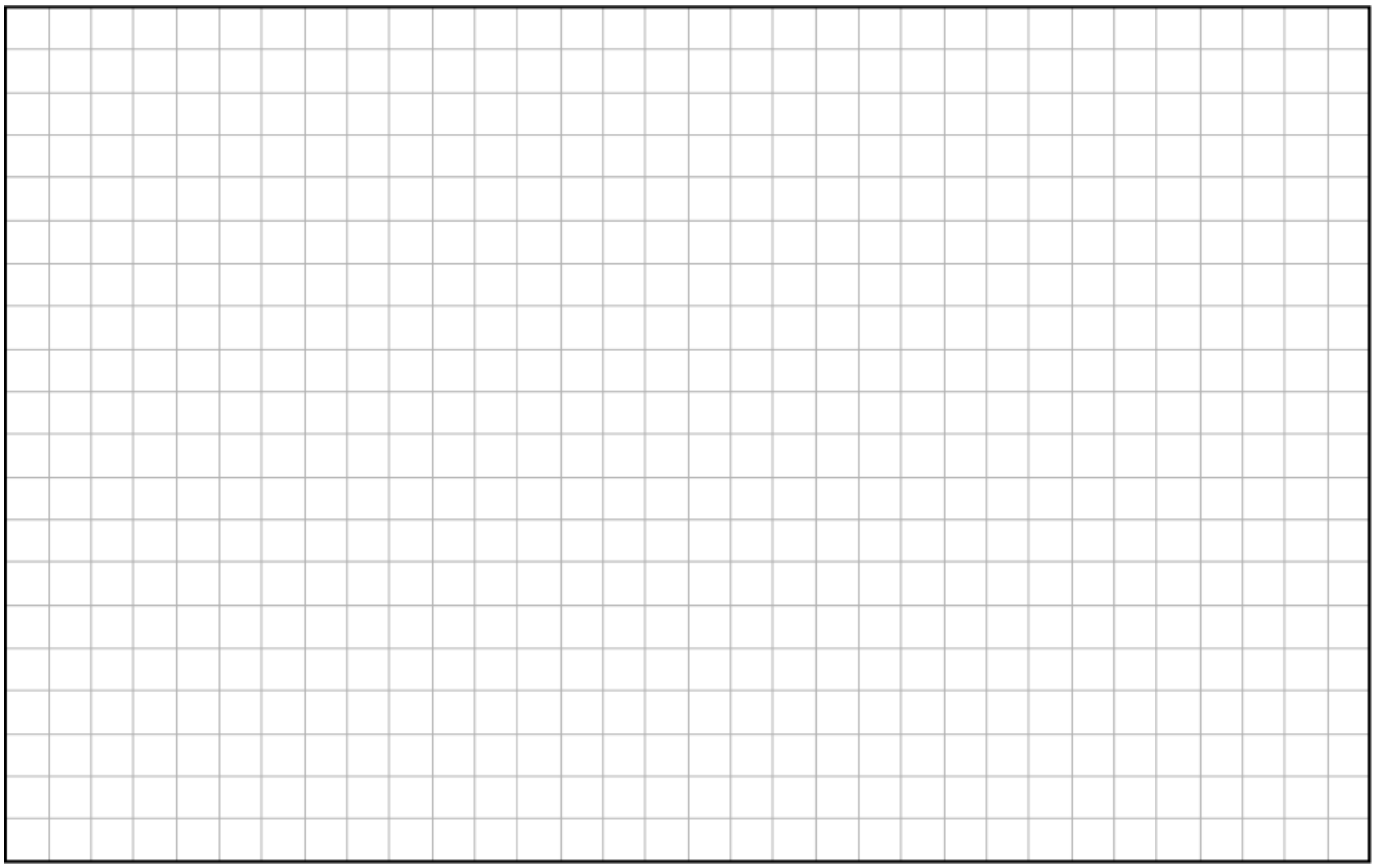

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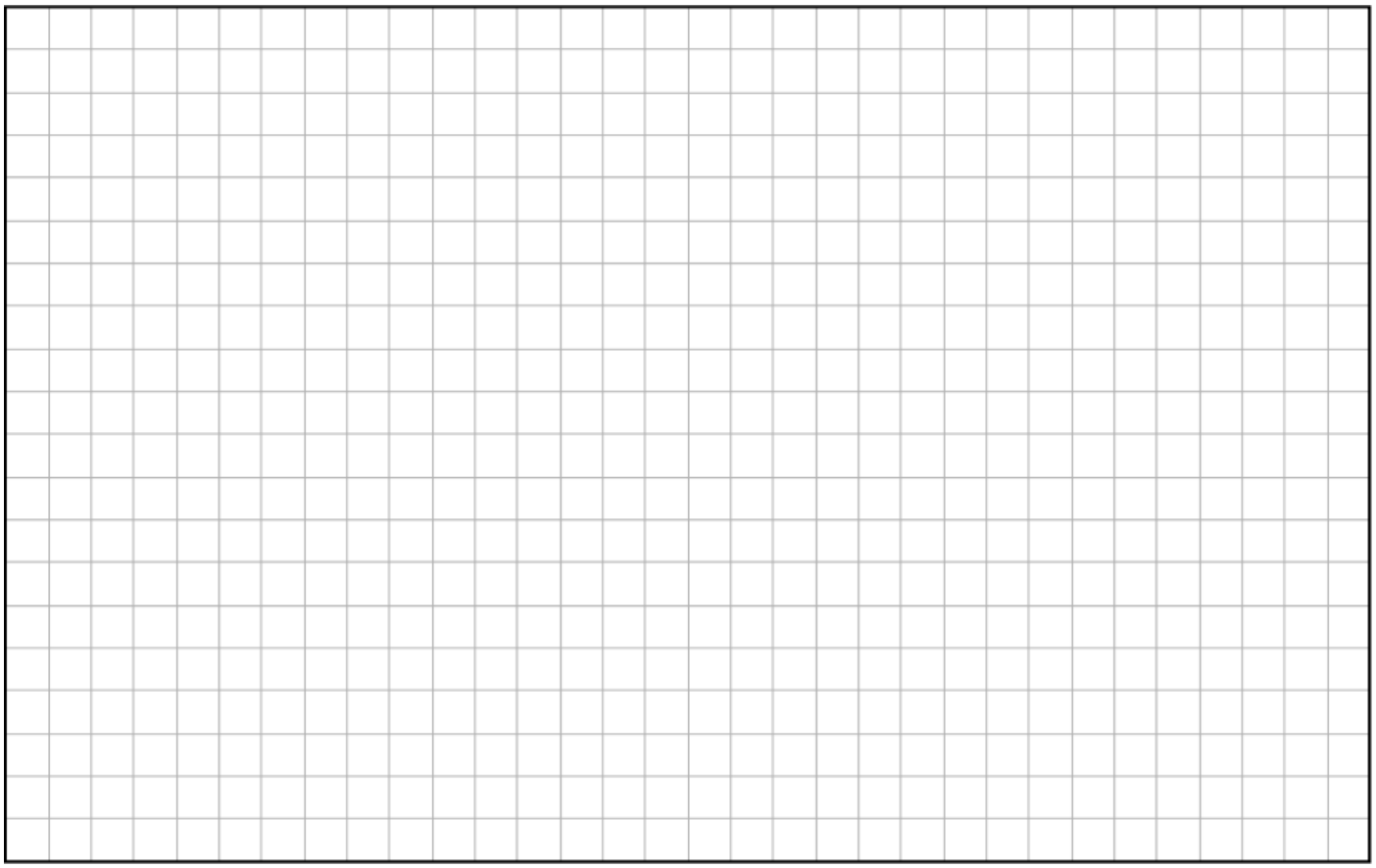

▶ 2024 LCHL Paper 1 – Question 8 (b)

The clothing company also sells scarves. The daily sales of scarves can be modelled using the following function, $S(t)$, which is a periodic function with a period of 365 days:

$$S(t) = 21 + 19\cos\left(\frac{2\pi t}{365}\right)$$

Here, $t \in \mathbb{R}$ is the time, in days, from 1 January 2024, $t \geq 0$, and $\frac{2\pi t}{365}$ is in radians.

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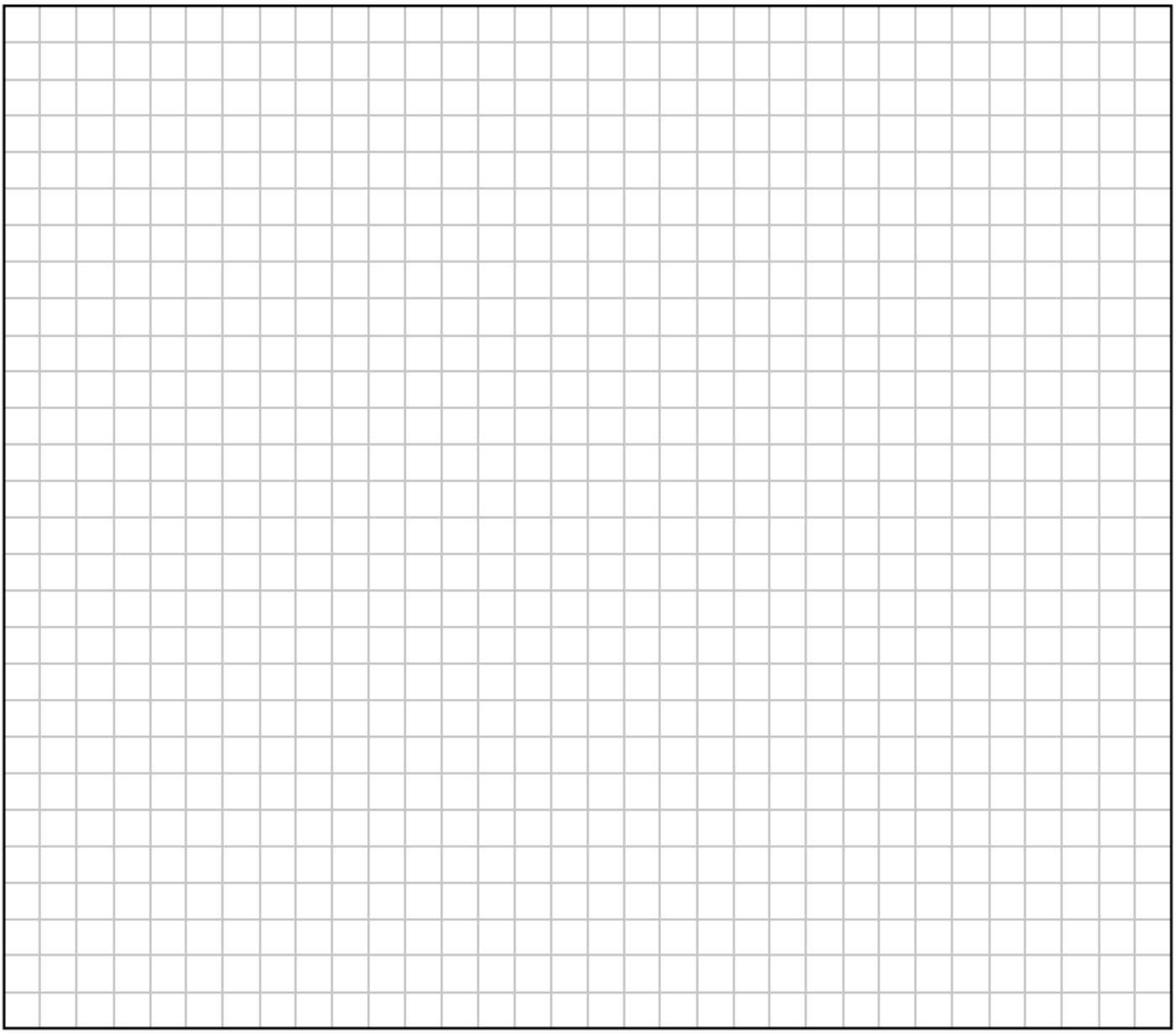

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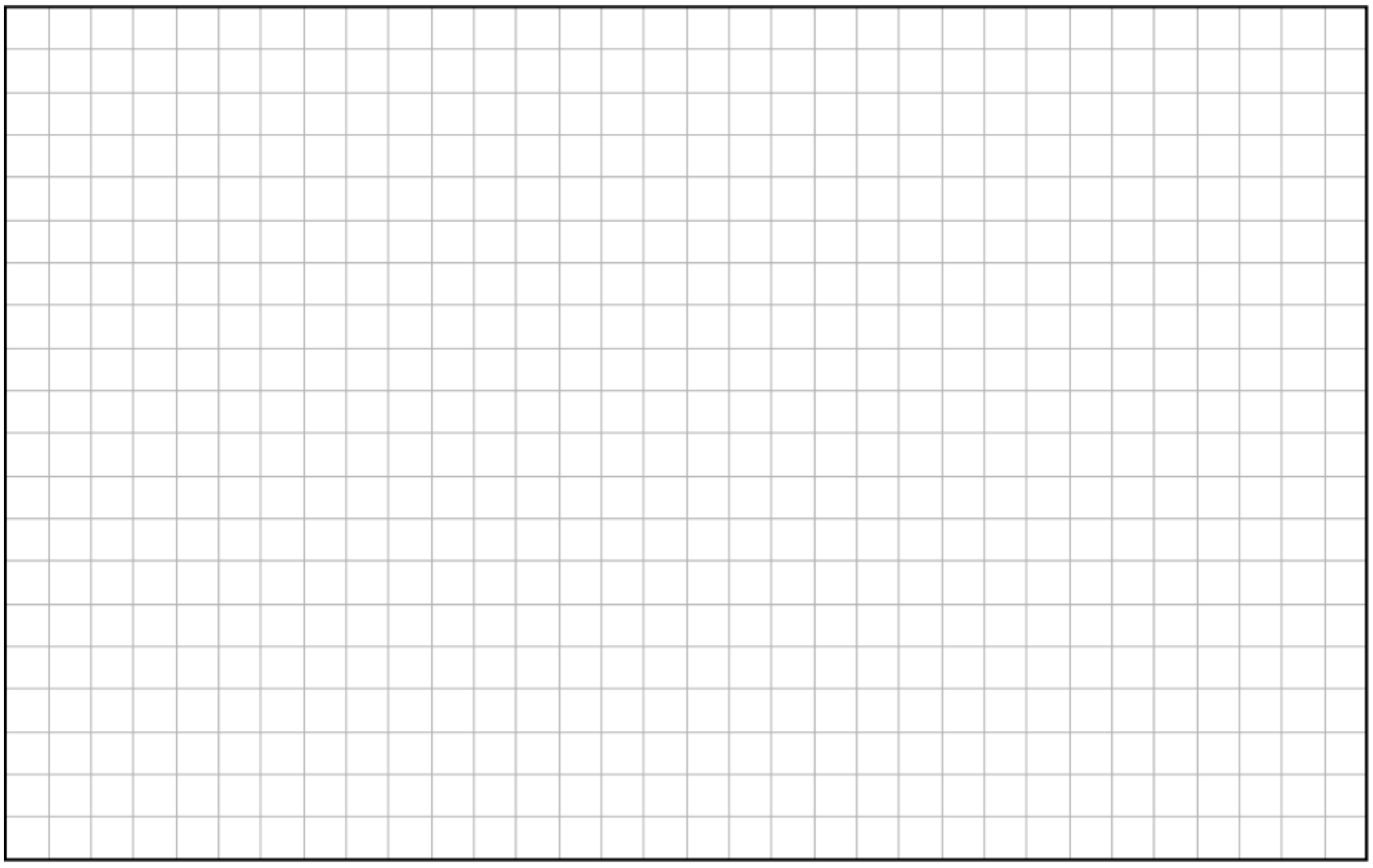

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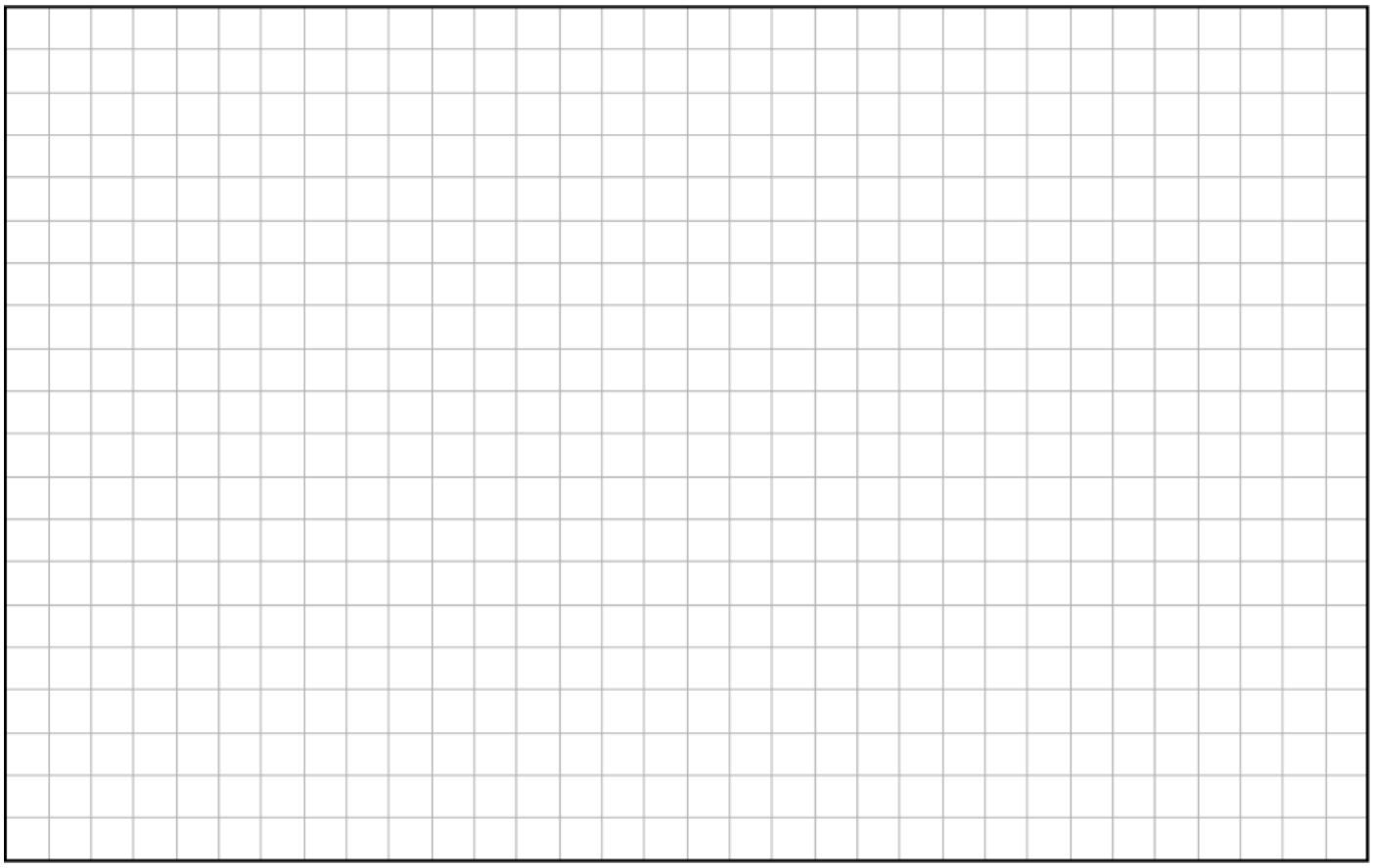

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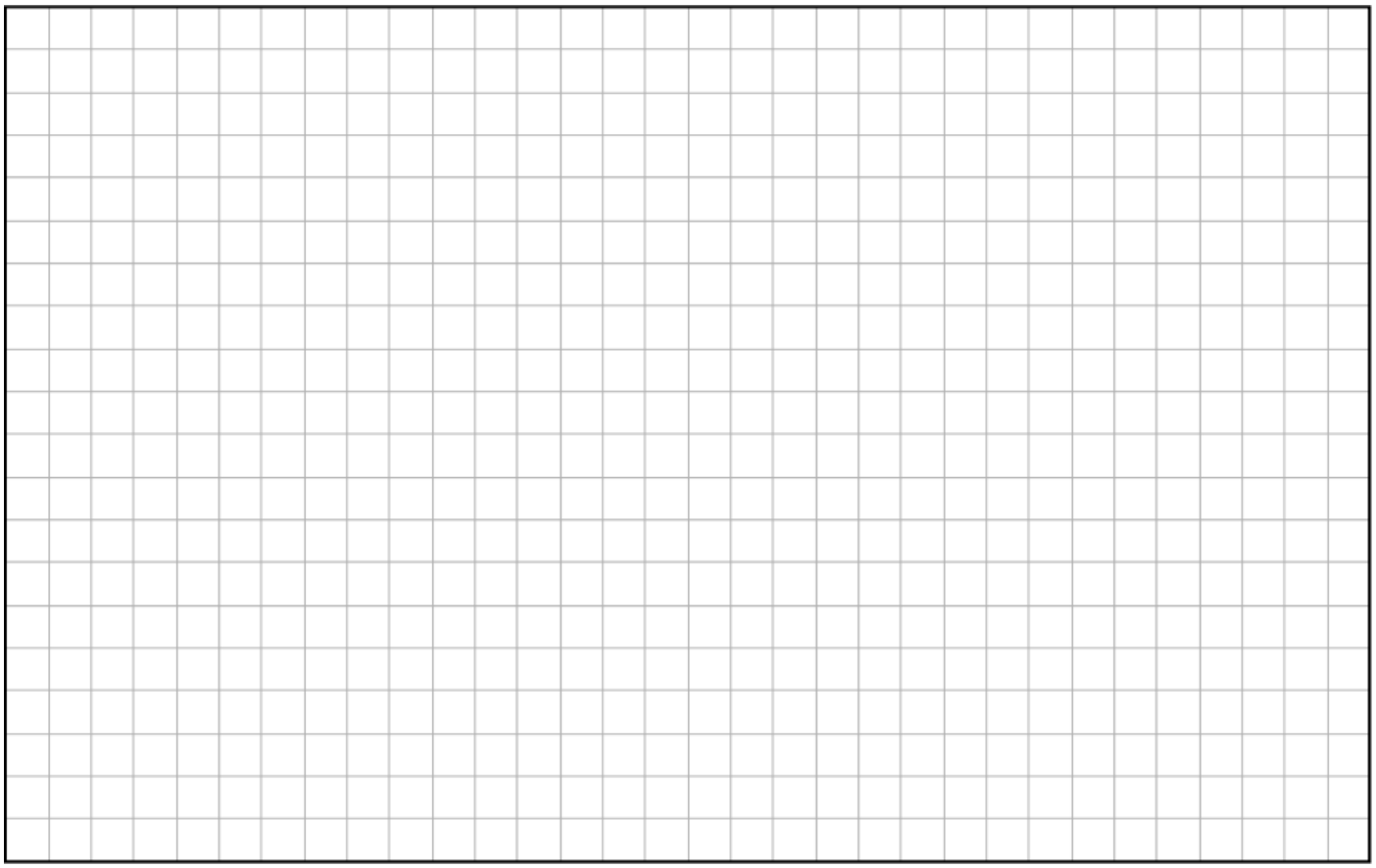

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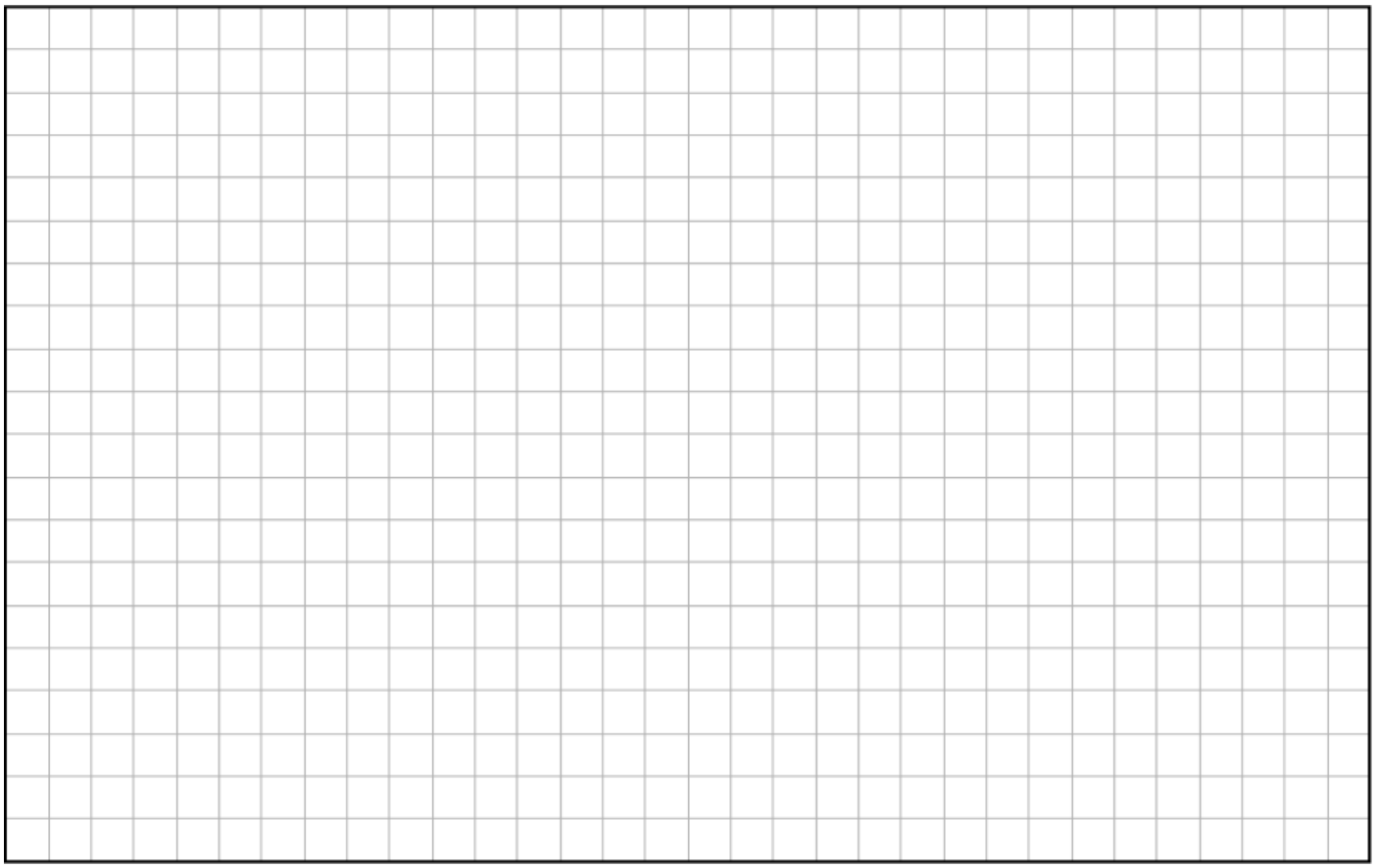


▶ 2024 LCHL Paper 1 – Question 10

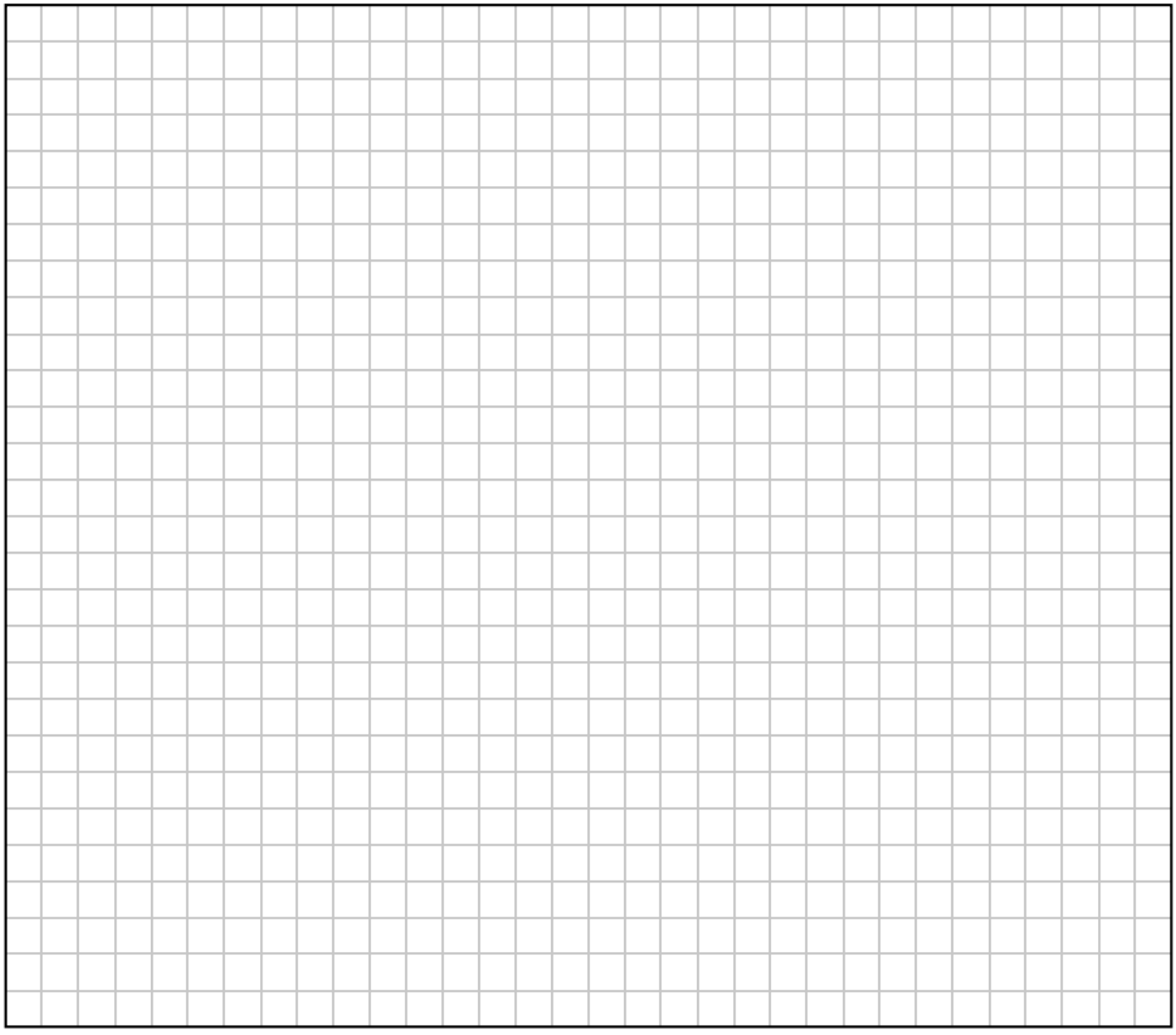
A company grows and sells plants.

The function $W(x)$ is defined below. It can be used to model the height, in mm, of a water spinach plant, for the first 35 days after it starts to grow.

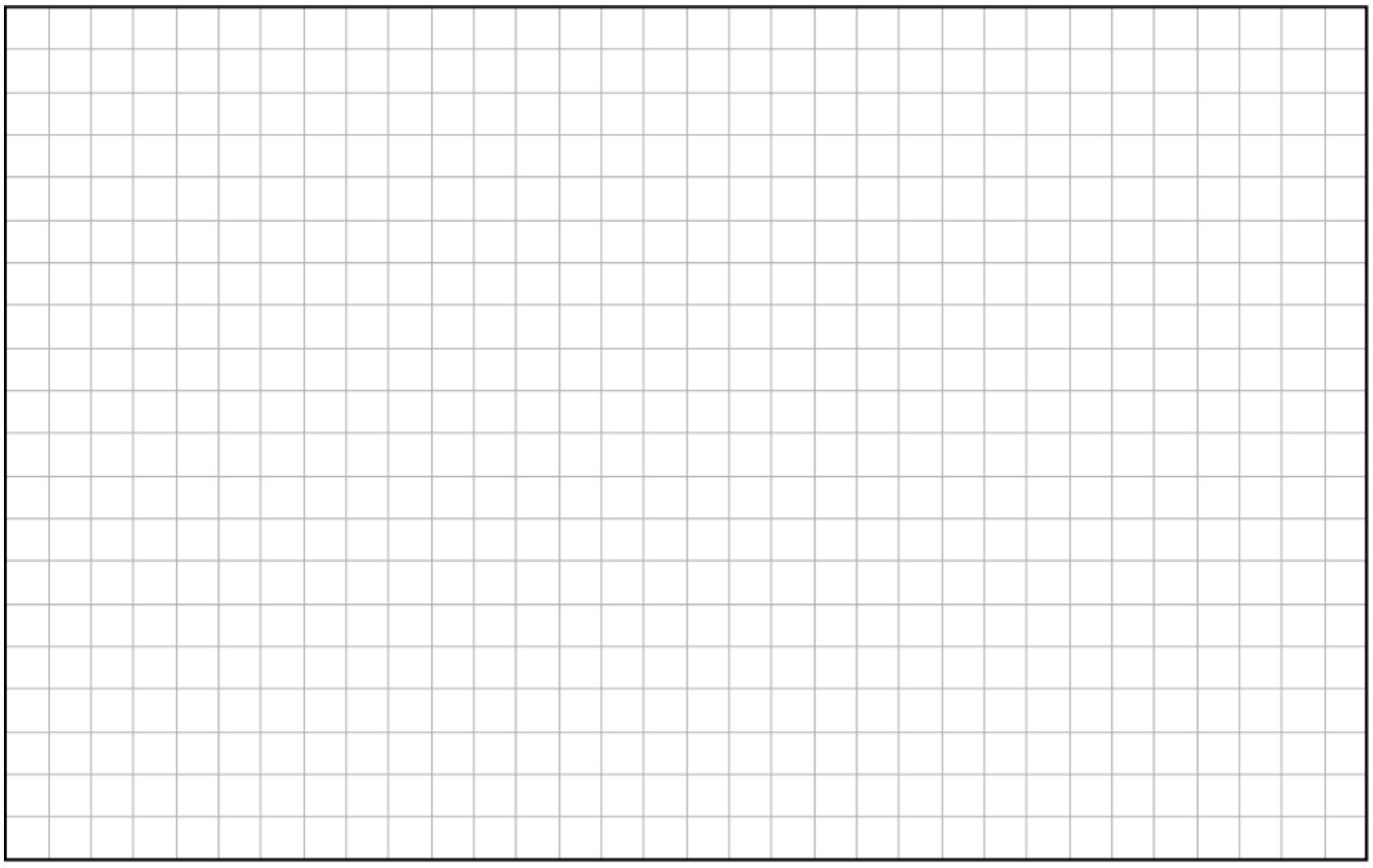
$$W(x) = 0.667x + 1.5x^2 - 0.025x^3$$

Here, x is the number of days after the plant starts to grow, where $0 \leq x \leq 35,\ x \in \mathbb{R}$.

1. Initial Exploration (Set the Scene)



2. **Visualise the Function**



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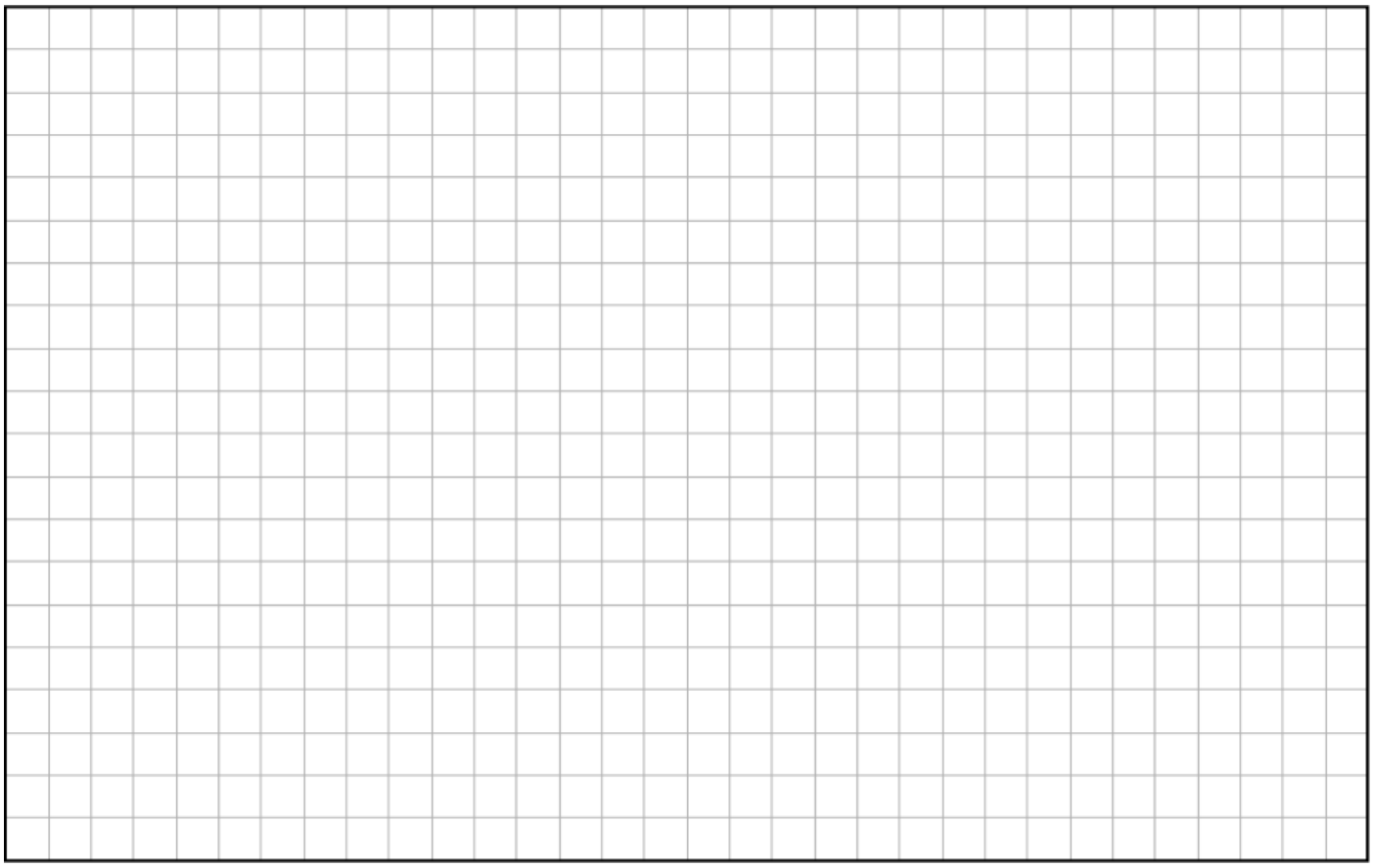

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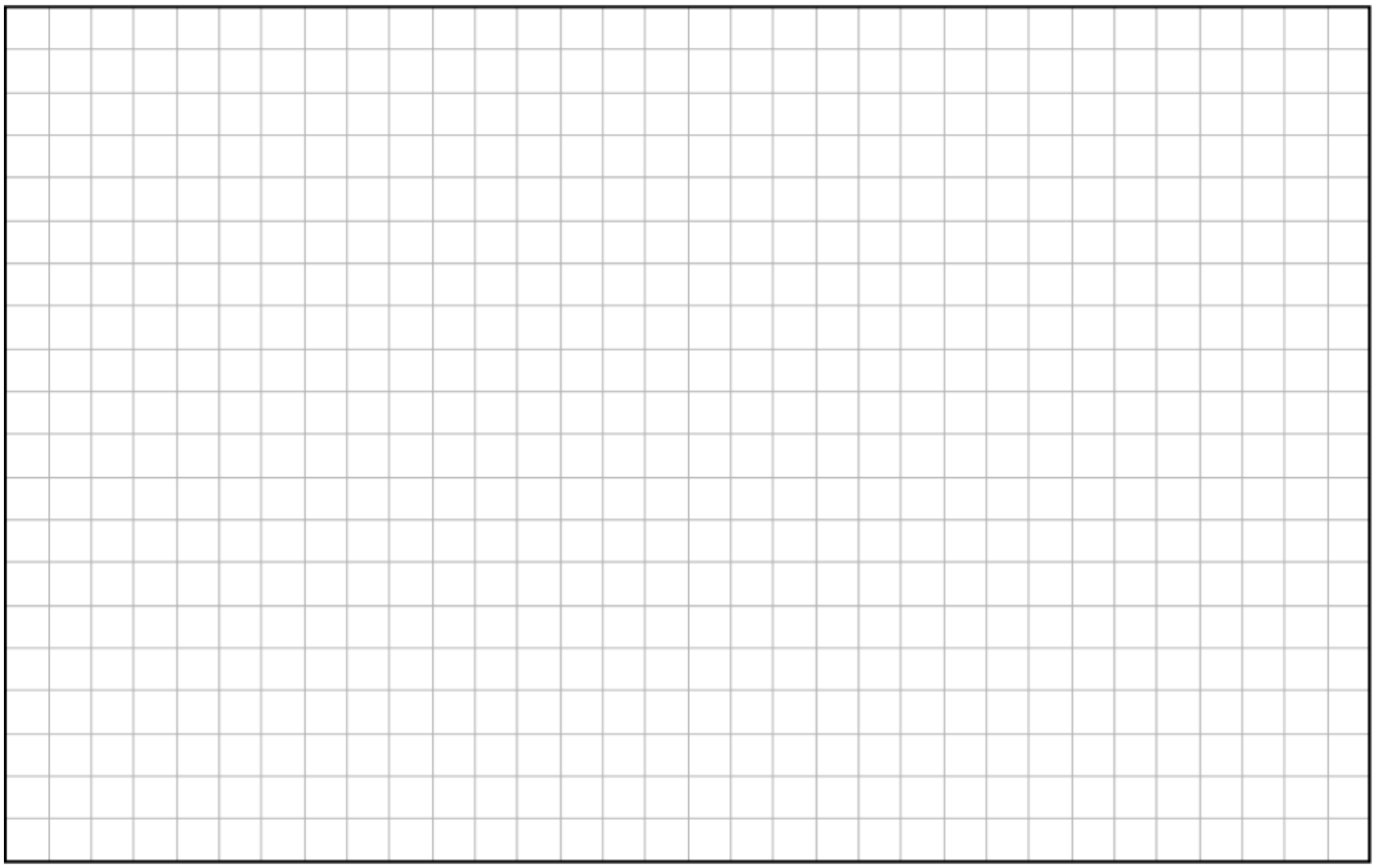

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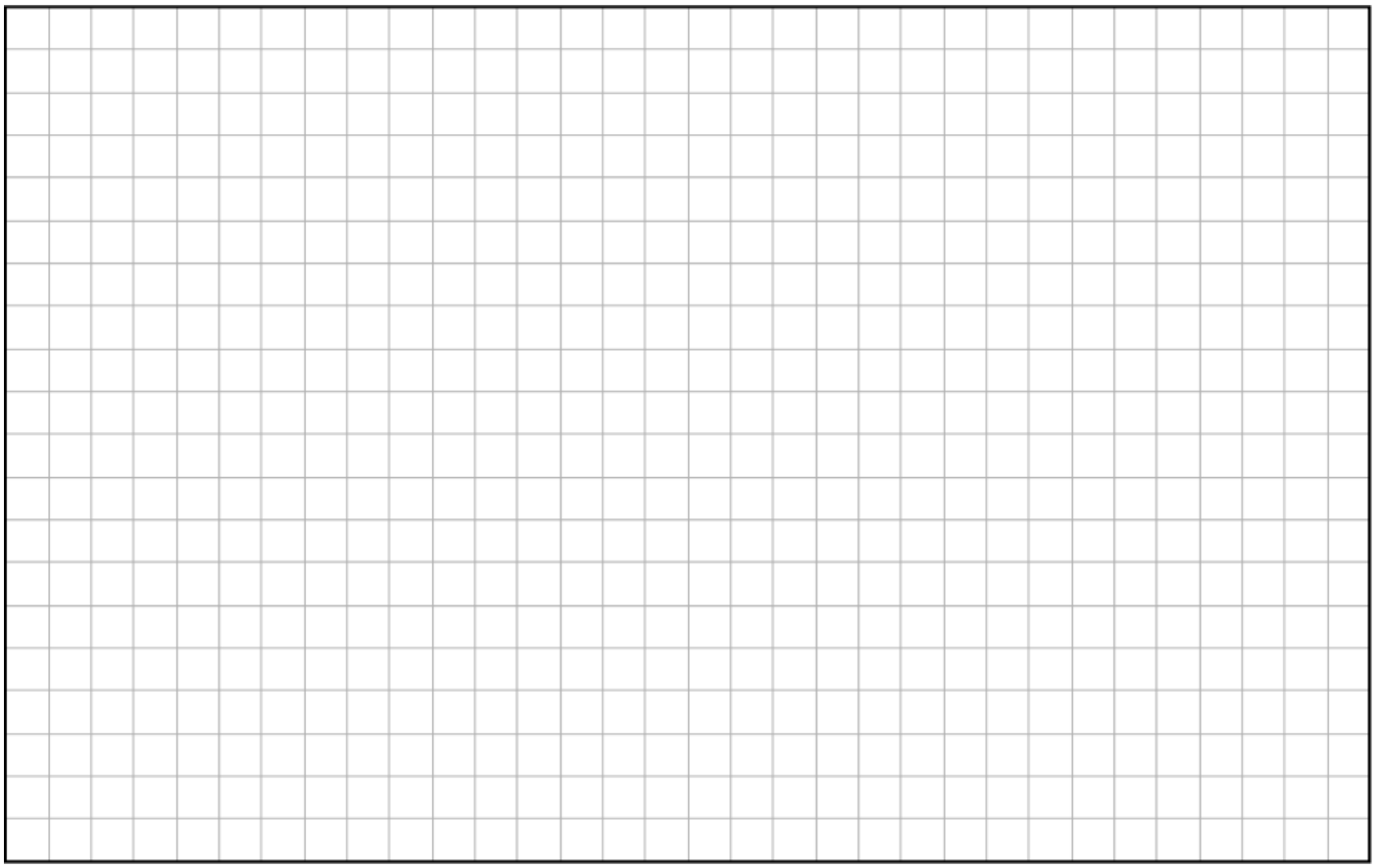


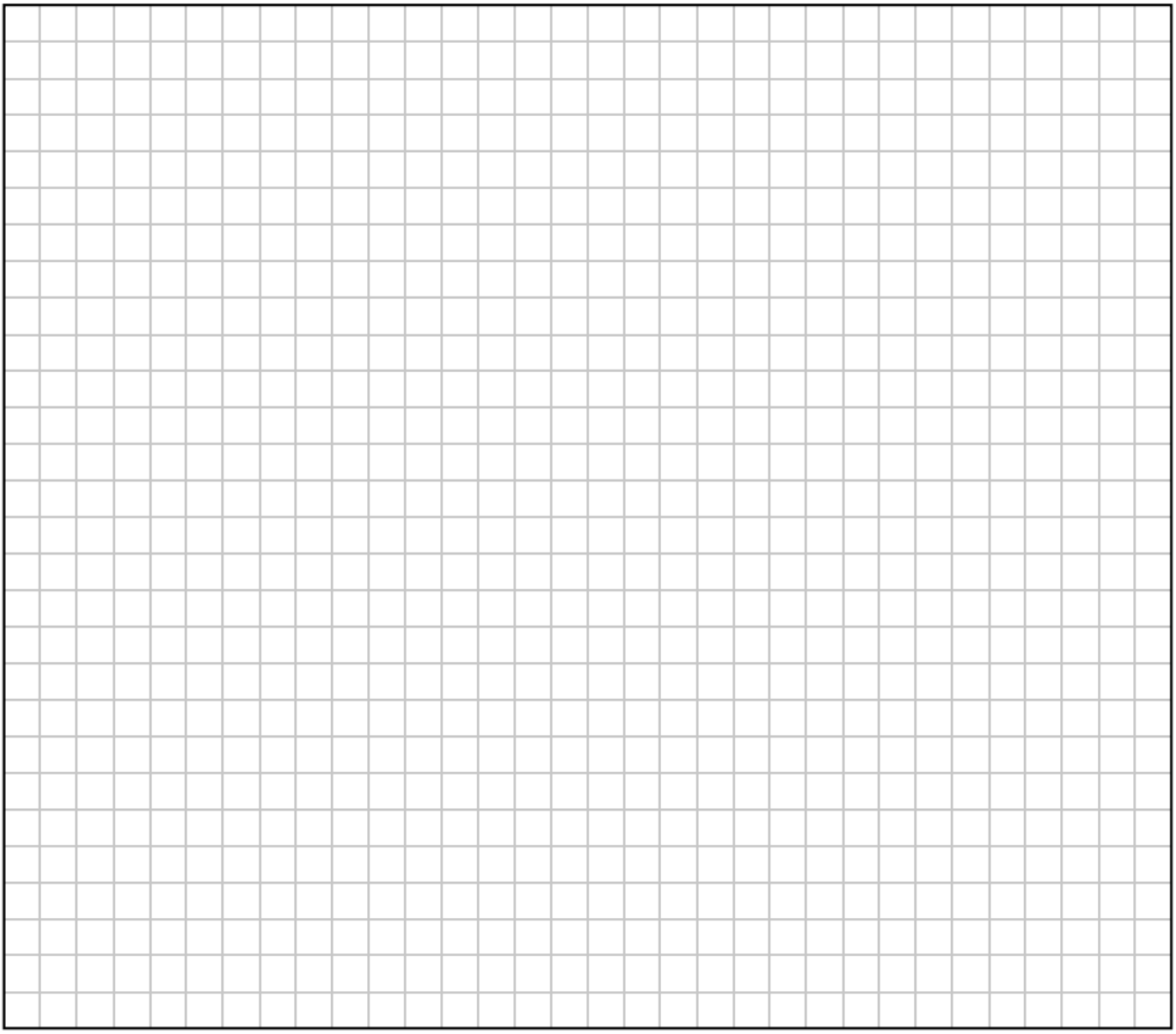
▶ 2023 LCHL Paper 1 – Question 7

Fiona is driving on a motorway. She passes a point **A** on the motorway. Her speed is given by:

$$v(t) = \frac{2}{3}t^3 - 6t^2 + 13t + 109$$

where v is her speed in km/hour t minutes after passing the point **A**, for $0 \leq t \leq 5$ and $t \in \mathbb{R}$.

