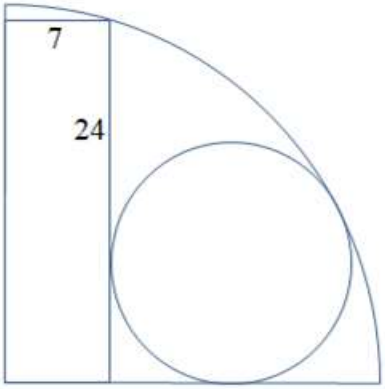
Problem No. 1
Three Thursdays of a month fall on even numbered dates. On what day of the week is the 24th of that month?

Solution to No. 1
Since adding 7 days turns an even number into an odd number any day which starts on an even date will only be on an even date again two weeks later. In order to have three even dates in a month the second even date must be 28 days later.. Thus the first even date must be on the 2nd of the month, the second on the 16th and the third on the 30th. Three weeks after the 2nd is the 23rd which is also a Thursday. Therefore the 24th must fall on a Friday.
Answer: Friday

Problem No. 2
2024 has 3 different prime factors. Which is the largest of these three numbers?

Solution to No. 2
The prime factors of 2024 are 2x2x2x11x23. The three different numbers are 2, 11, 23.
The largest is 23.
Answer: 23

Problem No. 3
a) Find the radius length of the quadrant shown and hence,
b) Using a scale diagram, find the integer radius length of the circle within the quadrant (in cm) in the following diagram. Measurements are given in cm. **Show all stages of your work.**
The scale diagram must be set within the Answer Sheet provided (A4 size) and **returned**.



Solution to No. 3
a): From the diagram we can see that the radius of the quadrant R = $\sqrt{24^2 + 7^2}$ = $\sqrt{625}$ =25.
Answer: Radius of Quadrant = 25cm.

b): The construction of the scale diagram should allow the radius of the circle to be measured at 8cm.
For example, in the diagram given the height of the rectangle is 48mm so the scale is 2mm:1cm which is 2:10 or 1:5. The radius of the quadrant is 25cm and is shown on the diagram as 50mm. The diameter length of the circle on the diagram is 32mm and thus the diameter length of the circle is 5x32 = 16cm. The radius is 8cm.
Answer: 8cm

Not required for Solution to No. 3

Further algebraic calculations show that 8cm is the correct integer answer for the radius length.

The radius |OA| = 25.

|CA| = radius r

|CB| = r

|OB| = r +7

|OC| = 25-r

In right-angled triangle OCB

$|OC|^2 = |OB|^2 + |CB|^2$

$(25-r)^2 = r^2 + (r+7)^2$

$625 - 50r + r^2 = r^2 + r^2 + 14r + 49$

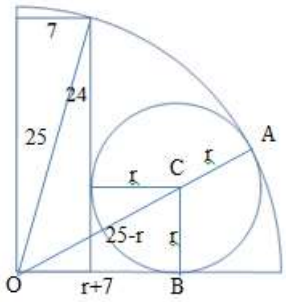
$r^2 + 64r - 576 = 0$

$(r-8)(r+72) = 0$

$r = 8$ or $r = -72$

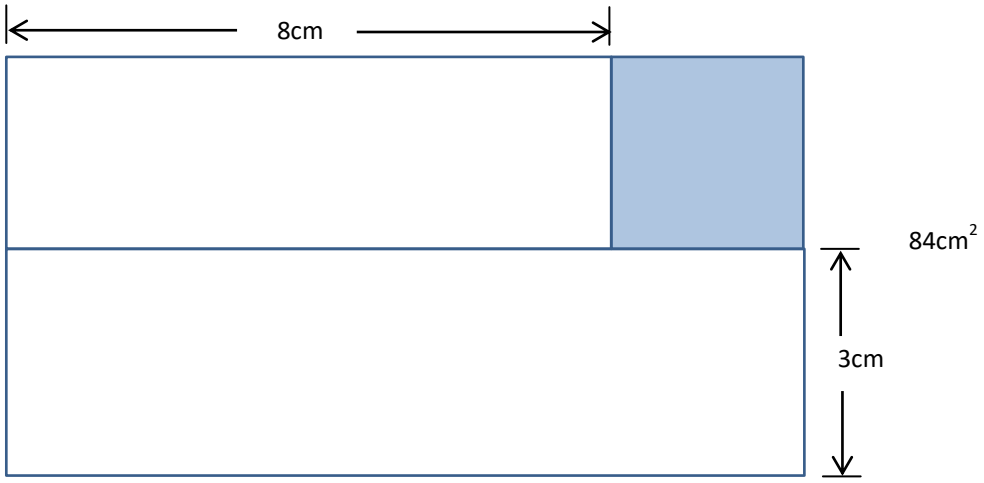
Since r is less than 25 we have r=8.

