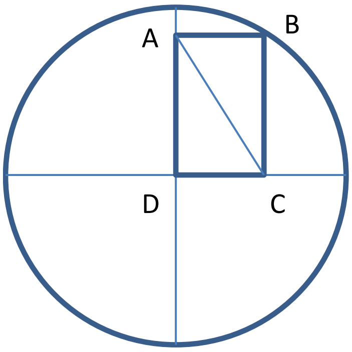
Peter's Problem 2023 -Suggested Solutions

Problem No. 1
How many degrees, if any, in the angle between the hour hand and the minute hand of a clock when the time is 3:15pm.

Solution to No. 1
At 3:15pm the minute hand forms an angle of 0^0 with where the hour hand was at 3pm. There are $\frac{360^0}{12} = 30^0$ between each hour on the clock. When the minute hand has moved to a quarter past three it has moved the hour hand on by $\frac{30^0}{4} = 7.5^0$.
Answer: 7.5^o.

Problem No. 2
A circle of radius 10cm has a rectangle ABCD inscribed in the first quadrant. Find |AC|



Solution to No. 2
The diagonal |AC| = |DB| = radius of the circle. Therefore |AC| = 10cm.
Answer: 10cm.

Problem No. 3
Working in a science laboratory Albert isolated a newly-identified bacterium. The bacteria triple in size every 30 minutes. If Albert placed one bacterium in a test tube at 8am then the test tube was full of bacteria at 4pm.
If he had placed 3 bacteria in the test tube at 8am, at what time would the test tube be full?

Solution to No. 3
Having 3 bacteria in the test tube at 8am is the same as putting one bacterium in the test tube at 7.30am. The means that the test tube will be filled half an hour earlier which is at 3.30pm.
Answer: 3.30pm (or 15:30)

Problem No. 4
300 is divided into 4 parts such that the first integer number is twice the second, three times the third and four times the fourth. What is the value of the largest of these numbers?

Solution to No. 4
Let the 4th number be 3*x*, then then the 1st is 12*x*; the 2nd is 6*x*; the 3rd is 4*x*.
We have the equation:

$$12x + 6x + 4x + 3x = 300$$
$$25x = 300$$
$$x = 12$$

Hence, the largest is 12*x* which is 12(12) = 144.
Answer: 144

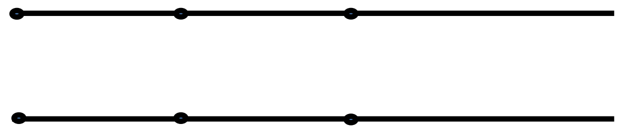
Problem No. 5
A 55-seater bus was 40% full of its capacity on leaving the depot. At the first stop only my friend and I boarded the bus. At the second stop one third of the passengers alighted and 9 passengers boarded. At the third stop I, together with 10 other passengers, got off the bus and 5 boarded.
How many passengers were on the bus as it pulled away from the stop?

Solution to No. 5
Leave depot: 40% of 55 = 22 on the bus.
First stop: 22+ 2 = 24 on bus.
Second stop: 24/3 = 8 alighted, 9 boarded; 24 + 1 = 25 on the bus.
Third stop: 11 alighted, 5 boarded, so 6 less; 25 – 6 = 19.
Answer: 19 passengers remain on the bus.

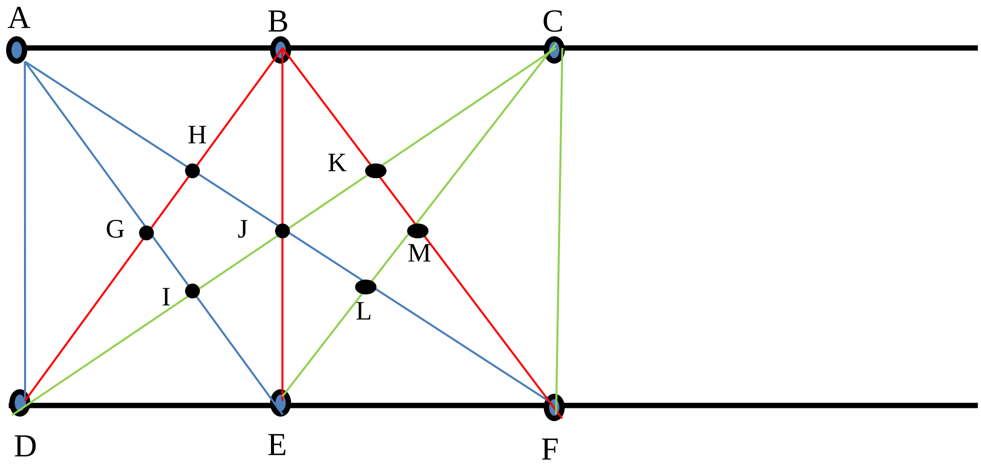
Problem No. 6
Tamsin is saving up for a smartwatch and puts all of her 50 cent and 20 cent coins in a jar each night. She thinks that she has saved enough and empties the 620 coins to count the money. 286 of the coins are 50 cent.
If the cost of the smart watch is €230, how much more must she save to allow her buy the watch?