1. Initial Exploration (Set the Scene)



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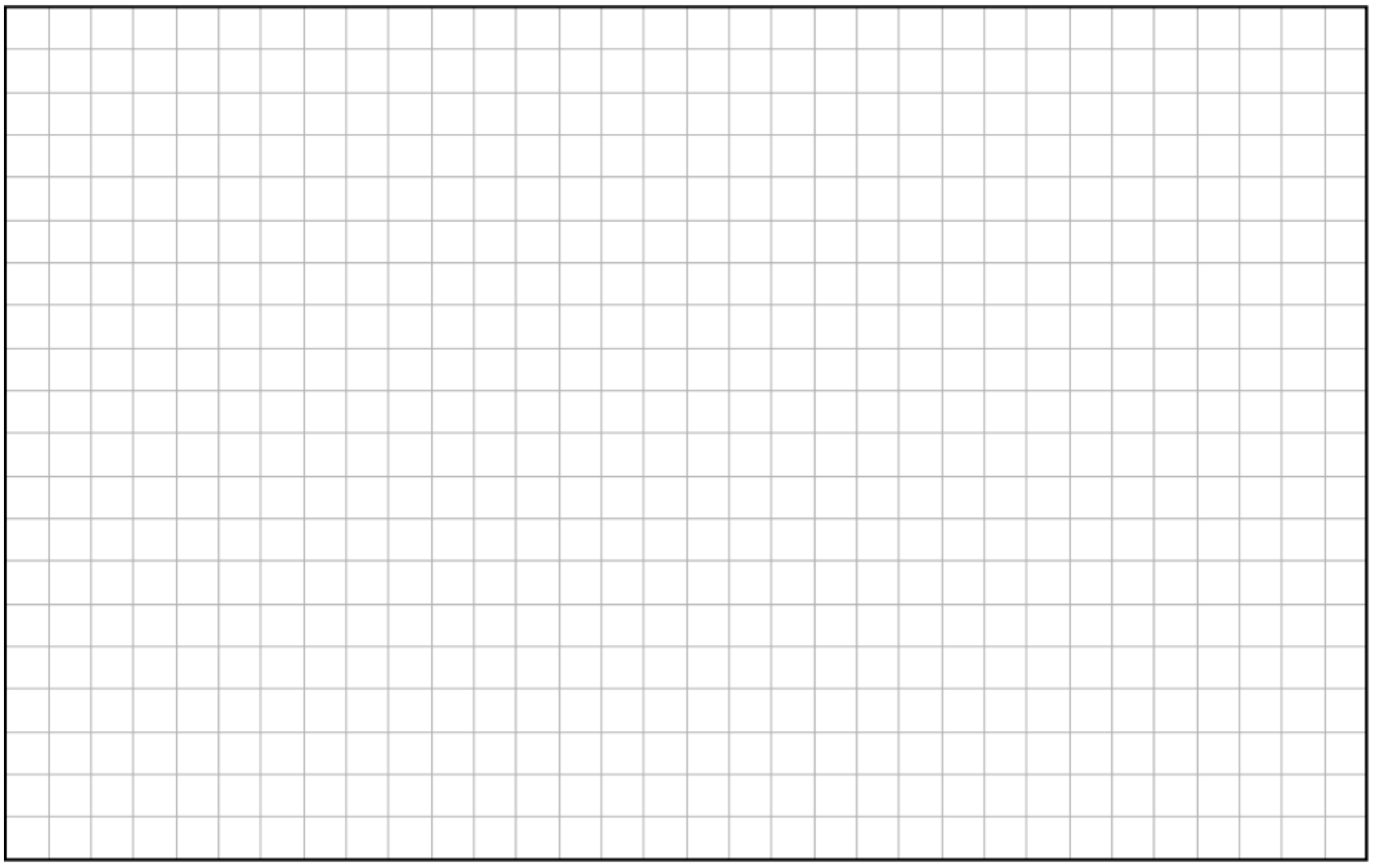

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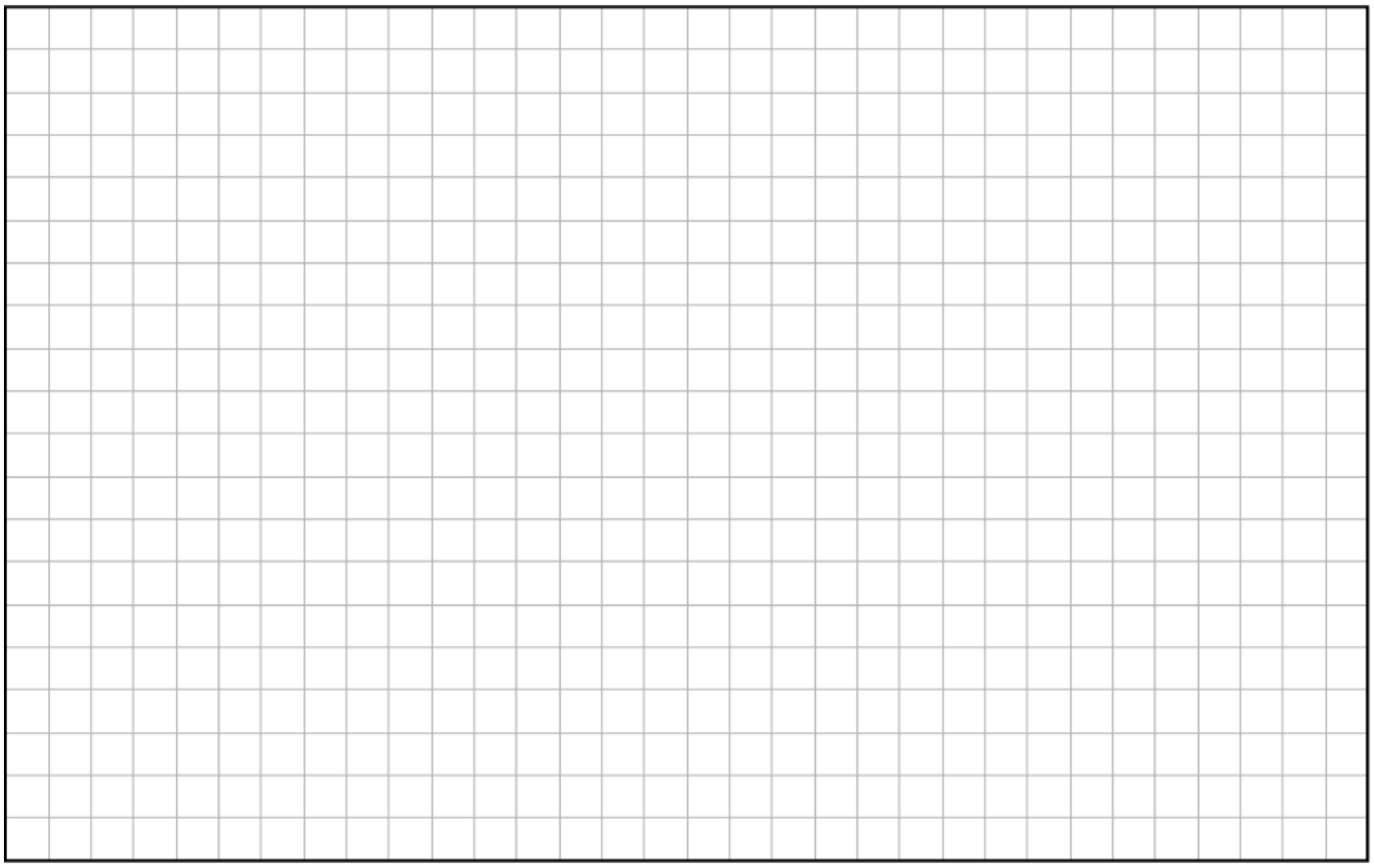

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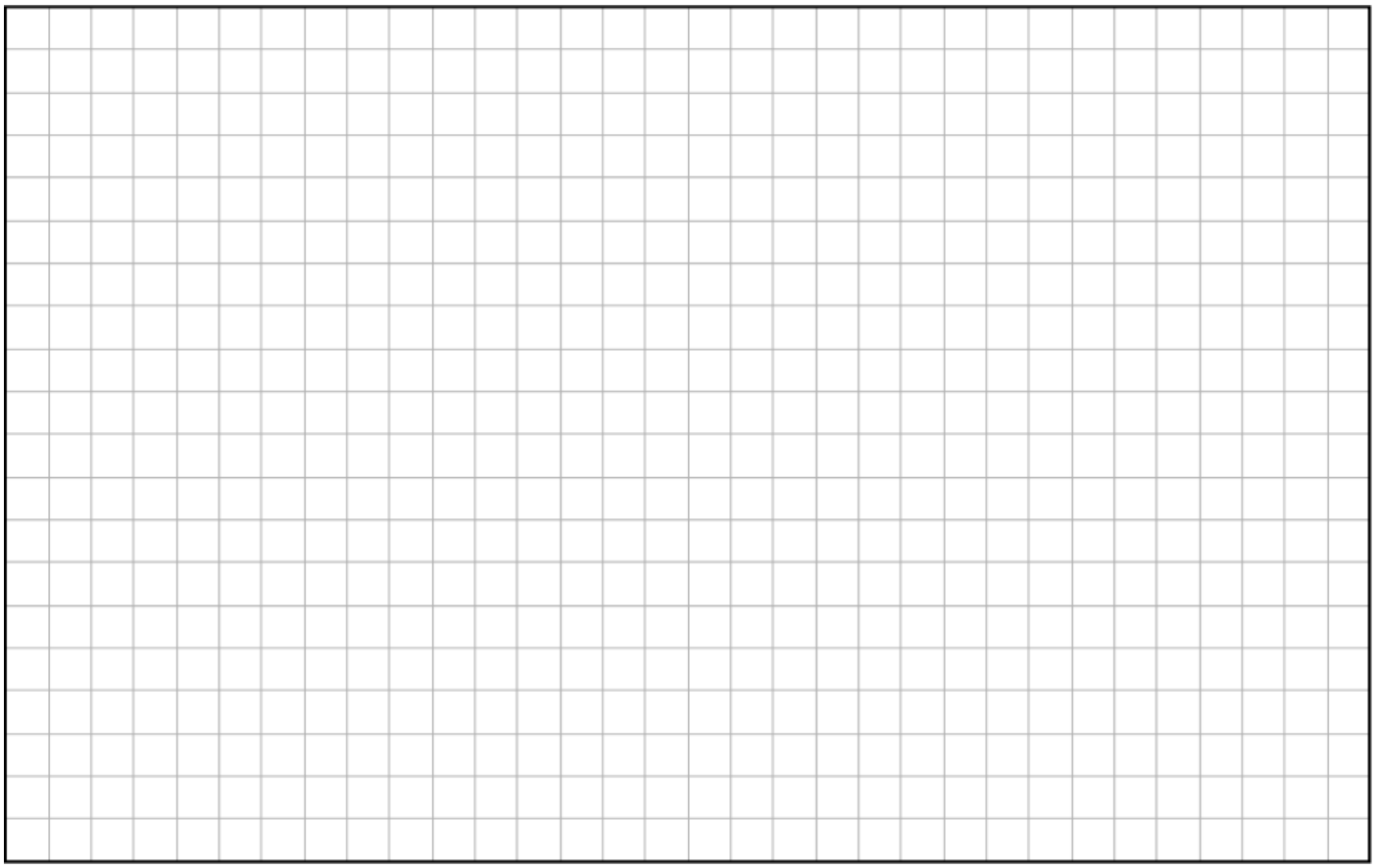

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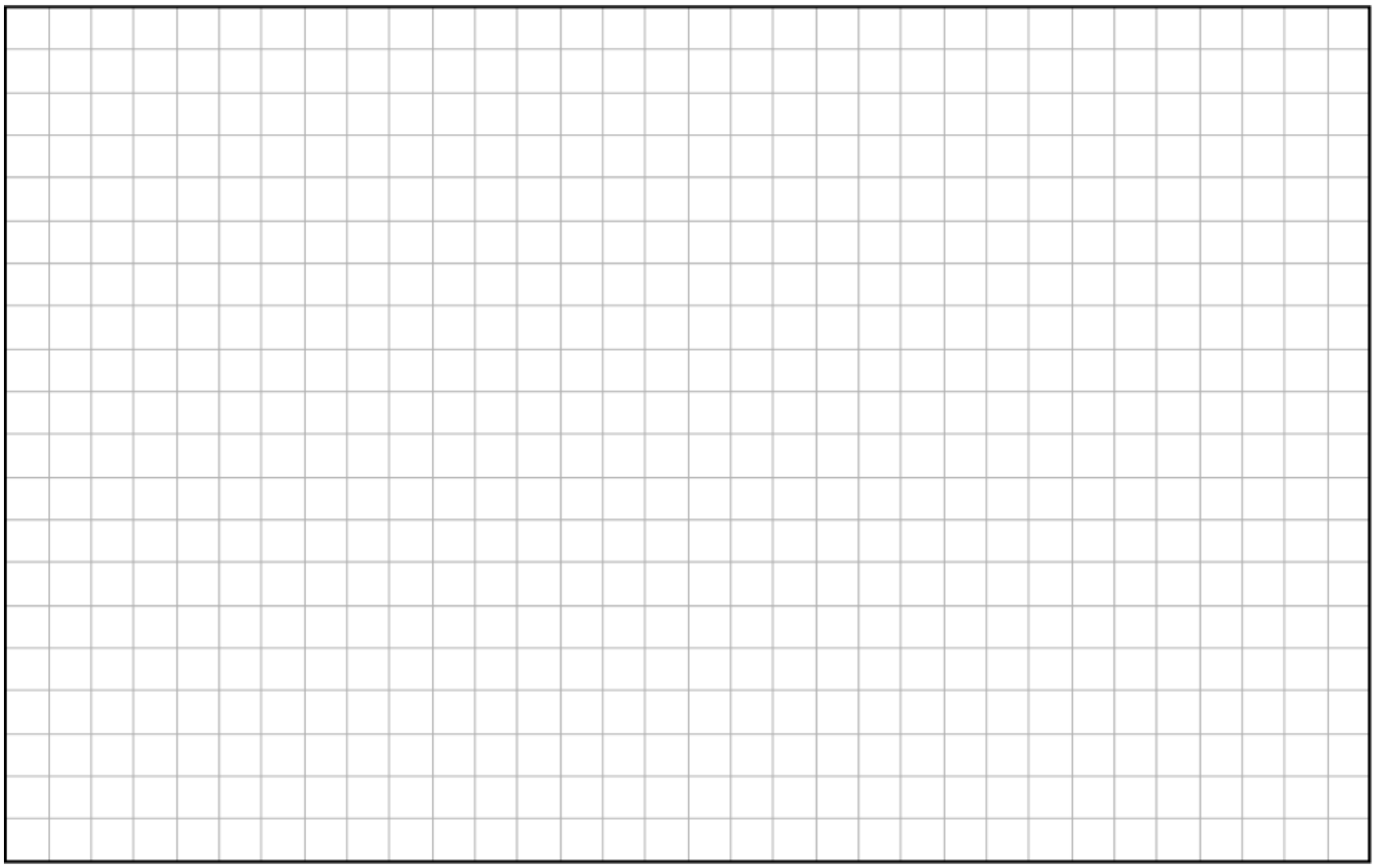


▶ 2022 LCHL Paper 1 – Question 7

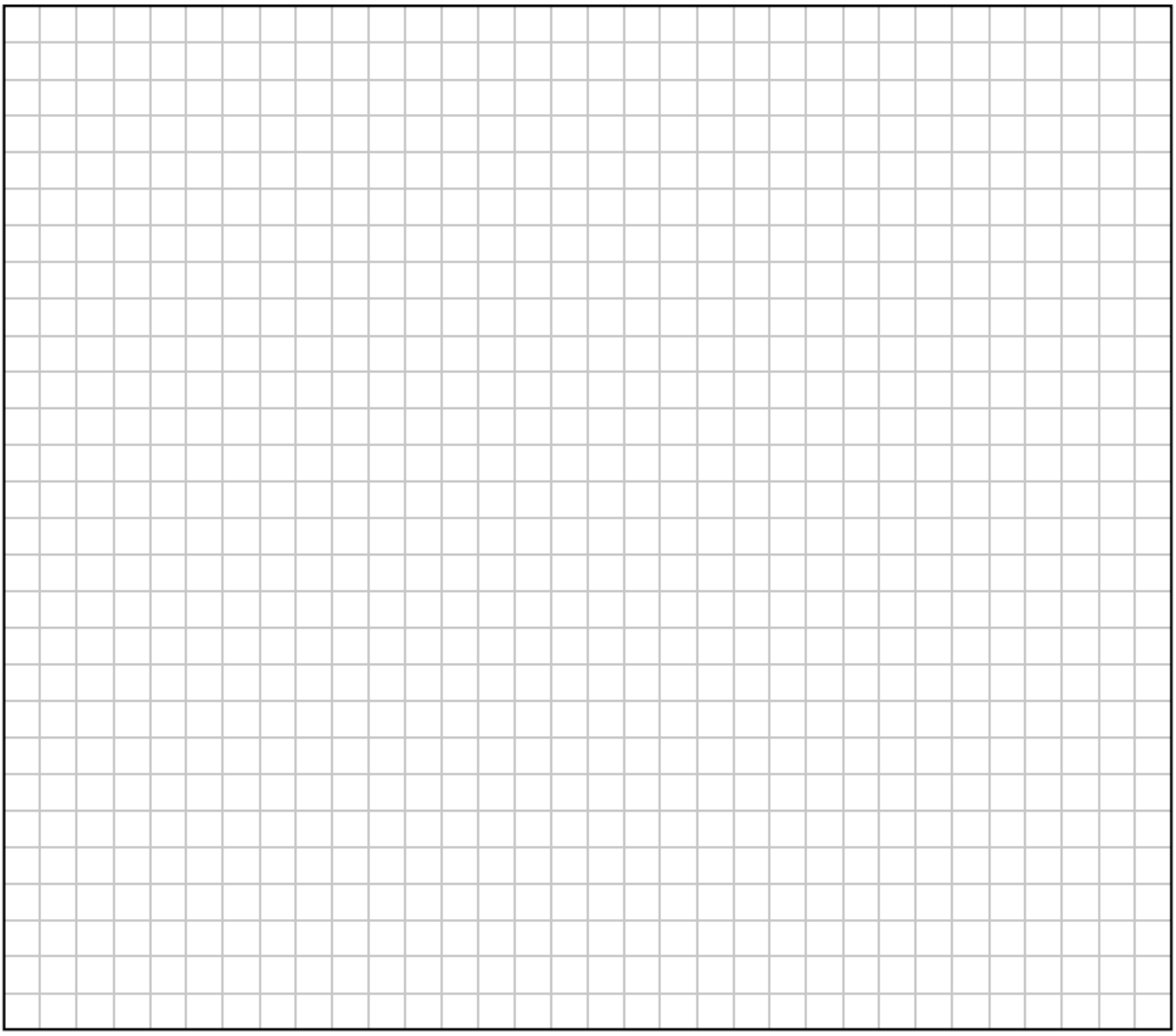
Hannah is doing a training session. During this session, her heart-rate, $h(x)$, is measured in beats per minute (BPM), where x is the time in minutes from the start of the session, $x \in \mathbb{R}$.

For the first 8 minutes of the session, Hannah does a number of exercises.

As she does these exercises, her heart-rate changes. In this time, $h(x)$ is given by:

$$h(x) = 2x^3 - 28.5x^2 + 105x + 70$$

1. Initial Exploration (Set the Scene)



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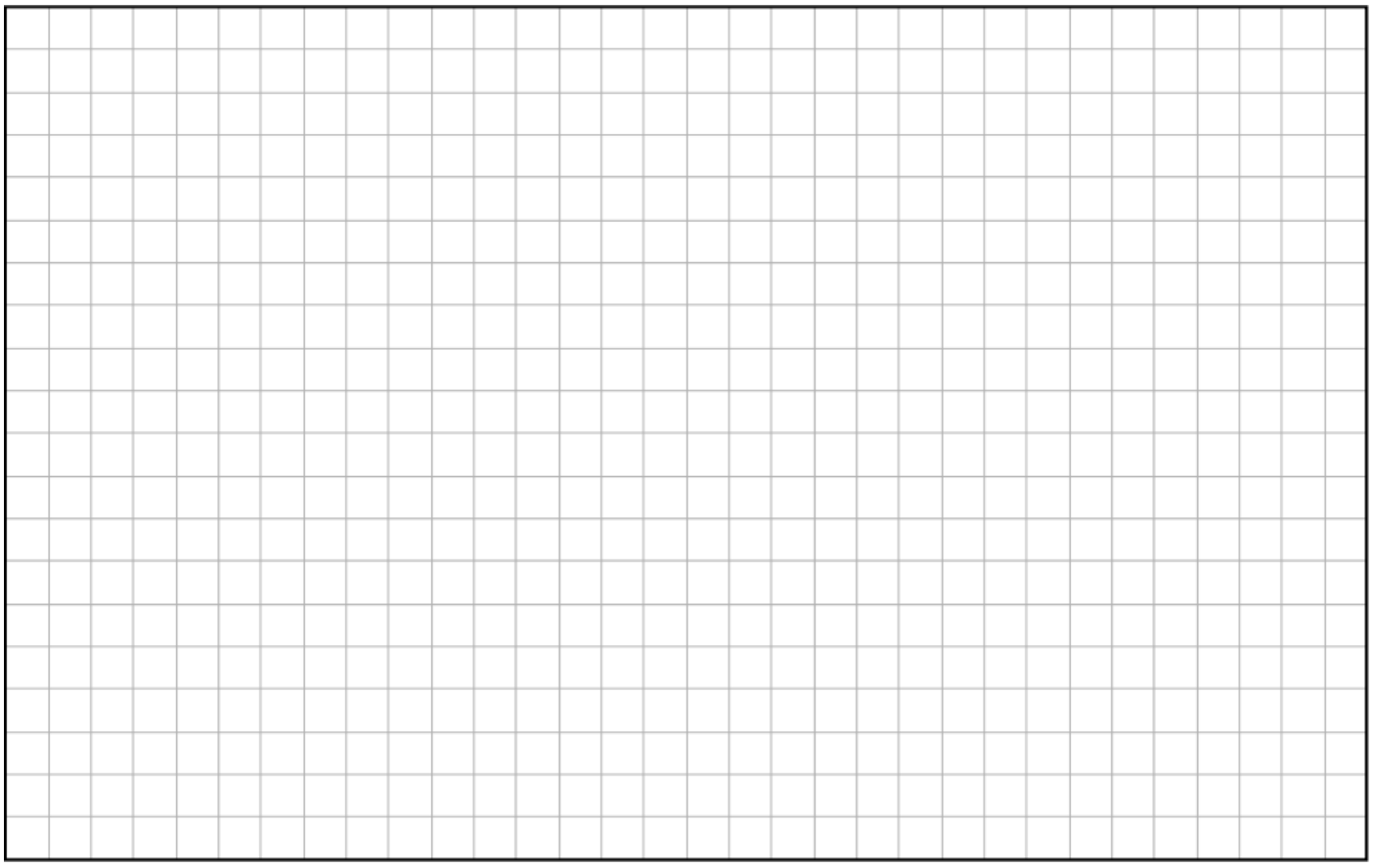

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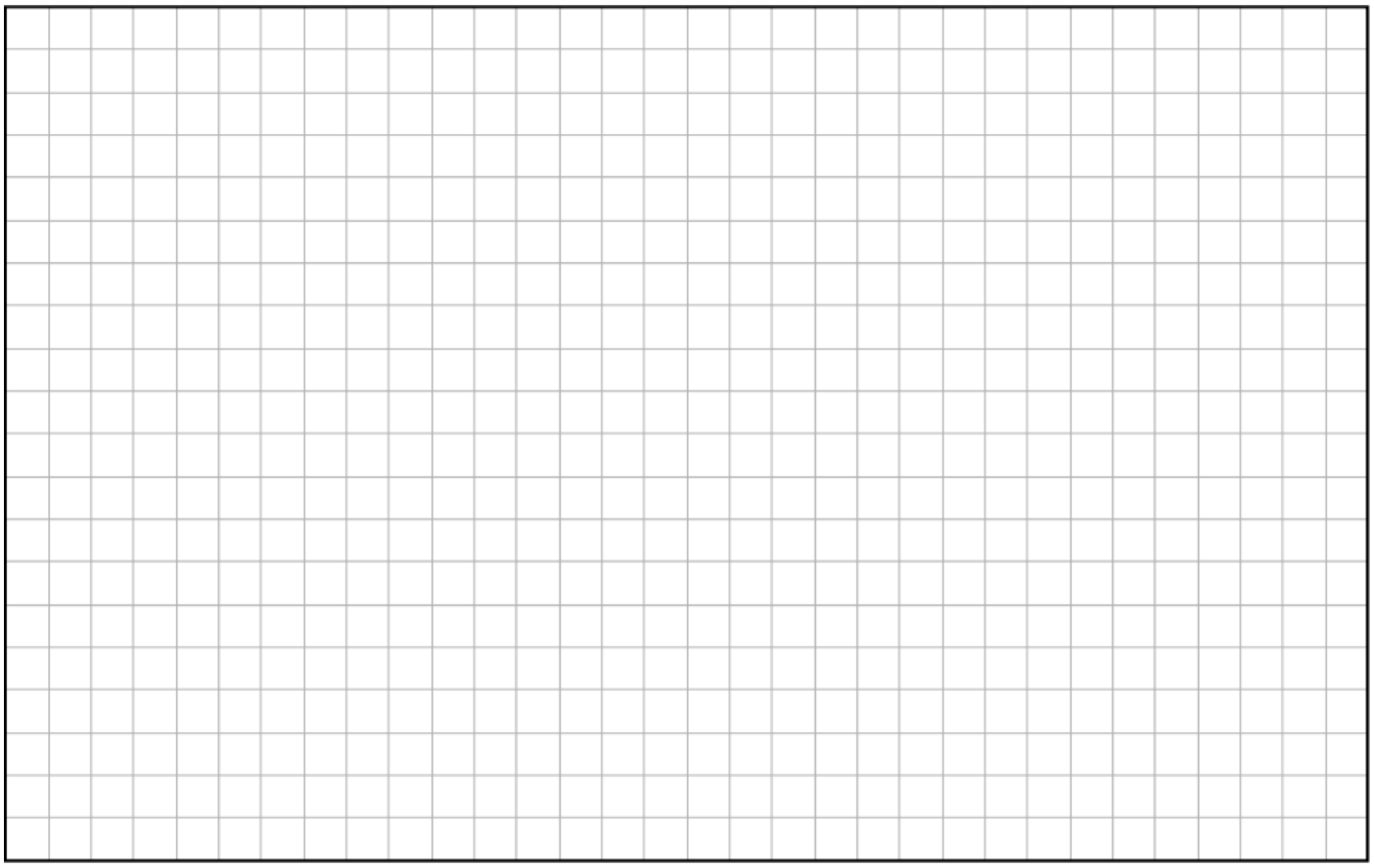

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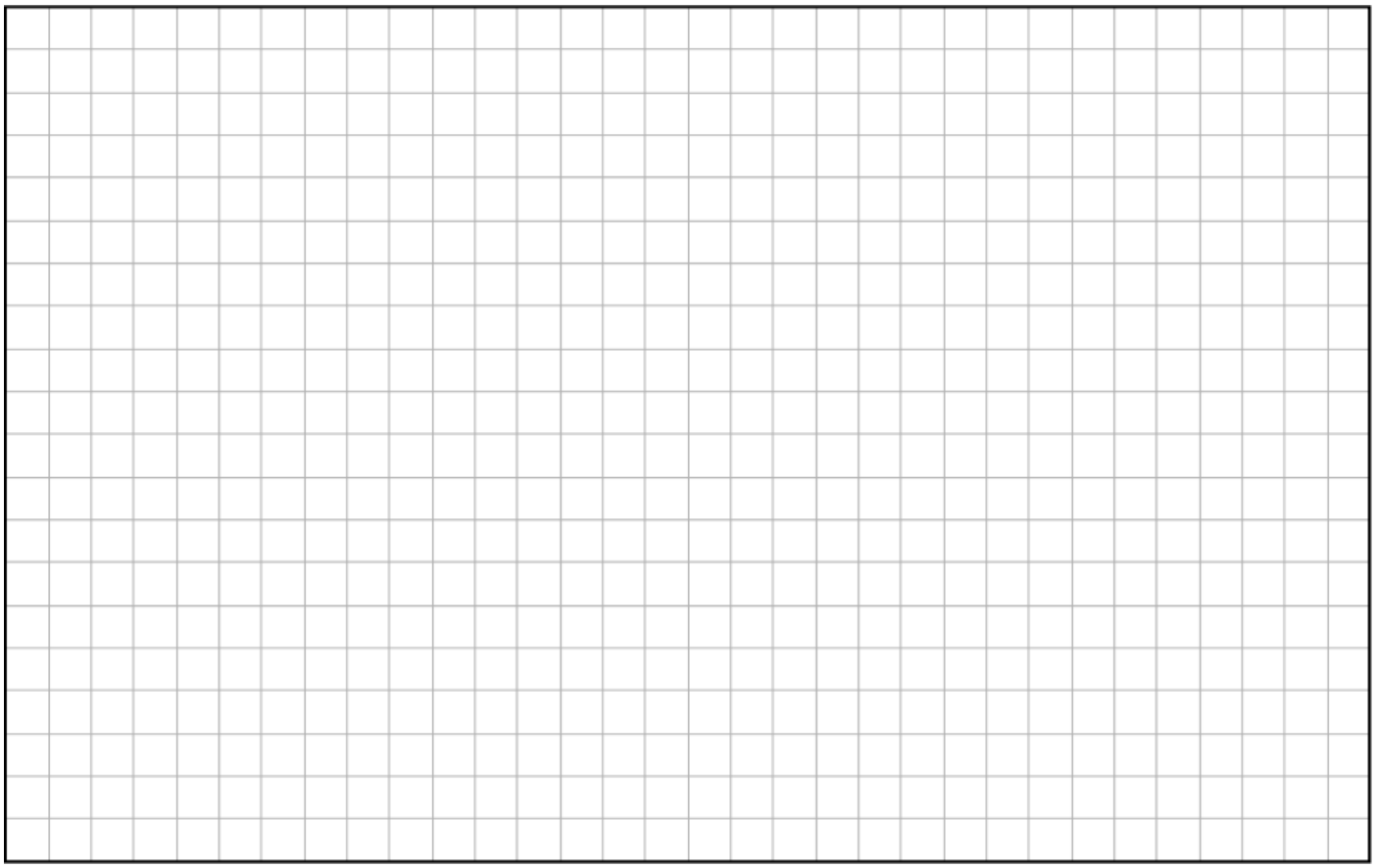

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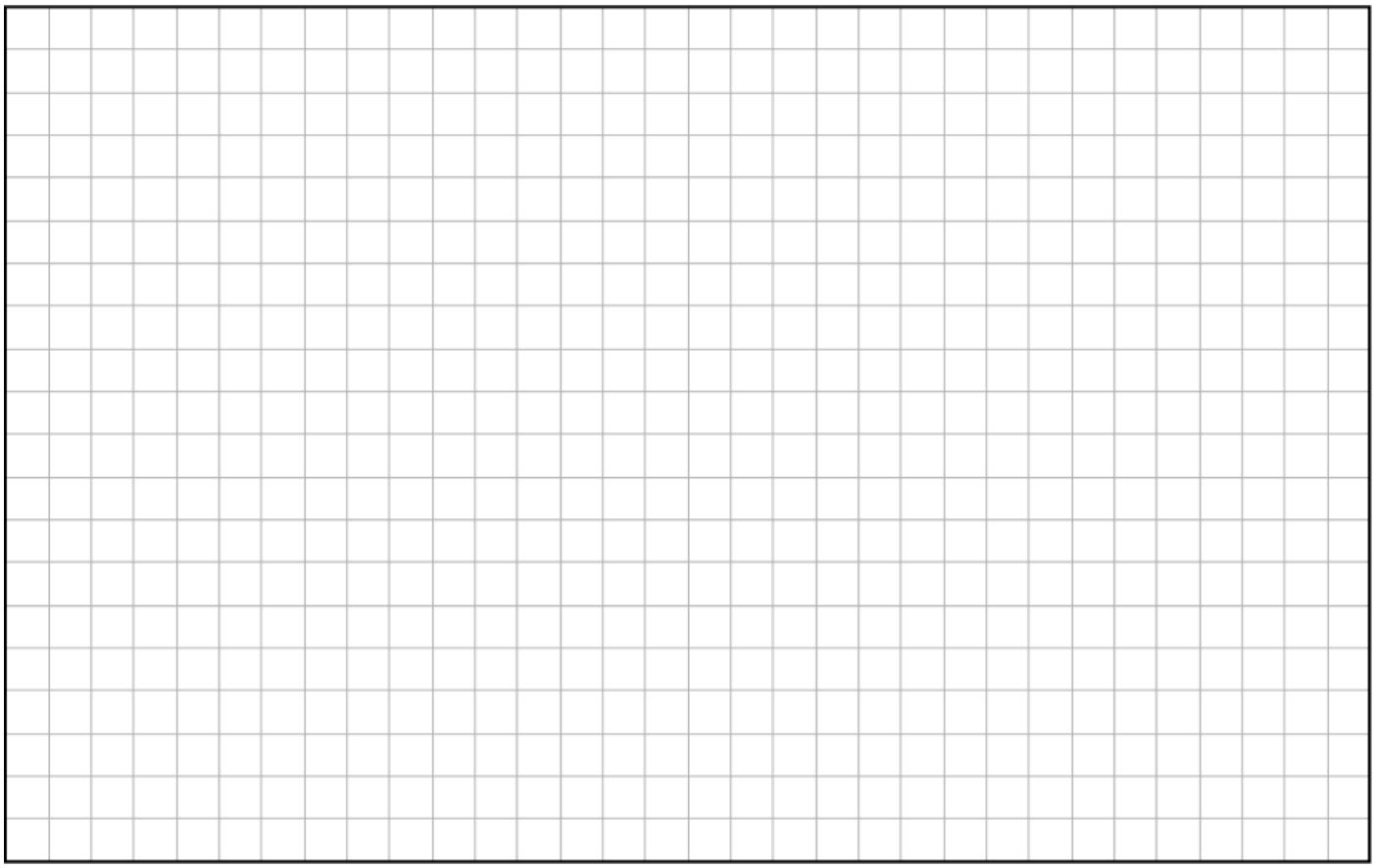


▶ 2022 LCHL Paper 1 – Question 10 (a)

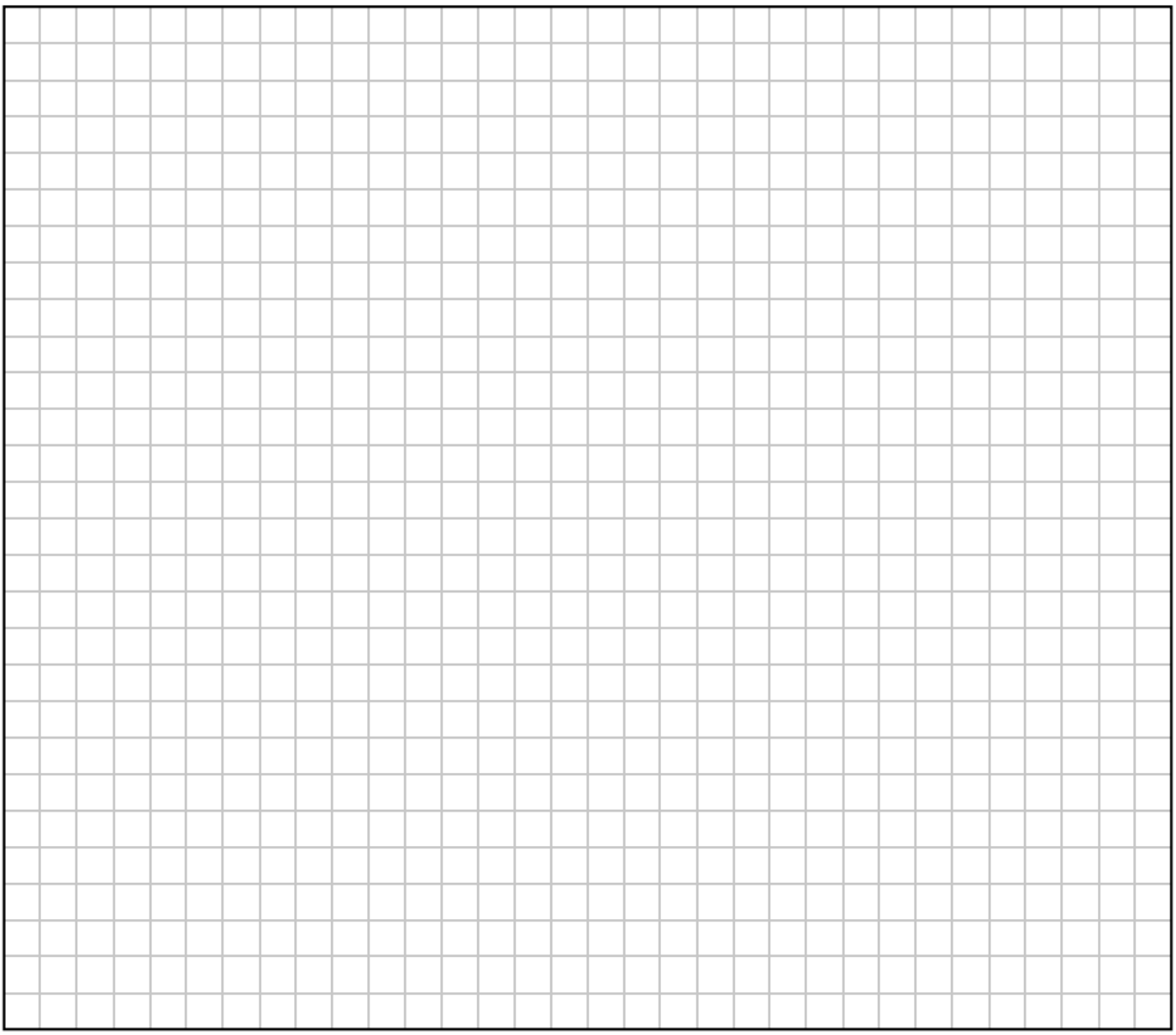
A student is asked to memorise a long list of digits, and then write down the list some time later.

The proportion, P, of the digits recalled correctly after t hours can be modelled by the function:

$$P(t) = 0.82 - 0.12\ln(t+1)$$

for $0 \leq t \leq 12$ and $t \in \mathbb{R}$.

