

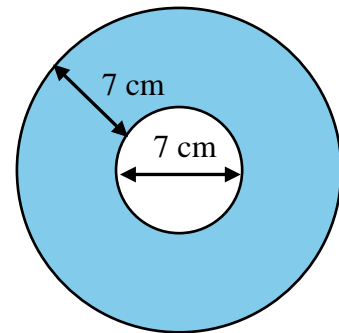
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Round 2

Question 1

Calculate the area of the grey region in terms of π .
The diagram is not to scale.



Solution

$$(10.5)^2\pi - (3.5)^2\pi = 98\pi \text{ cm}^2 \text{ (units not required)}$$

Question 2

In a competition, a school is awarded medals in different categories. 36 medals are awarded in Science, 12 are awarded in technology and 18 are awarded in Maths. If these medals had a combined total of 45 persons receiving medals, with only four students awarded medals in all of the three categories, how many students received medals in exactly two of the three categories?

Solution

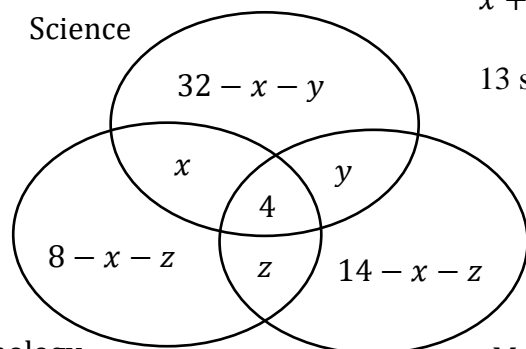
$$32 - x - y + 8 - x - z + 14 - y - z + x + y + z + 4 = 45$$

$$58 - x - y - z = 45$$

$$x + y + z = 58 - 45$$

$$x + y + z = 13$$

13 students received exactly medals in two categories



Technology

Maths

Round 3

Question 1

The diagram shows five equal semicircles and the lengths of some line segments in centimetres. What is the radius of the semicircles?

Solution

$$22 + x = 12 + x + y$$

$$22 - 12 = y$$

$$y = 10$$

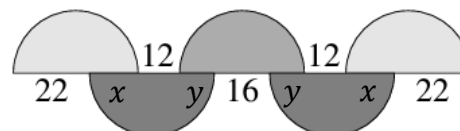
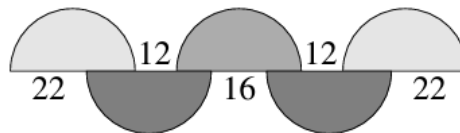
$$\text{Diameter} = 16 + y + y$$

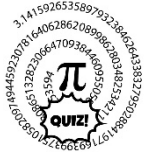
$$= 16 + 2y$$

$$= 16 + 2(10)$$

$$= 36$$

$$\Rightarrow \text{Radius} = 18$$





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Round 3

Question 2

AB is a straight line. The coordinates of A are $(-9, -4)$. The midpoint of AB is $(8, 1.5)$. Find the coordinates of B .

Solution

$A(-9, -4)$, midpoint $(8, 1.5)$. Translation $(+17, +5.5)$.

$$(8 + 17, 1.5 + 5.5) = (25, 7) \Rightarrow B \text{ is } (25, 7)$$

Round 4

Question 1

The current value of a smart watch is 20% less than when the smart watch was purchased. By what percentage must the current value of the smart watch increase in order to have its original value?

Solution

Assume the smart watch is worth €100 originally, and now is worth €80.

$$\text{Increase} = \frac{20}{80} = 25\%$$

Question 2

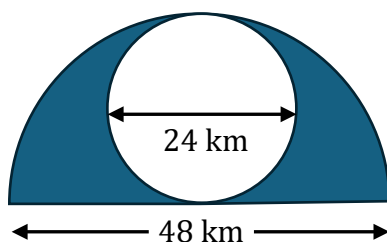
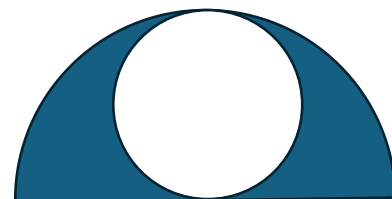
The diagram shows a circle inside a semi-circle. The circle has a diameter of 24 km. The semi-circle has a diameter of 48 km. Calculate the area of the shaded region to 1 decimal place.