Solution to No. 6
286 ×€0.5 = €143. This leaves 620 – 286 = 334 coins. 334×€0.2 = €66.80.
€143 + €66.80 = €209.80.
Then €230 - €209.80 = €20.20 ⇒ €20.20 more to save.
Answer: €20.20

Problem No. 7
Three equidistant dots on each of two parallel lines (as illustrated) are joined in all possible ways. How many triangles can be formed?



Solution to No. 7



ABH	ABJ	ABF	ACJ	ACE	
ABG	ABE		ACI		
ABD	AJE	ALE	ACD	ACF	
AHG	AJI	AED	ACL		
AHD	AJD	AFE			
AGD	AID	AFD			23
BJH	BKJ	BCM	BEG	BFJ	
BCK	BKD	BCE	BED	BFH	
BCJ	BJD	BCF		BFE	
BCD			BME	BFD	17
CMK	CEJ	CFM	CFL	CFE	
CLJ	CEI	CFK	CFJ	CFD	
	CED				11
DGI	DHJ	DJE	DKF		8
DGE	DHF	DJF			
DIE					
EIJ	EJL	ELF	EMF		
	EJF				5
FLM	FJK				2
				Total	66

Answer: 66 triangles

Problem No. 8

A rectangular schoolroom floor can be covered by 2400 square tiles. If the tiles are 5cm longer and 5cm wider, 1350 tiles are needed.
If none of the tiles are cut, what is the area of the floor in m^2?

Solution to No. 8

There are 2400 – 1350 = 1050 less tiles. Let x be the side length of the original tile. Then the new tile has side length $(x+5)$
Each tile has increased by $(10x + 25)\ cm^2$.
Therefore, the $1350(10x + 25)\ cm^2$ of the new tiles has covered the area of 1050 original tiles.
In proportion,$(90x + 225)cm^2$ is the area of 7 tiles. We must find the value of x which x^2 is the same area as $(90x + 225)/7$.
$(90x + 225)/7$ gives integer values for $x = 1, 8, 15, \ldots$
Testing, we find $x = 15$ gives the required value.
The area of the floor is $2400(15)((15) = 540\ 000cm^2 = 54m^2$.

Alternative Solution:

Let the smaller tile have an area of x^2, The larger tile has an area of $(x+5)^2$.
Thus $2400x^2 = 1350(x+5)^2$, Divide both sides by 150.
$16x^2 = 9(x+5)^2$.
⇨ $4x = 3(x+5) = 3x + 15$
⇨ $x = 15$
⇨ Area $= 2400(15)^2 = 540000m^2 = 54m^2$.
Answer: $54m^2$.

Problem No. 9

Each of the letters in the multiplication sum below represents a digit:

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        7A
 times  B5
       C90
      DE20
  =   F5GH
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What is the value of A+B+C+DE+F+G+H?

Solution to No. 9

H = 0; G = 1;
5(A) = _0 so A = 0, 2, 4, 6 or 8; and 5(7) = 35 so 35 + x = C9 implies x = 4 so A = 8 and C = 3;
2+9 = 11 give Carry(1) so 1+ 3(=C) + E = _5 implies E = 1;
B(8) = _2 implies B = 4 or 9. Then B = 9 gives 9(8) = 72 and 9(7) = 63; then DE = 63 + 7 = 70 and then E = 0 impossible.
So B = 4. Then DE = 31 and F = 3.
We have A = 8; B = 4; C = 3; DE = 31; F = 3; G = 1; H = 0
A+B+C+DE+F+G+H = 8+4+3+31+3+1+0 = 50
Answer: 50

Problem No. 10
Tammy's Taxis has rented an office in town and needs to get a landline for the premises. The area code is 07893 followed by five other digits, ie. 07893xxxxx where each 'x' stands for a digit. Tammy requires that the final six digits contain only two different digits. How many different telephone numbers are possible?