1. Initial Exploration (Set the Scene)



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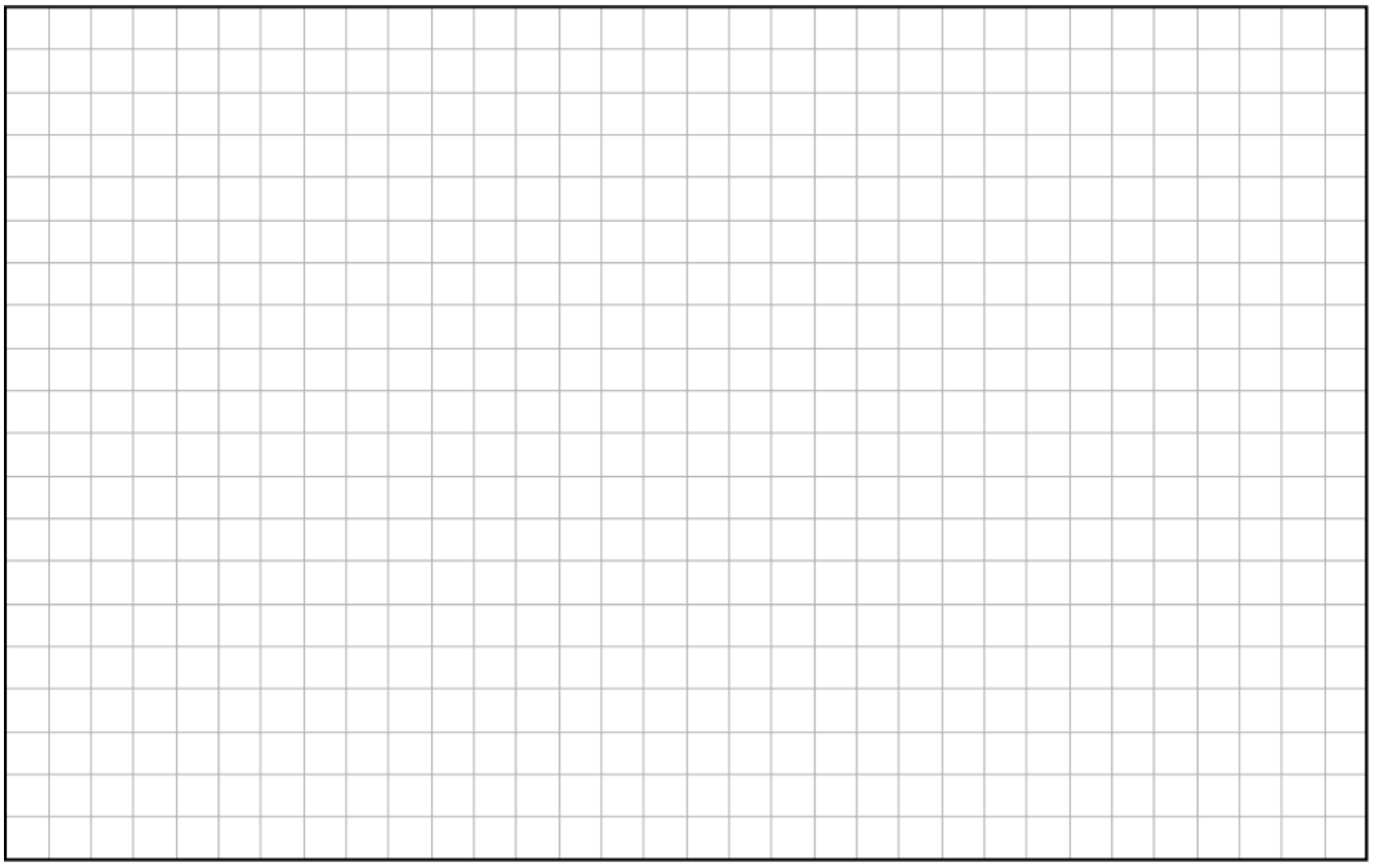

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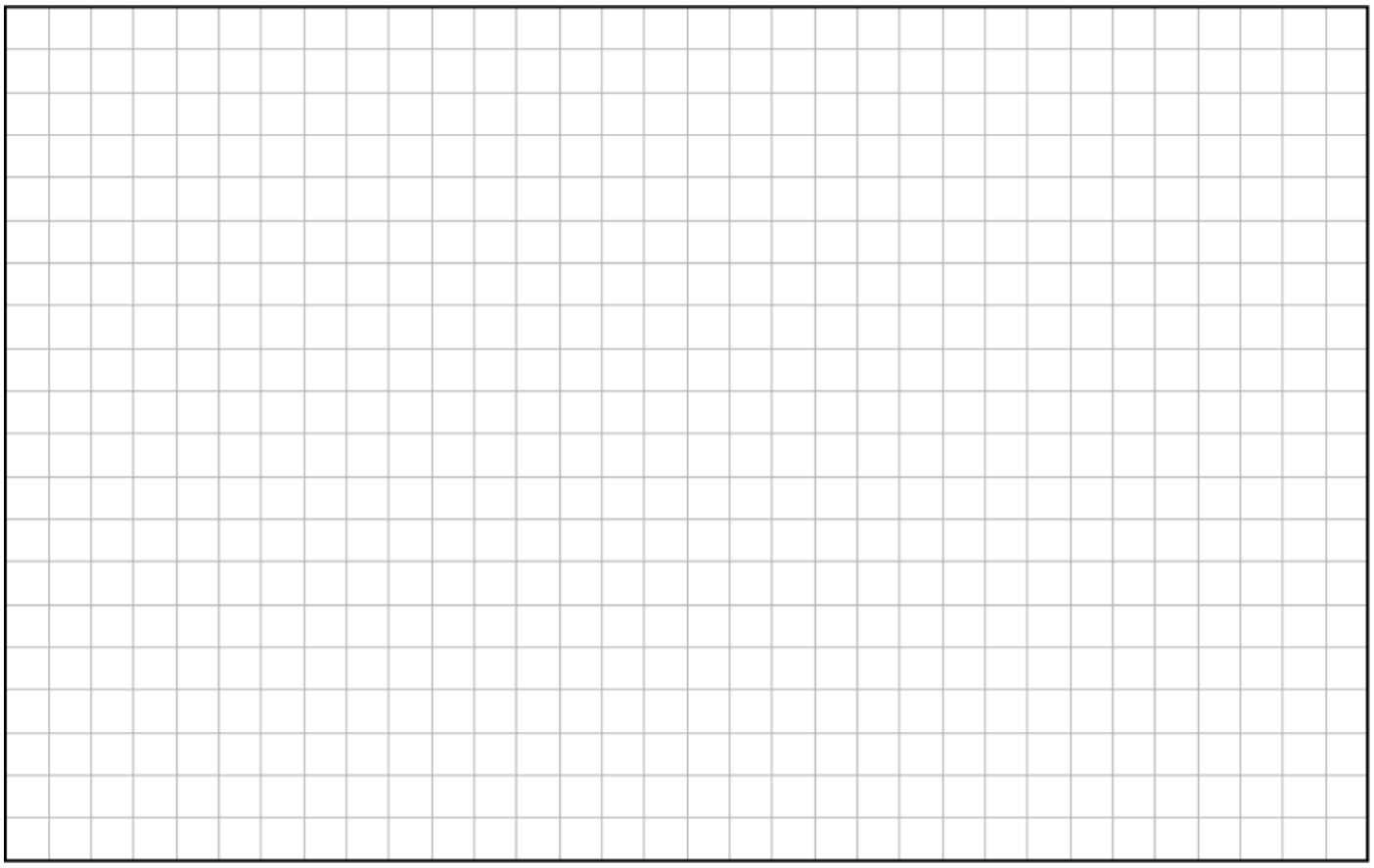

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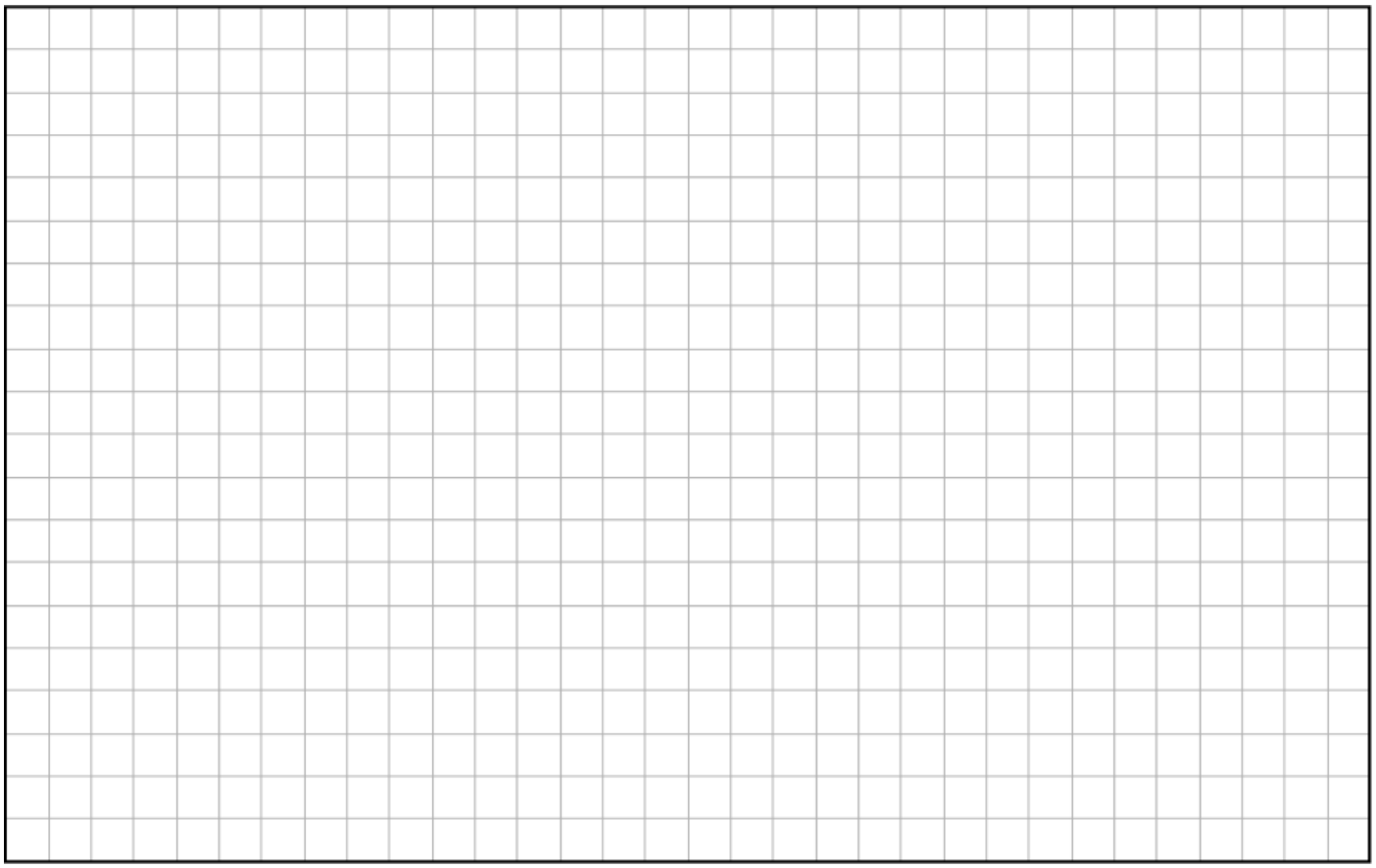

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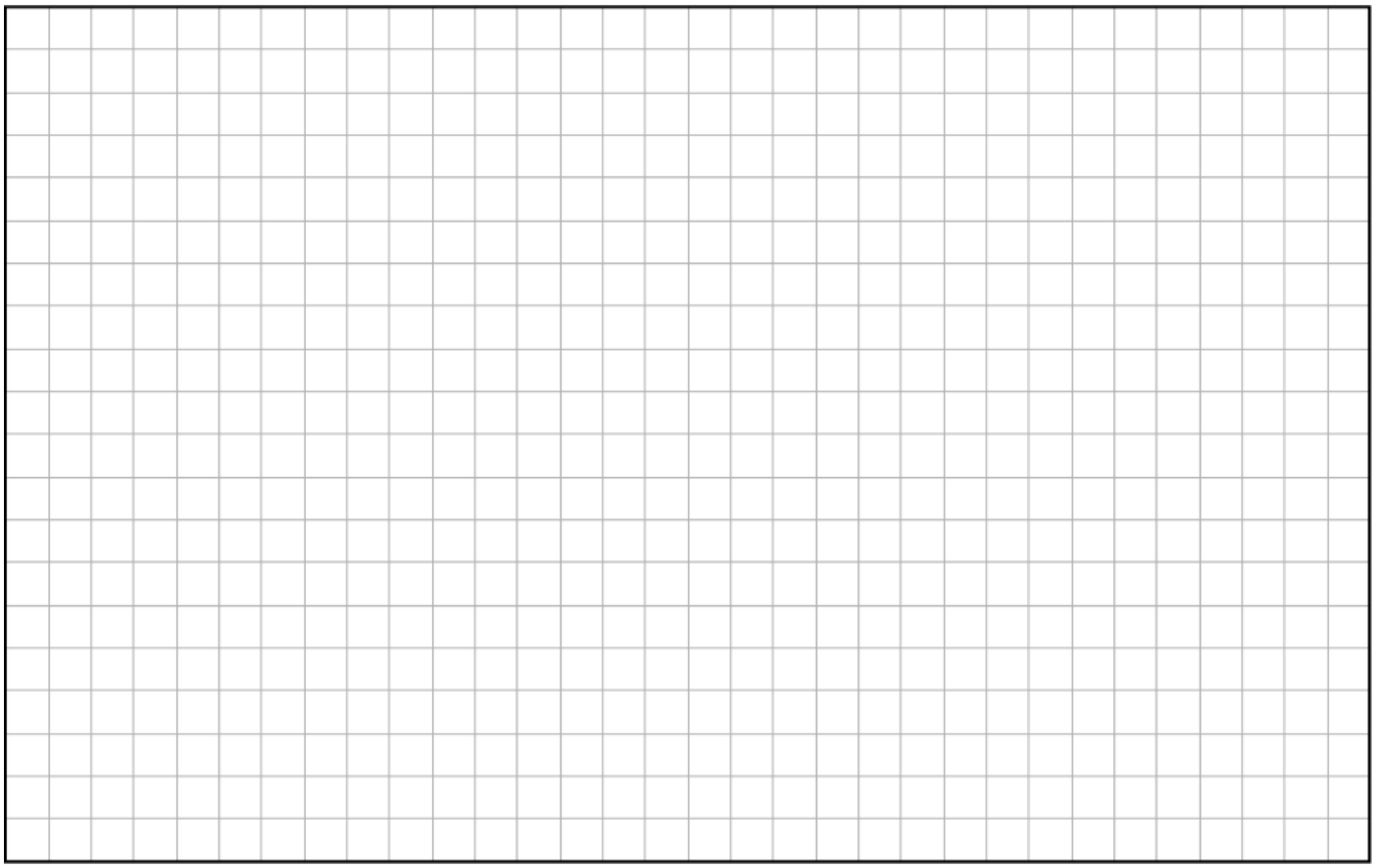
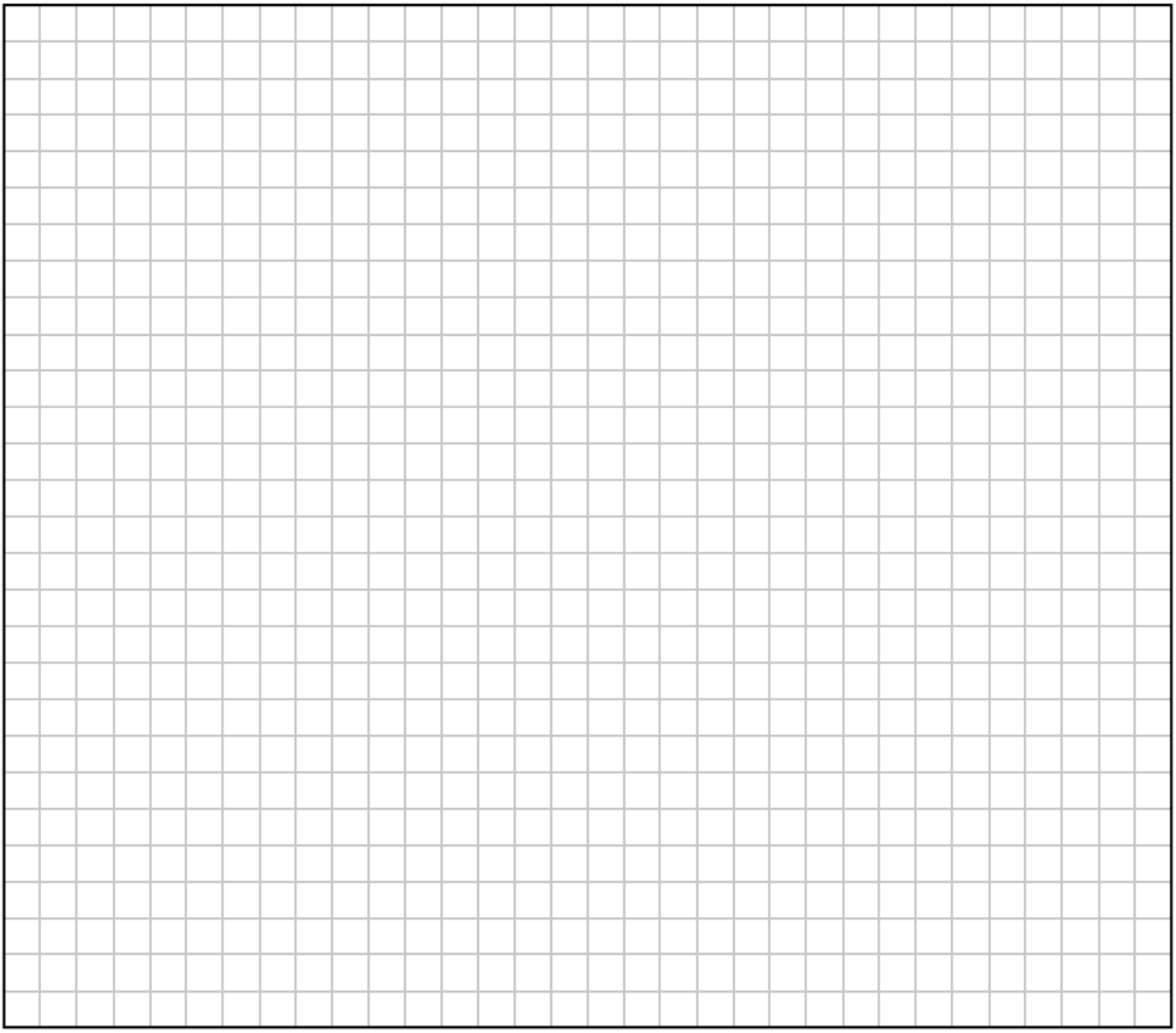


▶ 2021 LCHL Paper 1 – Question 8

$h(x) = 0.001x^3 - 0.12x^2 + 3.6x + 5, x \in \mathbb{R}$,

The function $h(x)$ can be used to model the height above level ground (in metres) of a section of the path followed by a rollercoaster track, where x is the horizontal distance from a fixed point.

1. Initial Exploration (Set the Scene)



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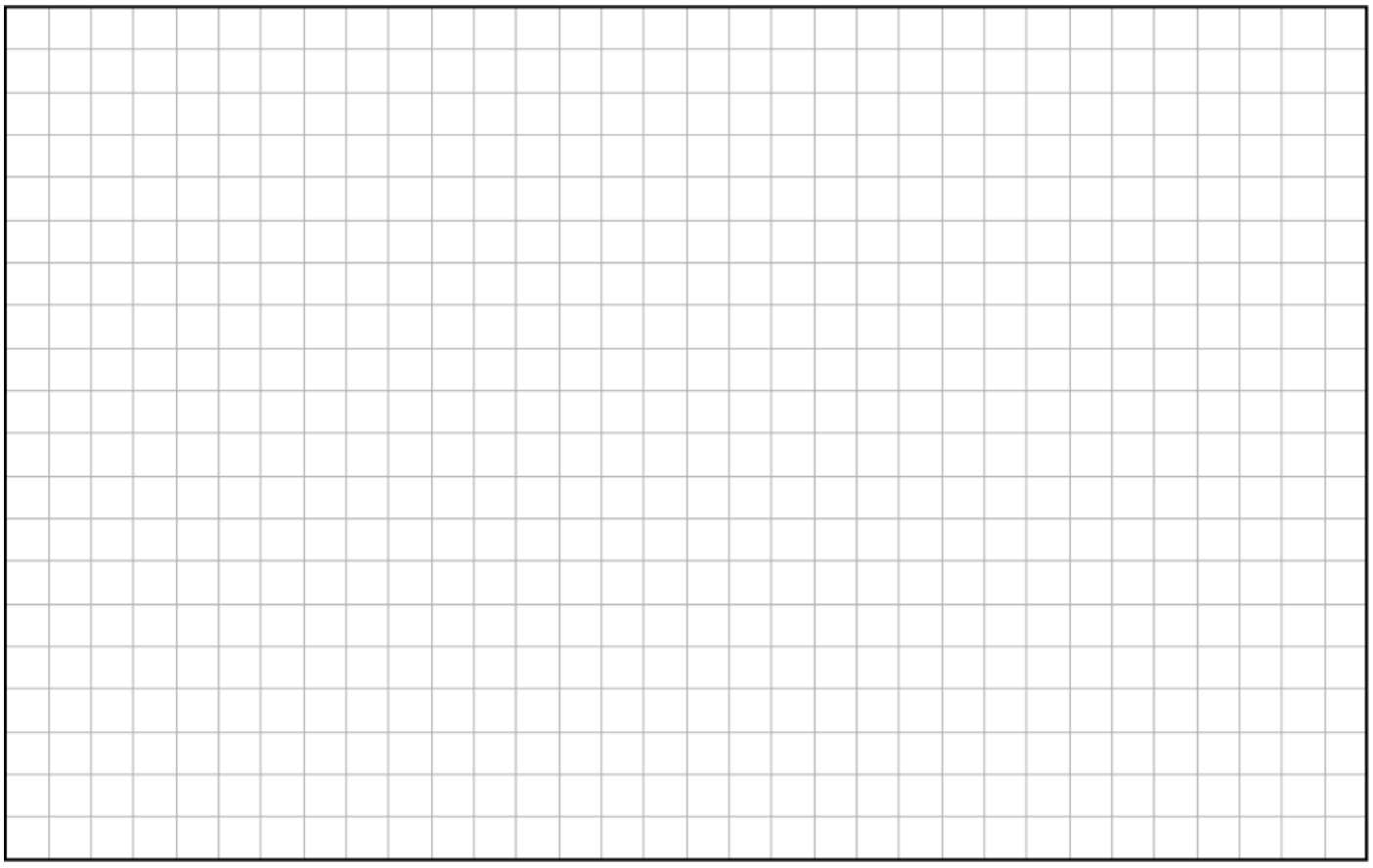

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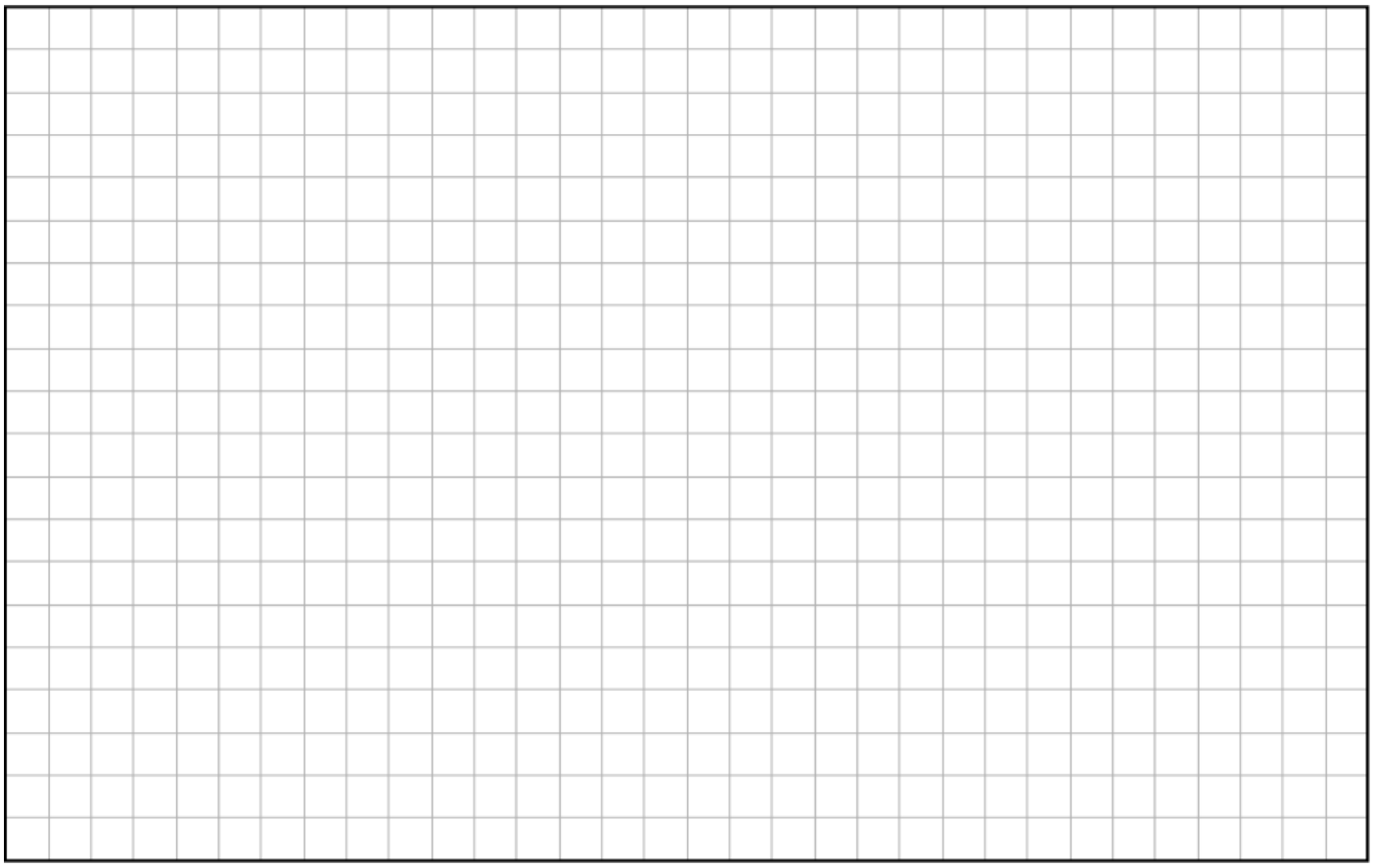

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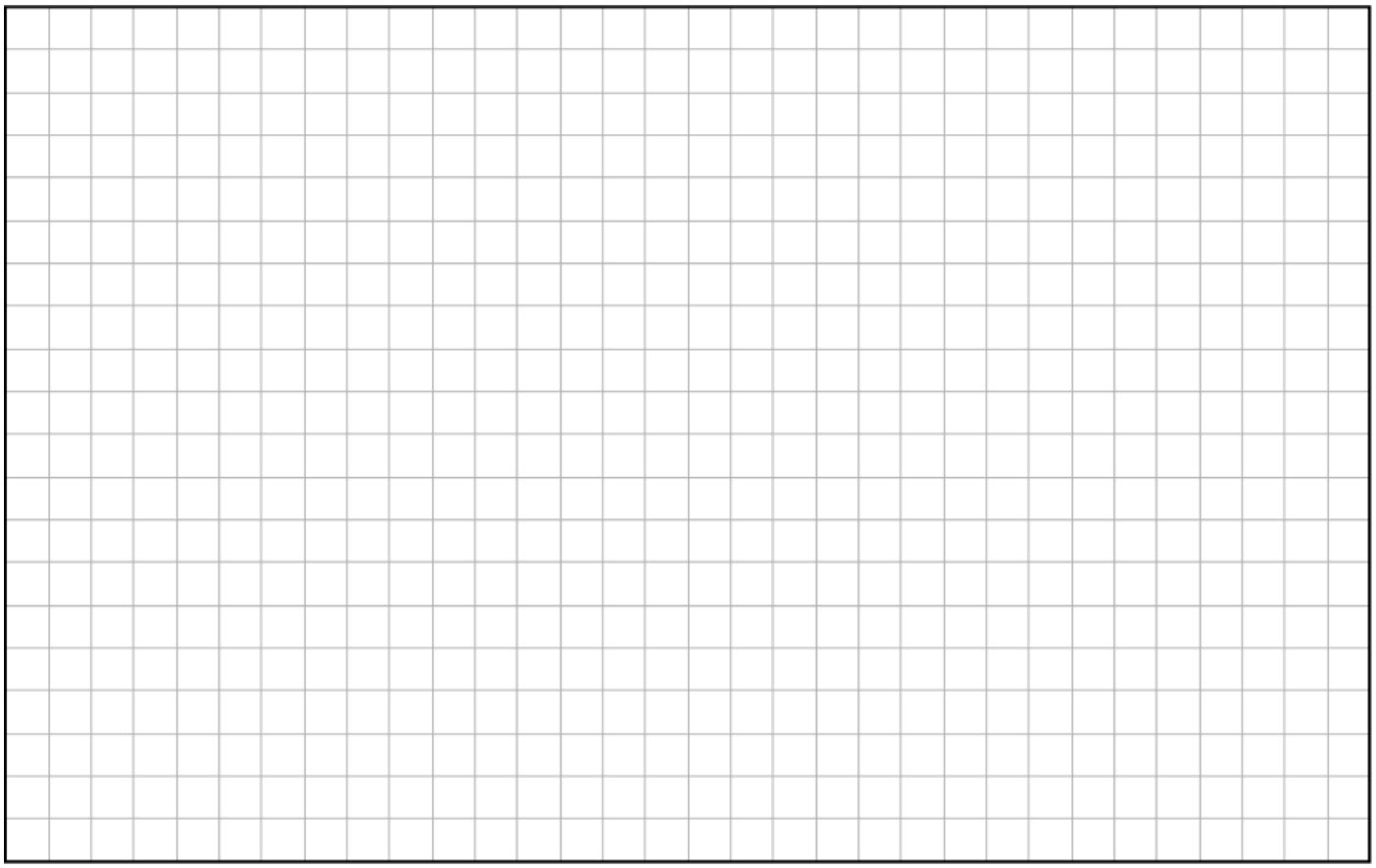

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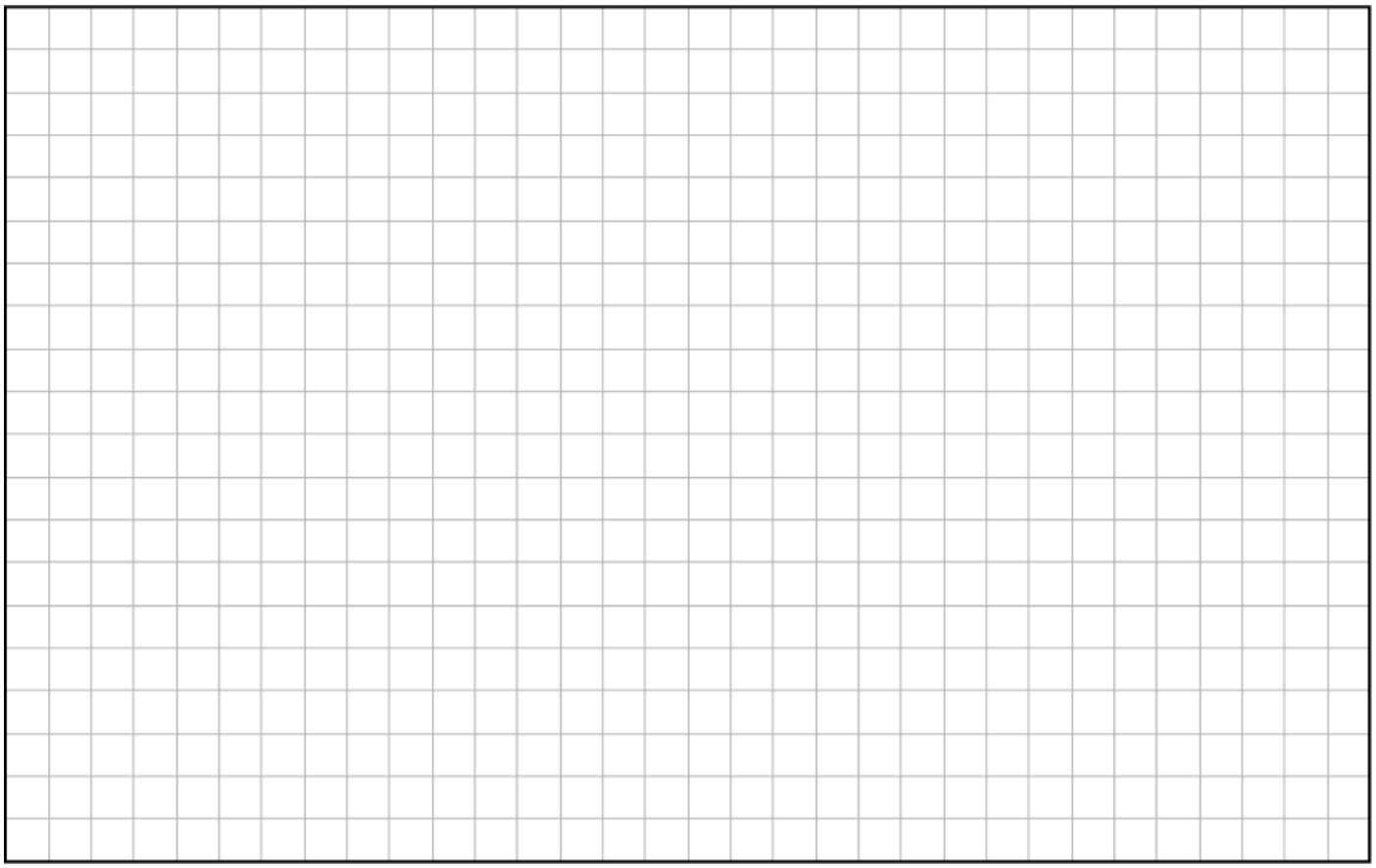


▶ 2021 LCHL Paper 1 – Question 9

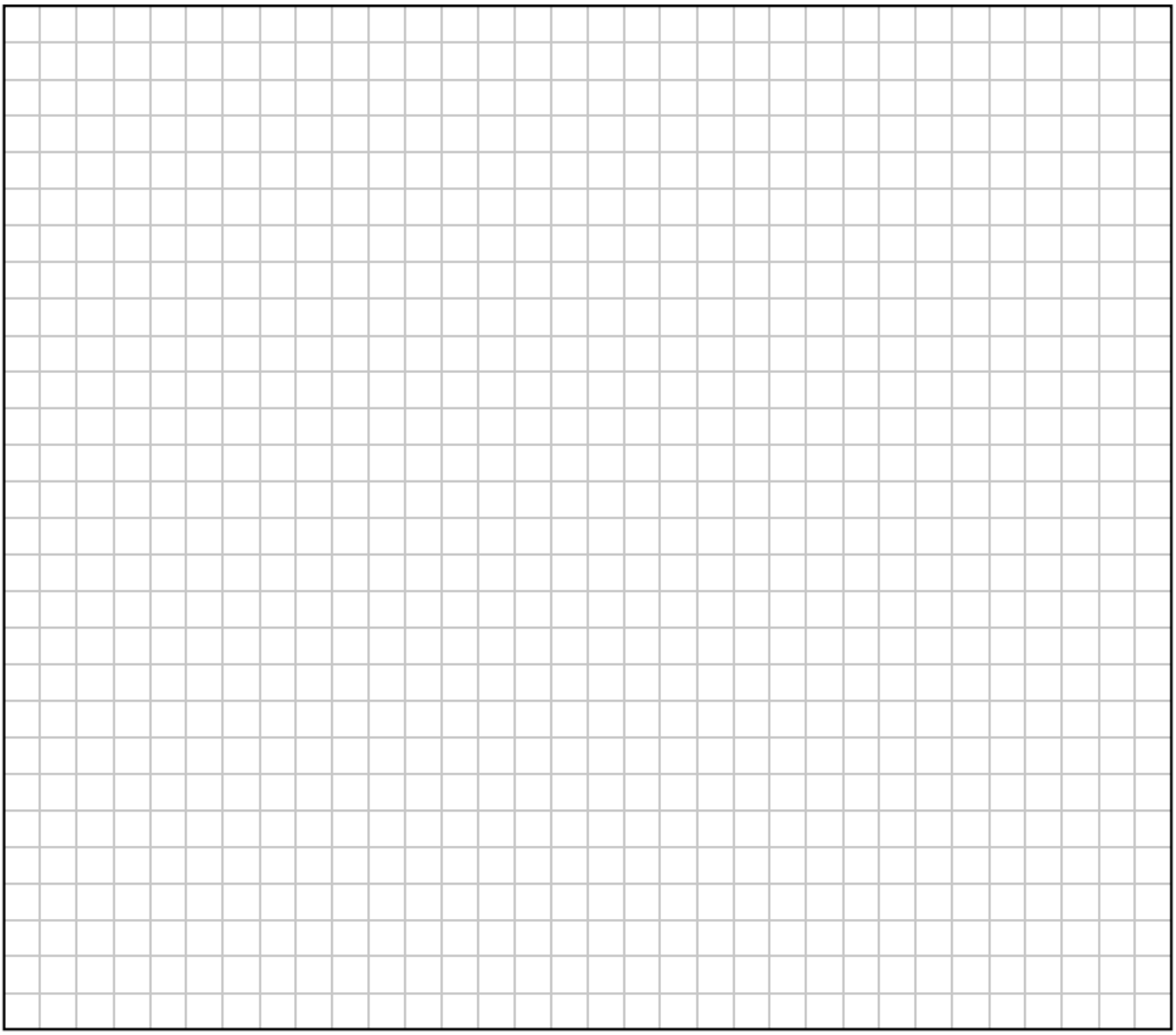
A cup of coffee is freshly brewed to 95° C.

The temperature, T, in degrees centigrade, of the coffee as it cools is given by the formula

$$T(t) = 75e^{-0.081t} + 20$$

where t is time measured in minutes from when the coffee was brewed.

1. Initial Exploration (Set the Scene)



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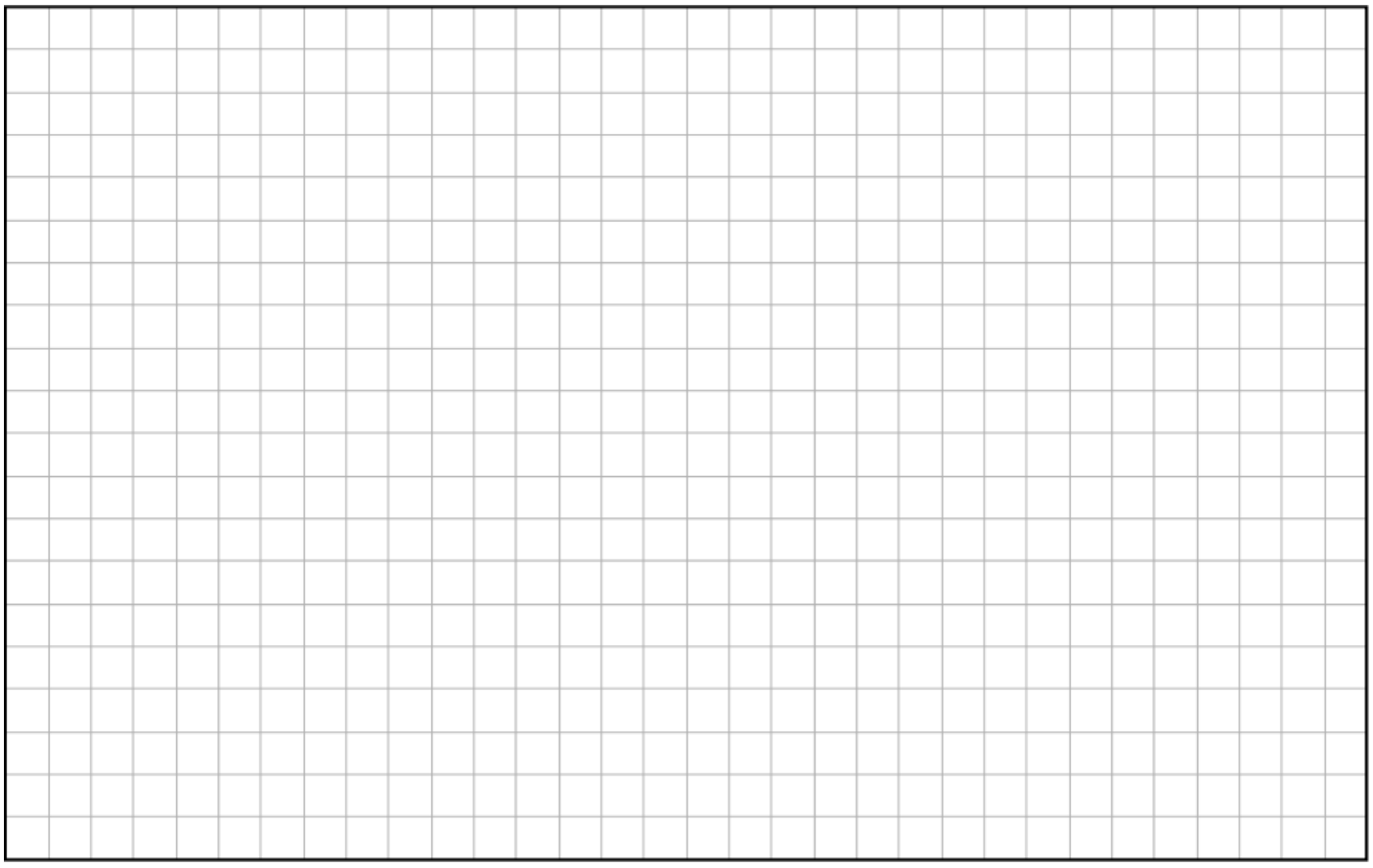

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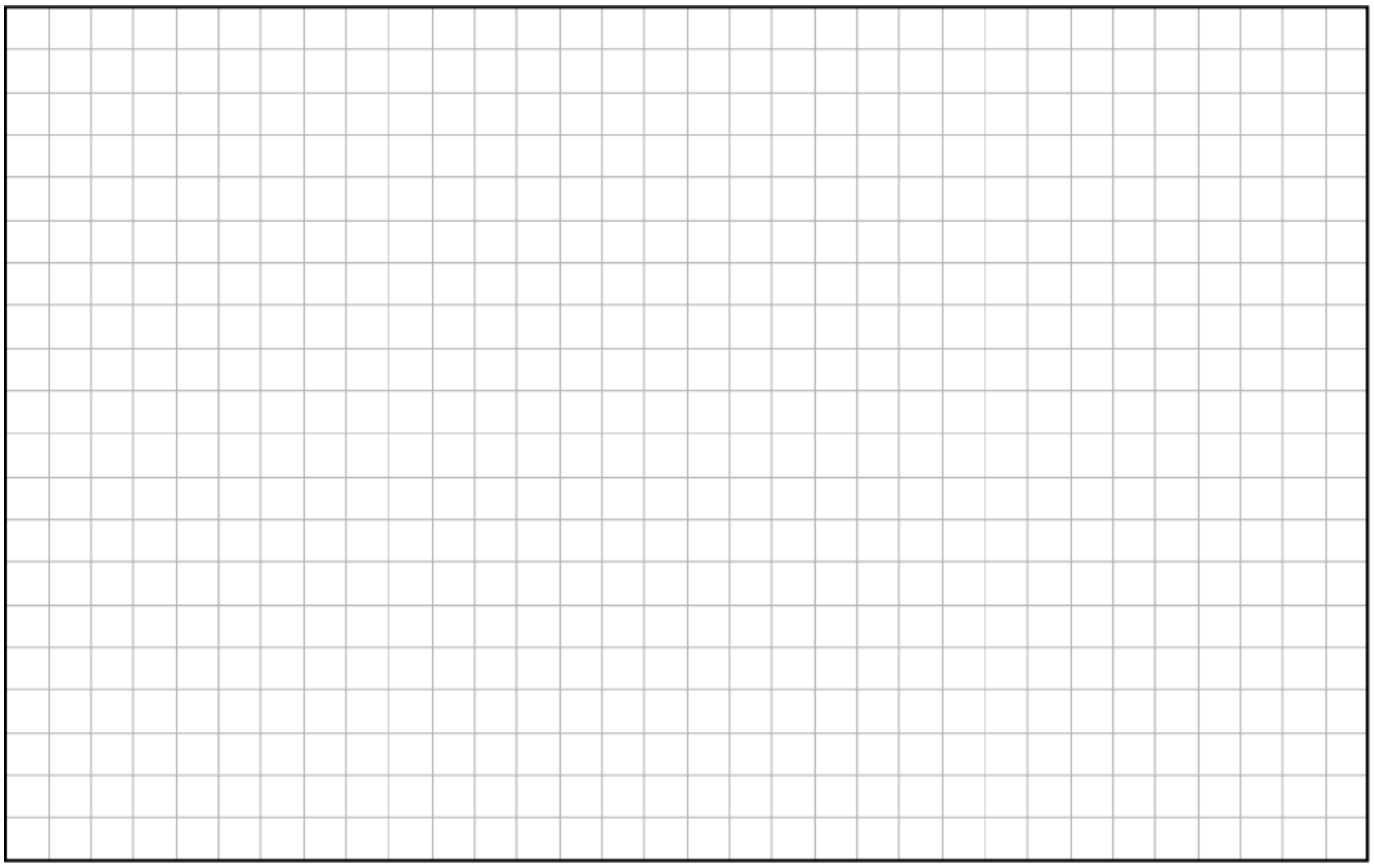

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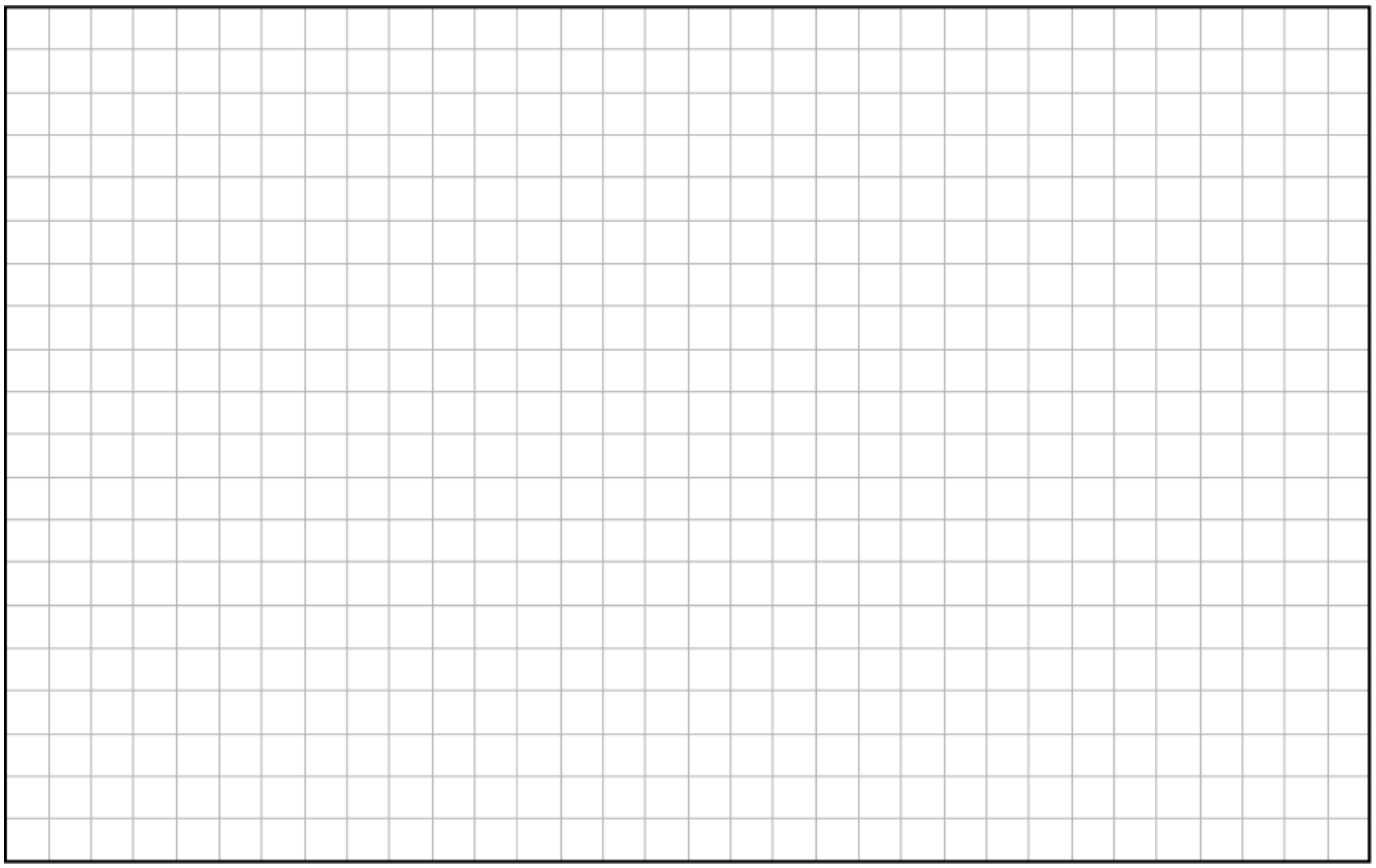

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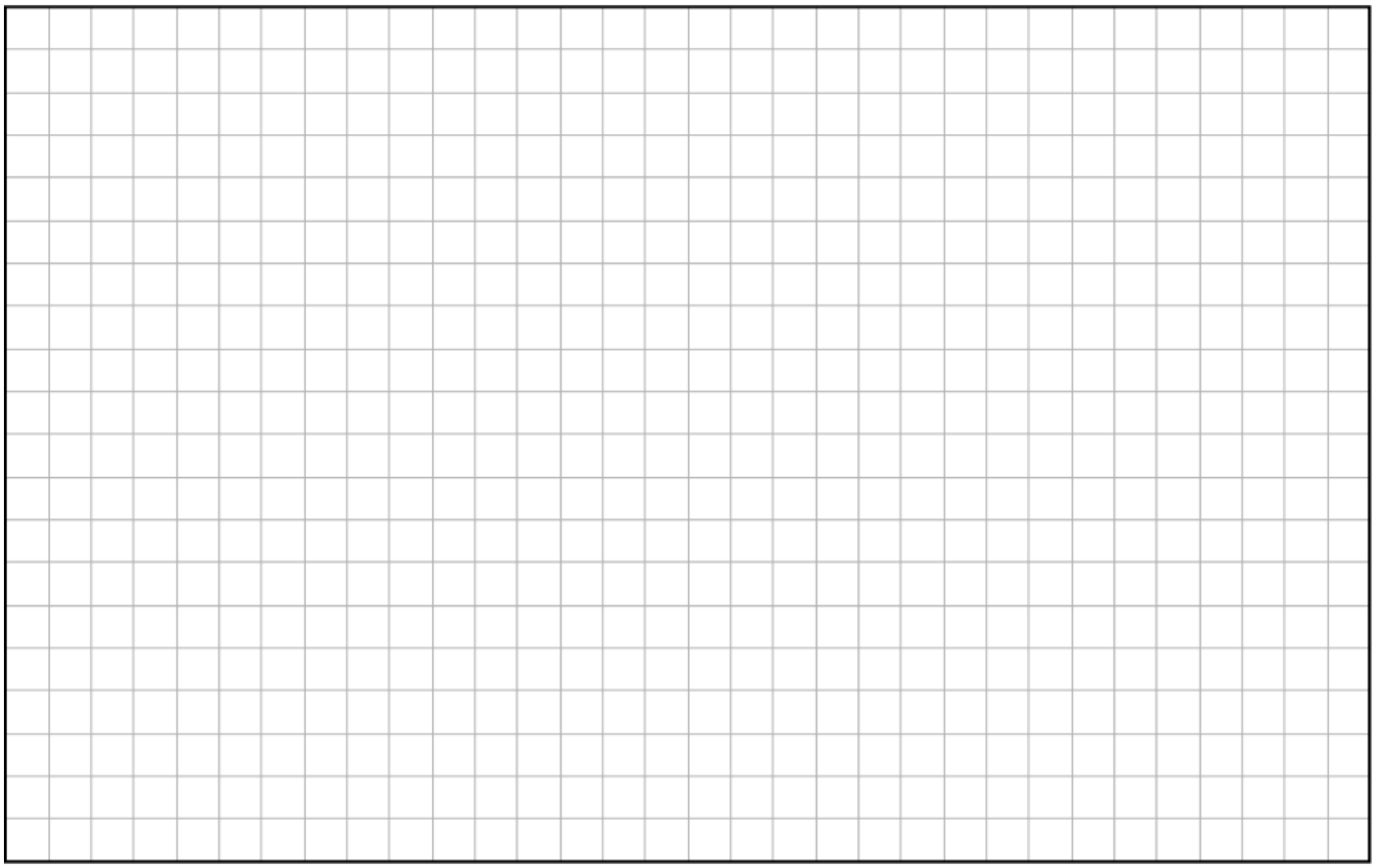


▶ 2021 LCHL Paper 1 – Question 10

Water is flowing into and being removed from a tank.