Answer: Quadrant radius = 25cm; Circle radius = 8cm.



Problem No. 4

Find the area of the shaded square in the following diagram if the total area of the diagram is $84cm^2$.



Solution to No. 4

Let the side of the square be x cm.

Then (8+x)(3+x) = 84.

$x^2 + 11x - 60 = 0$; $(x - 4)(x + 15) = 0$; $x = 4$ or $x = -15$.

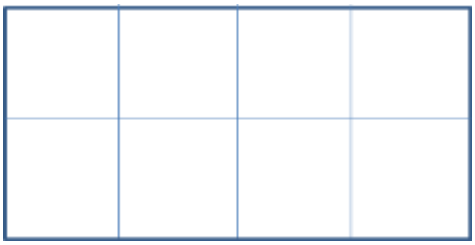
Since the problem relates to positive lengths we take the answer to be x = 4.

Thus the area of the square is $4^2 = 16cm^2$.

Answer: $16cm^2$.

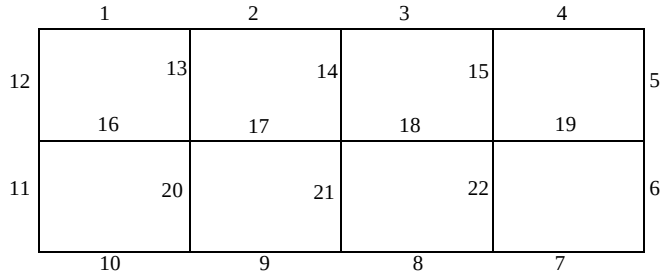
Problem No. 5

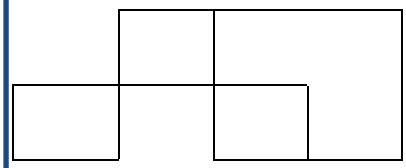
The diagram shows 22 matches forming 11 squares. Remove 5 matches to leave only 4 squares. Each remaining stick must be part of a square. In how many different ways can this be done?



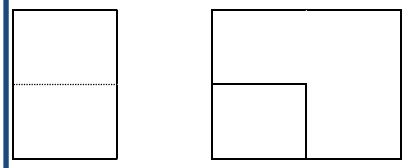
Solution to No. 5

Label the matchsticks from 1 to 22 as shown.

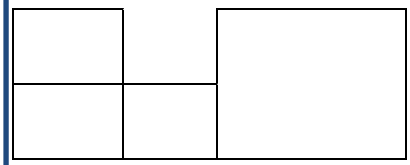




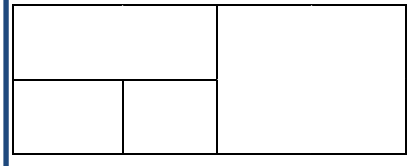
Choose 3 sections from the perimeter. This may be done in 4 ways: (1, 12, 9) or (11, 10, 2) or (4, 5, 8) or (6, 7, 3). In each case remove 2 interior sections from the remaining large square. This may also be done in 4 ways. E.g. Remove 1, 12, 9 and two of 15, 18, 19, 22. Thus there is a total of 4x4 = 16 ways to leave 4 squares in this arrangement.



Choose 2 sections from the perimeter. This may be done in 2 ways: (2, 9) or (3, 8). In each case remove 1 interior section from the centre between these two perimeter sections (only 1 way to do this) and remove 2 interior sections from the remaining large square as above which may be done in 4 ways. E.g. Remove 2, 17, 9, 15, 19. Thus there is a total of 2x1x4 = 8 ways to leave 4 squares in this arrangement.



Choose 1 section from the perimeter. This may be done in 4 ways: 2 or 3 or 8 or 9. In each case remove 4 interior sections from the remaining large square. This may be done in 1 way each time. Thus there are 4 ways to leave 4 squares with this arrangement. E.g. Remove 2, 15, 18, 19, 22.