Solution to No. 10
The last 6 digits of 07893xxxxx are 3xxxxx. We must pick two digits to fill in the 6 places. The first is a 3 (already picked for us). The second must be one of the other 9 digits – i.e. 9 choices. Having picked a digit we must fill in the 5 blank places with either a '3' or the other digit picked. We have 2 possibilities for each place giving 2^5 = 32ways of filling the places. But the combination '333333' is not allowed. This means that there are 31 different phone numbers for each of the 9 digits we can pick with the '3'. ie There are 31x 9 ways = 279 different phone numbers overall.
Answer: 279

Problem No. 11
Without repetition and using only addition show a selection of the ten numbers
25, 27, 3, 12, 6, 15, 9, 30, 21, 19
which makes a total of 50.

Solution to No. 11
50 leaves a remainder of 2 when divided by 3. Both 25 and 19 leave a remainder of 1 when divided by 3 and must be used since all the other numbers are divisible by 3. We have 25 + 19 = 44 and require another number to total 50. We have 50 – 44 = 6. Then 25 + 19 + 6 = 50.
Answer: The selection is 25, 19, 6.

Problem No. 12
In the 2022 Eurovision Song Contest, the entry from Ukraine was adjudged the winner with 631 points made up of the points awarded by the jury and the popular vote (televote) from the participating countries.
Considering the number of countries that actually awarded Ukraine top points in the televote what is the minimum number of other countries which **could** have awarded Ukraine their next highest number of points in the televote in order to obtain the same winning total?

Solution to No. 12
From https://eurovisionworld.com/eurovision/2022 Ukraine got 439 from Televoting and 192 from the jury. 40 countries took part so 39 countries could vote for Ukraine. Points awarded can be 12, 10, 8, 7, 6, 5, 4, 3, 2 or 1. The highest is 12 points and the next highest is 10 points.

In the Televote Ukraine got 336 points from first place votes from 28 countries and 439 in total. So the difference of 439-336 = 103 points came from (39-28) 11 other countries who gave them less than first place. All eleven countries could not have given them 10 points as 11x10 = 110 which is more than 103.
The possibilities to make 103 are: (10x10 + 1x3) or (9x10 + 1x7 + 1x6) or (8x10 + 2x8 + 1x7).
Seven countries could not have given them 10 points since 7x10 = 70 leaving 4 countries to give (103-70) 33 votes; but the highest possible from lower points is 4x8 = 32 and therefore impossible.
This leaves the minimum at 8.
Answer: The minimum number of countries is 8.

Problem No. 13
There are 21 women and 14 men in a choir. How many women should join the choir so that the number of women represents 72% of the membership of the choir, assuming that the number of men in the choir stays the same?

Solution to No. 13
21 + 14 = 35.
21 is 60% of 35. Let x be the number of extra women required. Then $\frac{21+x}{35+x} \times 100 = 72$.
This gives $x = 15$.
Answer: 15 extra women required.

Problem No. 14
I choose 10 consecutive numbers. If I exclude one of the numbers the remaining 9 sum to 2023. Which number did I exclude?

Solution to No. 14
The 9 numbers add to 2023.
2023÷9 = 224.7,
This means that the 10 numbers have 224 and 225 in the middle of the list.
This list total is 220+221+222+223+224+225+226+227+228+229= 2245
Then 2245 – 2023 = 222.
That is, 222 is the number to be omitted.
Answer: 222

Problem No. 15
A square field has an area of 1 hectare. The farmer decides to set aside 10% of the field as a uniform border all around the field to be managed for rewilding. What is the width of the border in metres correct to 2 decimal places?

Solution to No. 15
The area is 1ha = 10 000m^2 and the side length is 100m.
1000m^2 (10%) is set aside leaving 9000m^2 as the area of the remaining square. The side of the square is $\sqrt{9000}$ = 94.868m.
The total reduction in length is (100-94.868) = 5.132m.
The border is $\frac{5.132}{2} = 2.566$ = 2.57m correct to 2 decimal places.