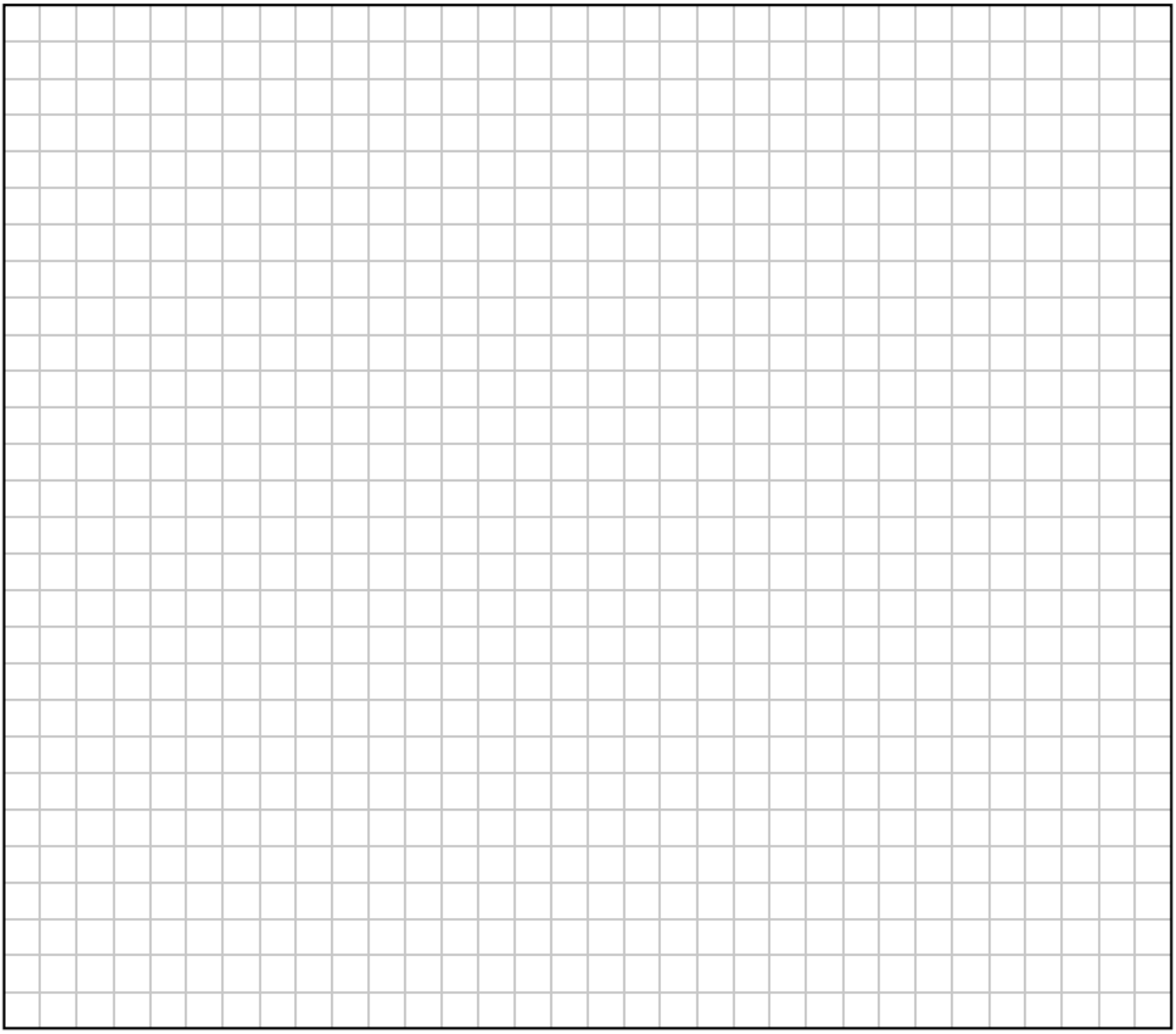
The volume of water in the tank is measured on a daily basis, starting at day 0.

The volume of water, in litres, in the tank can be modelled by the formula

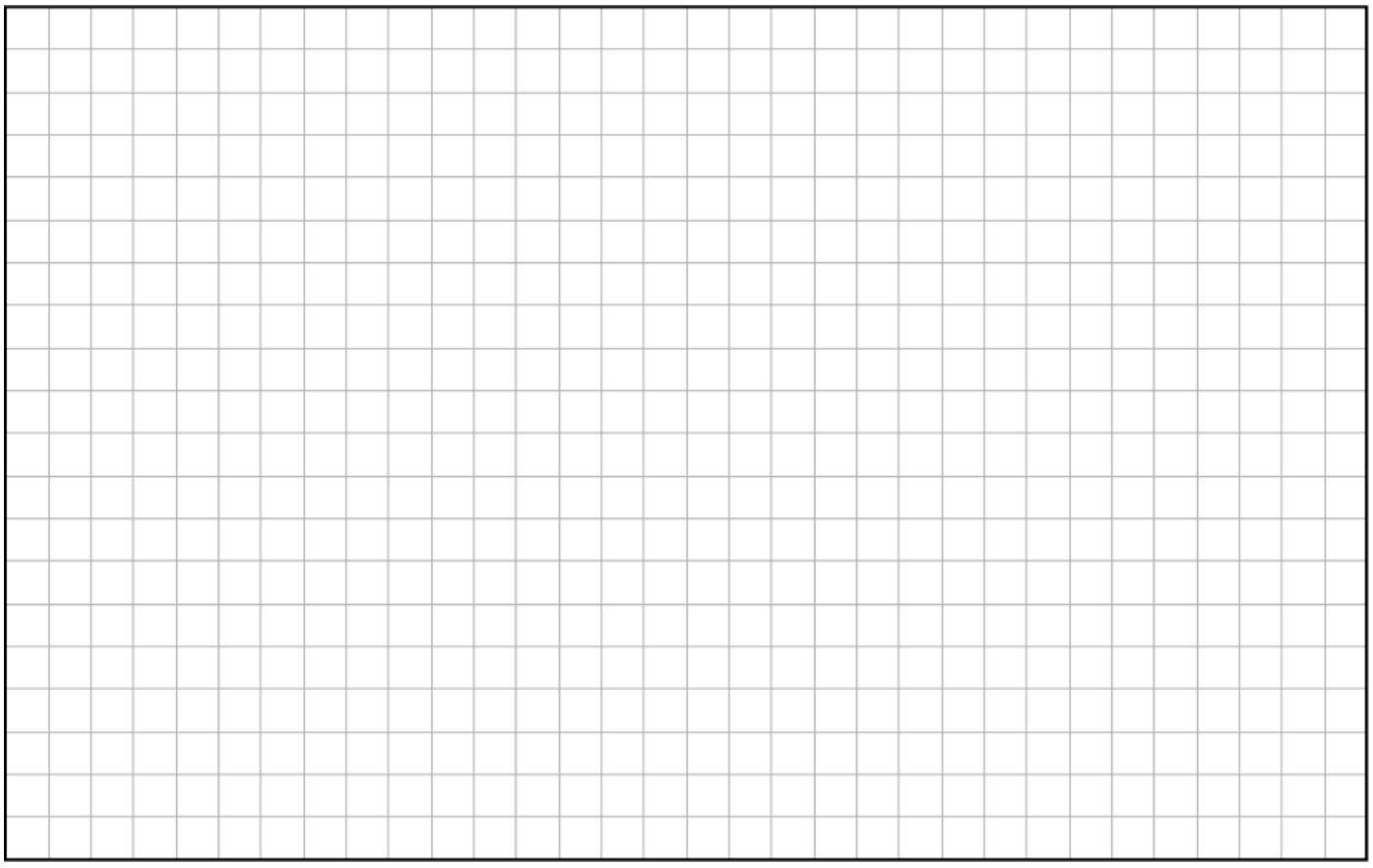
$$V(t) = 60 + 41t - 3t^2, \text{ while } V(t) \geq 0,$$

where $t \in \mathbb{R}$ is the time in days, starting at day 0.

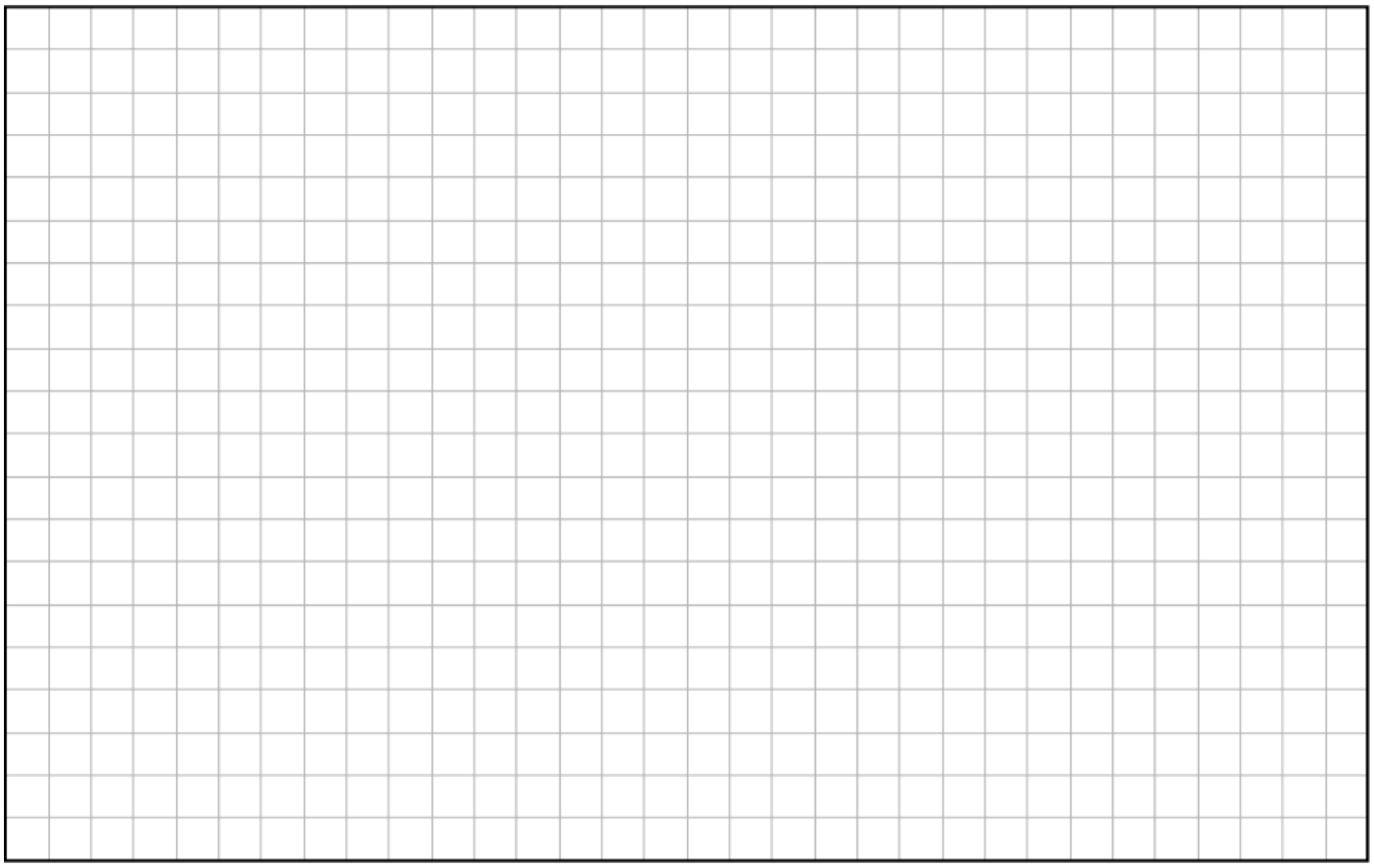
1. Initial Exploration (Set the Scene)



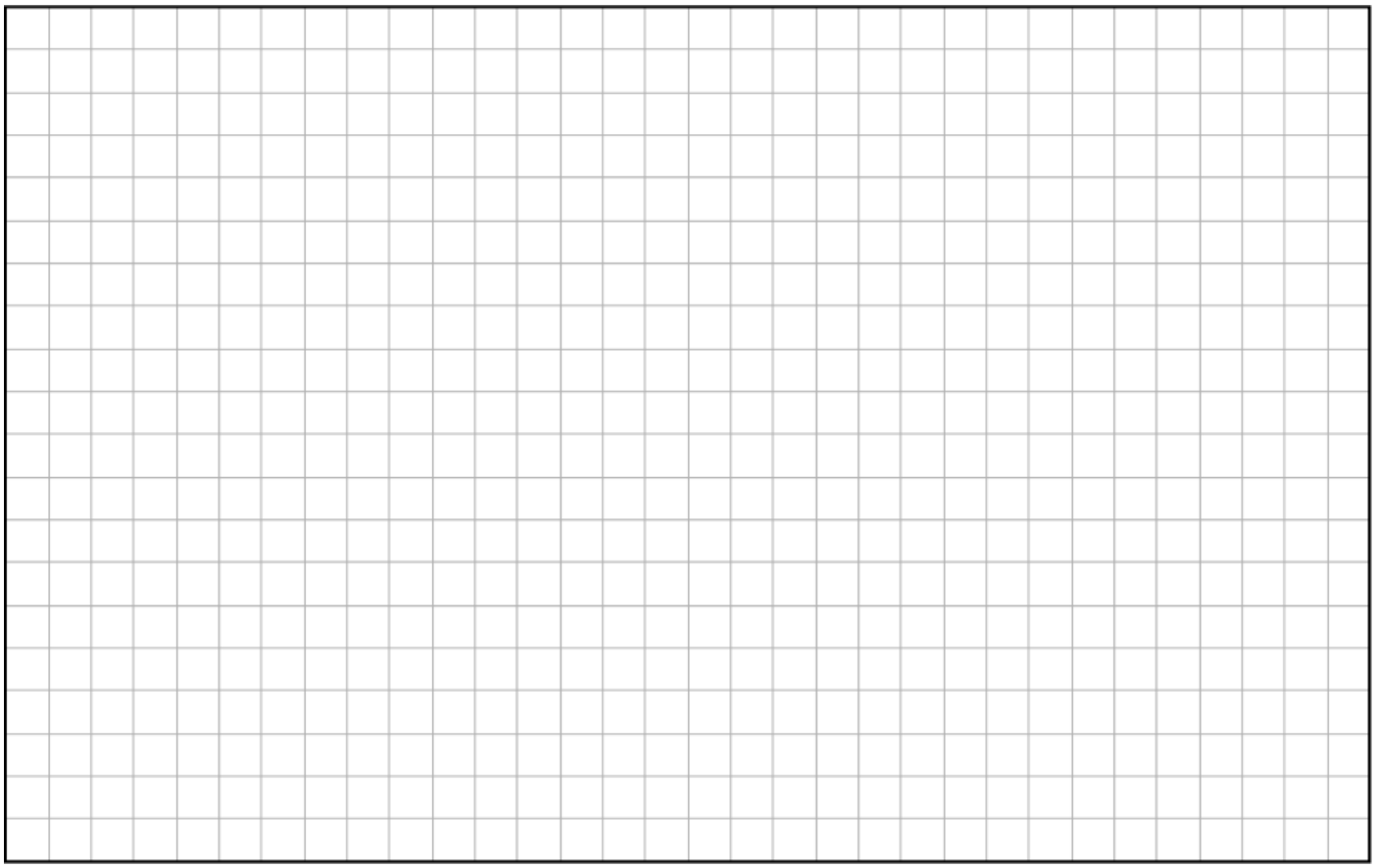
2. **Visualise the Function**



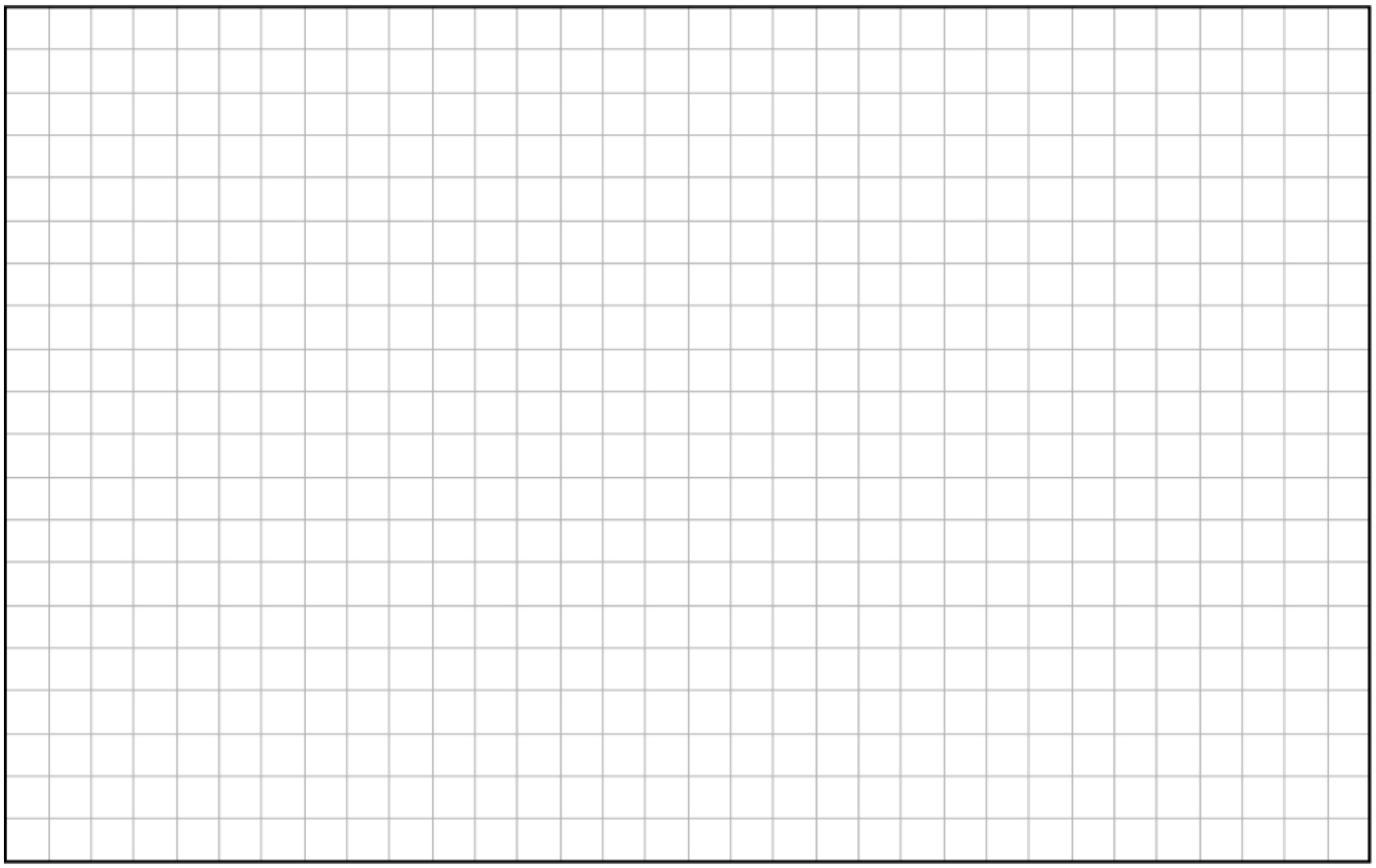
3. **Using outputs to find inputs!**



4. What does differentiation do?



5. What does integration do?



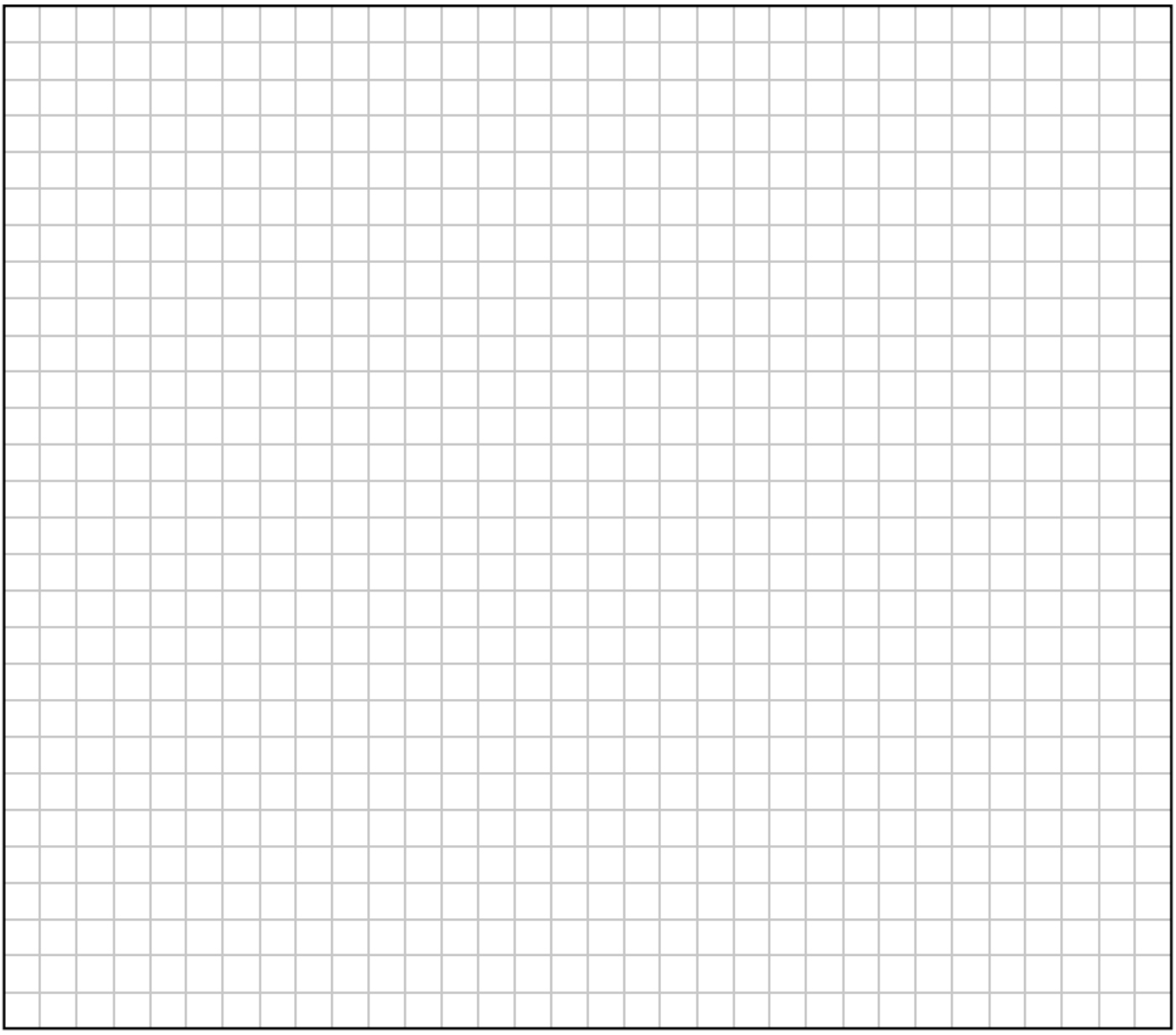
▶ 2020 LCHL Paper 1 – Question 9

The number of bacteria in the early stages of a growing colony of bacteria can be approximated using the function:

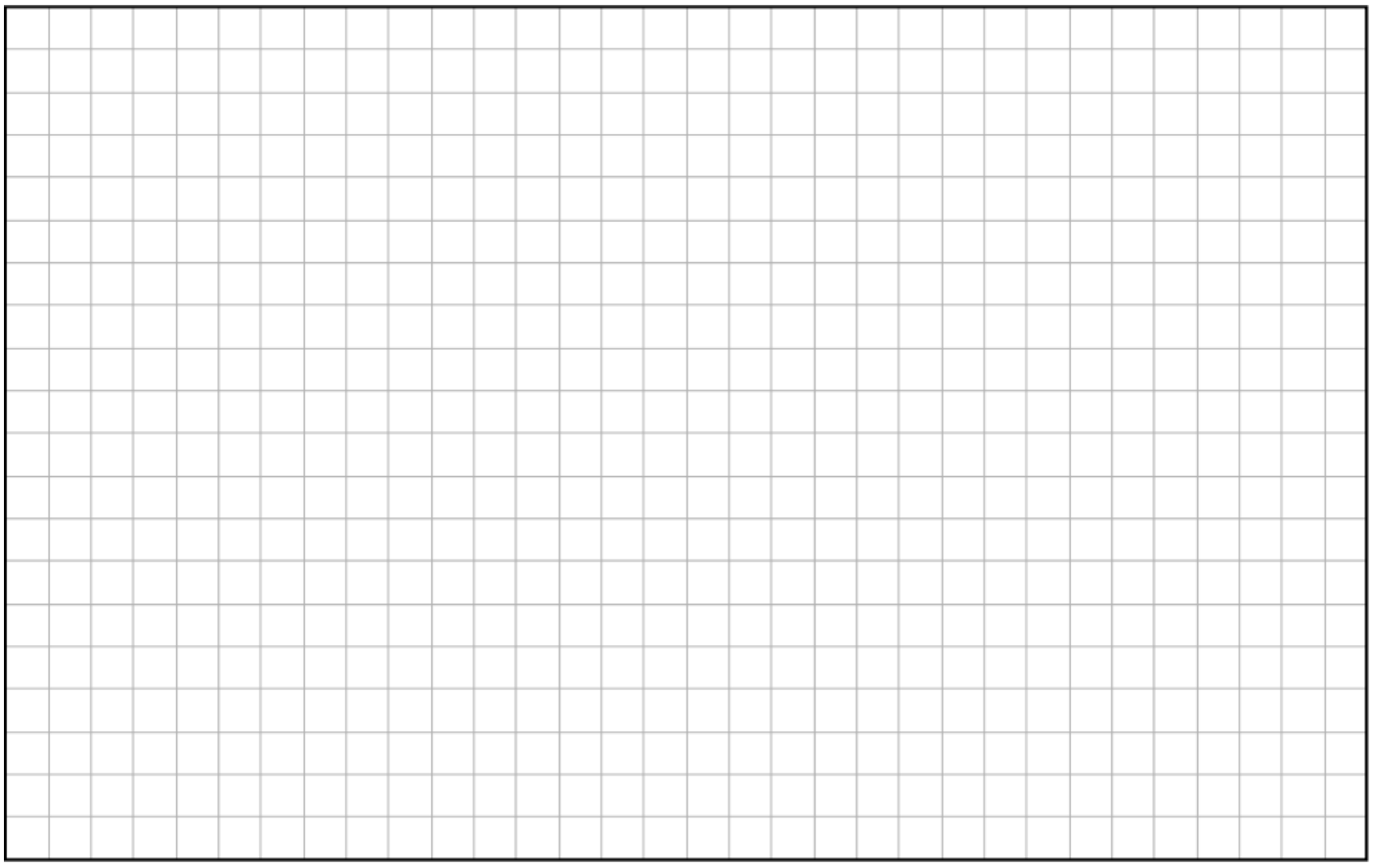
$$N(t) = 450e^{0.065t}$$

where t is the time, measured in hours, since the colony started to grow, and $N(t)$ is the number of bacteria in the colony at time t.

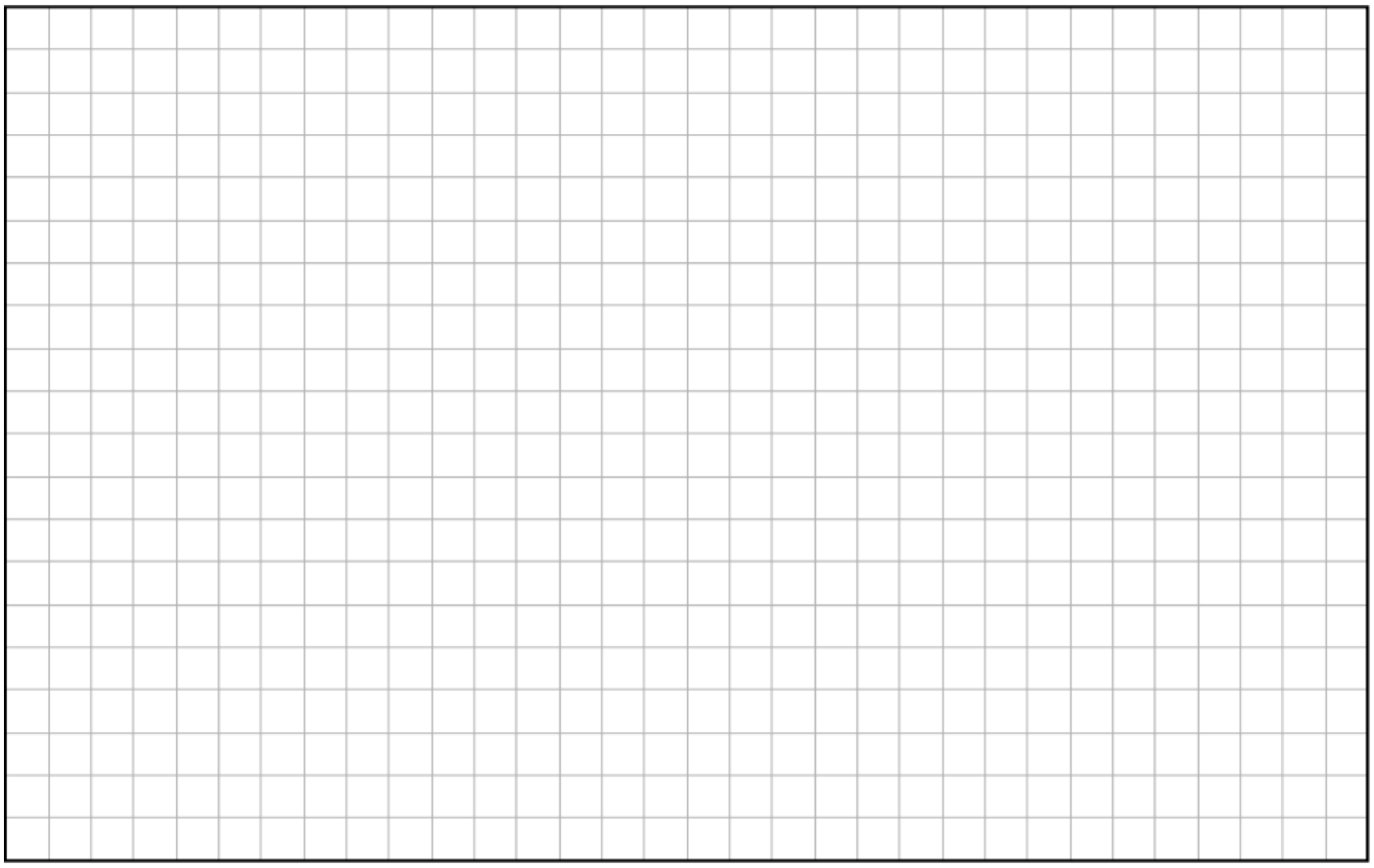
1. Initial Exploration (Set the Scene)



2. **Visualise the Function**



3. **Using outputs to find inputs!**



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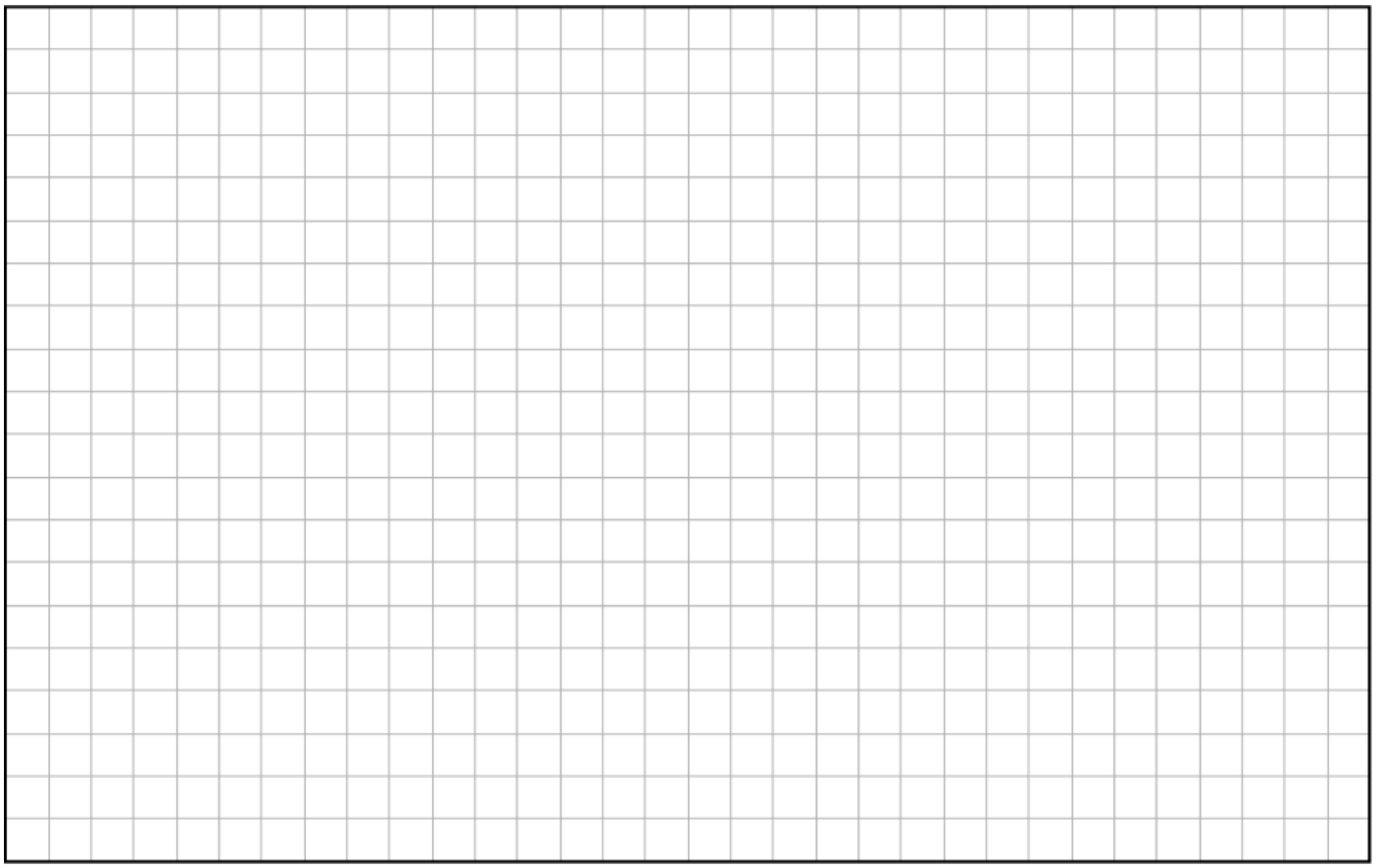

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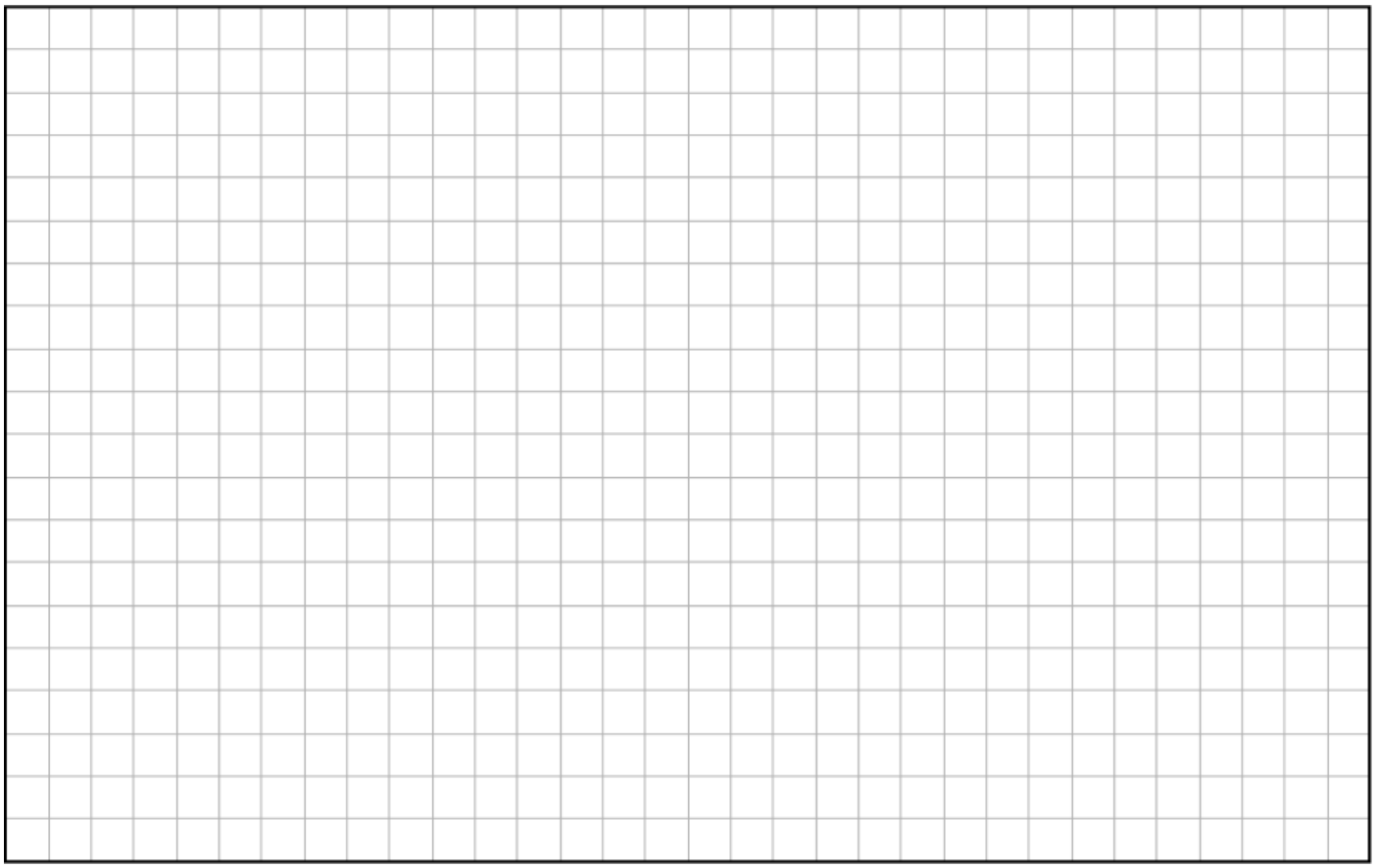


▶ 2019 LCHL Paper 1 – Question 8

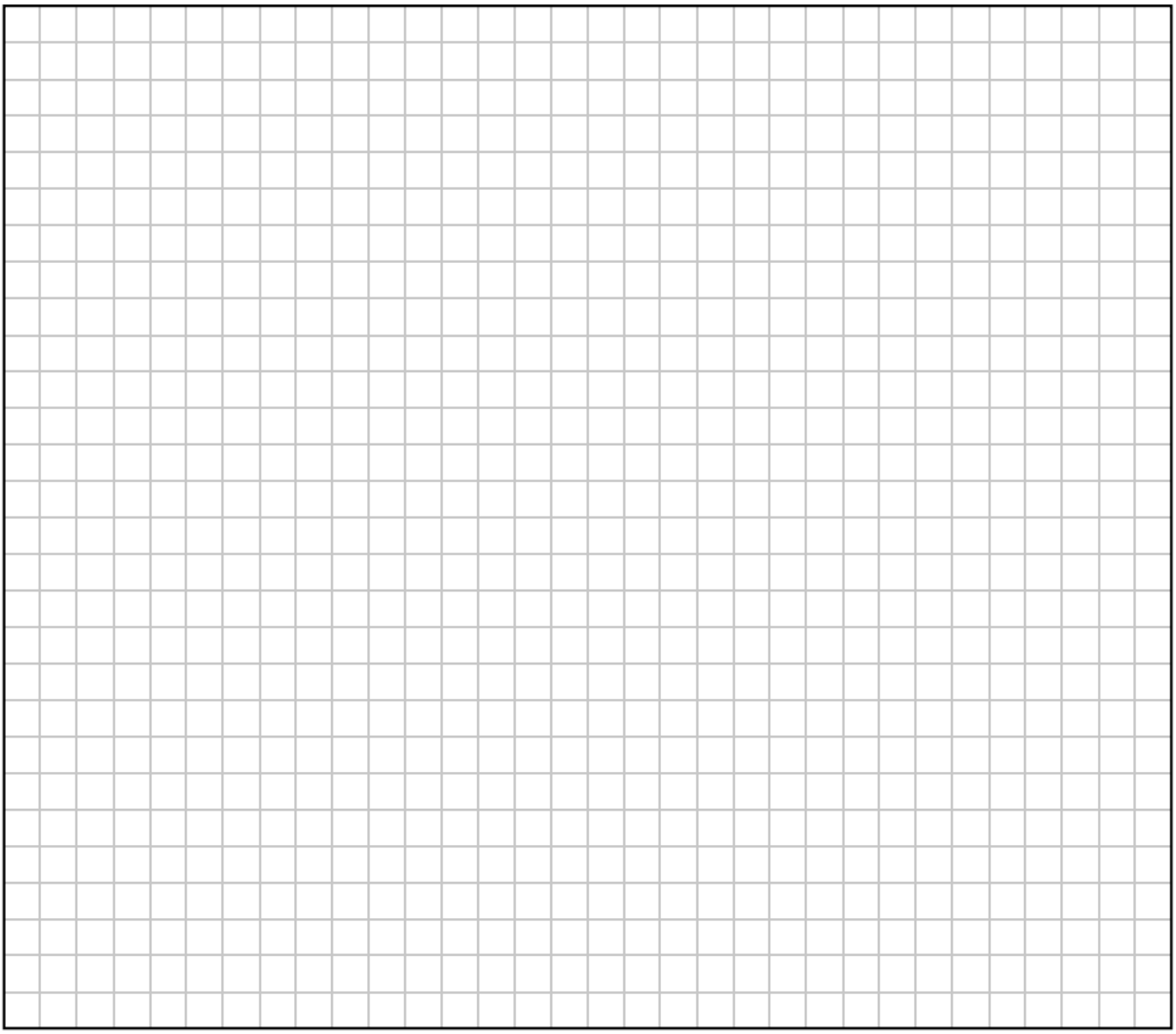
The weekly revenue produced by a company manufacturing air conditioning units is seasonal.

The revenue (in euro) can be approximated by the function:

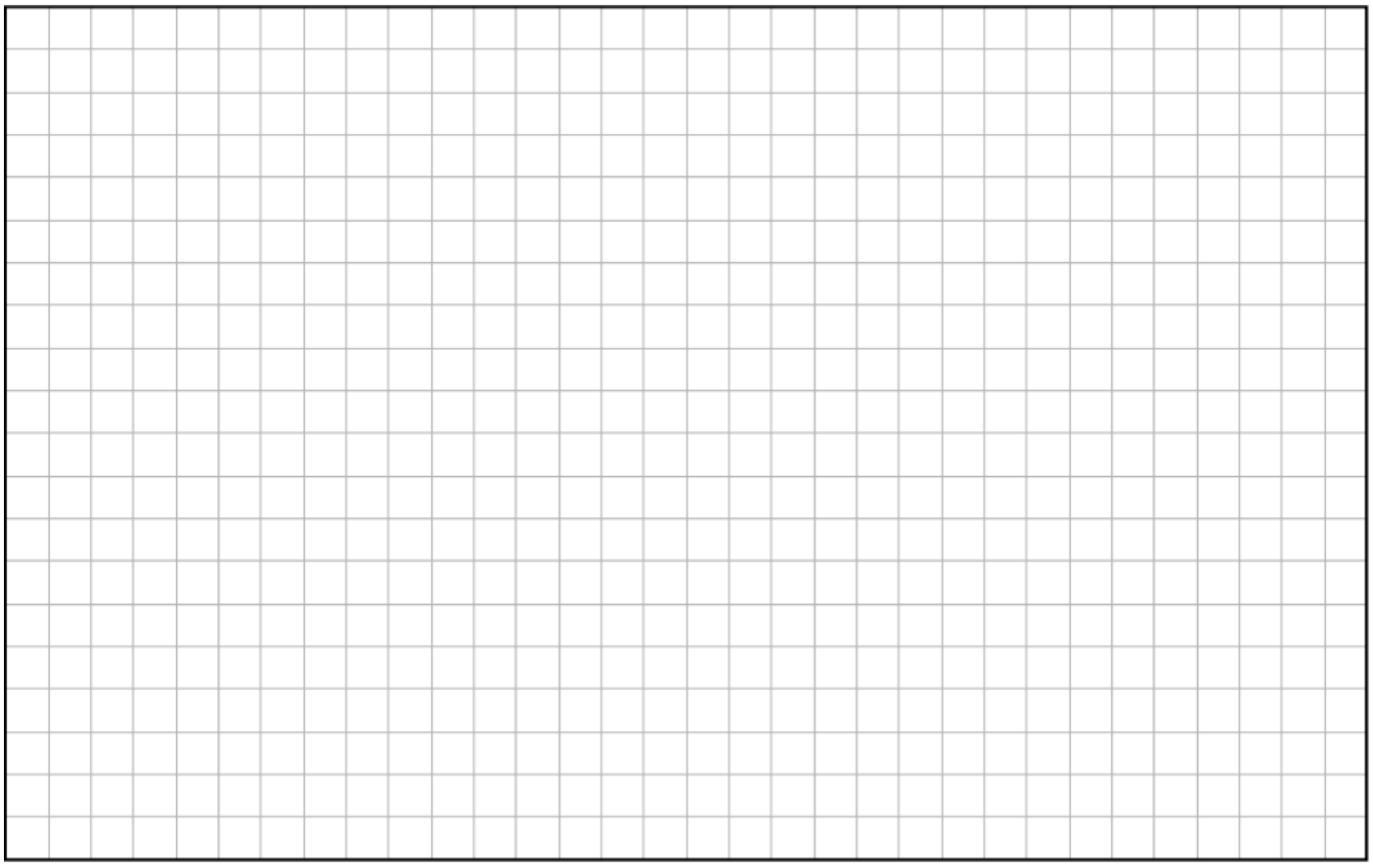
$$r(t) = 22\,500\cos\left(\frac{\pi}{26}t\right) + 37\,500,\ \ t \geq 0$$

where t is the number of weeks measured from the beginning of July and $\left(\frac{\pi}{26}t\right)$ is in radians.