Remove 5 sections from the interior. Remove 4 sections from a large square either on the right or the left. (2 ways). In each case remove 1 section from the remaining large square. This may be done in 4 ways. Thus there are 2x4 = 8 ways to leave 4 squares with this arrangement. E.g. Remove 15, 18, 19, 22, 13.

The total number of ways is 16 + 8 + 4 + 8 = 36 ways in total.

Answer: 36

Problem No. 6
How many 3-digit numbers are there such that the Tens digit is smaller than both of the other digits?

Solution to No. 6
The numbers will be in the form xTy where T varies from 0 to 8.
For each value of T both x and y can vary from (T+1) to 9. That is, they vary from 9 elements each when T= 0 to one element each when T = 8.
For each value of T the number of 3-digit numbers under this condition will be the product of the number of elements in x and in y which are both the same. Thus the total number of 3-digit numbers will be the sum of the square numbers from 9 to 1. This is 1+4+9+16+25+36+49+64+81 = 285.
[e.g. For T=0 we have 101, 201, …, 901, and 102, ..., 202, …902, and etc. …, …, 109, 209, …909. This set has 81 numbers.]
Ans: 285.

Problem No. 7
The flight time between Dublin, Ireland, and Vancouver, Canada, is 9 hours and 35 minutes. A plane left Dublin on the 1st of December at 10.45 am local time. At what local time did the plane land in Vancouver? Give your answer as on the 24-hour clock display.

Solution to No. 7
The time difference between Dublin and Vancouver is 8 hours unless there is an overlap of the semi-annual time changes. The 1st of December is not part of that overlap zone. Therefore, the local time in Vancouver will 8 hours earlier than Dublin. Allowing for the flight time the local time in Vancouver will be 1 hour and 35 minutes later than the take-off time. Thus, it will be 12:20.
Answer: 12:20.

Problem No. 8
Eight people rented a boat. If there were two more people to share the expense it would have cost each of them €1.50 less. What was each person's share of the rent?

Solution to No. 8
Let €x be the cost per person when the 8 people rented the boat. Then €8x is the total rent.
(€x - €1.5) is the cost per person when 10 people rent the boat.d
So, 10(x-1.5) = 8x; That is 2x = 15 and thus x = 7.5.
Ans: €7.50

Problem No. 9
What is the median of the six consecutive odd numbers that multiply together to give 6235515?

Solution to No. 9
The prime factors of 6235515 are 3, 3, 3, 5, 11, 13, 17, 19.
The six consecutive odd numbers are 9, 11, 13, 15, 17, 19.
The median is (13+15)/2 = 14.
Ans: 14

Problem No. 10
The first two numbers called in a game of Bingo (traditional Irish or British version) are 11 and 1. What is the probability that the next number called has 10 as the sum of its digits.
Write your answer in the form $\frac{A}{B}$ in its simplest form.

Solution to No. 10
In the traditional game of Bingo played in Ireland (or Britain) numbers from 1 to 90 may be called. There are 8 numbers whose digits sum to 10 (19, 28, 37, 46, 55, 64, 73, 82). Since two numbers are used, neither of which has digits which sum to 10, there are 88 numbers remaining of which 8 have digits which sum to 10. Thus the probability that one of these numbers is called is $\frac{8}{88} = \frac{1}{11}$.
Answer: $\frac{1}{11}$

Problem No. 11

a, b, c and d are integer numbers such that

$$a + \frac{1}{b+\frac{1}{c+\frac{1}{d}}} = \frac{17}{11}$$

What is a+b+c+d?

Solution to No. 11

$a + \frac{1}{b+\frac{1}{c+\frac{1}{d}}} = \frac{17}{11} = 1 + \frac{6}{11}$ and therefore a = 1.

$\frac{1}{b+\frac{1}{c+\frac{1}{d}}} = \frac{6}{11}$ and so, inverting, $\frac{b+\frac{1}{c+\frac{1}{d}}}{1} = \frac{11}{6} = 1 + \frac{5}{6}$ and so b = 1.

$\frac{1}{c+\frac{1}{d}} = \frac{5}{6}$ and so, inverting, $\frac{c+\frac{1}{d}}{1} = \frac{6}{5} = 1 + \frac{1}{5}$ and so c = 1.

$\frac{1}{d} = \frac{1}{5}$ and so d = 5.

Then, a + b + c + d = 1 + 1 + 1 + 5 = 8.