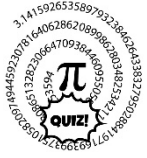
Solution

$$\text{Area of circle} = \pi(12)^2 = 144\pi$$

$$\text{Area of semicircle} = \frac{1}{2}\pi(24)^2 = 288\pi$$

$$\begin{aligned} \text{Shaded area} &= 288\pi - 144\pi = 144\pi \\ &= 452.3893421 \\ &= 452.4 \text{ km}^2 \end{aligned}$$





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Round 5

Question 1

Find h in cm the height of the trapezium below.

Solution

Pythagoras' Theorem

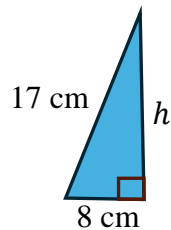
$$(17)^2 = (h)^2 + (8)^2$$

$$(17)^2 - (8)^2 = h^2$$

$$h^2 = 225$$

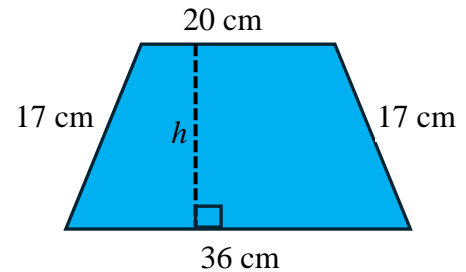
$$h = \sqrt{225}$$

$$h = 15 \text{ cm (units not required)}$$



$$36 - 20 = 16$$

$$16 \div 2 = 8$$



Question 2

The first five terms of a sequence are shown below.

$$-17, -30, -49, -74, -105, \dots$$

Work out an expression for the n th term of this sequence in the form of $an^2 + bn + c$.

Solution

$$-17 \quad -30 \quad -49 \quad -74 \quad -105$$

$$-13 \quad -19 \quad -25 \quad -31$$

$$-6 \quad -6 \quad -6$$

$$\text{Second difference} = 2a = -6 \Rightarrow a = -3$$

$$T_1 = -3(1)^2 + b(1) + c = -17$$

$$-3 + b + c = -17 \Rightarrow b + c = -14$$

$$T_2 = -3(2)^2 + b(2) + c = -30$$

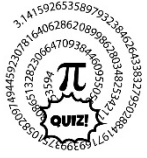
$$-12 + 2b + c = -30 \Rightarrow 2b + c = -18$$

$$2b + c = -18$$

$$b + c = -14$$

$$b = -4 \Rightarrow c = -10$$

$$\text{Sequence is } -3n^2 - 4n - 10$$



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Round 6

Question 1

What is the missing total?

Solution










Let pentagon = p , circle = c , and hexagon = h

$$3c = \frac{3}{8} \Rightarrow c = \frac{1}{8}$$

$$2p + c = \frac{5}{8} \Rightarrow 2p + \frac{1}{8} = \frac{5}{8} \Rightarrow p = \frac{2}{8}$$

$$h + 2p = 1 \Rightarrow h + \frac{4}{8} = 1 \Rightarrow h = \frac{4}{8}$$

$$\Rightarrow p + c + h = \frac{2}{8} + \frac{1}{8} + \frac{4}{8} = \frac{7}{8}$$

			$= \frac{5}{8}$
			$= \frac{3}{8}$
			$= 1$
$= ?$	$= \frac{1}{2}$	$= \frac{5}{8}$	

Question 2

Find the probability of getting exactly two heads when five coins are tossed.

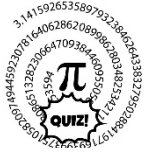
Give your answer as a **rational number**.

Solution

HHTTT, HTHTT, HTTHT, HTTTH, TTTTH,

THTTH, THTTT, THTHT, TTHHT, THHTT

$$P = 10 \left(\frac{1}{2}\right)^5 = \frac{10}{32} \text{ or } \frac{5}{16}$$



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Round 7

Question 1

What is the shaded area? Give your answer in terms of π .