1. Initial Exploration (Set the Scene)



2. **Visualise the Function**



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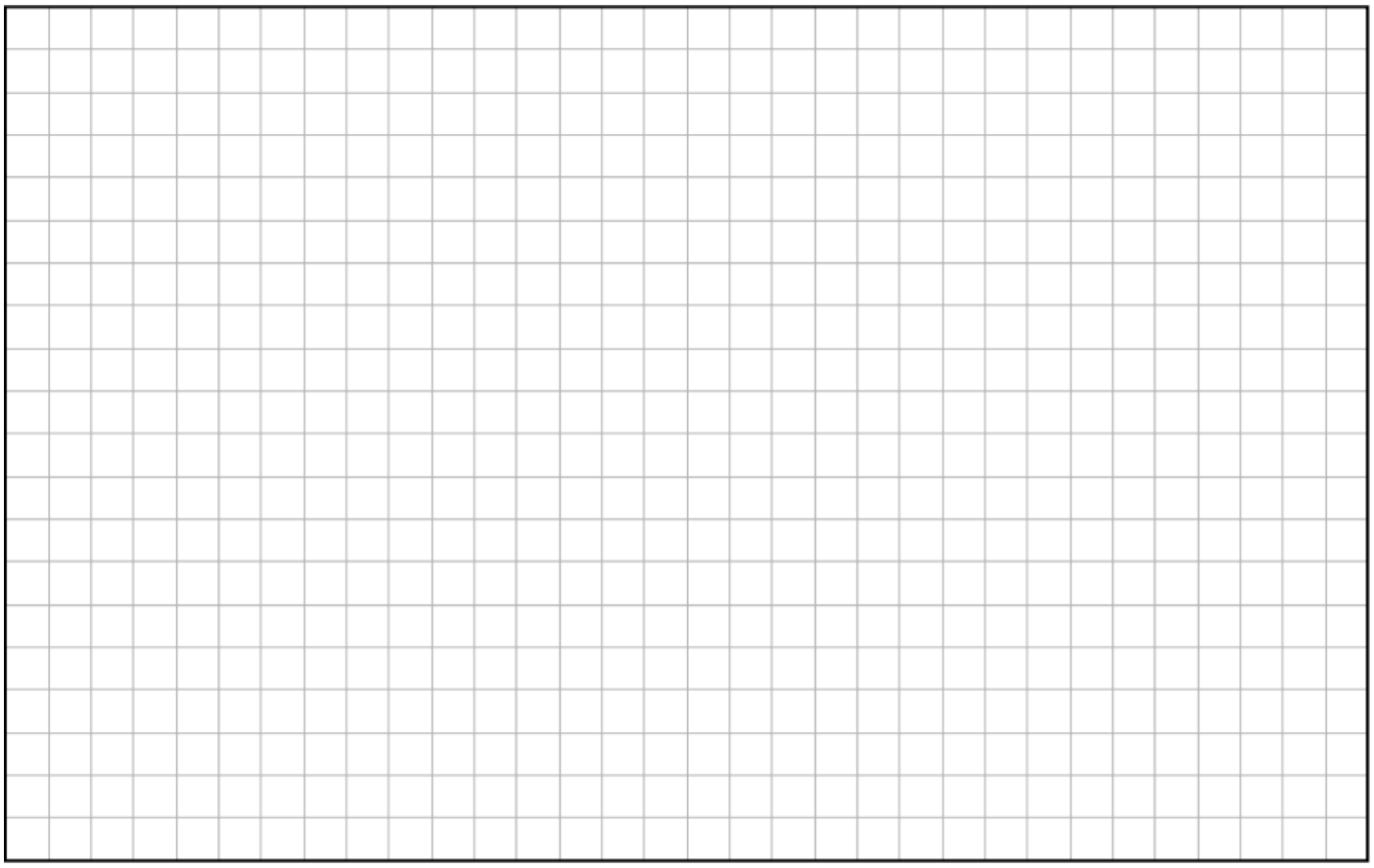

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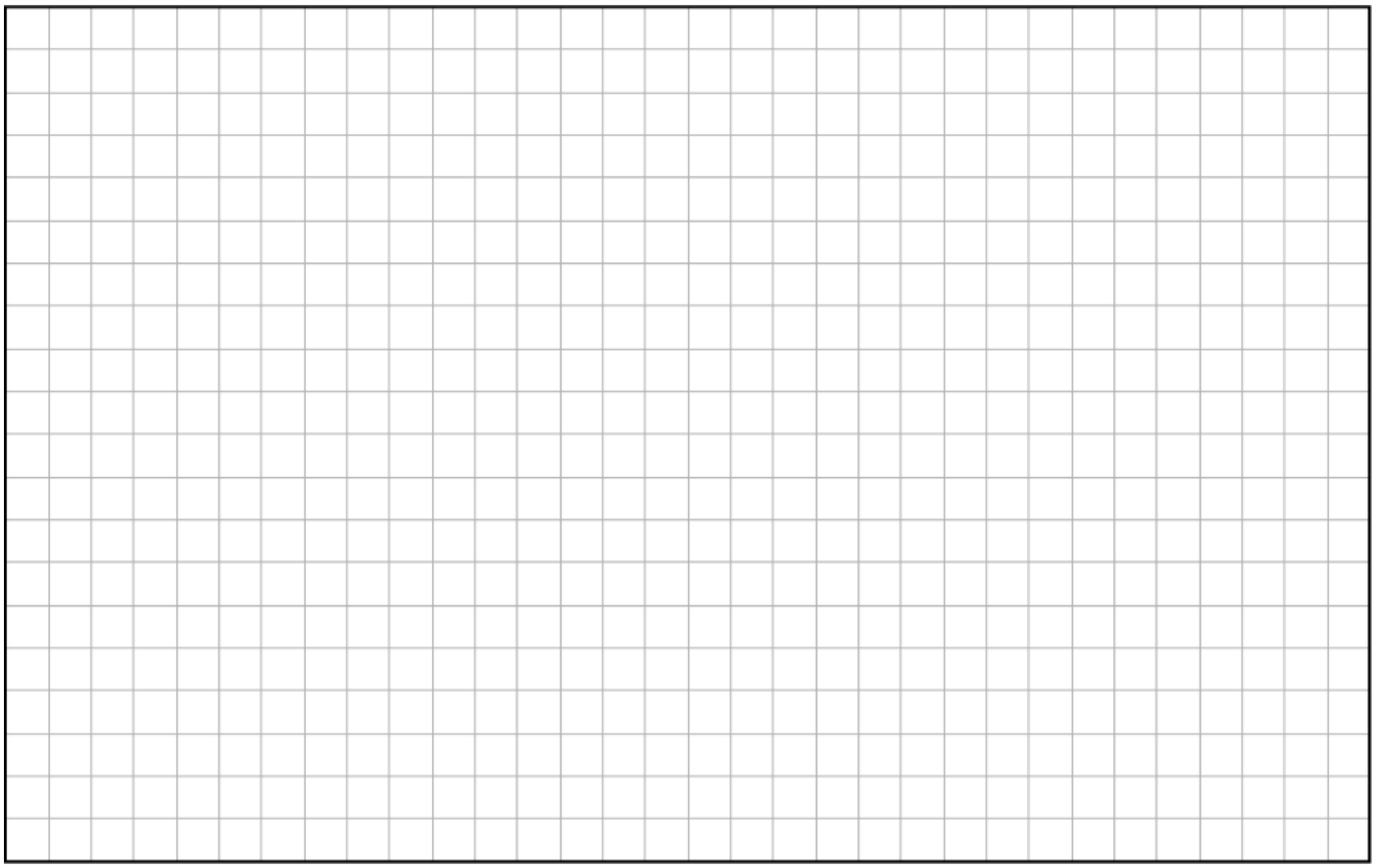

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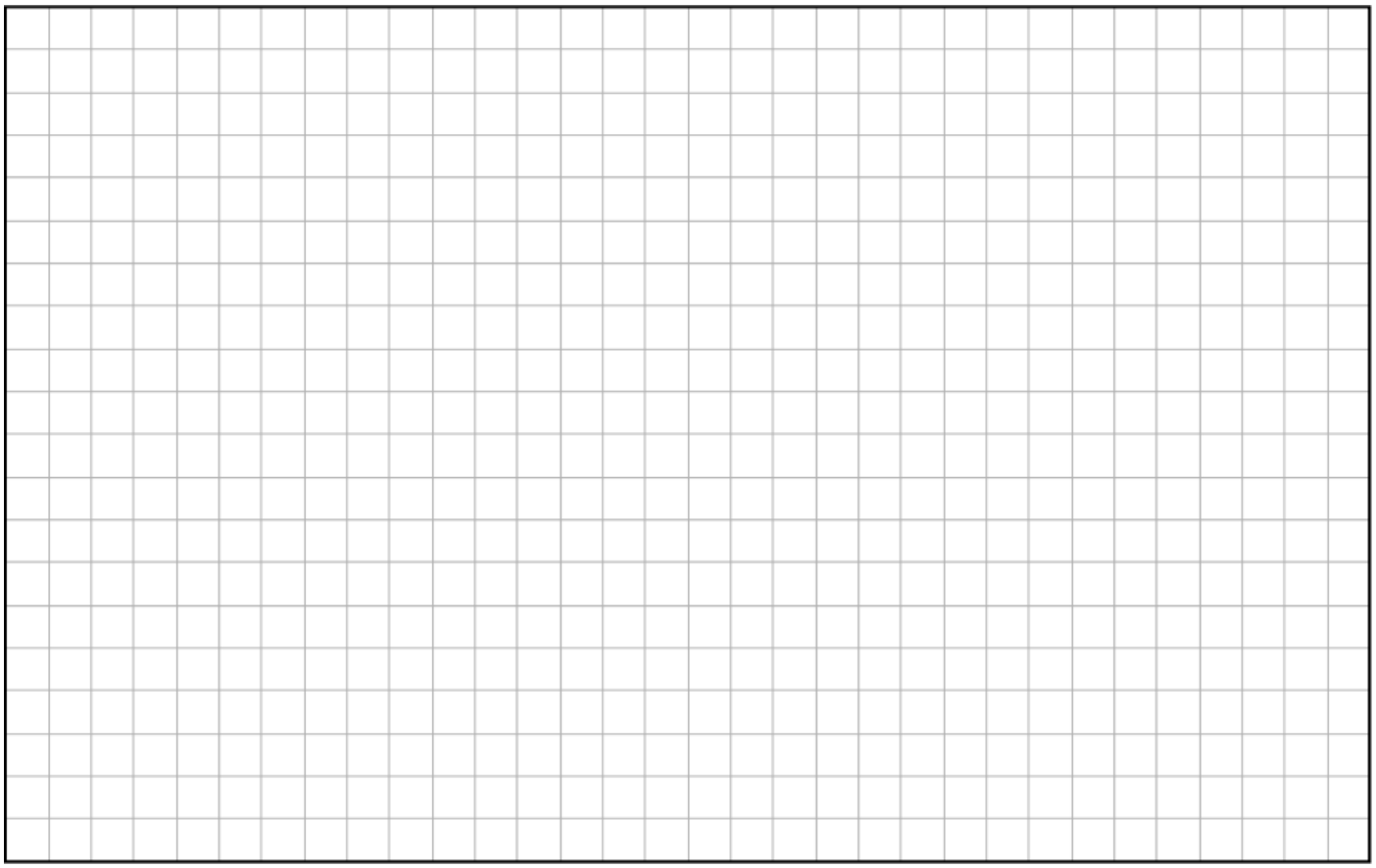
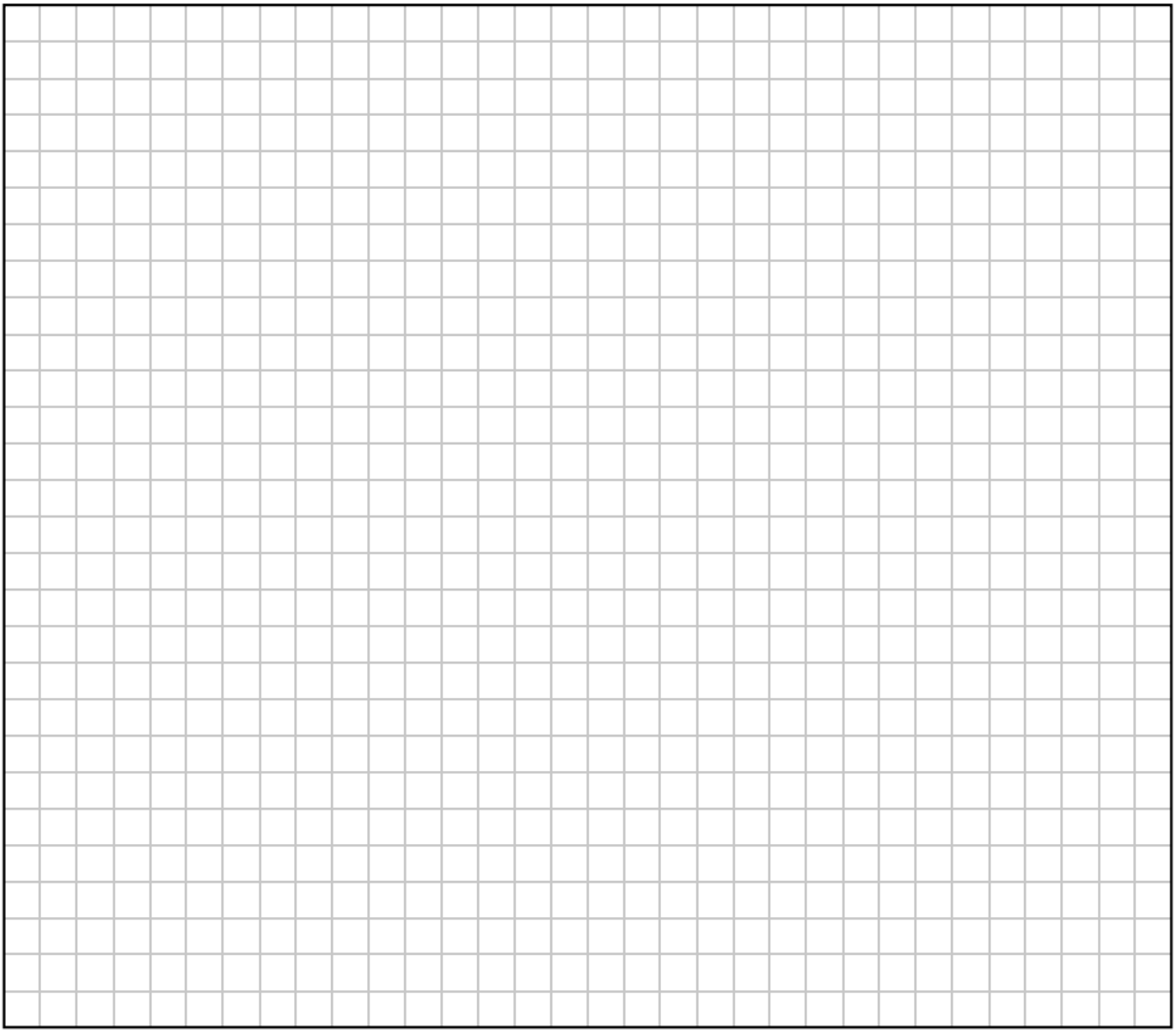


▶ 2018 LCHL Paper 1 – Question 7

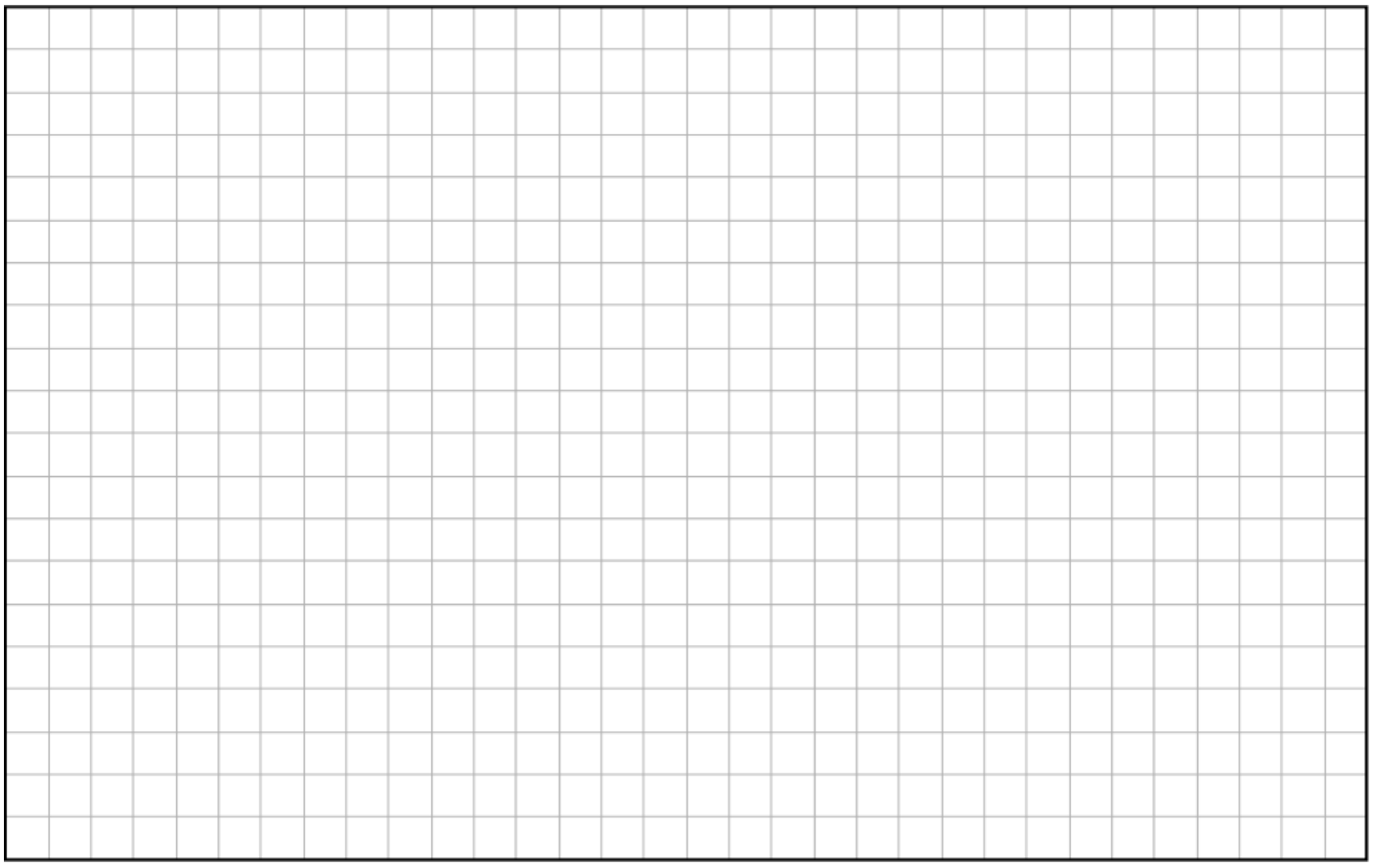
The time, in days of practice, it takes Jack to learn to type x words per minute (wpm) can be modelled by the function:

$t(x) = -62.5\left[\ln\left(1 - \frac{x}{80}\right)\right]$, where $0 \leq x \leq 70, x \in \mathbb{R}$.

1. Initial Exploration (Set the Scene)



2. **Visualise the Function**



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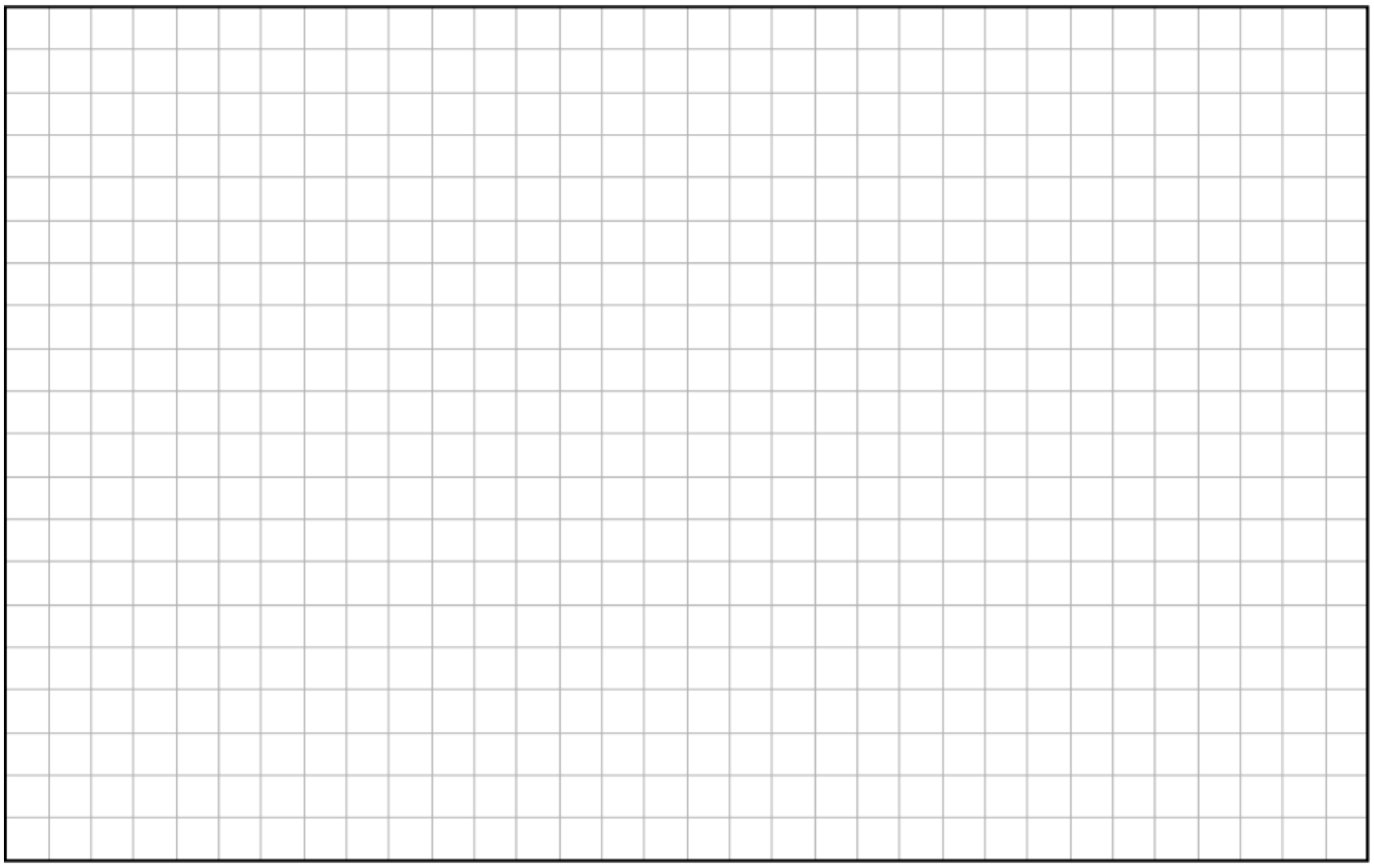

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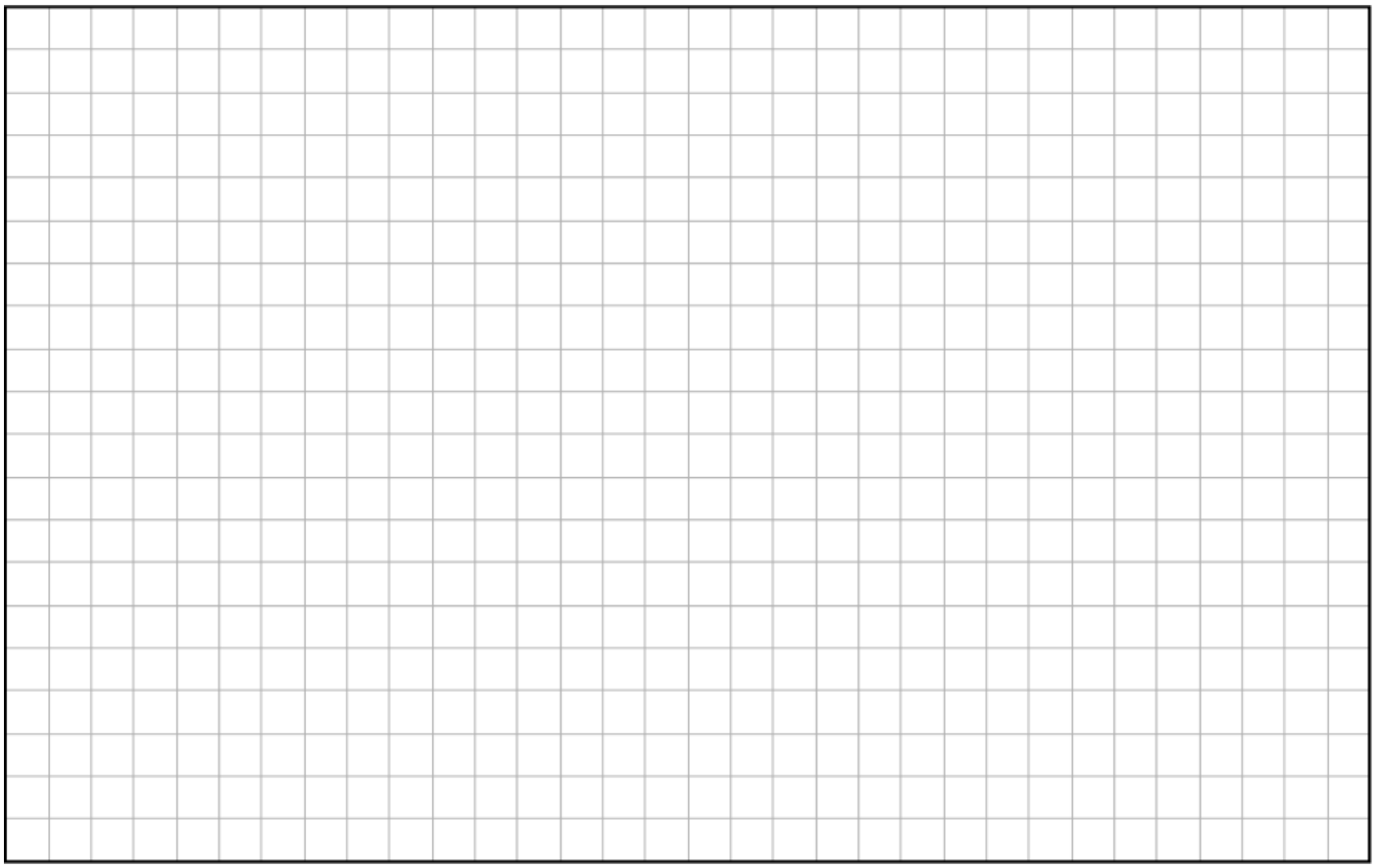

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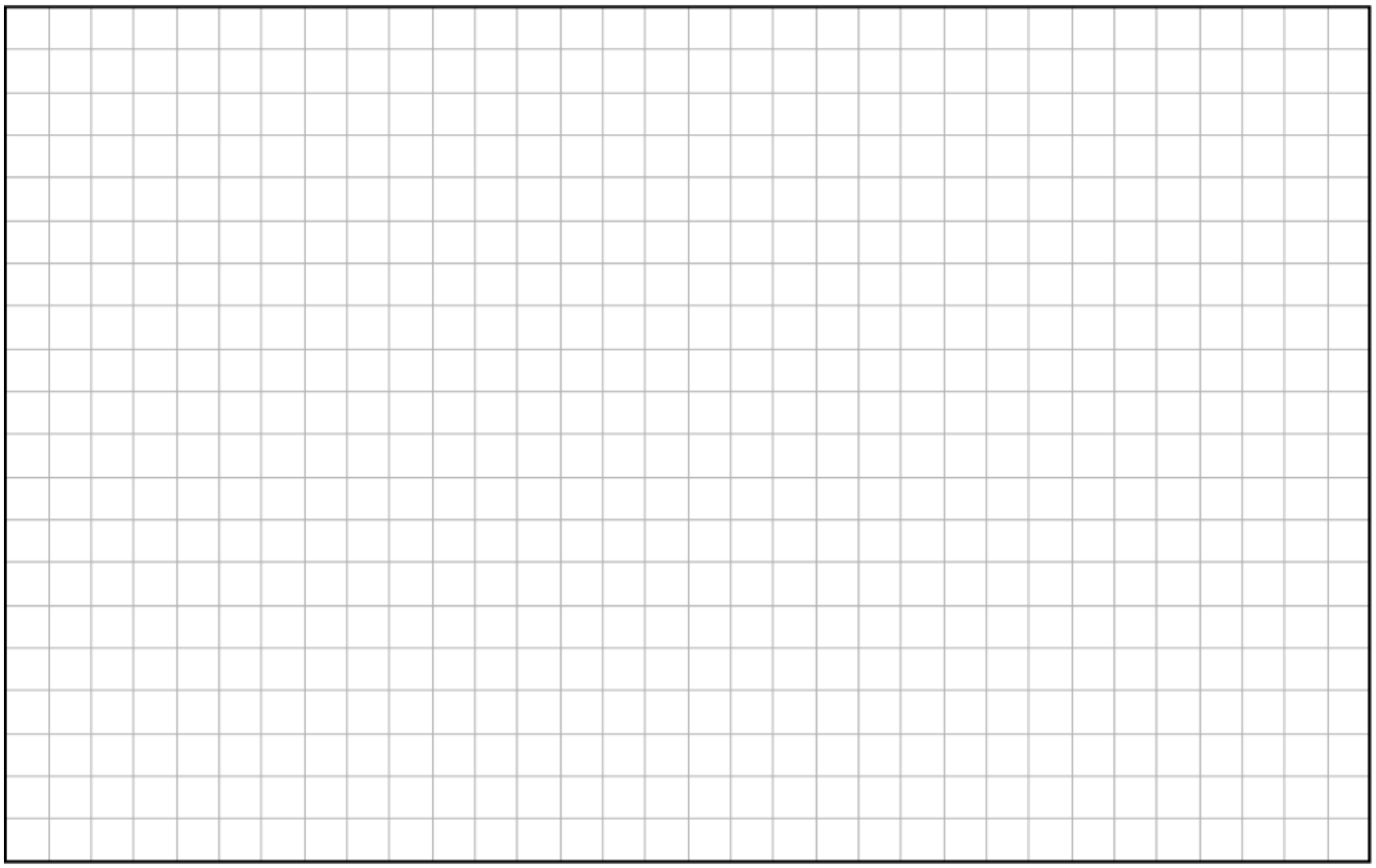


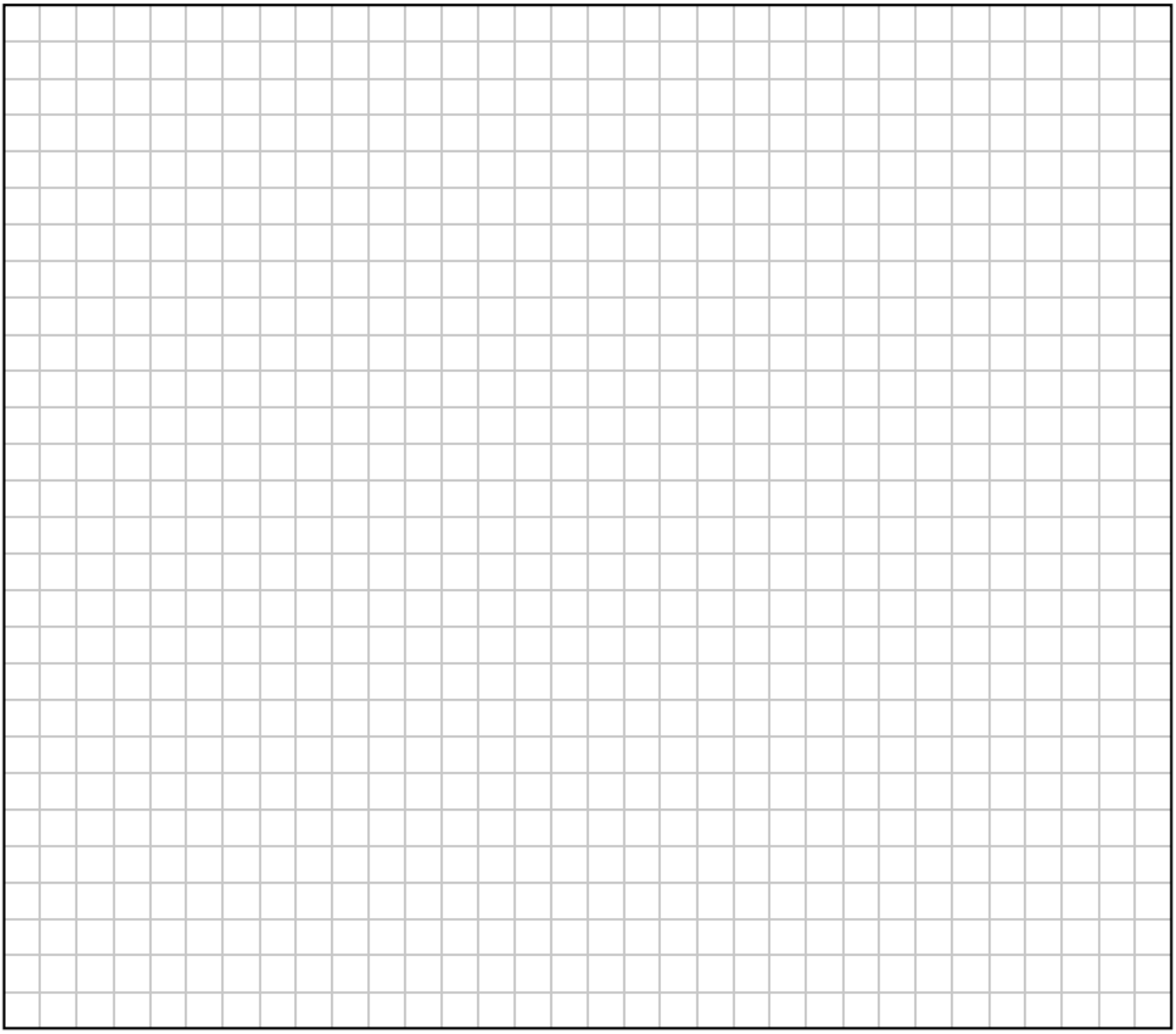
▶ 2024 LCOL Paper 1 – Question 7 (a)

During an experiment, the number of bacteria in a dish can be modelled by the function:

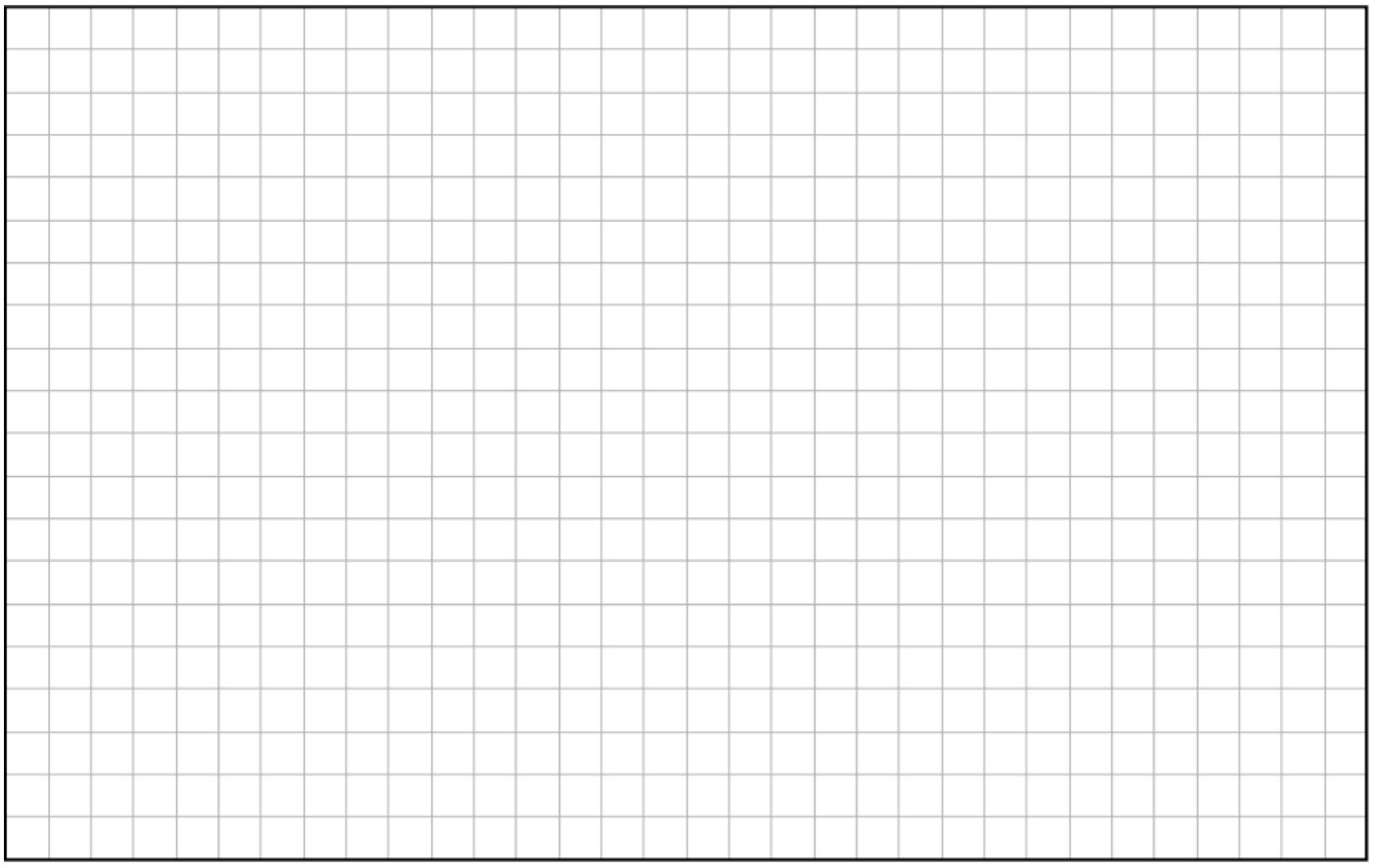
$$p(t) = t^3 - 6t^2 + 6t + 20$$

where $p(t)$ is the number of bacteria, in thousands, and t is the time in hours since the start of the experiment, with $0 \leq t \leq 5,\ t \in \mathbb{R}$.