Ans: 8 [Note: A clarification was issued to state that the solution required positive integers only]

Problem No. 12

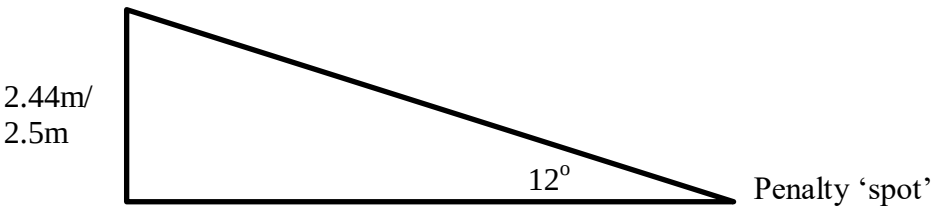
While practising taking penalties for my football team I drive the ball from the 'spot' to the roof of the net in the centre of the goal.

What is the greatest integer angle at which the ball should travel to ensure I score a goal? (The size of the ball may be ignored).

Solution to No. 12

The penalty 'spot' is directly out from the mid-point between the goal-posts at a distance of 11m (in all codes of football that use a 'spot' for taking a penalty). The crossbar is at a height of 2.44m in FAI football and 2.5m in Gaelic sports.

The angle of trajectory of the ball that will hit the crossbar is given by $\tan^{-1}\frac{2.44}{11} = 12.506^o$ or $\tan^{-1}\frac{2.5}{11} = 12.802^o$. In both cases the greatest integer angle which may be used is 12^o.



Answer: 12^o

Problem No. 13
Petrol and oil are to be mixed in a ratio of 40:1 for a hedge-trimmer with a 2-stroke engine. A mixing bottle is provided with a 500ml mark drawn at 123mm above the base to indicate the level for the completed mixture. A second mark is drawn below the 500ml mark to indicate the maximum level of petrol to be used. How far below the 500ml mark should this be?
Give your answer in mm correct to one decimal place.

Solution to No. 13
Divide 123mm in ratio 40:1 ie 1 unit = 123/41 = 3mm, exactly. Therefore draw the mark 3.0 mm below the 500ml mark. (Accept 3mm as answer also).
Answer: 3.0mm.

Problem No. 14
€20240 is bequeathed to 5 members of a family, none of which are twins, such that each sibling gets €200 more than the next youngest. How much does the eldest get?

Solution to No. 14
Let €x be the amount given to the youngest. Then the total amount is given by x + (x+200) + (x+400) + (x + 600) + (x+800) = 20240
5x + 2000 = 20240
5x = 18240
x = 3648. The eldest gets 800 more so the eldest gets €4448.

Alterative solution:
There are 4 siblings older than the youngest so there is a difference of €800 between what the youngest and eldest get. Therefore the eldest must get €400 more than the average. The average is €20240/5 = €4048 so the eldest gets €4448.
Ans: €4448

Problem No. 15
Tommy arranges books on his bookshelves going from the bottom shelf to the top. He puts half of the books on the bottom shelf and two thirds of what remains on the second shelf. He then divides the rest of his books on the other two shelves so that there are 4 more books on the third shelf than on the top shelf.
If there are 3 books on the top shelf how many books are on the bottom shelf?

Solution to No. 15
$\frac{1}{2}$of books are on the bottom shelf so ½ of books remain. $\frac{2}{3}x\frac{1}{2} = \frac{1}{3}$ of the books go on the 2nd shelf. $\frac{1}{3}+\frac{1}{2} = \frac{5}{6}$ so $\frac{1}{6}$ remain for the 3rd shelf and the top shelf. There are 3 books on the top shelf and, therefore, 7 books on the 3rd shelf. Thus, there are 10 books for these two shelves and therefore 6x10 = 60 books altogether.
Thus, there are 30 books on the bottom shelf.
Ans: 30

Problem No. 16

A puzzle starts with 9 numbers in a grid as shown:

7	5	4
11	10	16
22	19	8

On each move you are asked to swop any two numbers. The aim is to arrange for the sum of the numbers in each **row** to be a multiple of 3.
What is the minimum number of moves needed?

Solution to No. 16

Each number leaves a remainder on division by 3 as given in the following table:

7 R1	5 R2	4 R1
11 R2	10 R1	16 R1
22 R1	19 R1	8 R2

In order for the total of each row to be a multiple of 3 the sum of the remainders on that row must equal 3. This can only be achieved by have the three numbers with R2 all on the same row (5, 11 and 8). Since they are on three different rows the minimum number of swops required to place them on the same row is 2.