Answer: 2.57m

Problem No. 16
I write the numbers from 1 to 10 inclusive on a whiteboard. I erase two of the numbers at random and add 1 to their sum and jot this number into a notepad. I repeat the exercise, erasing two numbers in turn until all 10 numbers are erased. I then add up the numbers in the notepad and always get the same total. What is this total and why is it so? (Give your explanation on the separate sheet)]

Solution to No. 16
The numbers are paired and there are 5 pairs no matter which way the pairing is done. The sum of the total from all the pairs is the same as the sum of all the numbers so the total is always the sum of the numbers from 1 to 10 inclusive which is 55 and since 1 is added to each pair there is an extra 5 in the final total which gives 55 + 5 = 60.
Answer: 60

Problem No. 17
A number of teenagers are playing with their calculators. One of them multiplies their ages (in whole numbers) together and finds that the product is eighteen million seven hundred and twenty seven thousand two hundred.
How many teenagers are in the group and how did you reach that answer? (Give your explanation on a separate sheet)

Solution to No. 17
Eighteen million seven hundred and twenty seven thousand two hundred
is written as 18 727 200. The thirteen factors of 18 727 200 are 5,5,2,2,2,2,2,3,3,3,3,17,17.

From the factors two of the teenagers are aged 17. Selections from the other eleven factors must multiply to numbers which lie between 13 and 19 since these are the ages of teenagers.
This is achieved by 5x3 = 15; 5x3 = 15; 2x2x2x2 = 16; 2x3x3 = 18.
No other combination of the factors will satisfy the problem.
The ages are 15, 15, 16, 17, 17, 18.
These are the 6 factors. 15x15x16x17x17x18 = 18 727 200.
Answer: There are 6 teenagers

Alternative solution:
18727200 is divisible 15 (1 teen); Remainder 1248480;
1248480 is divisible by 15 (2 teens); Remainder 83232;
83232 is an even number which must be divisible by an even number between 13 and 19;
83232 divided by 16 (3 teens)= 5202 an even number;
5202 divided by 18 (4 teens) = 289;
289 = 17×17 (making 6 teens). In total there are 6 teenagers.
(Similarly we could have 83232/18 = 4624; 4624/16 = 279 and 279 = 17x17.)
Answer: There are 6 teenagers.

Alternative solution:
Using highest age: 19^5 = 2 476 099 < 18 727 200. Therefore there are more than 5 teenagers.
Using lowest age: 13^7 = 62 748 517 > 18 727 200. Therefore there are less than 7 teenagers.
By the Squeezing Principle of Integers there must be 6 teenagers.
Answer: There are 6 teenagers.

Problem No. 18
At a scout jamboree, dinner is served from tinned food. Each tin of soup is shared between 2 scouts. Each tin of frankfurters is shared between 3 scouts and each tin of fruit is shared between 4 scouts. If 117 tins are opened to feed all the scouts, how many scouts attended the jamboree?

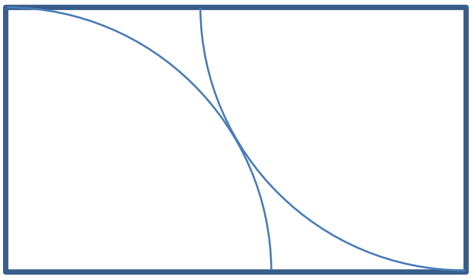
Solution to No. 18
12 scouts would consume 6 tins of soup, 4 tins of frankfurters and 3 tins of fruit. This is 13 tins in total. 117/13 = 9 so there are 9 groups of 12 scouts which is 108 scouts.
Alternative solution:
Let x be the number of scouts
The number of tins used is $\frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 117$

$$\frac{6x+4x+3x}{12} = 117$$

$$13x = 1404 \quad \text{and } x = 108$$

Answer: 108 scouts

Problem No. 19
Two quarter circles each of radius 5cm fit inside a rectangle. The centres of the circles are at opposite vertices of the rectangle. What is the area of the rectangle in the form $a^2\sqrt{b}$?