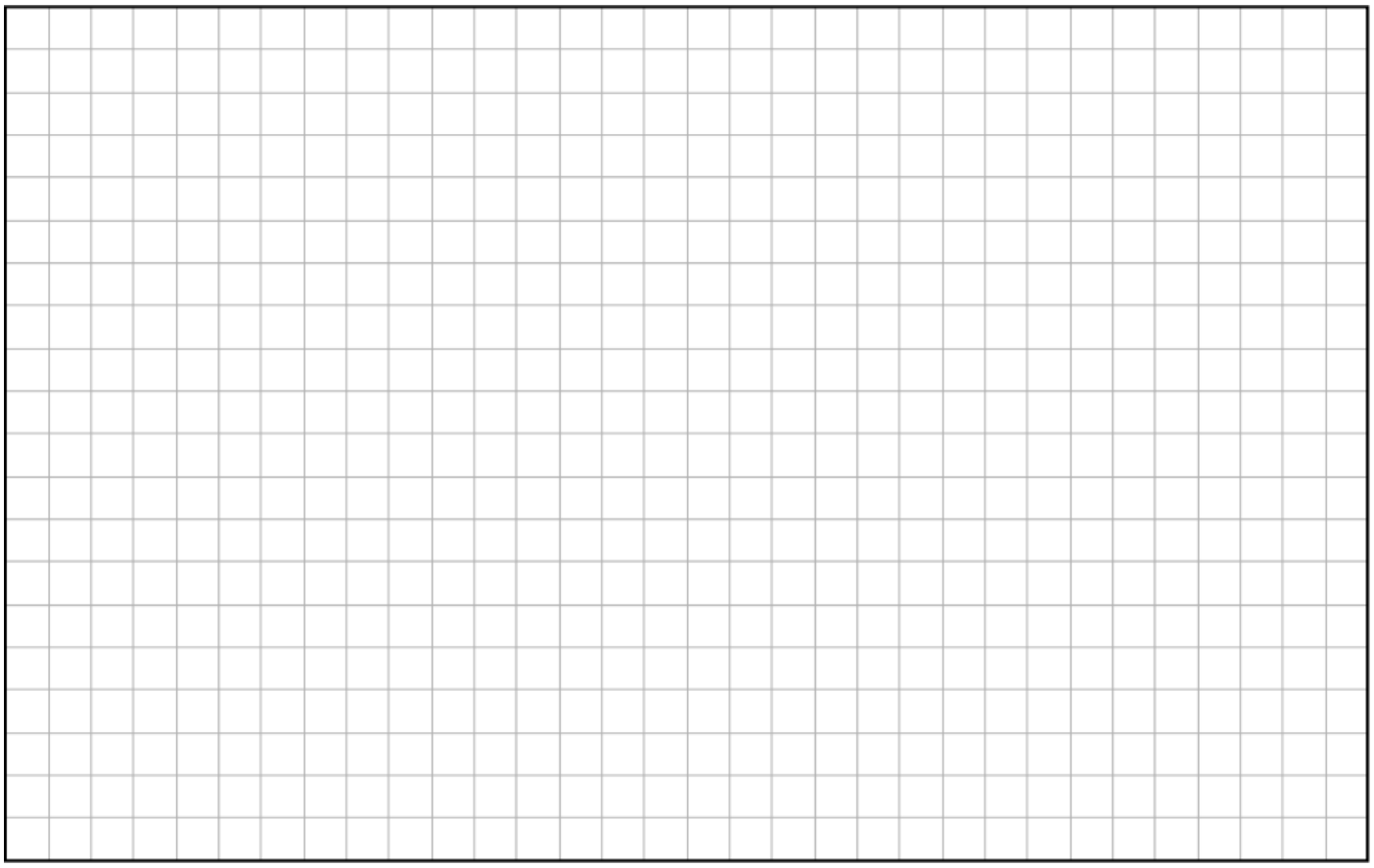
1. Initial Exploration (Set the Scene)



2. **Visualise the Function**



3. **Using outputs to find inputs!**



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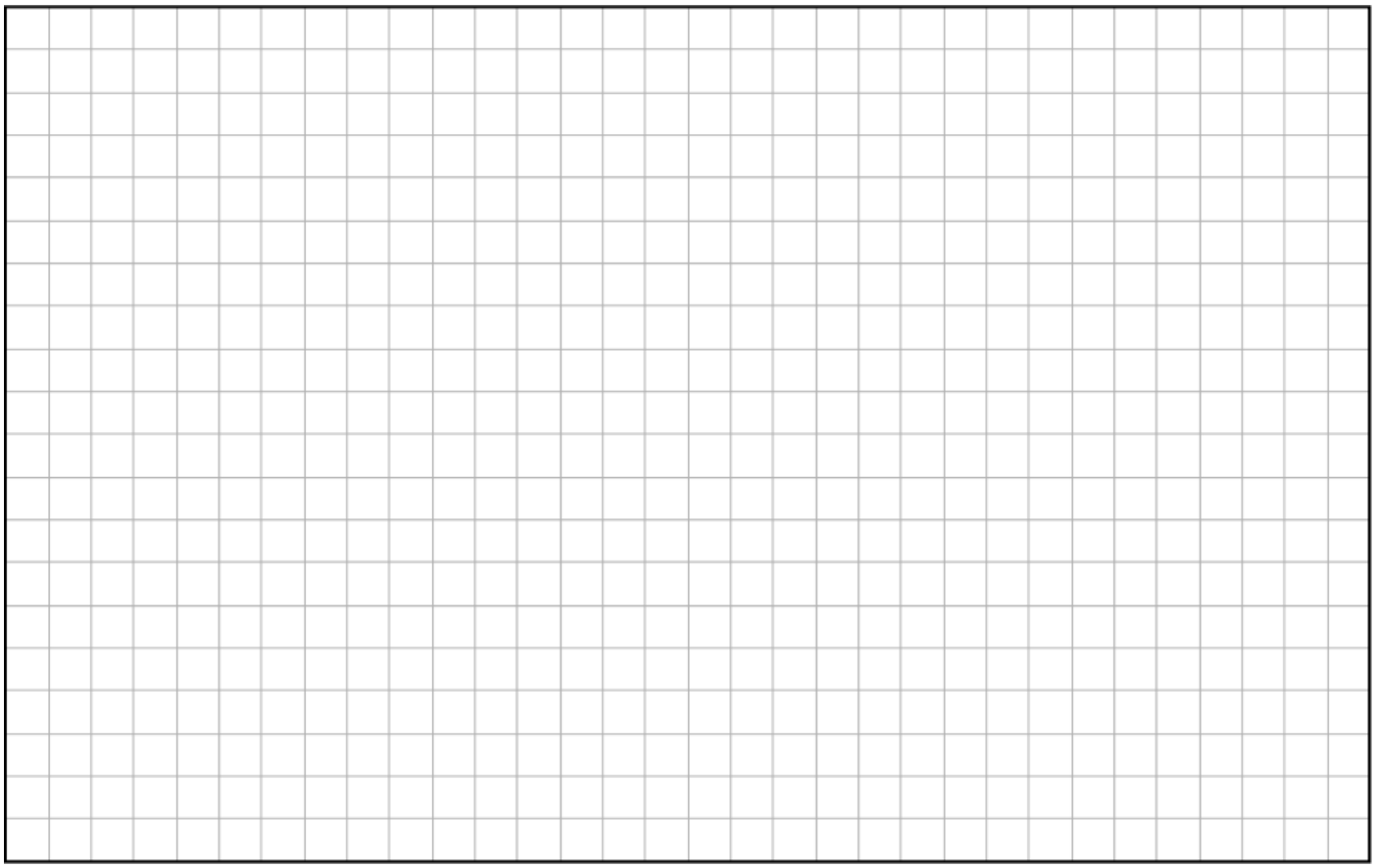

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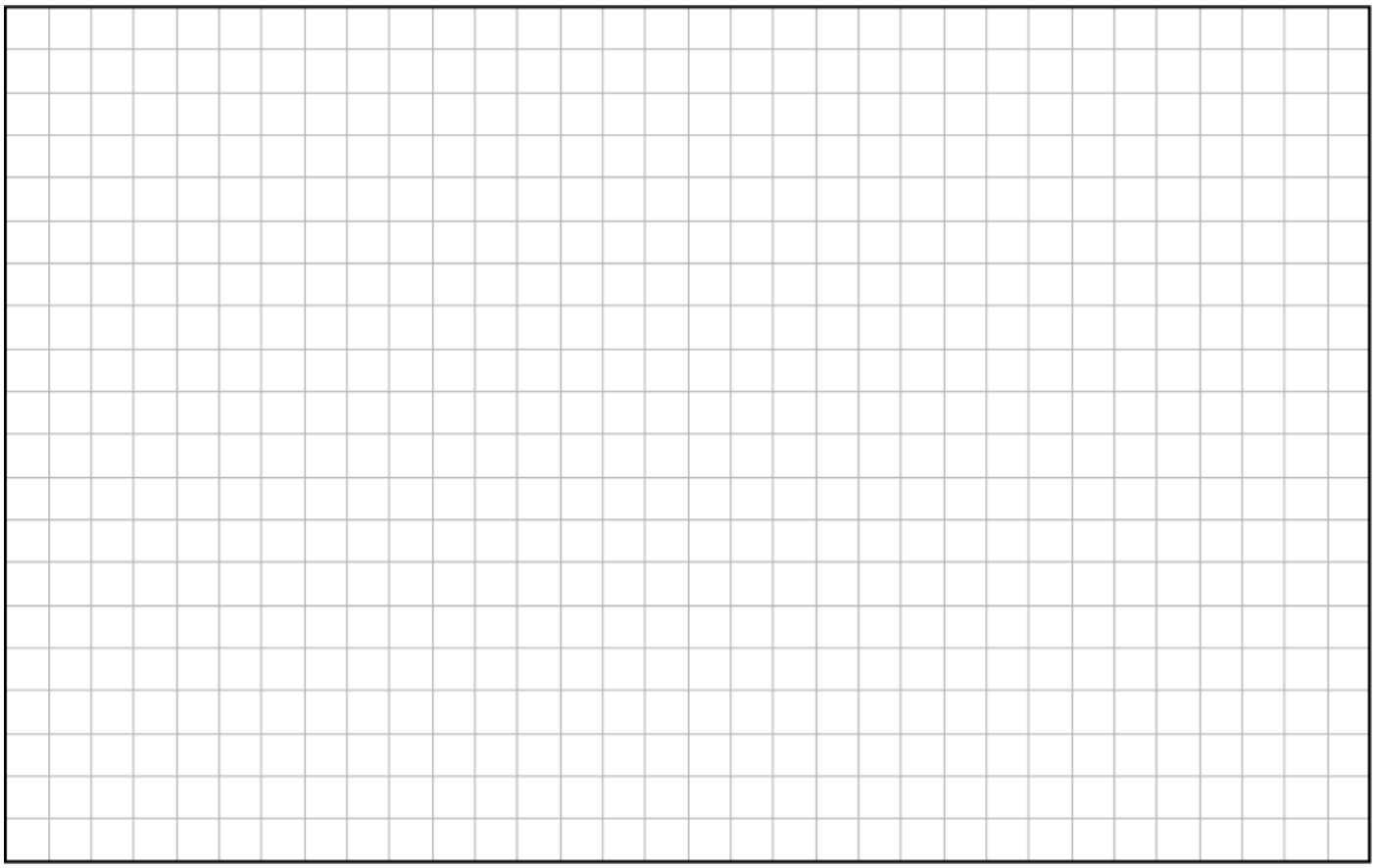


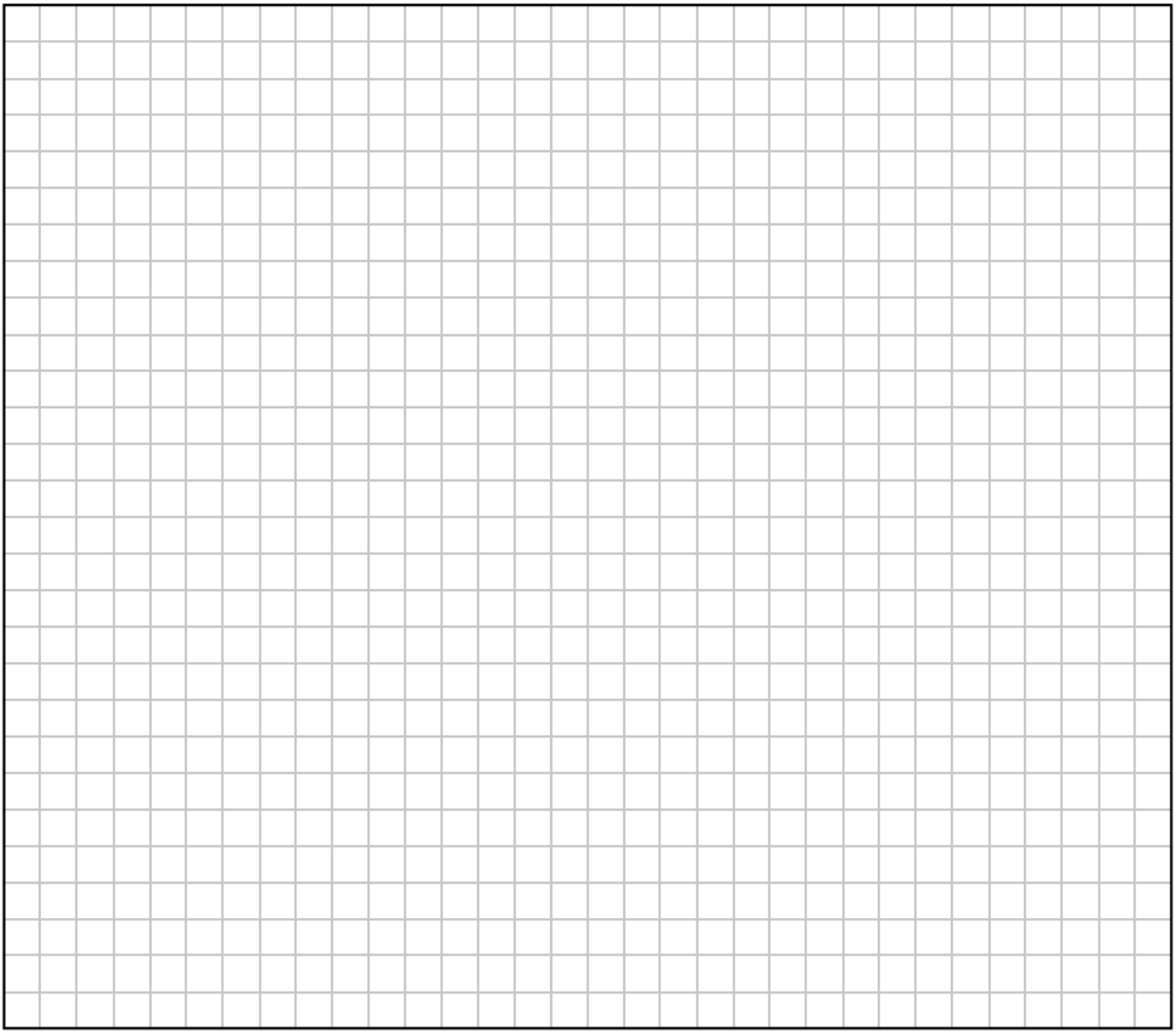
▶ 2024 LCOL Paper 1 – Question 7 (b)

Over the course of one day (24 hours), the probability that it is raining in Waterville, $P(t)$, could be modelled by the following function:

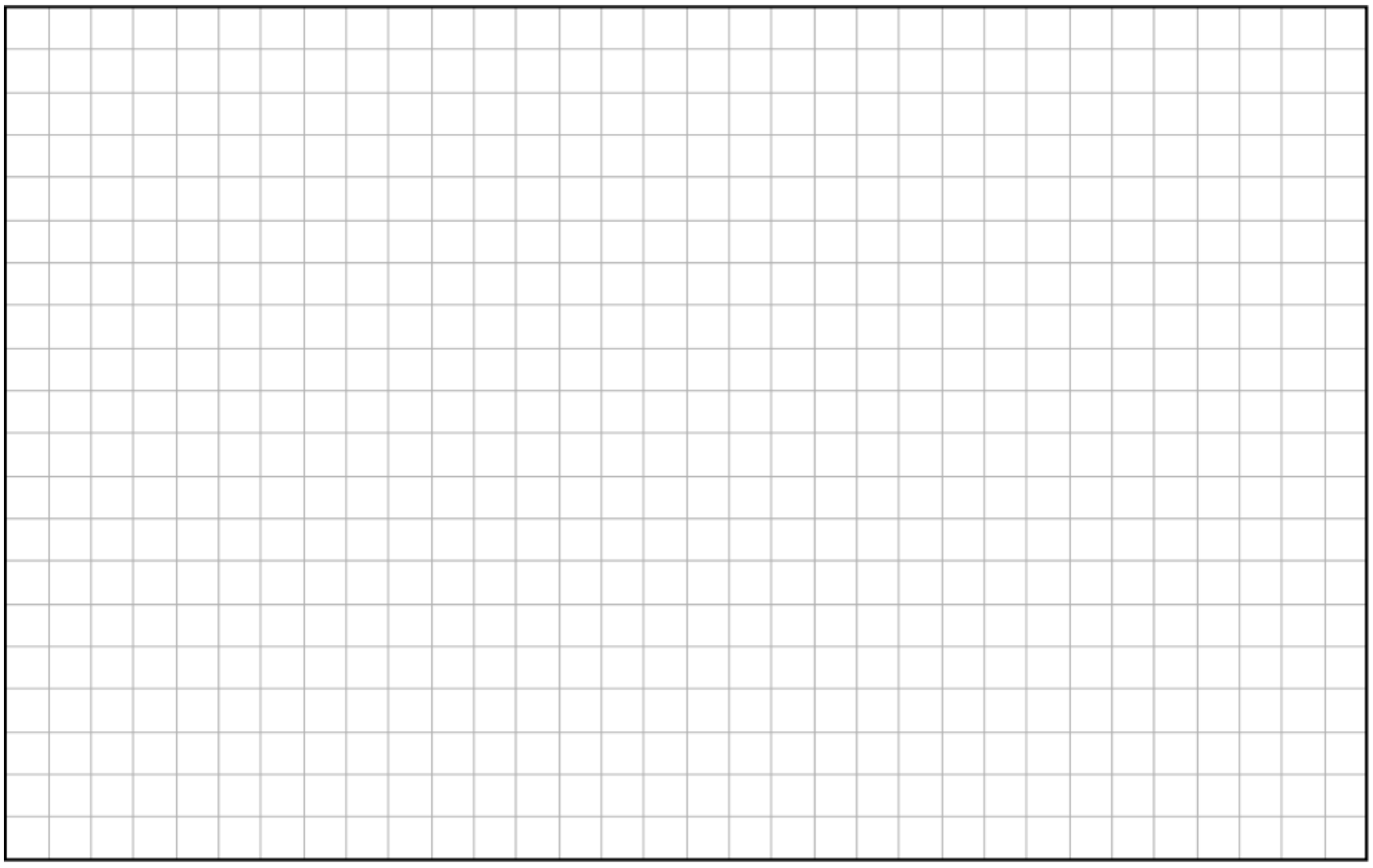
$P(t) = 0.3 + 0.02t$.

Here, $P(t)$ is the probability that it is raining t hours after the start of the day, with $0 \leq t \leq 24$, $t \in \mathbb{R}$.

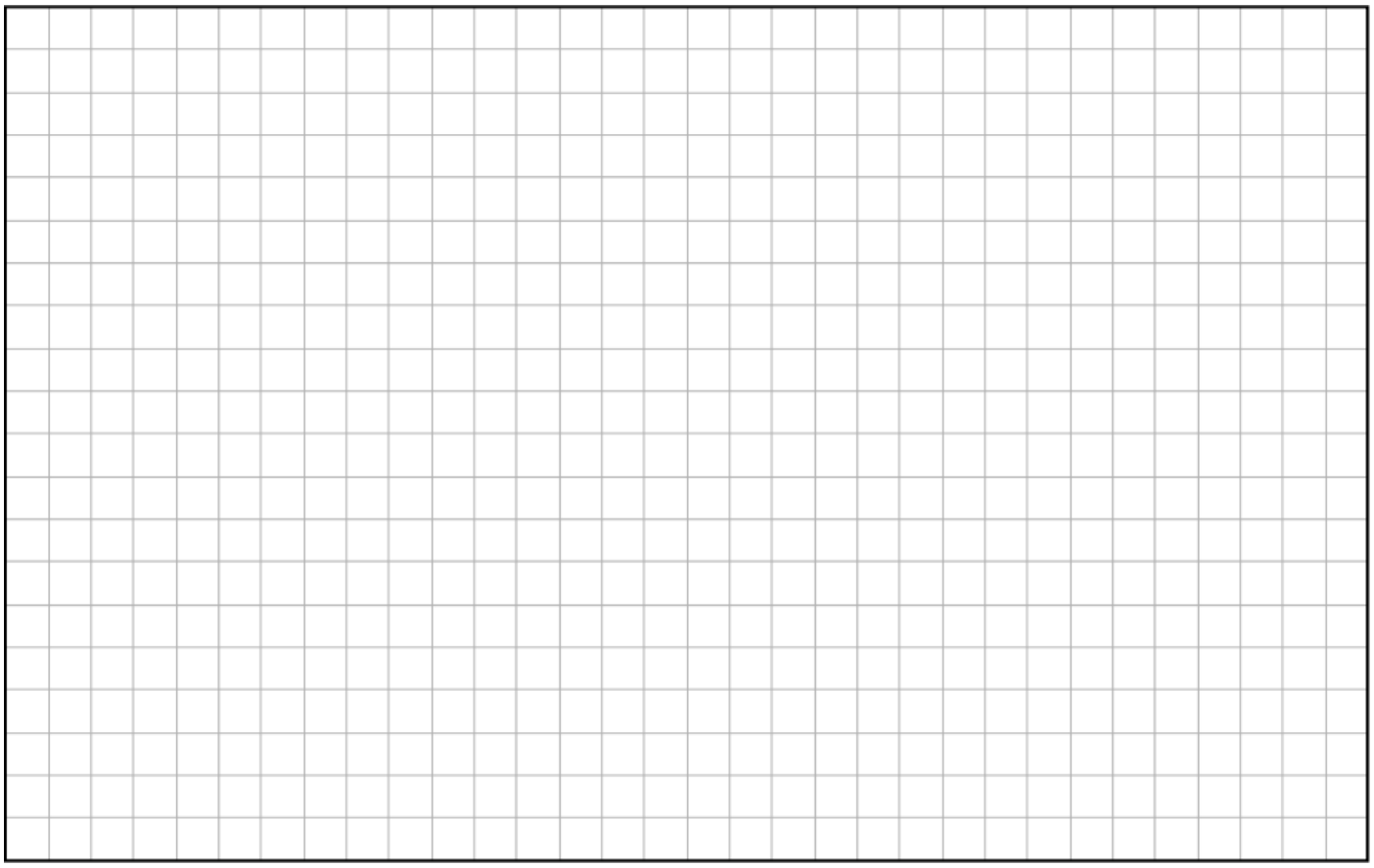
1. Initial Exploration (Set the Scene)



2. **Visualise the Function**



3. **Using outputs to find inputs!**



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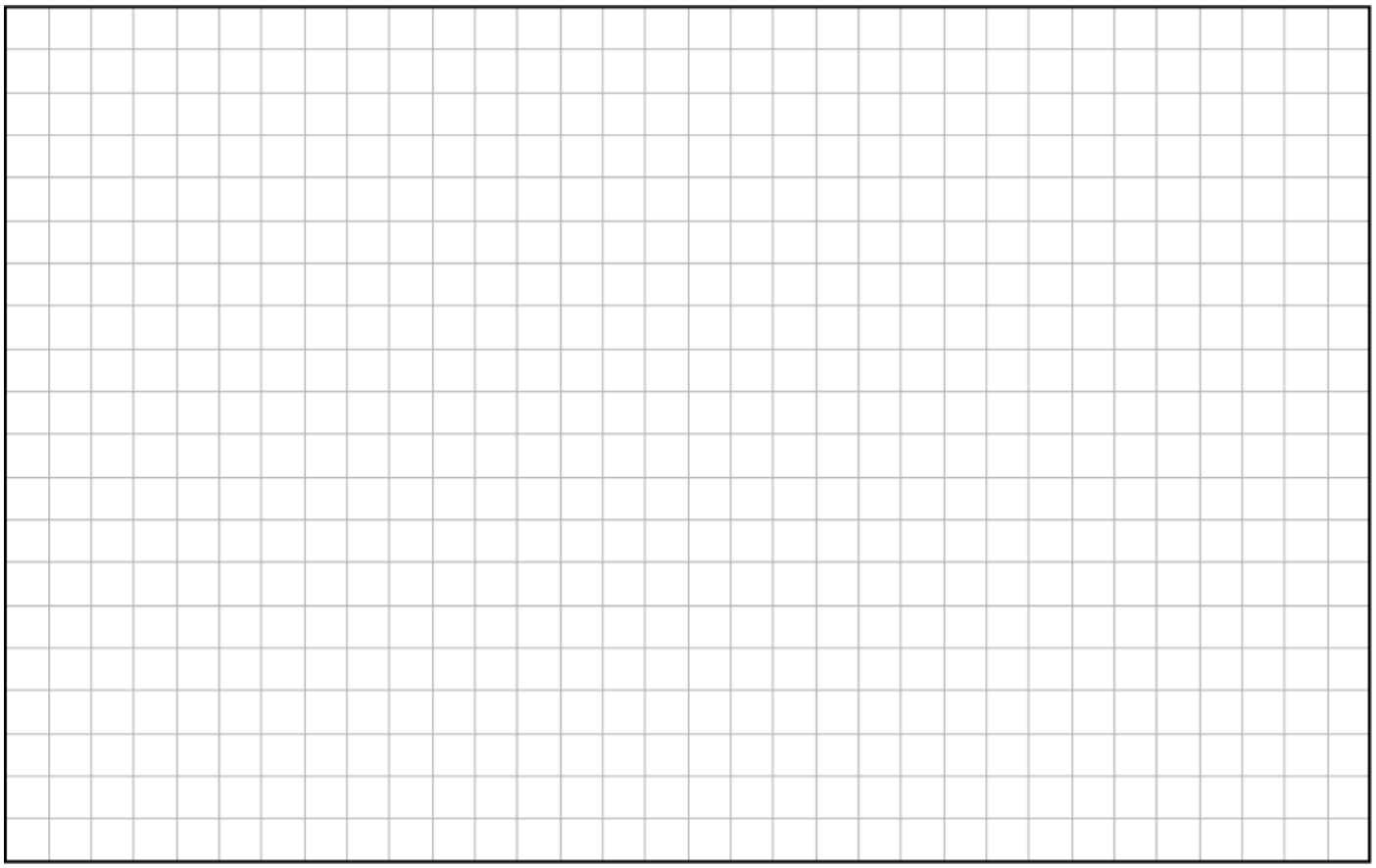

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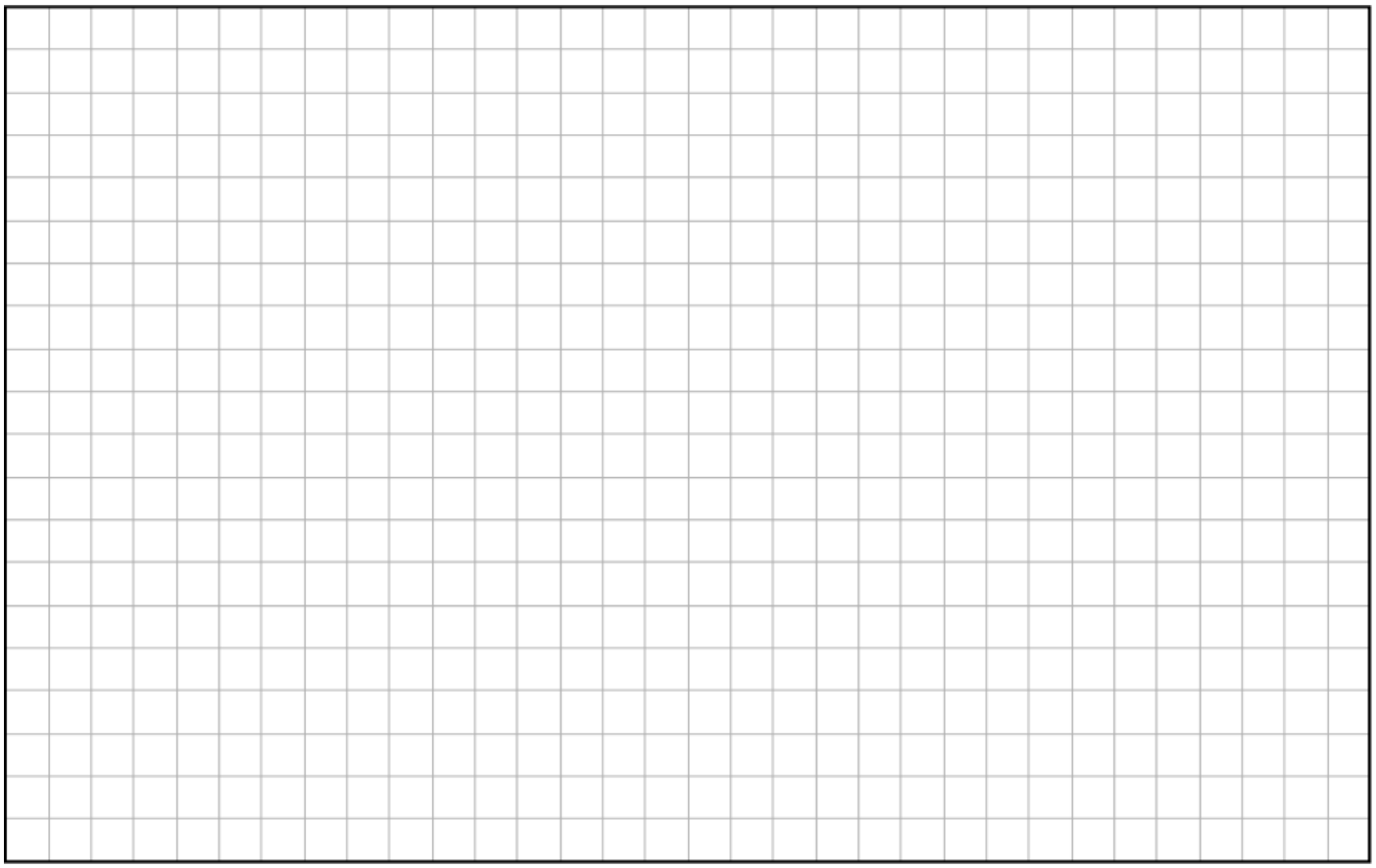


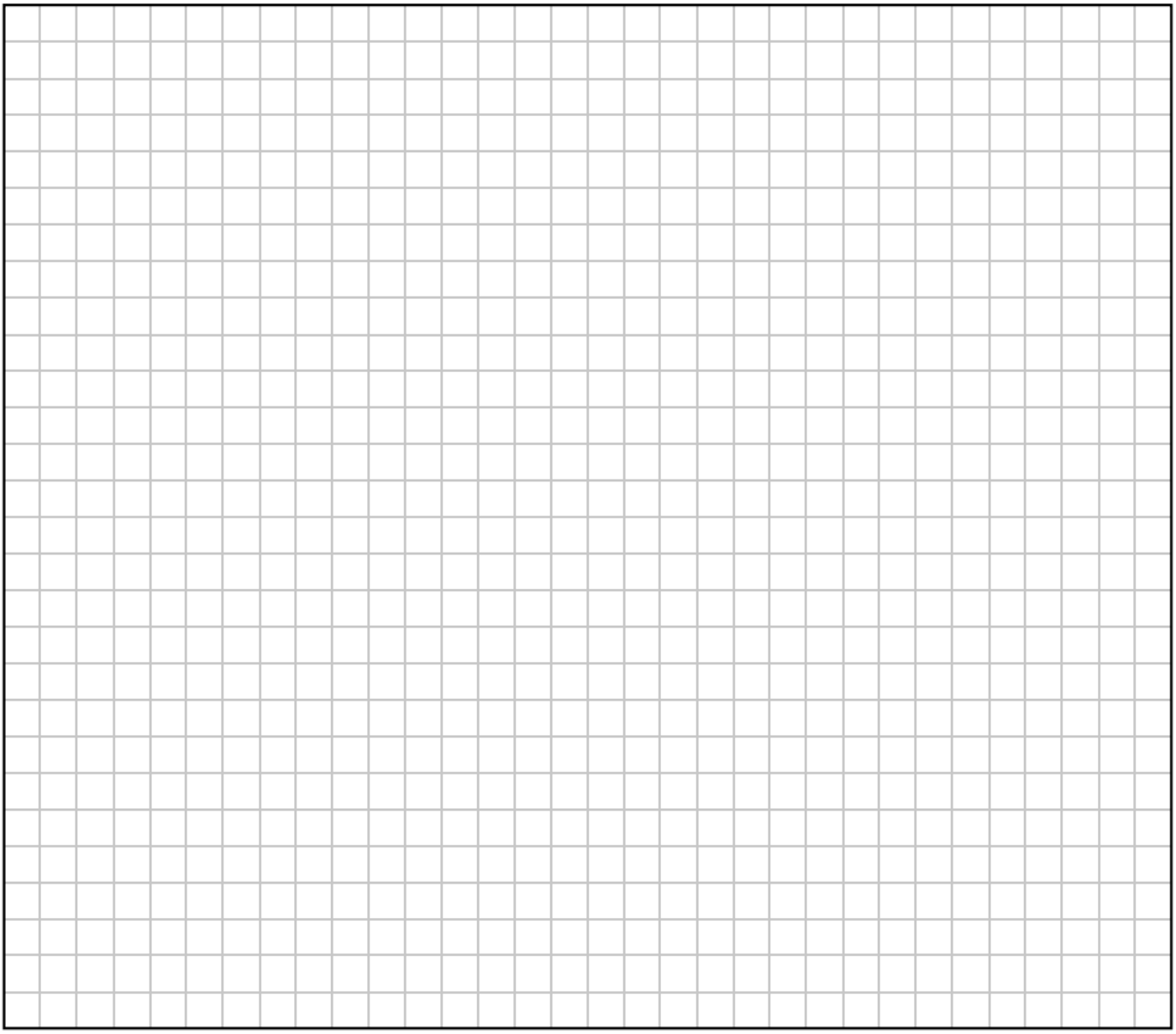
▶ 2022 LCOL Paper 1 – Question 7

Joseph is doing a training session. During the session, his heart-rate, $h(x)$, is measured in beats per minute (BPM). For part of the session, $h(x)$ can be modelled using the following function:

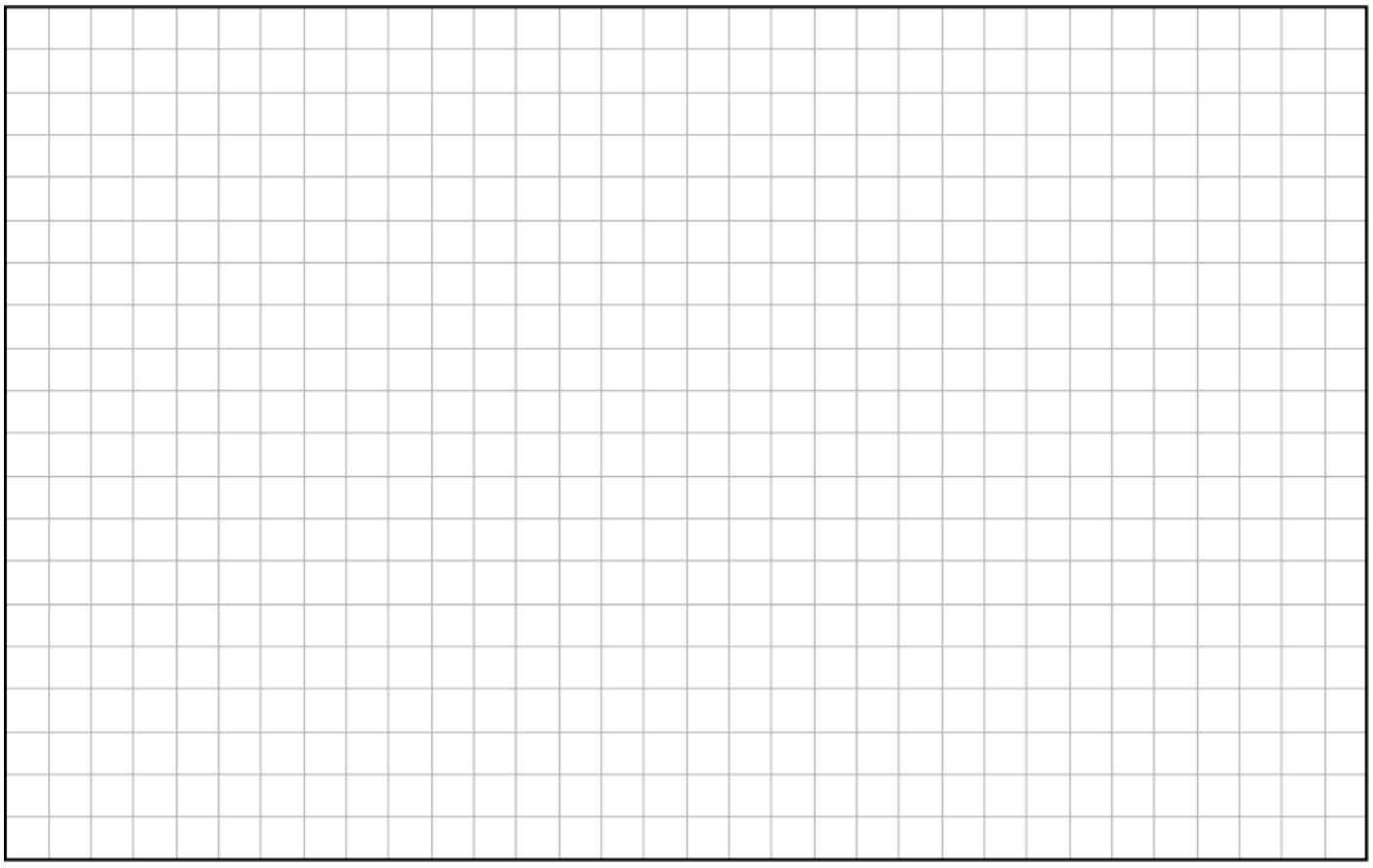
$$h(x) = -0.38x^3 + 2.6x^2 - 0.13x + 158$$

where x is the time, in minutes, from the start of the session, and $0 \leq x \leq 6, x \in \mathbb{R}$.

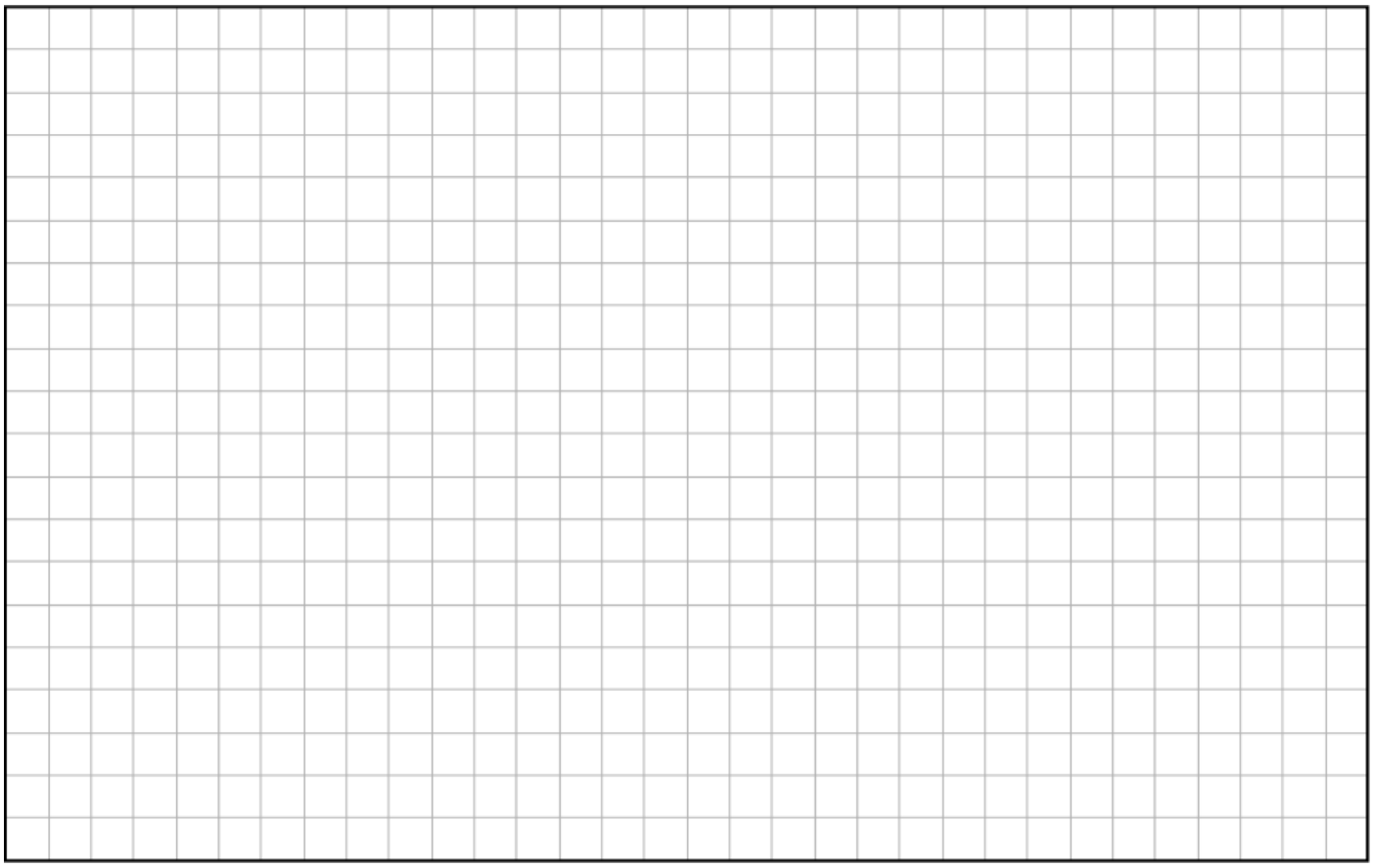
1. Initial Exploration (Set the Scene)



2. **Visualise the Function**



3. **Using outputs to find inputs!**



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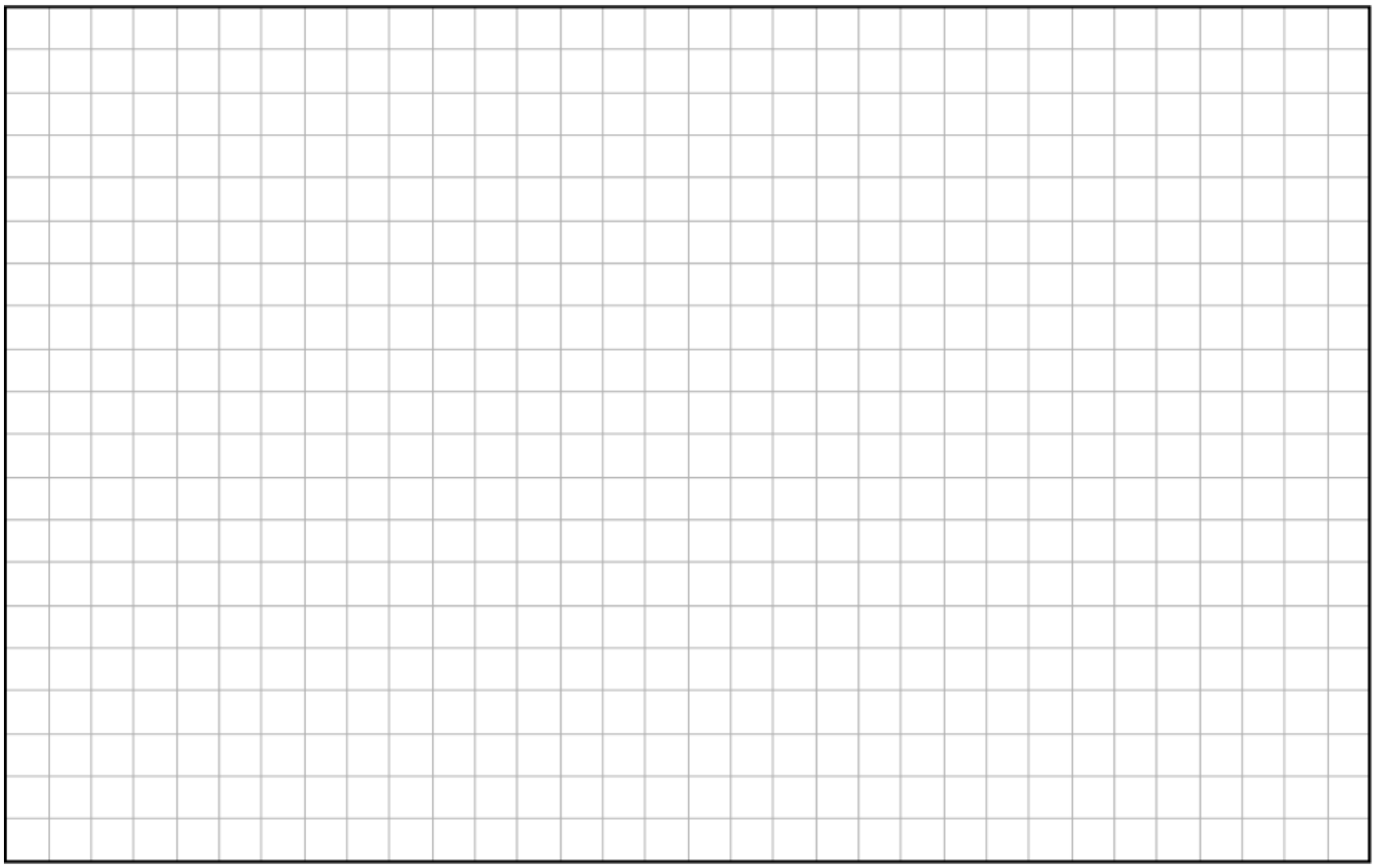

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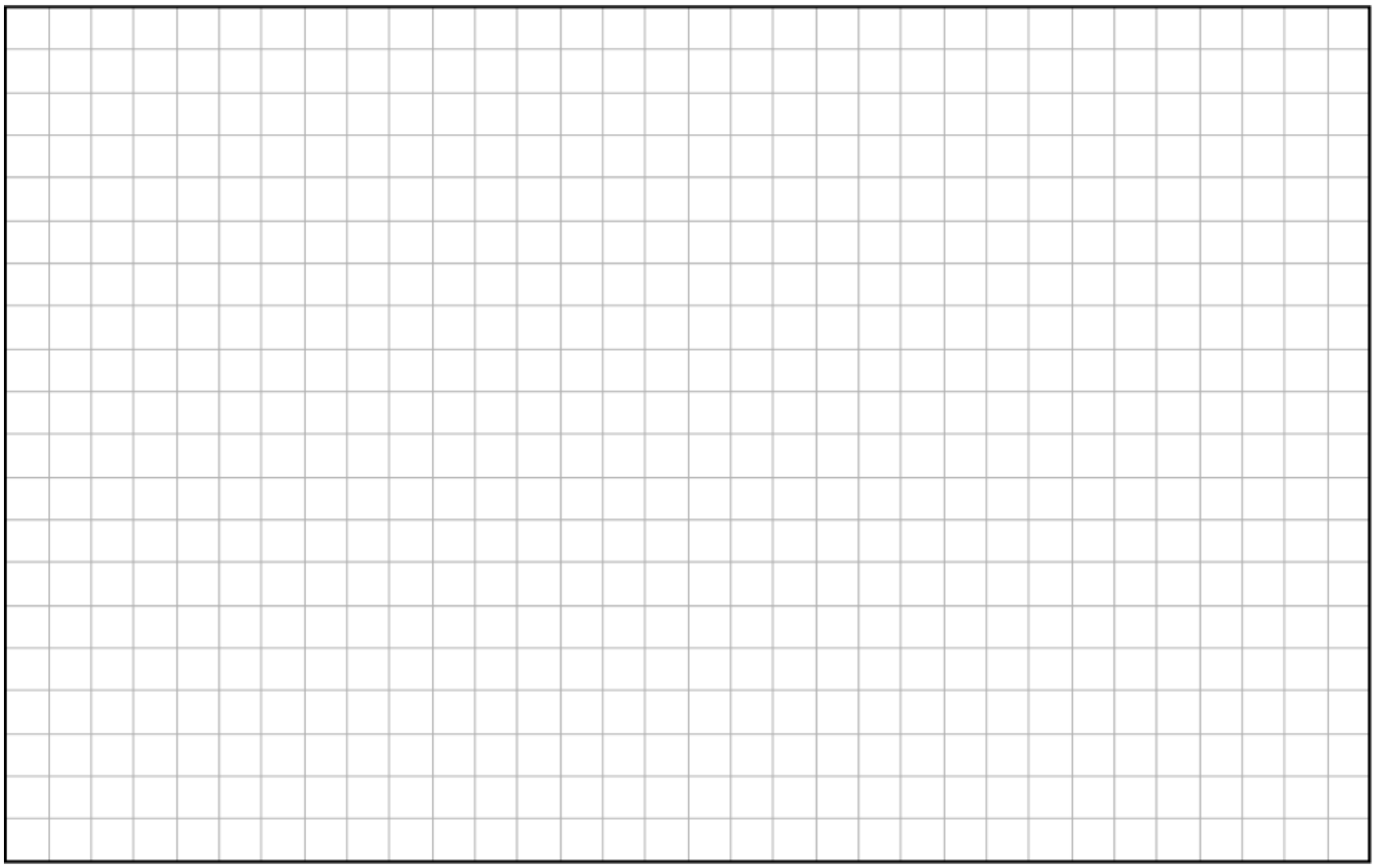


▶ 2022 LCOL Paper 1 – Question 10

Keith plays hurling.

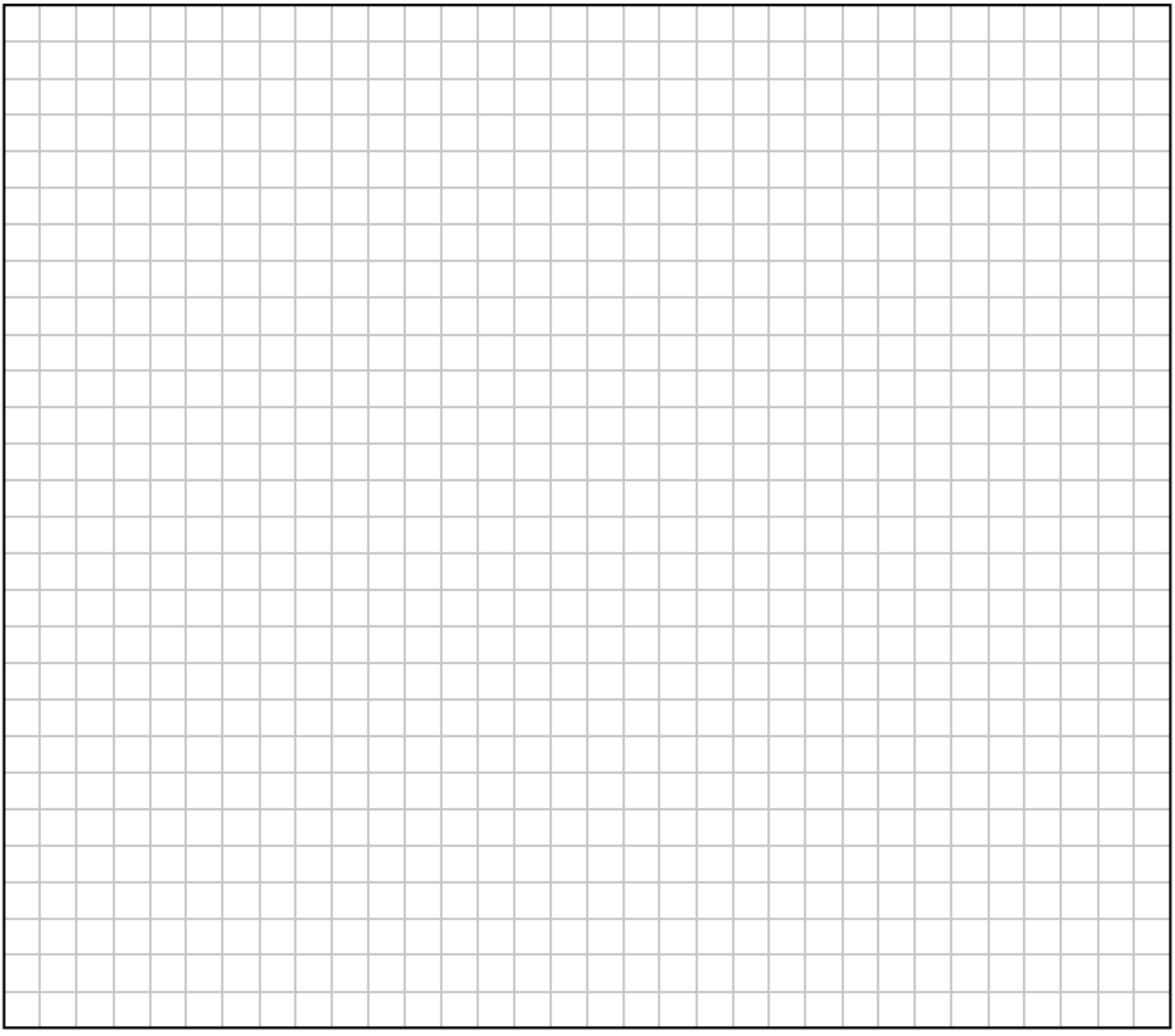
During a match, Keith hits the ball with his hurl.