This may be done in a number of different ways.

For example, swop 5 with 10; swop 8 with 16.

			Totals
7 R1	10 R1	4 R1	21 = 3x7
11 R2	5 R2	8 R2	24 = 3x8
22 R1	19 R1	16 R1	57 = 3x19

Answer: The minimum number of moves is 2.

Problem No. 17

Some time ago TII (Transport Infrastructure Ireland) installed an Average Speed System on Irish motorways. In this system two cameras are placed at intervals on the motorway. The times taken for vehicles to pass the cameras are noted and their average speeds are calculated to determine if speed limits have been exceeded.

In this problem one such pair of cameras was placed 25 km apart on a motorway and the times taken to pass the cameras were noted for 4 vehicles as follows:

A truck (> 3500kg): 18mins 4secs;

An all-seater bus: 16 mins 32secs;

A car 13 mins 59secs;

A car towing a caravan: 18mins 24secs.

How many vehicles exceeded the appropriate speed limit?

Use rules-of-the-road-.pdf (rsa.ie) to guide your answer.

Copy and paste the following to your browser if the link does not automatically work:

https://www.rsa.ie/docs/default-source/services/s1.8-learner-driver-resources/rules-of-the-road-.pdf

Solution to No. 17

The required information on speed limits is on p118 at rules-of-the-road-.pdf (rsa.ie) [See below] showing speed limits for vehicles.

	Truck > 3500kg	Bus	Car	Car towing
Time	18min 4sec = 18.667 min = 0.301hr	16min 32sec = 16.533min = 0.275hr	13min 59sec = 13.983min = 0.233hr	18min 24sec = 18.4min = 0.307hr
Distance	25k	25k	25k	25k
Speed= D/T	25/0.301 = 83.026km/hr	25/0.275 = 90.726km/hr	25/0.233= 107.27km/hr	25/0.307 = 81.522km/hr
Speed limit on motorway	90km/hr for HGV > 3500kg	100km/hr for Bus	120km/hr for Car	80km/hr for Car towing
Under speed limit	Yes	Yes	yes	No

Answer: 1

Speed limits for vehicles

Some drivers **must** obey speed limits for their vehicles as well as speed limits for the roads on which they are travelling.

The table below outlines the speed limits that apply to different vehicles.

Vehicle speed limit	Type of vehicle to which it applies	
65 kilometres an hour (65kmh) On all roads	A single or double deck bus or coach designed for carrying standing passengers	
80 kilometres an hour (80kmh) On all roads except motorways	A goods vehicle with a maximum authorised mass (MAM) of more than 3,500 kilograms	
90 kilometres an hour (90kmh) On motorways	A goods vehicle with a maximum authorised mass (MAM) of more than 3,500 kilograms	
80 kilometres an hour (80kmh) On all roads	Any vehicle towing a trailer, caravan, horsebox or other attachment	
80 kilometres an hour (80kmh) On all roads except motorways or dual carriageways	A single or double deck bus or coach that is not designed for carrying standing passengers	
100 kilometres an hour (100kmh) On motorways or dual carriageways where no lower speed limit is in place	A single or double deck bus or coach that is not designed for carrying standing passengers	

Problem No. 18
Every day, Carol uses a smooth step-less escalator on her way to work. If she stands still, her journey on the escalator takes 60 seconds. One day the escalator is not working and it takes her 90 seconds to walk the length of the escalator.
How long would it take her to travel up the escalator when it is working if she were to walk at the same speed as before? Give your answer in seconds.

Solution to No. 18
The journey takes 60 seconds with machine help and 90 seconds without.
Let D = the distance waked in metres.
The speed without the machine is $\frac{D}{90}$ metres per second and with the machine is $\frac{D}{60}$ metres per second. If she walks at the same speed on the machine then both speeds are added giving D/90+D/60 = D/36$\frac{D}{90}+\frac{D}{60}=\frac{D}{36}$ m.p.sec. The time taken then is $\frac{D}{\frac{D}{36}}=\frac{36D}{D}=36$ seconds.
Alternative: Suppose the length of the escalator is 90m. Then the machine is moving at a rate of 1.5m/sec and Carol walks at a rate of 1m/sec. Combining these, Carol moves at 1.5 + 1 = 2.5m/sec walking on the machine. This distance then takes her 90/2.5 = 36 seconds.
Answer: 36 seconds

Problem No. 19
Using a restricted set of data a mathematician estimated that a person's height was connected to their length of step by the relationship
$SL=\frac{13}{15}H-75$
where SL = Step Length, H = Height and all measurements are in cm.