Solution to No. 19
Let y = the length of the longer side of the rectangle. The radius of each quarter circle is 5cm. The diagonal is 10cm since the circles touch at the intersection point with the diagonal. Using Pythagoras Theorem $10^2 = 5^2 + y^2$ and we then have
$y = \sqrt{100-25} = \sqrt{75} = 5\sqrt{3}$. The area of the rectangle is $5(5\sqrt{3}) = 5^2\sqrt{3}$.

Answer: $5^2\sqrt{3}$ cm^2.

Problem No. 20
In a 10 digit number, the first digit is the number of zeros in the number. The second digit is the number of ones in the number. The third digit is the number of twos in the number and so on with the tenth digit showing the number of nines in the number. What is the number?

Solution to No. 20
There are ten slots to be filled. Slots '6', '7', '8' and '9' cannot hold more than the digit 1 as this means some digit appears that many times. If a 9, say, appeared twice then two different digits have to each appear nine times which is impossible.
If either of the slots '7', '8' or '9' hold the digit 1 then it leads to contradictions. These slots can only hold the digit 0.
Thus, the digits 3, 4, 5 or 6 can appear in the first slot for the number of zeroes.
Using either 3, 4 or 5 in the first slot leads to contradictions.
When the digit 6 is used in the first slot we require six zeroes and hence the digit 1 in the slot for 'six'. This brings us to 6_ _ _ _ _ 1000. By filling the other three zeroes in the '3', '4' and '5' slots we reach 6 _ _ 0001000. The problem is then solved by putting the digit 1 in the '2' slot and hence the digit 2 in the '1' slot. The number is 6210001000.
Answer: 6210001000

Problem No. 21
Shown below is a magic square where all the rows and columns and both diagonals add to the same sum. Calculate the value of N

18			
9	16		
	17	12	18
	N	21	5

Solution to No. 21

18	z		
9	16		
x	17	12	18
y	N	21	5

On the diagonal 18 + 16 + 12 + 5 = 51. On the 3rd row 18 + 12 + 17 + x = 51 implies x = 4. On the first column 18 + 9 + 4 + y = 51 implies y = 20. On the last row 20 + N + 21 + 5 = 51 implies N = 5.

Answer: N = 5;

Problem No. 22
How many different pairs of prime numbers add to 90?

Solution to No. 22
The primes up to 90 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89.
Pairs which add to 90: (83, 7); (79, 11); (73, 17); (71, 19); (67, 23); (61, 29); (59, 31); (53, 37); (47, 43). That is, 9 pairs.
Answer: 9 pairs.

Problem No. 23
If Sam drives to work at an average speed of 60km/hr, she arrives in work two minutes early. If she travels at 45km/hr she arrives three minutes late,
How far, in km, does she live from work?

Solution to No. 23
Let D = the distance in km. The difference between travelling at 60km/hr and 45km/hr is 5 minutes.
Then D/45 – D/60 = 5/60 hr. Then, 60D – 45D = 45(5) implies D = 15km.
Answer: 15km.

Problem No. 24
There are 20 students in a TY class. As a fundraiser, they decided that, between them, they would sit on every seat which could be allocated for a rugby match in the Aviva stadium and raise sponsorship in doing so. They estimated that it would take each student an average of 2.4 seconds to sit on each seat, If they began their challenge at 10.30am, at what time, to the nearest minute will they expect to finish the challenge?

Solution to No. 24
https://en.wikipedia.org/wiki/Aviva_Stadium gives a capacity of 51700.
(Note: https://www.avivastadium.ie/ gives 51 711 but it does not cause a variation in the answer given here);
$\frac{51700}{20} = 2585$ seats per student. $2585 \times 2.4 = 6204$ seconds which is 1hr 43mins 24 seconds or 1hr 43min to the nearest minute.
They can expect to finish at 10:30 + 1hr 43min = 12:13pm
Answer: 12.13pm

Problem No. 25
As given on the STEPS Engineers Ireland website (Secondary School section) how many free engineering activities are available to download by teachers for classroom use?

Solution to No. 25
From https://www.engineersireland.ie/Schools/Engineers-Week/Secondary-Schools we quote:

> ***STEPS Engineers Week 2023 Activities and Resources***
> *STEPS has over 20 free secondary level engineering activities available for teachers to download and carry out in classrooms.*

Answer: Over 20.