Solution

$$\text{Shaded area} = \frac{1}{2}\pi r^2 + \frac{1}{4}\pi(2r)^2$$

$$= \frac{1}{2}\pi r^2 + \frac{1}{4}\pi(4r^2)$$

$$= \frac{1}{2}\pi r^2 + \pi r^2$$

$$= \frac{1}{2}\pi(\sqrt{2})^2 + \pi(\sqrt{2})^2$$

$$= \frac{1}{2}\pi(2) + \pi(2)$$

$$= 3\pi$$

Pythagoras' Theorem

$$(3r)^2 = (r)^2 + (4)^2$$

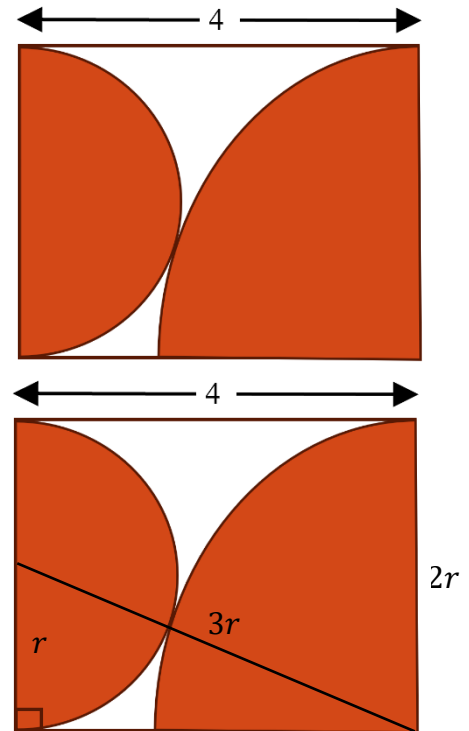
$$9r^2 = r^2 + 16$$

$$9r^2 - r^2 = 16$$

$$8r^2 = 16$$

$$r^2 = 2$$

$$r = \sqrt{2} \text{ units}$$



Question 2

Find the possible values of m , such that these coordinates $(m, 3)$ and $(1, m)$ are 10 units apart.

Solution

$$\sqrt{(1 - m)^2 + (m - 3)^2} = 10$$

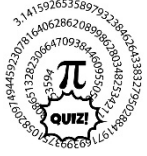
$$1 - 2m + m^2 + m^2 - 6m + 9 = 100$$

$$2m^2 - 8m + 10 - 100 = 0$$

$$m^2 - 4m - 45 = 0$$

$$(m - 9)(m - 5) = 0$$

$$m = 9 \text{ and } m = -5$$



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Round 7

Question 3

The total perimeter of the three rectangles below is 90 m.

Calculate the value of x in metres.

Solution

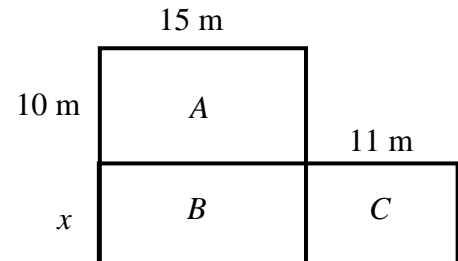
$$(15 + 10 + 11 + x + 11 + 15 + x + 10) = 90$$

$$72 + 2x = 90$$

$$2x = 90 - 72$$

$$2x = 18$$

$$x = 9 \text{ m (units not required)}$$



Question 4

Let $g(x) = \frac{1}{x^2} - \frac{1}{2x}$ and $h(x) = 1 - \frac{2}{x}$ where $x \neq 0$ and $x \in \mathbb{R}$.

Find the values of x for which $g(x) = h(x)$.