Two companions went on a walk together. The first person was 150cm tall and walked at a rate of 2 steps per second. Their FitBit recorded that this person had taken 5000 steps when they both stopped. The second person walked at a rate of 1.5 steps per second. How tall was the second person? Show all stages of your work. (Give your explanation on a separate sheet)
Give the answer in cm correct to 1 decimal place.

Solution to No. 19
5000 steps at 2 steps per second means that they were both walking for 2500 seconds.
At a rate of 1.5 steps per second the second person took 2500x1.5 = 3750 steps.
Using $SL=\frac{13}{15}H-75$ the first person with a height of 150cm has a step length of 55cm.
This first person took 5000 steps so the distance walked was 5000x55 = 275000cm = 2.75km.
Thus the second person walked 275000cm with 3750 steps and they have a step length of $\frac{275000}{3750}=$ 73.33cm.
Using $SL=\frac{13}{15}H-75$ we find that $73.33=\frac{13}{15}H-75 \Rightarrow H=171.15$ which is 171.2 correct to 1 decimal place.
The second person is 171.2cm tall.
Answer: 171.2cm.

Problem No. 20
Tickets for a school play cost €15 for adults and €5 for children. Receipts for the show were €6600. The hall has seating for 600 people but not all the tickets were sold. What is the smallest number of adults at the play?

Solution to No. 20
Let x be the number of adult tickets sold and y be the number of child tickets sold.
We have the equation $15x + 5y = 6600$ which is $3x + y = 1320$. Also $x + y < 600$.
The whole number values of x and y which satisfy this are: (3 more children per 1 less adult)
(440, 0), (439, 3), (438, 6) … (361, 237), (360, 240). As $x+y < 600$ then (360, 240) is discounted.
Thus the smallest number of adults is 361.
Ans: 361

Problem No. 21
The World Athletics Championships 2023 took place in Budapest. The final of the Women's 4x440m Relay was on 27th August. At the beginning of the final 100m of the 4th leg Stacey Ann Williams of Jamaica was 10m ahead of Femke Bol of the Netherlands. However, she was passed by Bol 5m from the end line.
See: 4X400m Relay Women Final World Championships Budapest 2023 - YouTube
(15:36min:sec. video; The full 4x400m race starts after 6:30; the last lap starts after 9:16; finish at 10:20).
Given that Stacey Ann Williams ran this short section of the race (while she was in the lead, as described above) in 13.1 seconds what average speed did Femke Bol have to run in the same section of the race in order to catch up with her?
Give your answer in kilometres per hour (km/hr) correct to one decimal place. (All given measurements have been estimated from the video.)

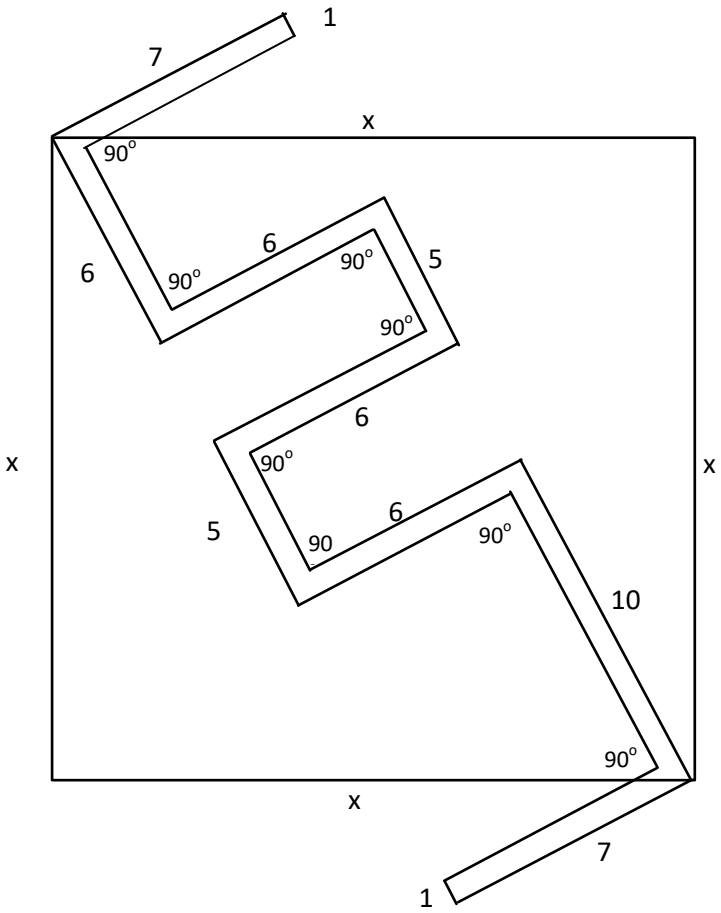
Solution to No. 21
Williams ran 95m in the section measured but Bol ran 95+10 = 105m.
Bol had to run this in the same 13.1 seconds.
Thus Bol's speed was 105x60x60/(13.1x1000)km/hr = 28.8549km/hr which is 28.9km/hr correct to 1 decimal place.
Answer: 28.9km/hr

