Round 1 Babtha 1

Question 1

2024 is a tetrahedral number and, as such, can be written in the form:

$$2024 = \frac{n(n+1)(n+2)}{6}$$

Solve for $n \in \mathbb{N}$.

Question 2

Given that $z = 3 + i$, where $i^2 = -1$, find the value of $k \in \mathbb{R}$ such that:

$$z + \frac{k}{z}$$

is real.

Round 2 Babtha 2

Question 1

James is the penalty taker for his soccer team. The probability James scores his first penalty is $\frac{7}{10}$. If subsequent penalties in a match are awarded, the probability of James scoring depends on his previous success or failure. If James scored the previous penalty, the probability of scoring again is $\frac{3}{4}$. If he missed, the probability is $\frac{1}{2}$.

On a particular day three penalties are awarded.

What is the probability James scores exactly two penalties? Give your answer as a fraction written in its simplest form.

Question 2

Let $f(x) = -\frac{5}{2}x + 1$. Solve for $k \in \mathbb{Q}$, for which:

$$f\big(x + f(x)\big) = kf(x)$$

Round 3 Babtha 3

Question 1

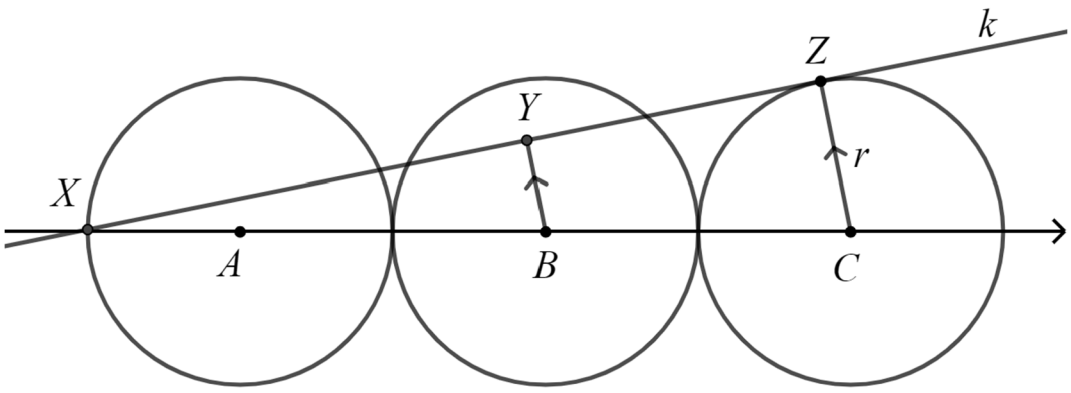
A box contains discs of equal size, coloured red, white or black. There are 11 red discs, 7 white discs and x black discs. Three discs are selected together at random from the box, without replacement.

If the probability that all three discs are all different in colour is equal to the probability that they are all black, find the value of x.

Question 2

3 circles of equal radius r, have centres A, B, and C respectively, on the x-axis. They touch externally as shown in the diagram. The line k is a tangent to the third circle at the point Z. The line k intersects the first circle on the x-axis at the point X. BY is drawn parallel to CZ.

Express $|BY|$ in terms of r. Your answer must be given in its simplest form.