Solution

$$g(x) = h(x)$$

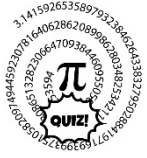
$$\frac{1}{x^2} - \frac{1}{2x} = 1 - \frac{2}{x} \quad \text{LCM is } 2x^2$$

$$2 - x = 2x^2 - 4x$$

$$2x^2 - 3x - 2 = 0$$

$$(2x + 1)(x - 2)$$

$$x = 2 \text{ and } x = -\frac{1}{2}$$



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Round 8

Question 1

A right triangle has one side that is 7 cm longer than the other side. The hypotenuse is 13 cm long. Find the lengths of the other sides of the triangle.

Solution

Using Pythagoras' Theorem

$$(13)^2 = (x)^2 + (x + 7)^2$$

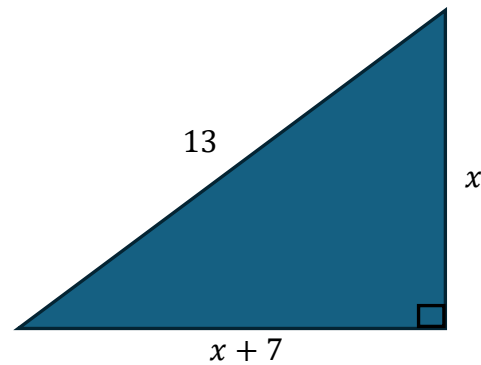
$$169 = x^2 + x^2 + 14x + 49$$

$$2x^2 + 14x - 120 = 0$$

$$x^2 + 7x - 60 = 0$$

$$(x + 12)(x - 5) = 0$$

$$x = 5 \text{ cm (units required)}$$

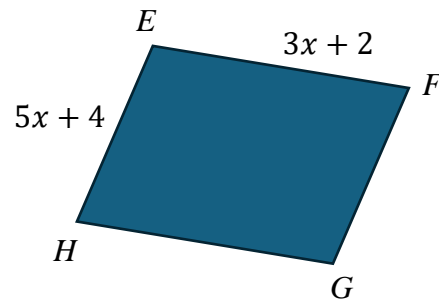


The other sides 5cm and 12cm

Knowledge of Pythagorean Triples would have given students a quick solution 5 cm, 12 cm, 13 cm

Question 2

A parallelogram has side lengths $3x + 2$ and $5x + 4$. The perimeter of the parallelogram is 44 cm and the area is 64 cm^2 . Find $|\angle EHG|$, to the nearest degree.



Solution

$$2(3x + 2) + 2(5x + 4) = 44$$

$$16x + 12 = 44 \Rightarrow x = 2$$

Sides

$$5(2) + 4 = 14 \text{ cm, and } 3(2) + 2 = 8 \text{ cm}$$

$$\text{Area} = ab \sin C = 64$$

$$(8)(14) \sin C = 64$$

$$\sin C = \frac{64}{112}$$

$$C = 34.84990458$$

$$C = 35^\circ \text{ (degree symbol not necessary)}$$



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Round 8

Question 3

Solve the simultaneous equations.

$$2x - y = 5$$

$$x + 3y = \frac{x - 4}{2}$$

Solution

$$2x - y = 5$$

$$x + 3y = \frac{x - 4}{2} \quad (\times 2) \Rightarrow 2x + 6y = x - 4$$

$$2x - y = 5 \quad (\times 6)$$

$$x + 6y = -4$$

$$12x - 6y = 30$$

$$x + 6y = -4$$

$$13x = 26 \Rightarrow x = 2$$

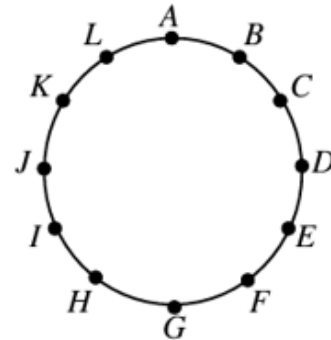
$$2(2) - y = 5 \Rightarrow y = -1$$

$x = 2$ and $y = 1$ (both required)

Question 4

Twelve balloons are arranged in a circle. Counting clockwise, every third balloon is popped. C is the first balloon to be popped. This continues until two balloons remain not popped.

Which balloons are the remaining two balloons not popped?



Solution

Order of popping of the balloons

$C, F, I, L, D, H, A, G, B, K$

Remaining two balloons are E and J