The height of the ball could be modelled by the following quadratic function:

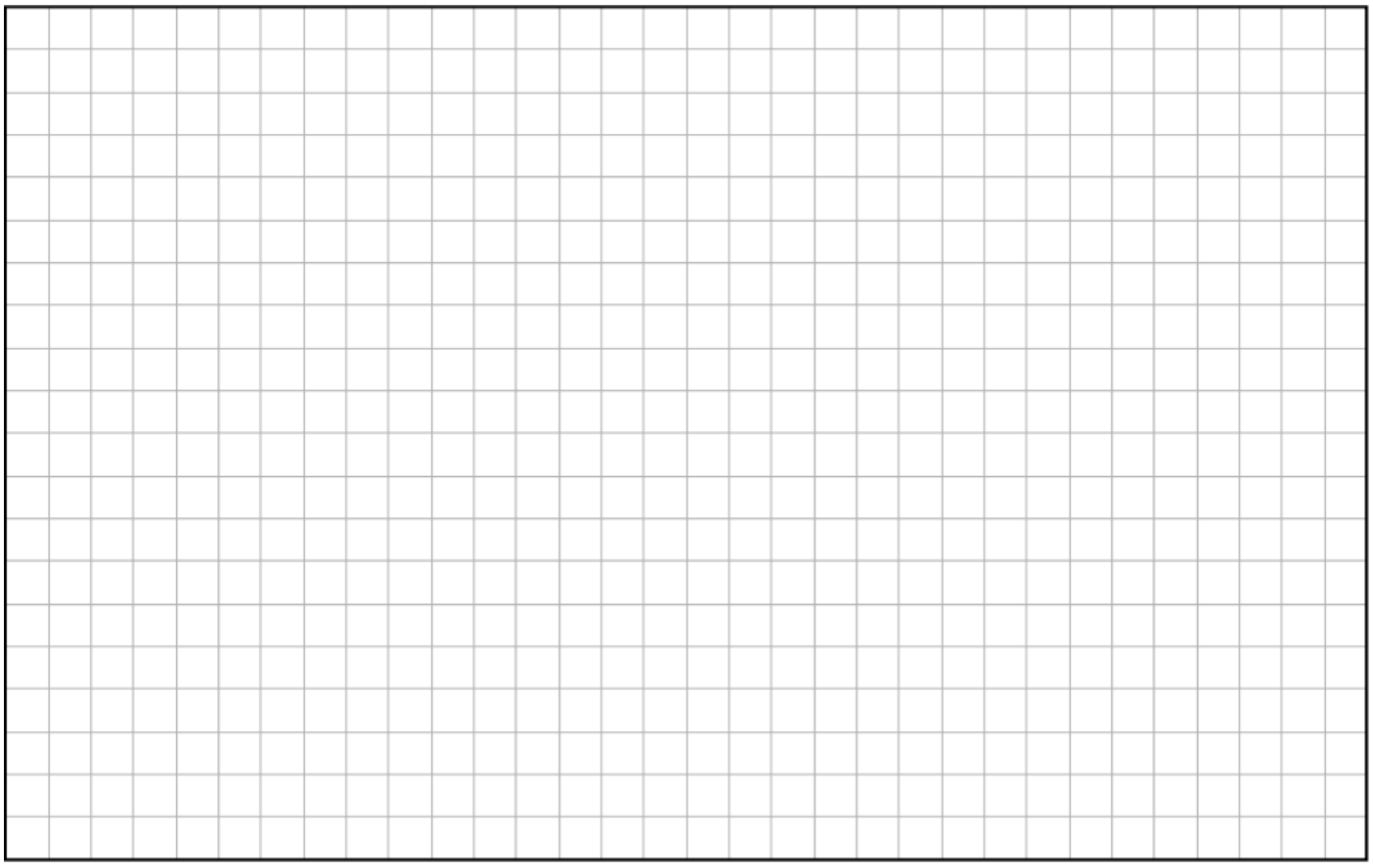
$$h = -2t^2 + 5t + 1.2$$

where h is the height of the ball, in metres, t seconds after being hit, and $t \in \mathbb{R}$.

1. Initial Exploration (Set the Scene)



2. **Visualise the Function**



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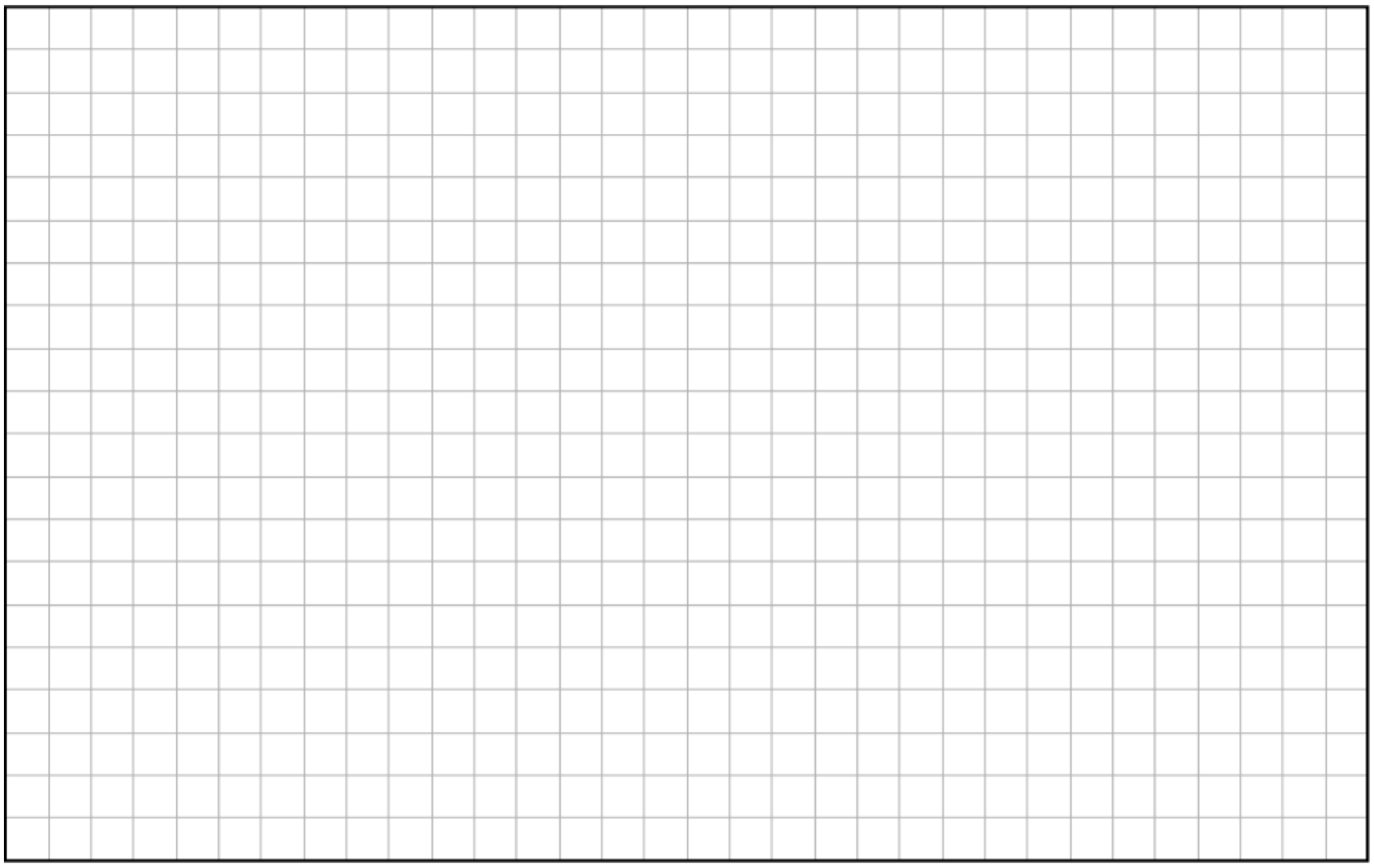

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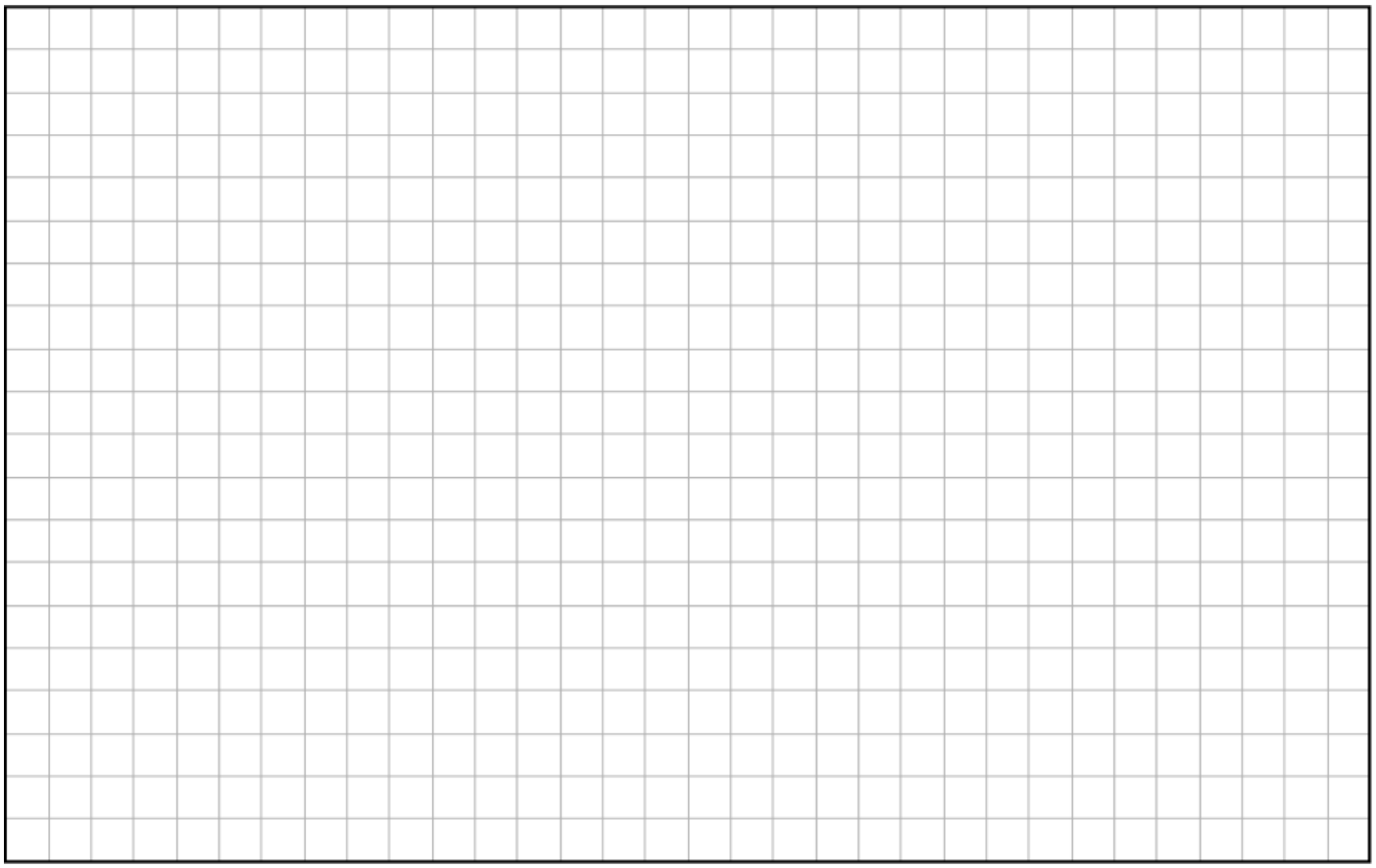

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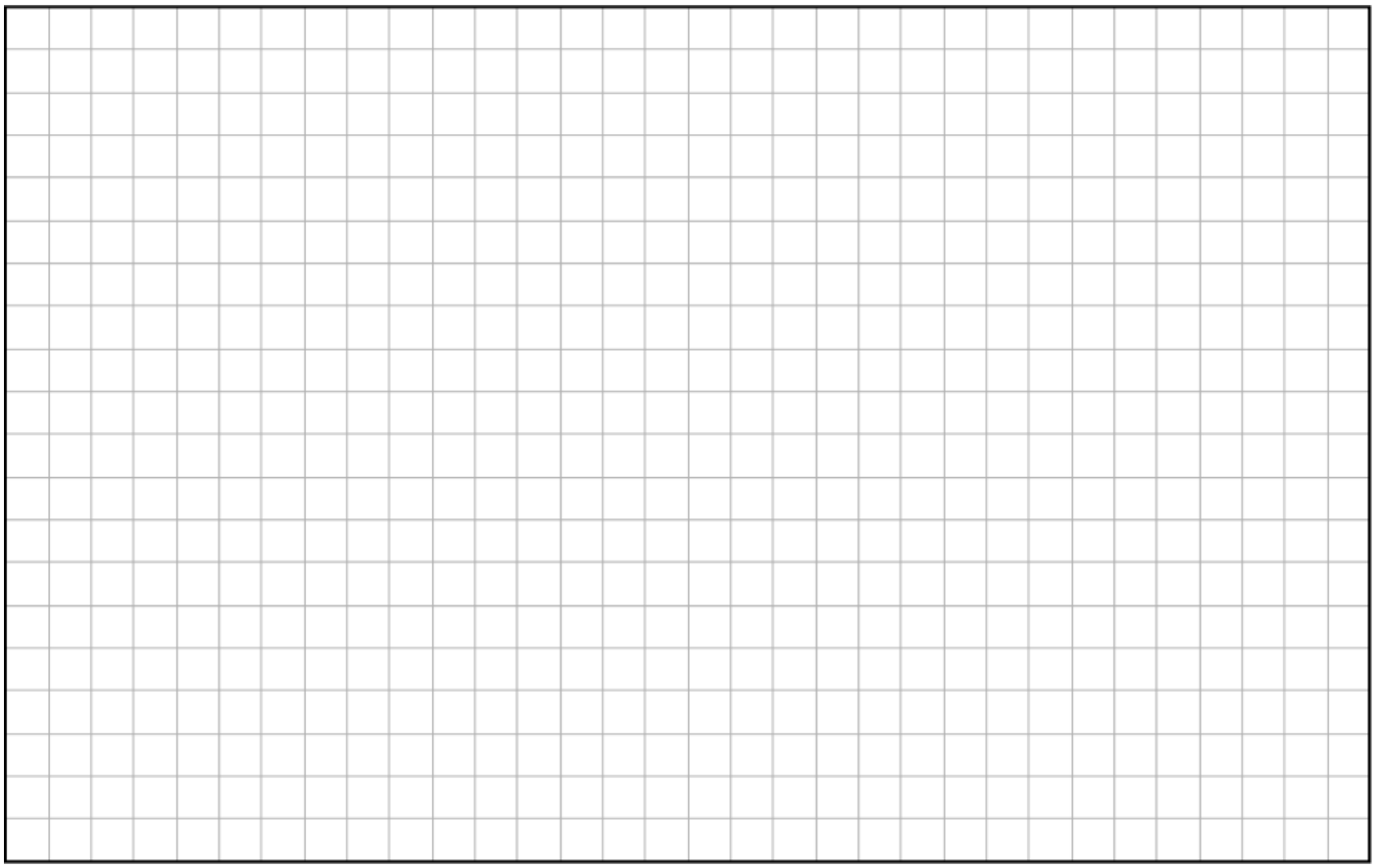


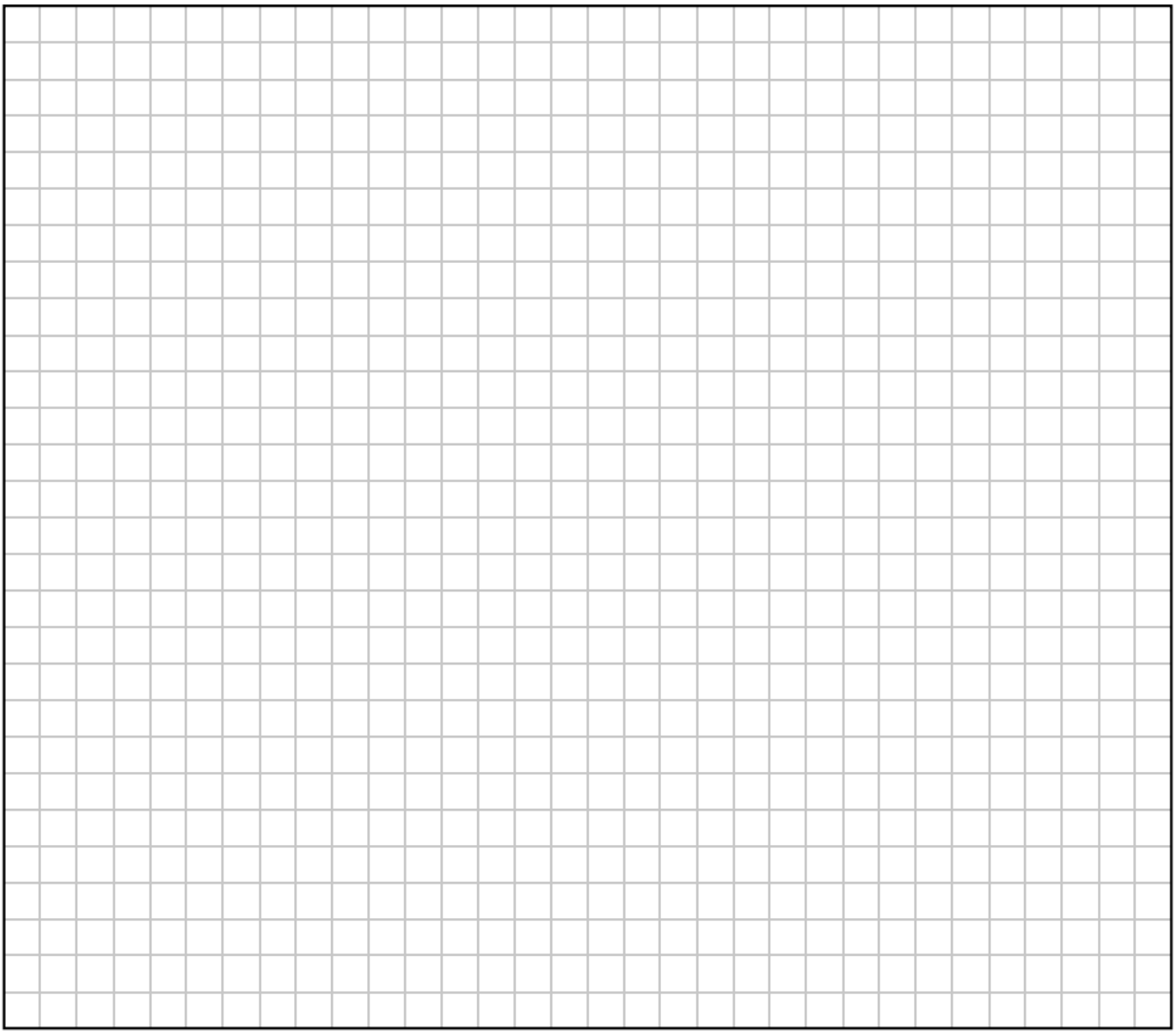
▶ 2020 LCOL Paper 1 – Question 8

A swimmer is on a starting block at the beginning of a race. When she dives off the block until she resurfaces, the level of the swimmer relative to the level of the water is given by the function:

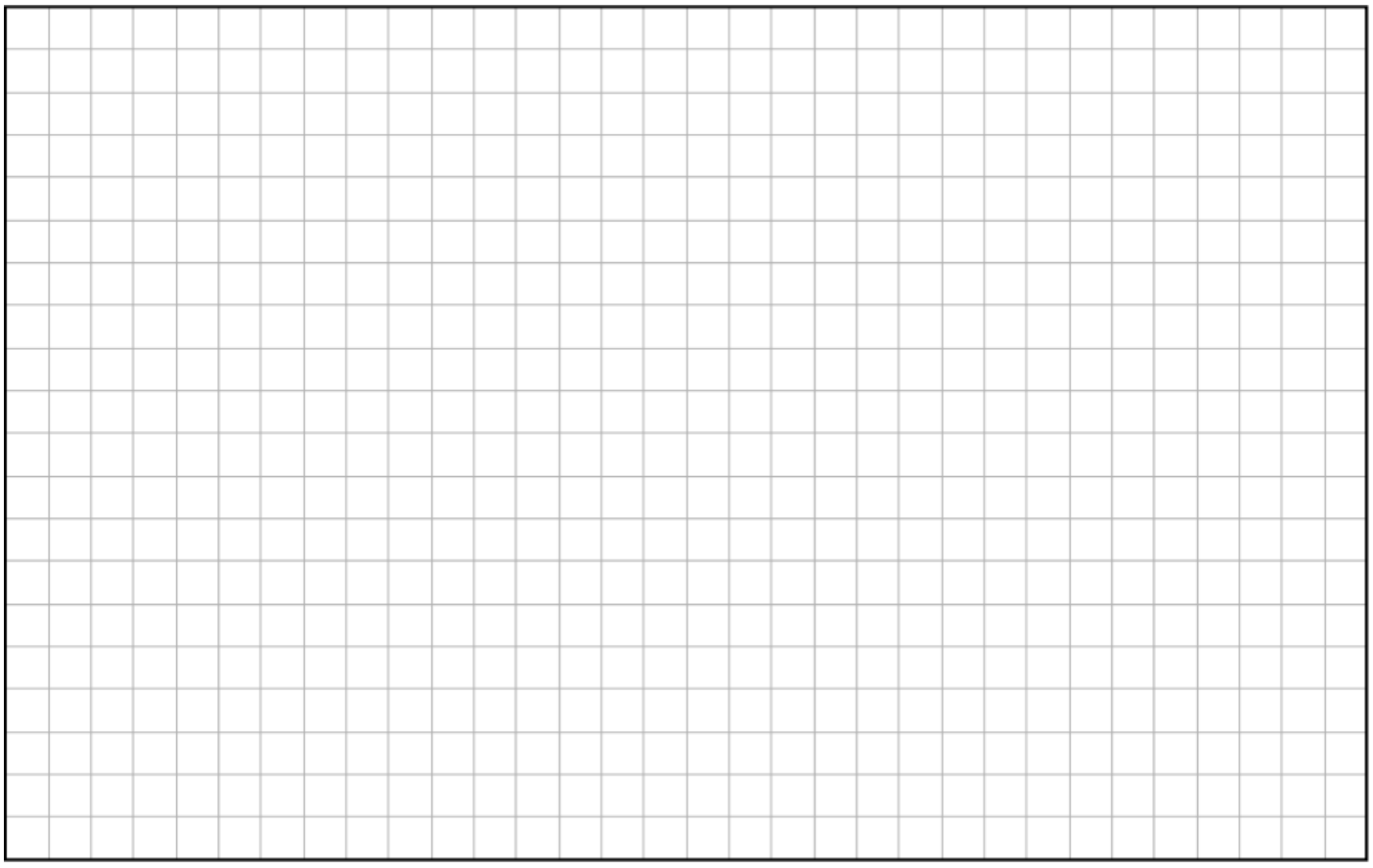
$$h(x) = \frac{1}{60}x^2 - \frac{1}{4}x + \frac{3}{5}$$

In the function, x is the horizontal distance in metres of the swimmer from the block, $0 \leq x \leq 12$, where $x \in \mathbb{R}$ and $h(x)$ is measured in metres.

1. Initial Exploration (Set the Scene)



2. **Visualise the Function**



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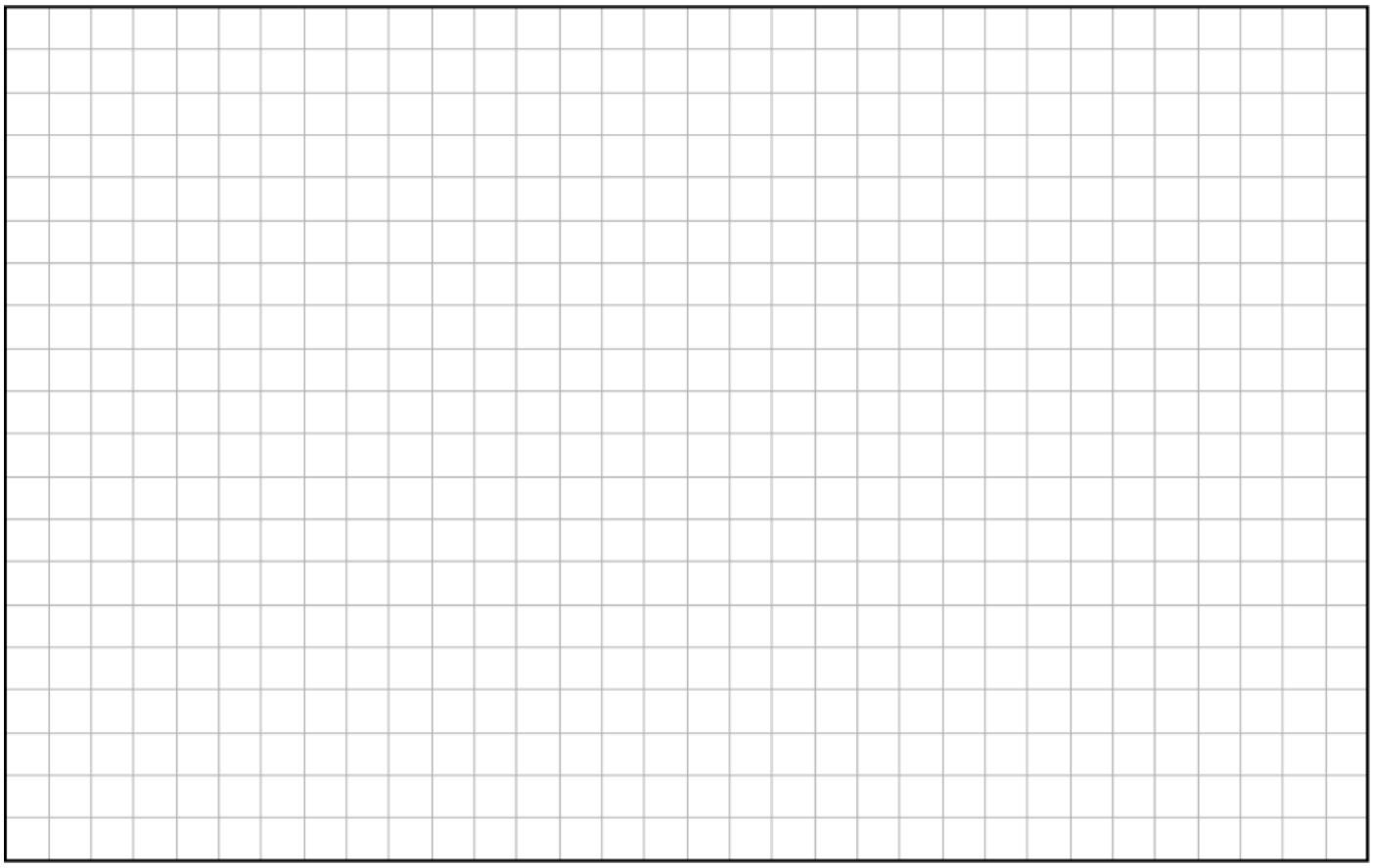

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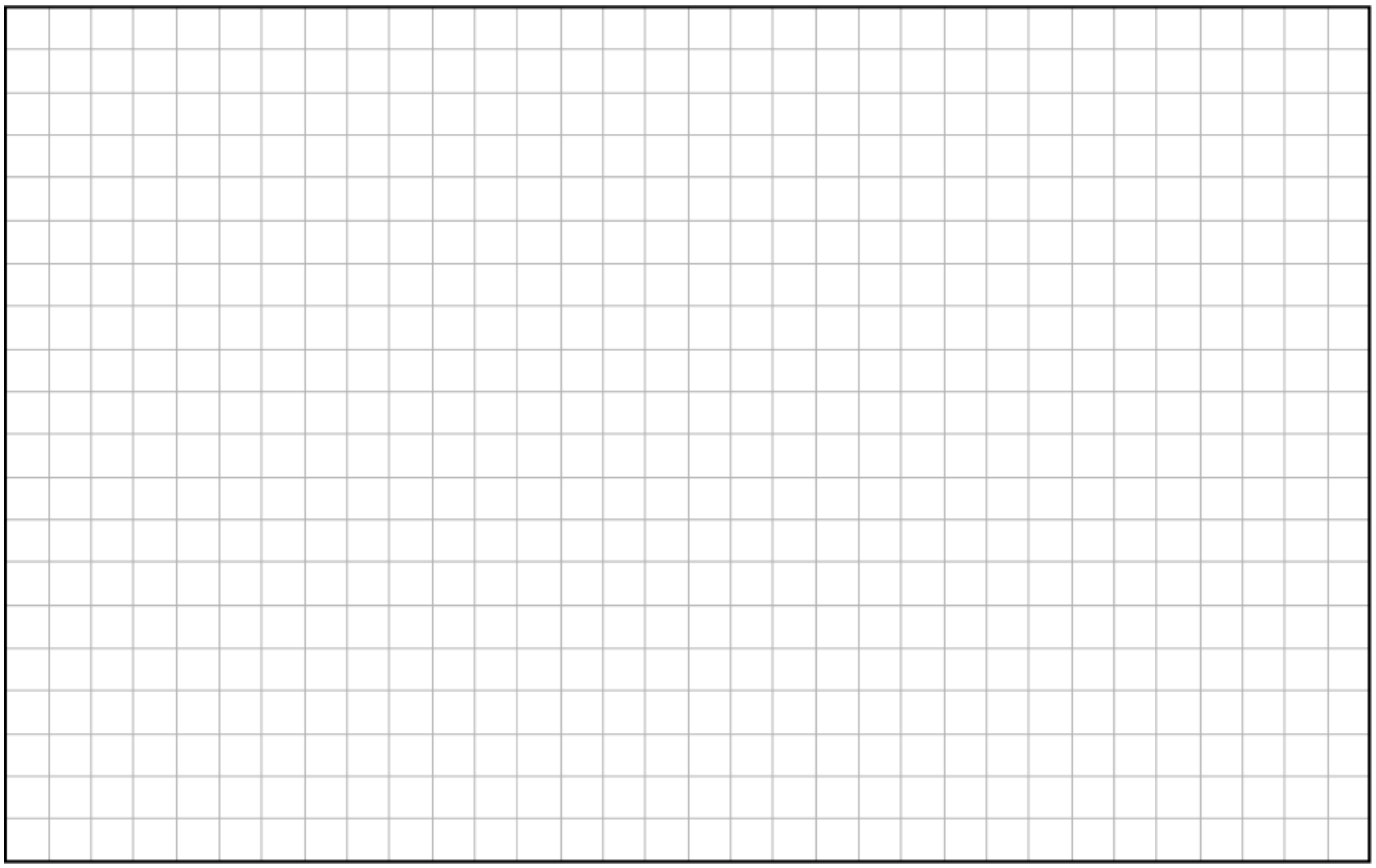

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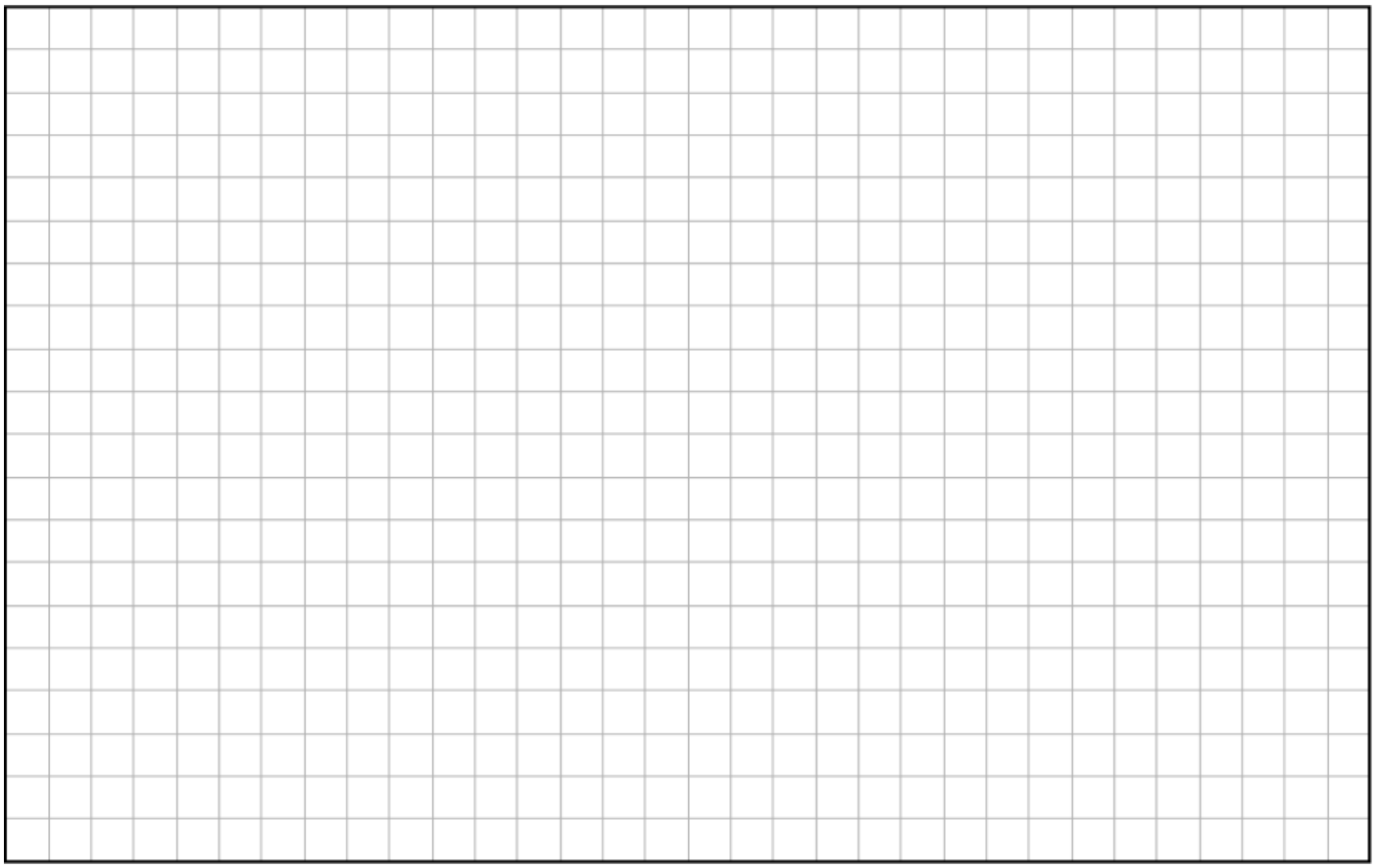
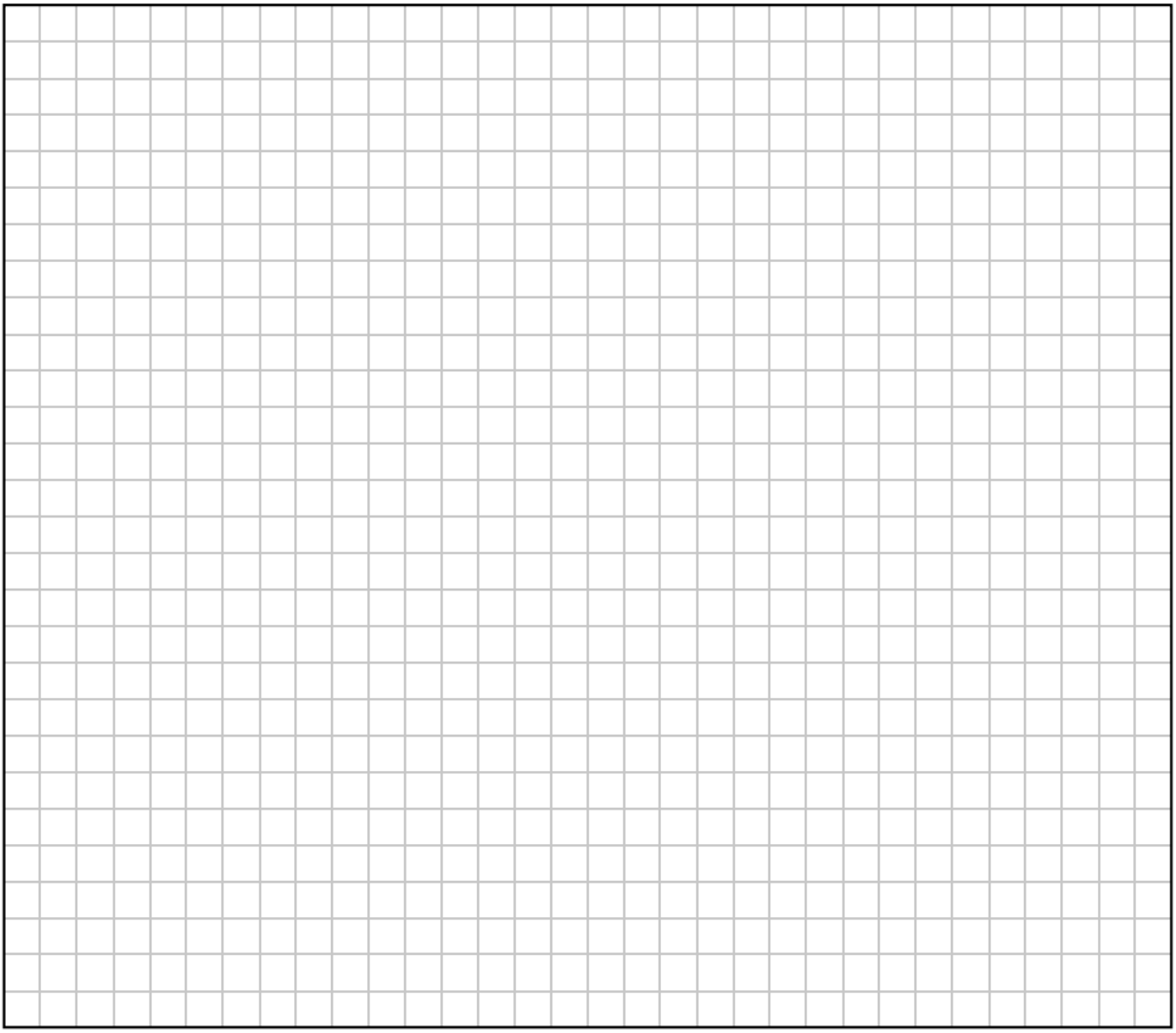


▶ 2018 LCOL Paper 1 – Question 8

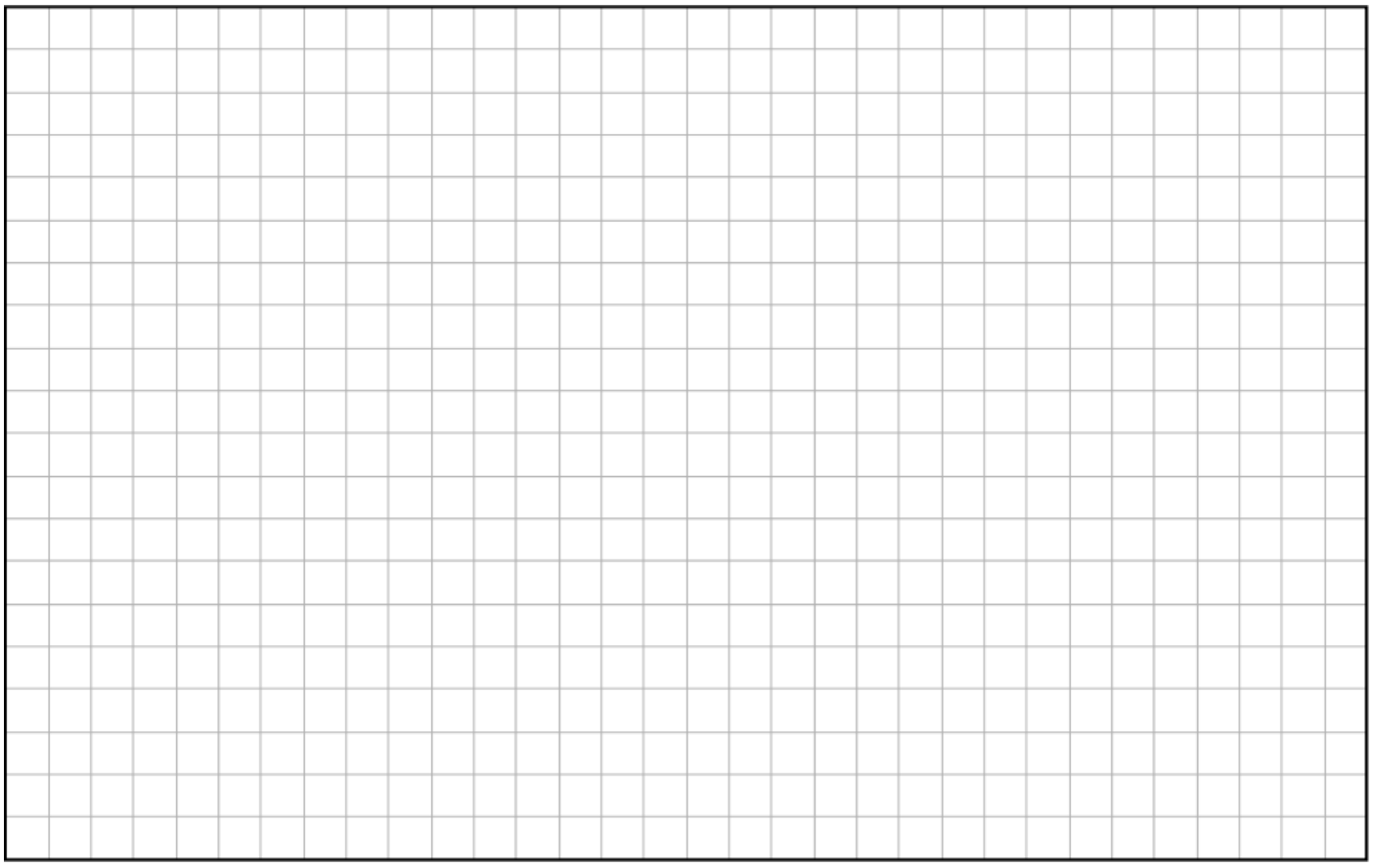
The amount, in appropriate units, of a certain medicinal drug in the bloodstream t hours after it has been taken can be estimated by the function:

$C(t) = -t^3 + 4.5t^2 + 54t$, where $0 \leq t \leq 9,\ t \in \mathbb{R}$.