Problem No. 22
The seven numbers 2, 3, 12, 14, 15, 20 and 21 can be divided into 2 sets such that the product of the numbers in each set is the same.
What is the product?

Solution to No. 22
The prime factors are 2, 3, 2, 2, 3, 2, 7, 3, 5, 2, 2, 5, 3, 7.
If the products are the same each set must have the same prime factors. So choose matching prime factors for each set. Each set must have 2, 2, 2, 3, 3, 5, 7.
Multiplying these we get 2520.
Alternative solution:
Multiply all the numbers given: 2x3x12x14x15x20x21 and get the square root.
We have $\sqrt{6350400} = 2520$.
Ans: 2520

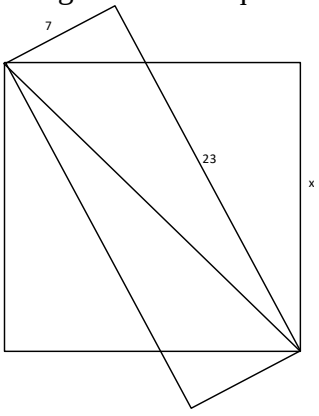
Problem No. 23

A wrought-iron worker cut out a straight-edged shape of uniform width, 1cm, from a flat plate of metal. She placed it so that it overlaid a square support base as shown. What is the length of the side, x, of the square? (All measurements are in cm.) Give your answer in centimetres.



Solution to No. 23

Complete the inner rectangle as shown. Allowing for the width (1cm) of the piece the left side length of the rectangle will be (from the top) 6 + (5-2) + 5 + (10-2) + 1 = 23 and the width of the rectangle is 7. Draw the diagonal of this rectangle. The diagonal length will be $\sqrt{(23^2 + 7^2)} = \sqrt{578}$. This is also the diagonal of the square. Therefore $2x^2 = 578$; $x^2 = 289$ and $x = \sqrt{289} = 17$.



Answer: x = 17cm.

Problem No. 24
A shopkeeper buys 10 items for a total of €P. Eight are sold for a profit of 50% and the rest in a sale at a profit of only 10%. What is the total profit in the form € $\frac{a}{b}P$, where *a* and *b* are relatively prime?

Solution to No. 24
The cost of 10 items is €P so the cost per item is €$\frac{P}{10}$.
The selling price for 8 of these is €$\frac{8P}{10}$+ €$\frac{4P}{10}$ = €$\frac{12P}{10}$. (Profit 50%)
The selling price for 2 of these is €$\frac{2P}{10}$+ €$\frac{1P}{50}$ = €$\frac{11P}{50}$. (Profit 10%)
The total selling price is €$\frac{12P}{10}$+ €$\frac{11P}{50}$ = €$\frac{71P}{50}$.
The profit is €$\frac{71P}{50}$- €$\frac{50P}{50}$ = €$\frac{21P}{50}$.
The numbers 21 and 50 are relatively prime. So the profit is €$\frac{21P}{50}$.
Answer: €$\frac{21P}{50}$.

Problem No. 25
Use the Engineers Ireland site Schools | Engineers Ireland to find an answer to this question: What is the 3-letter acronym for a programme that STEPS gives as *a hands-on, fun and practical insight into engineering at third level and as a career*?

Solution to No. 25
From Engineering Your Future | Engineers Ireland we find that the STEPS EYF – STEPS Engineering Your Future - is the programme referred to. The 3-letter acronym is EYF.
Answer: EYF