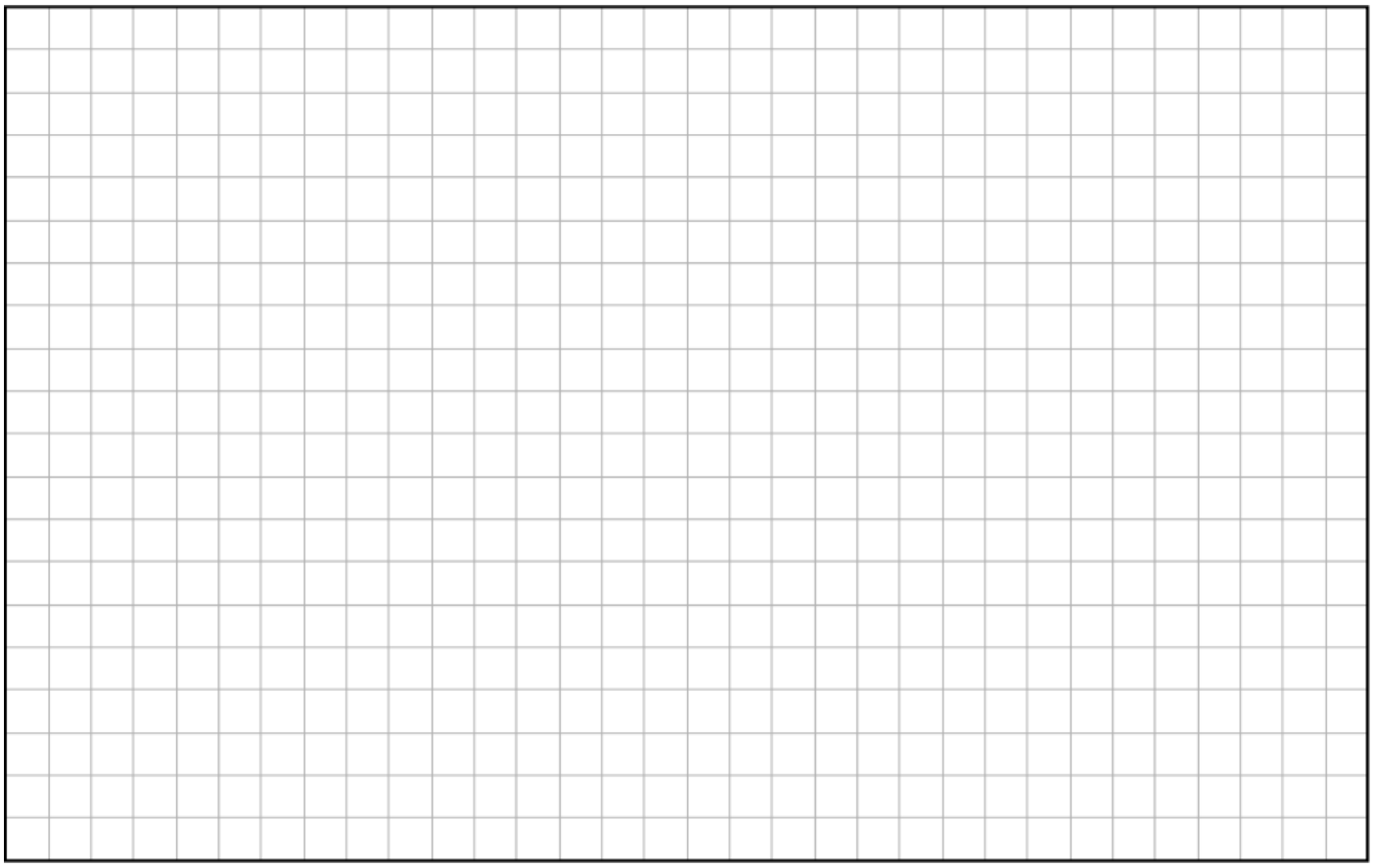
1. Initial Exploration (Set the Scene)



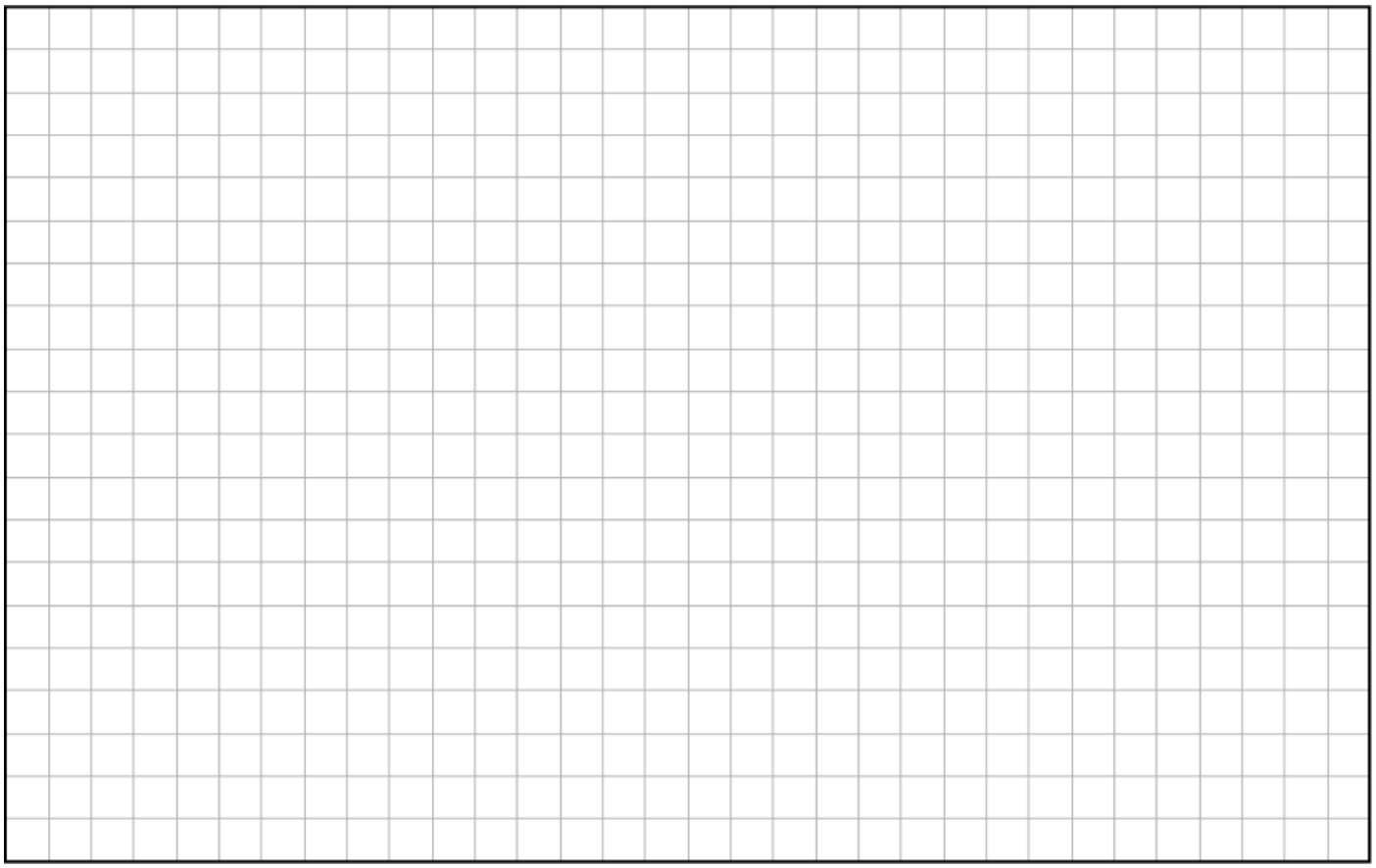
2. **Visualise the Function**



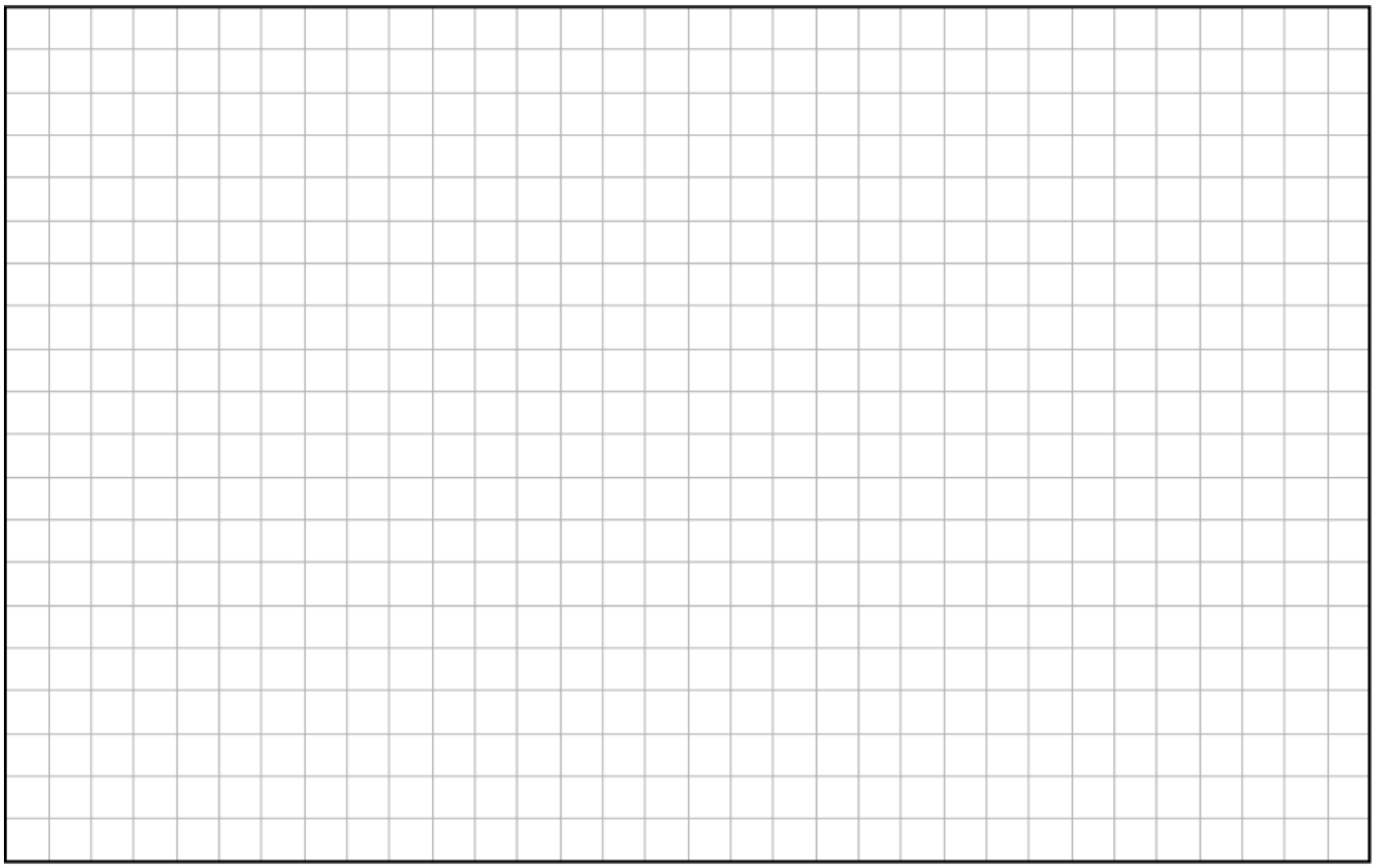
3. **Using outputs to find inputs!**



4. What does differentiation do?



5. What does integration do?



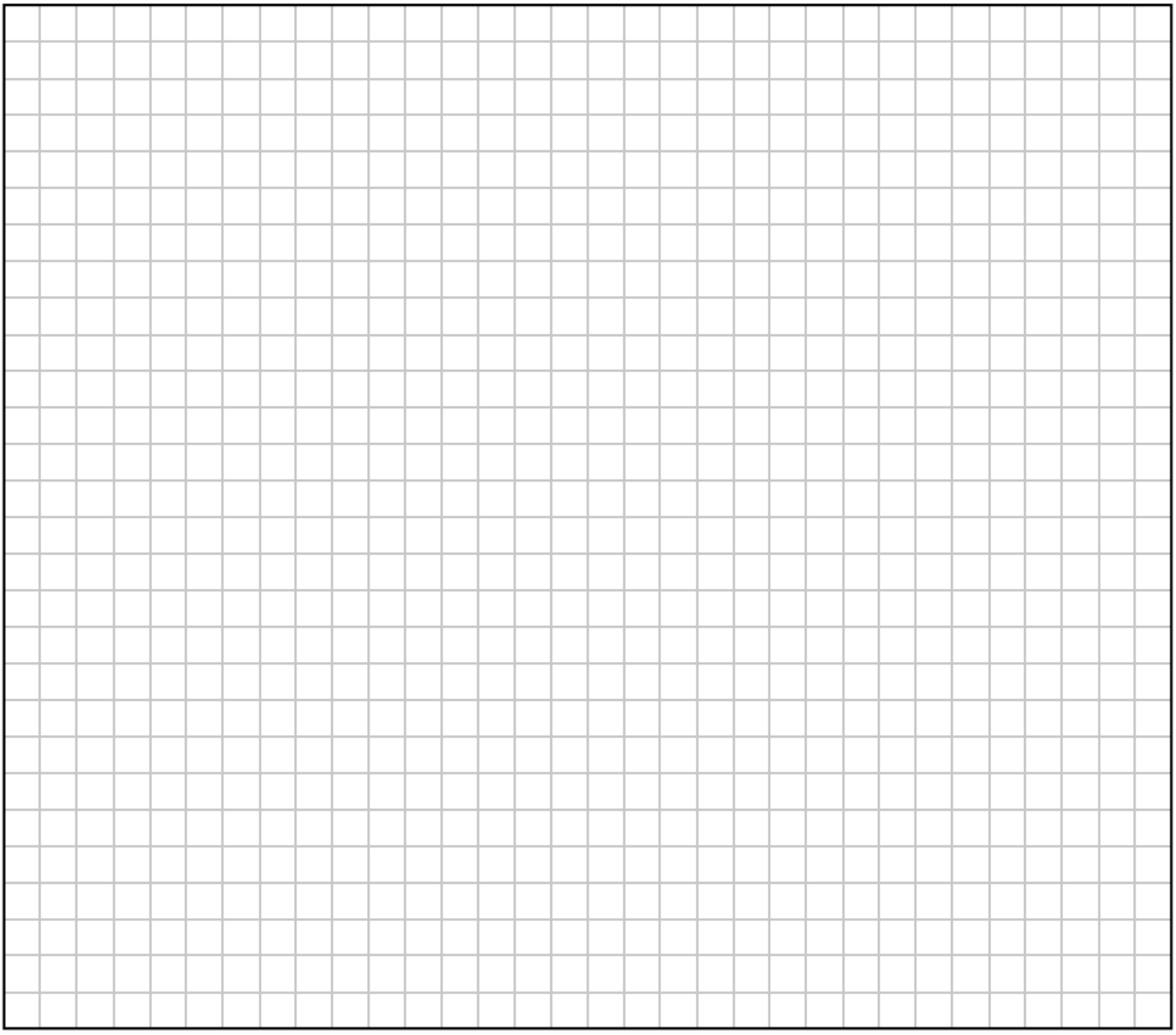
▶ 2017 LCOL Paper 1 – Question 8

A company makes and sells fibre optic cable. It can sell, at most, 200 kilometres of cable in a week. For a certain range of its production the company has found that profit can be modelled using the function:

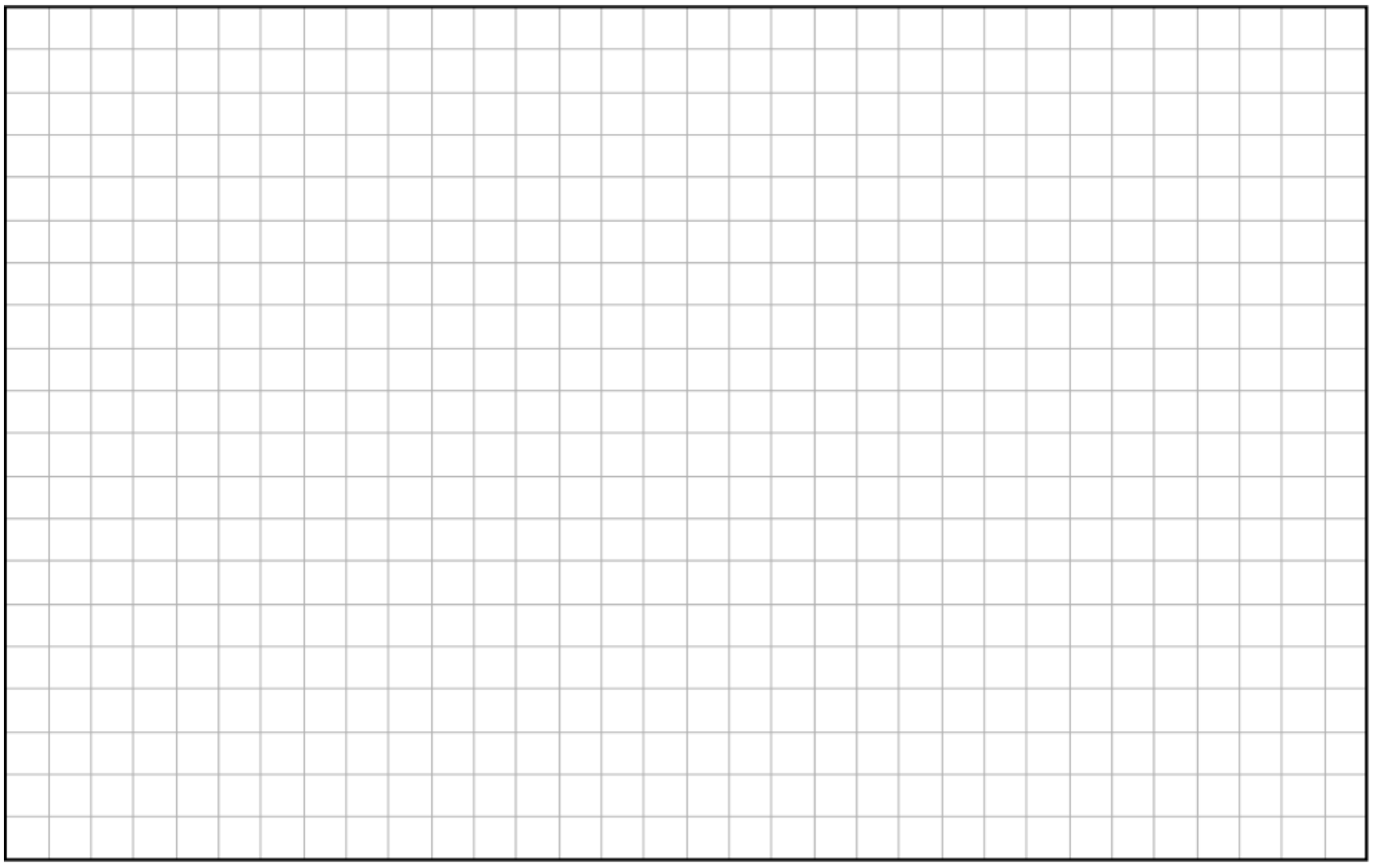
$$P(x) = 275x - x^2 - 2000, \text{ where } x \leq 200.$$

In the function, x is the number of kilometres of fibre optic cable sold and $P(x)$ is the profit in euro.

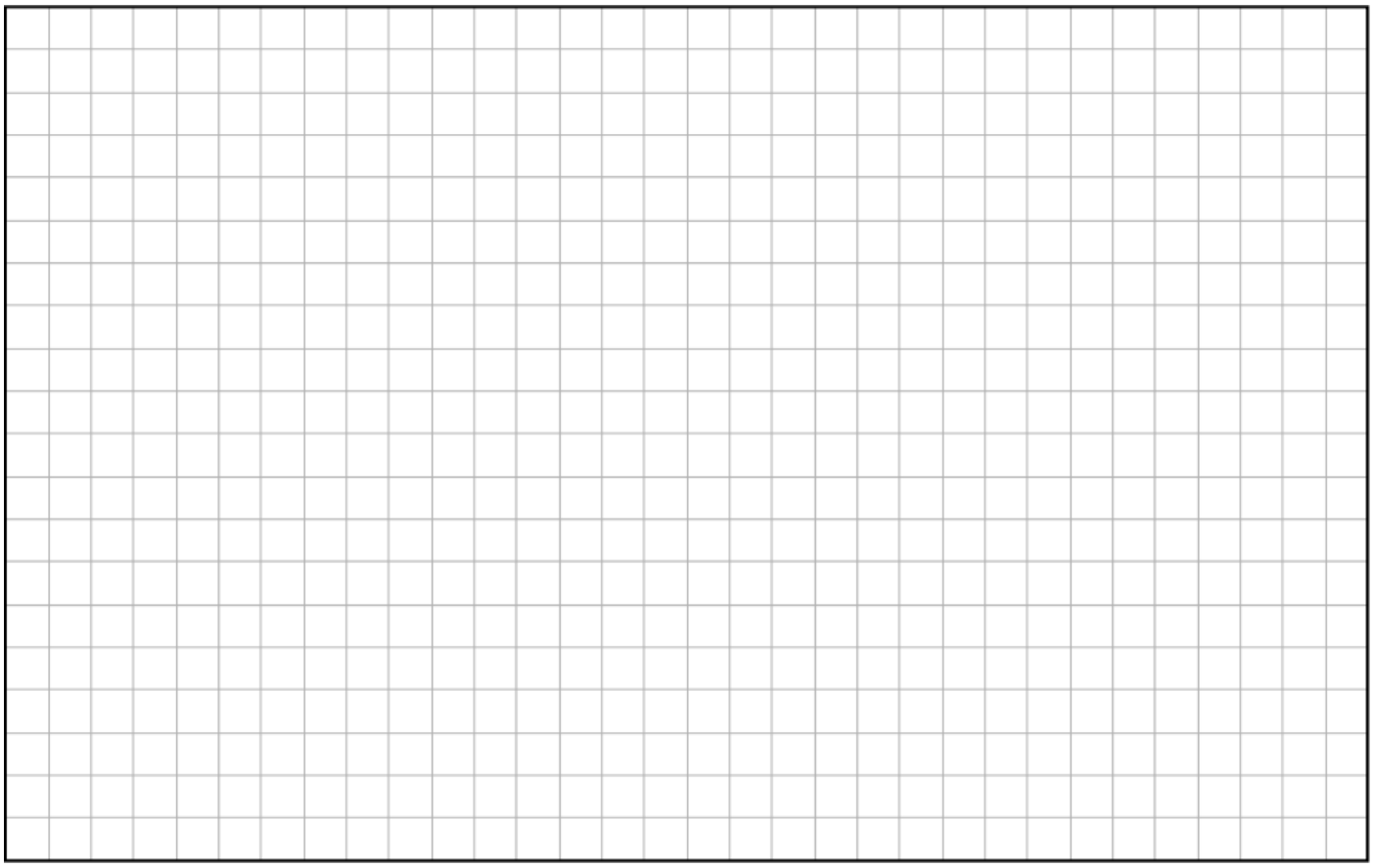
1. Initial Exploration (Set the Scene)



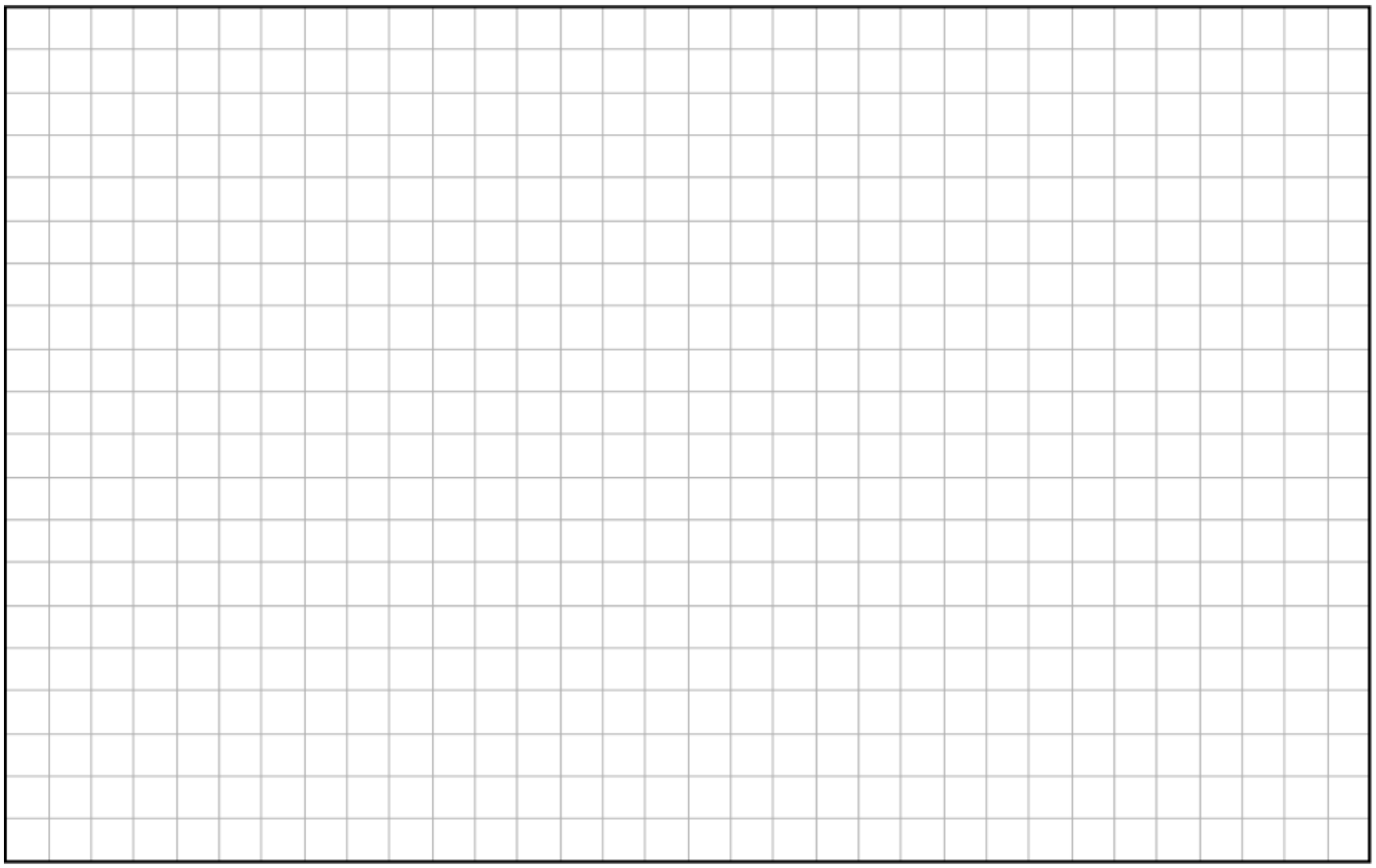
2. **Visualise the Function**



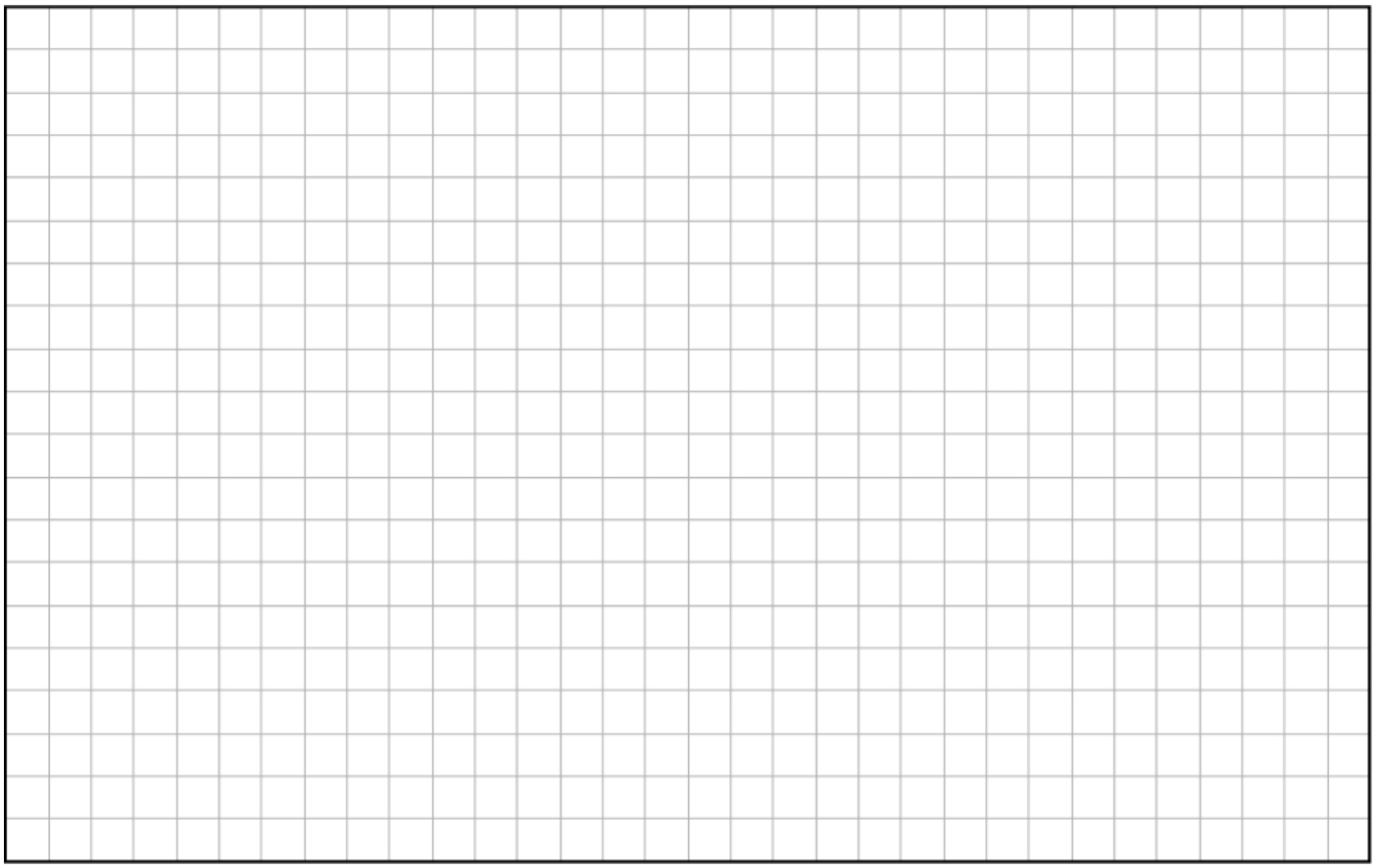
3. **Using outputs to find inputs!**



4. What does differentiation do?



5. What does integration do?



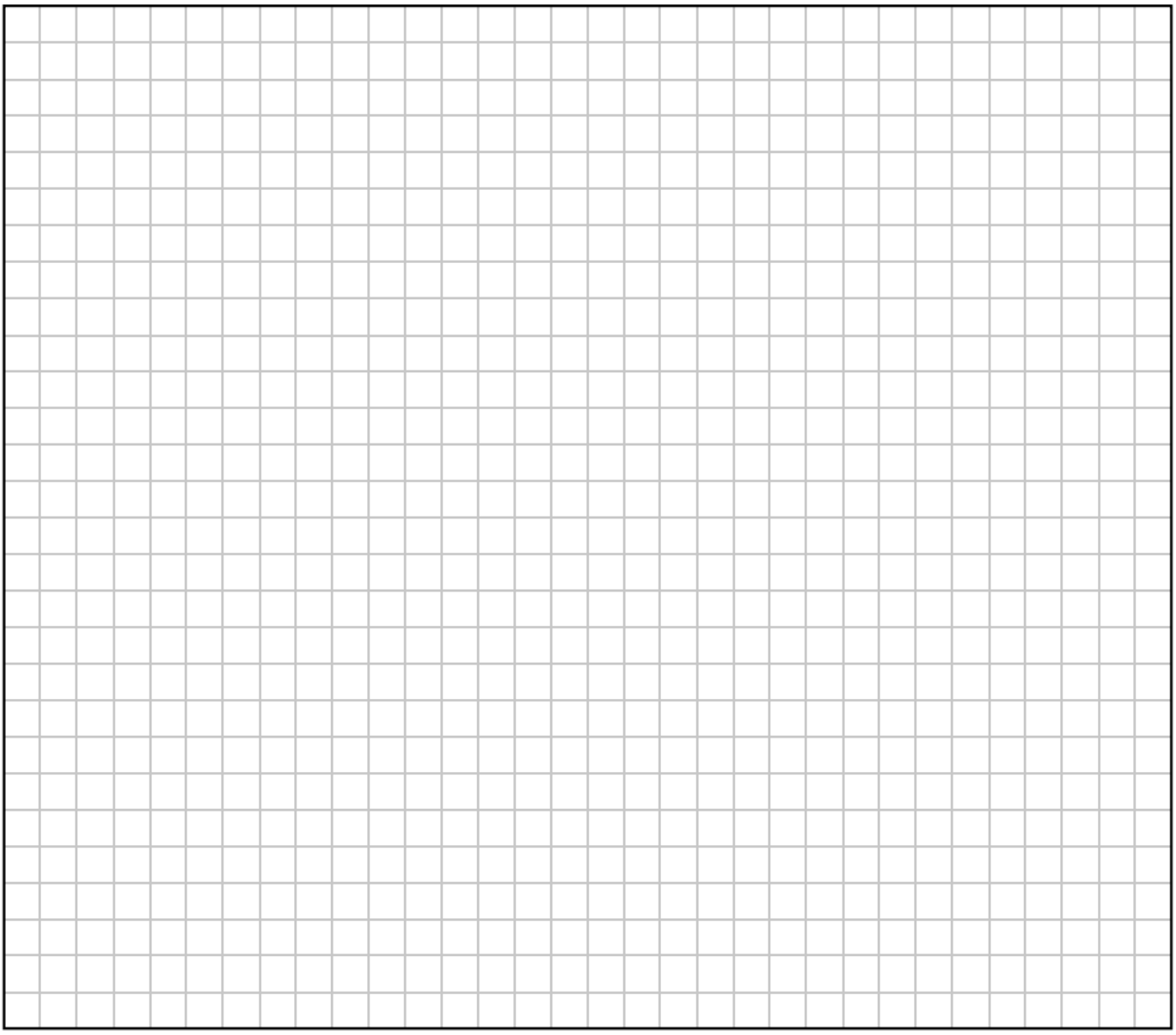
▶ 2014 LCOL Sample Paper 1 – Question 9

A farmer is growing winter wheat. The amount of wheat he will get per hectare depends on, among other things, the amount of nitrogen fertiliser that he uses. For his particular farm, the amount of wheat depends on the nitrogen in the following way:

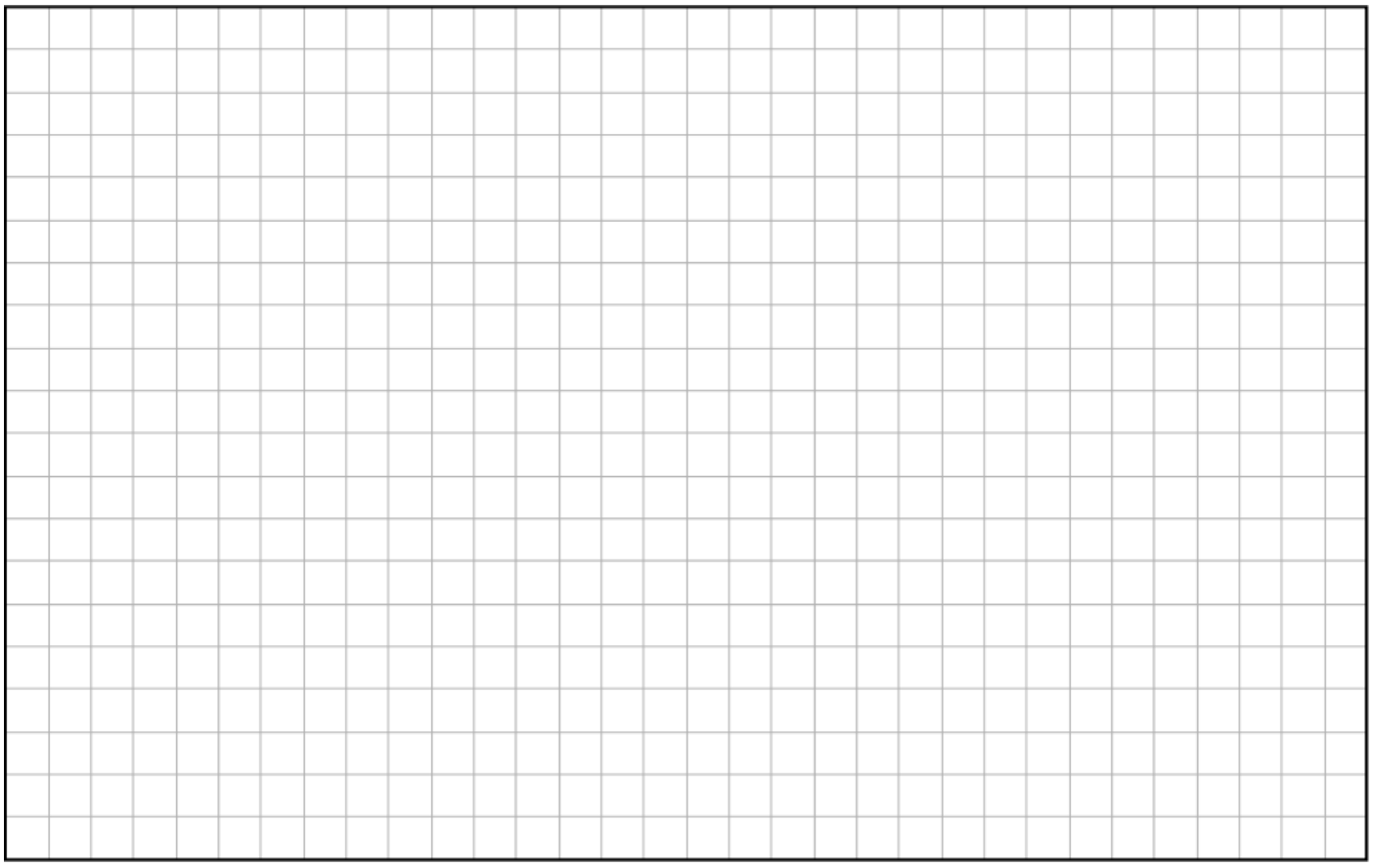
$$Y = 7000 + 32N - 0.1N^2$$

where Y is the amount of wheat produced, in kg per hectare, and N is the amount of nitrogen added, in kg per hectare.

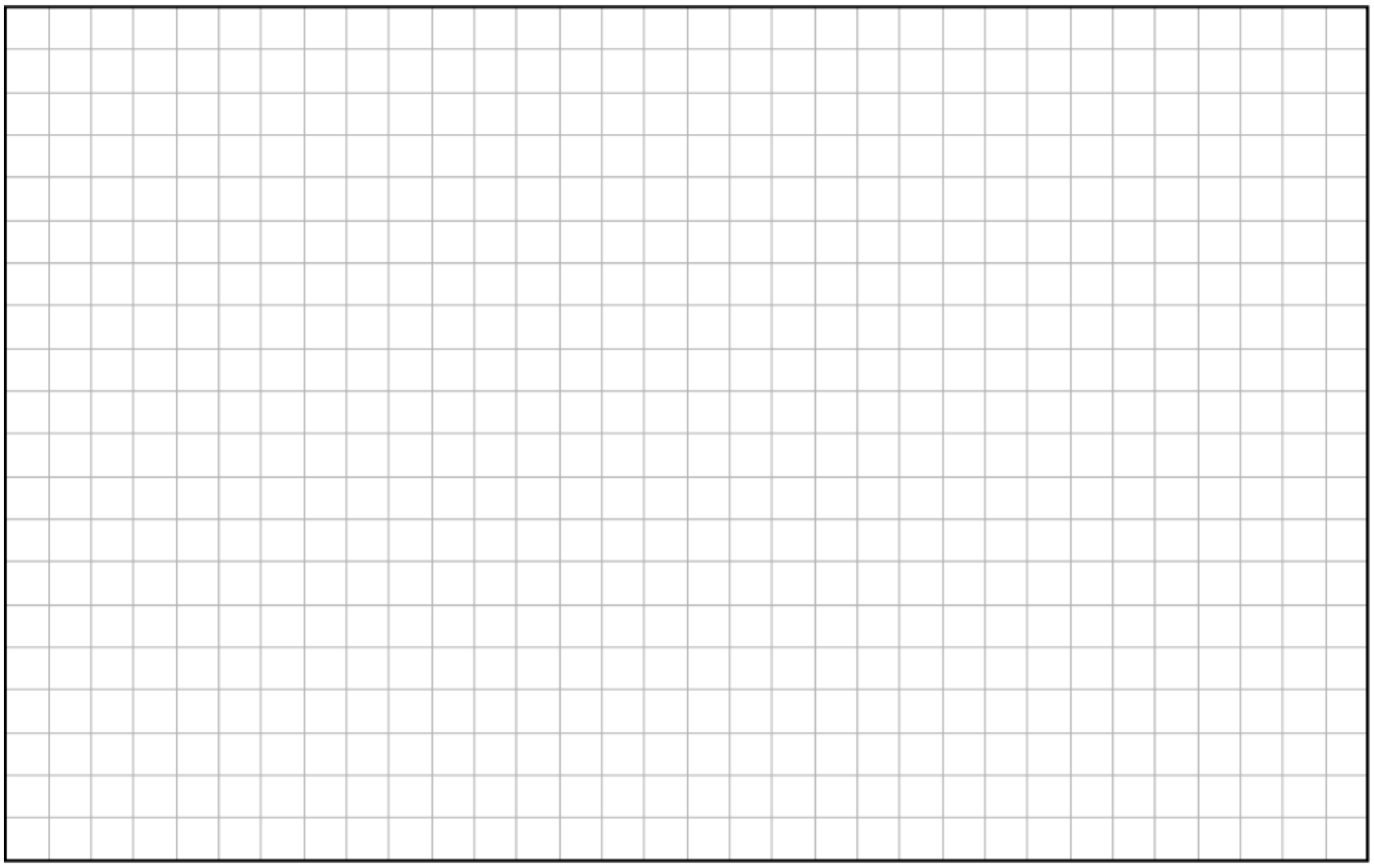
1. Initial Exploration (Set the Scene)



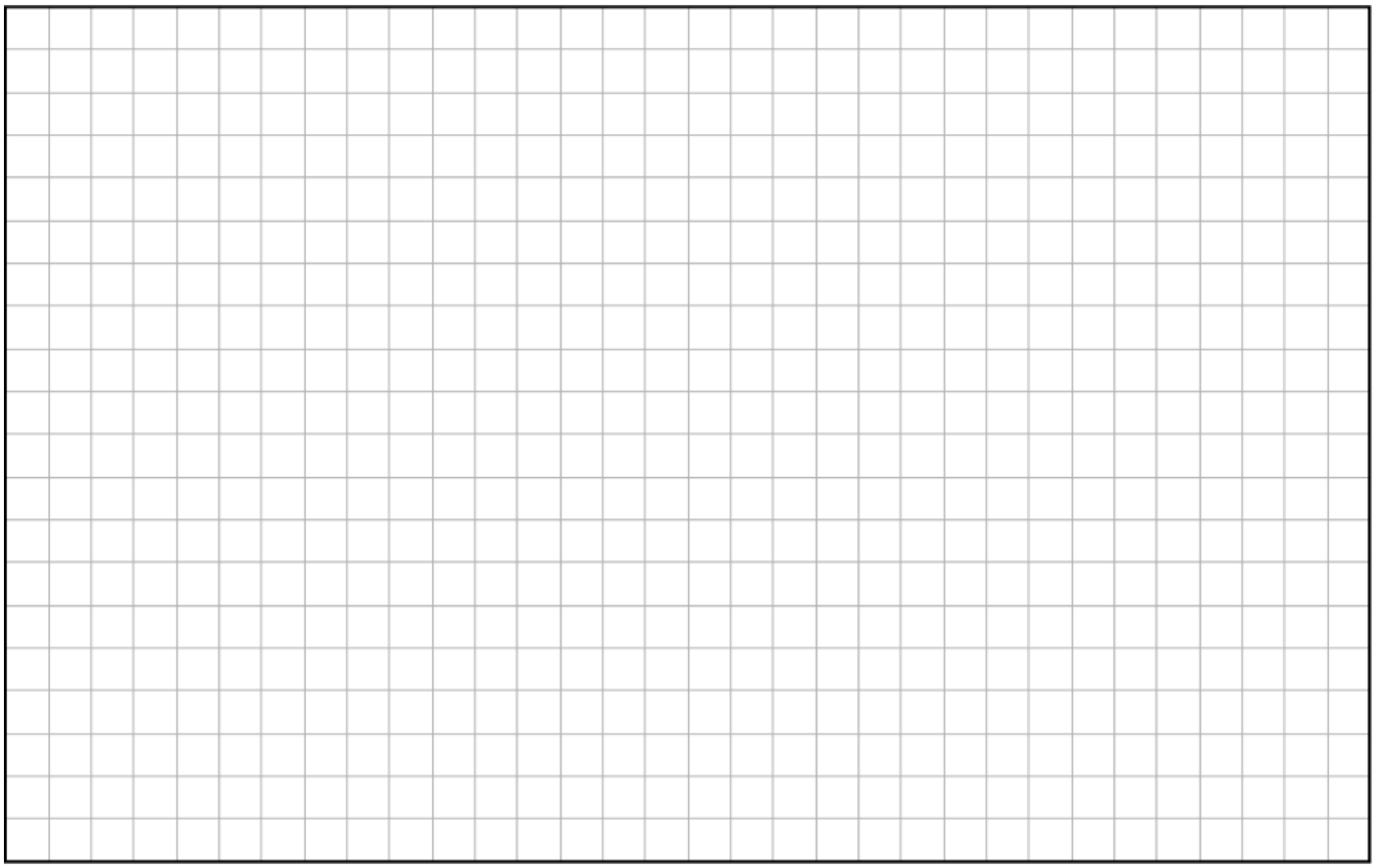
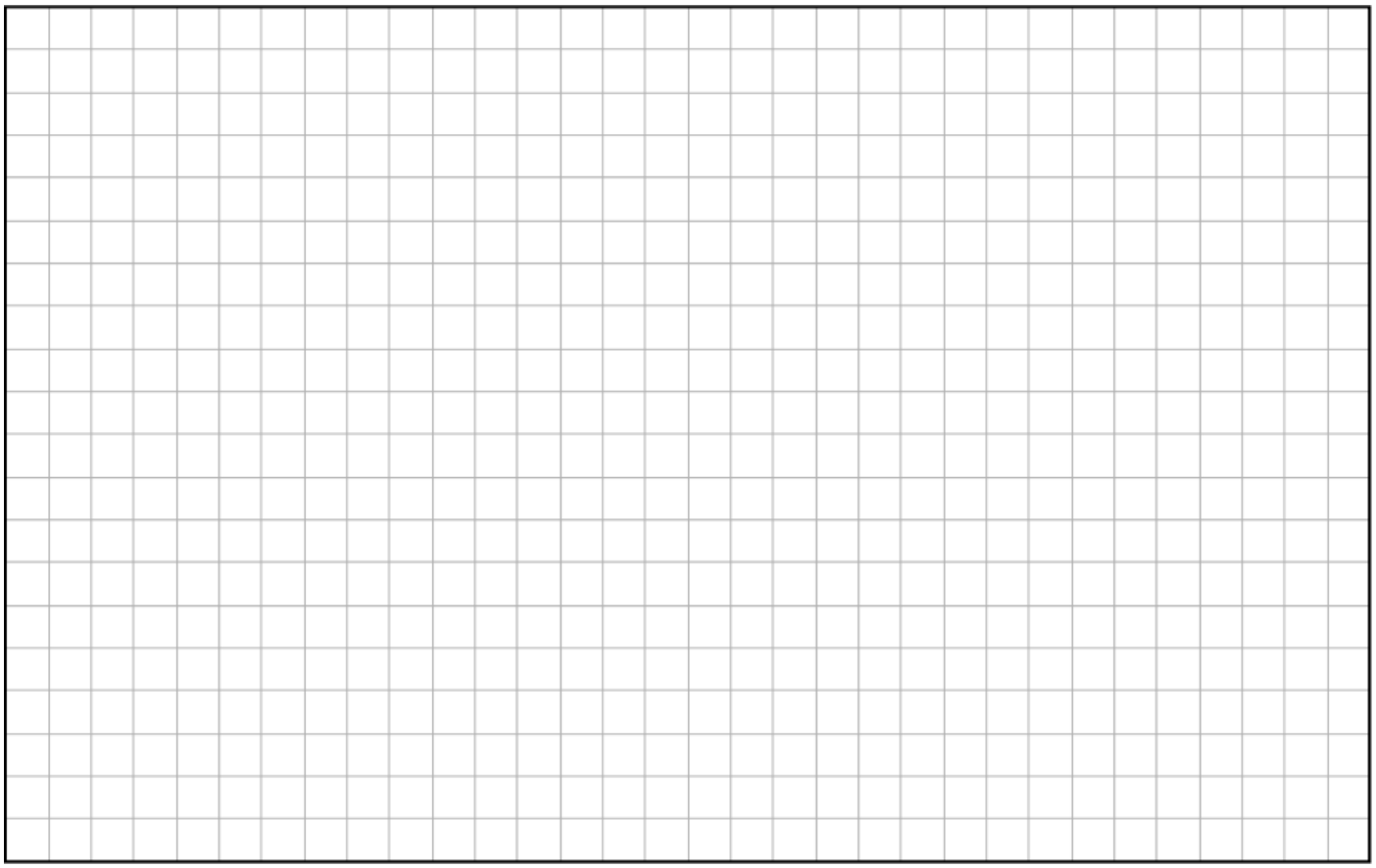
2. **Visualise the Function**



3. **Using outputs to find inputs!**



4. What does differentiation do?



5. What does integration do?