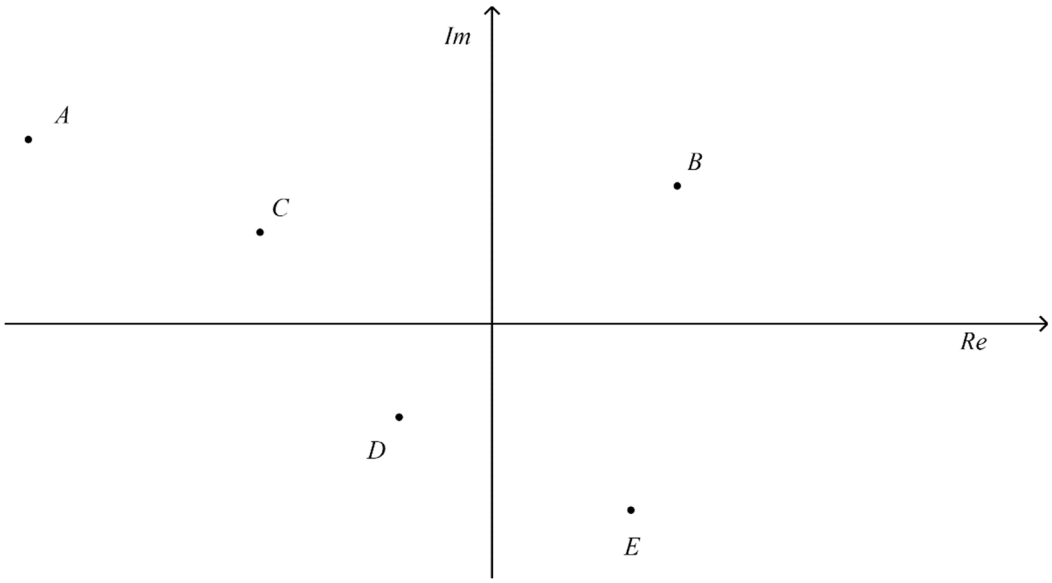
Round 4 Babtha 4

Question 1

5 complex numbers are shown with the same scale on each axis. They satisfy the following conditions:

$z_2 = 2z_1$; $z_5 = z_1 + z_4$; $z_4 = -iz_3$

Identify each point A, B, C, D, E correctly as to their z_n equivalent.



Question 2

A battery manufacturer claims a mean battery life of 2190 minutes and standard deviation 76 minutes for a particular battery. They conducted a random sample of 200 batteries, found the mean battery life of the sample and calculated a p-value of 0.037.

The sample mean battery life was lower than the previously established mean. Calculate the sample mean battery life from their sample. Give your answer in minutes, correct to 4 significant figures.

Round 5 Babtha 5

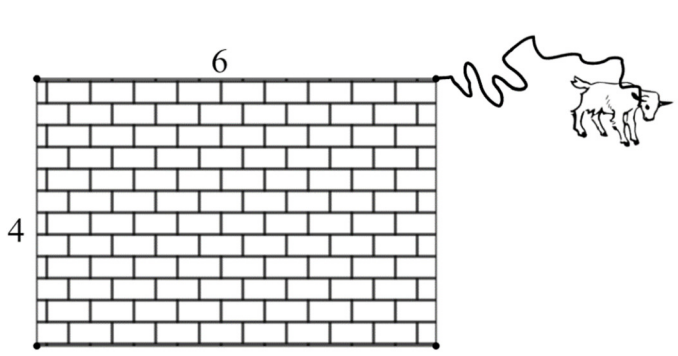
Question 1

Gertie, the mathematical goat, is tied by a rope to the corner of a 4x6 metre solid brick shed that is surrounded entirely by grass. The length of the rope is 8 metres. The goat can move freely in all directions that the rope and shed do not impede.

Calculate the area of grass that the goat can eat.

Give your answer in terms of π.

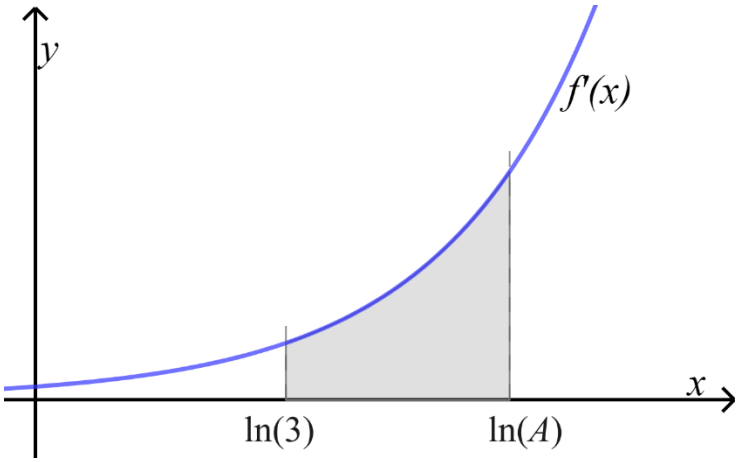
Diagram NOT to scale.



Question 2

Let $f(x) = \frac{2}{3}(1 + e^x)^{\frac{3}{2}} + C$, where C is a constant.

The diagram shows part of the graph of the function $f'(x)$. The shaded region in the diagram is between the graph and the x-axis, from $x = \ln(3)$ to $x = \ln(A)$.



This shaded region has an area of $\frac{38}{3}$ square units.

Find the value of A.

Round 6 Babtha 6

Question 1

Find the values of k for which the four numbers below have standard deviation $\sigma = \sqrt{20}$

$$1, \qquad k, \qquad 3k-2, \qquad 9$$

Question 2

Let $f(x) = \frac{e^{3x}+e^{-3x}}{3}$. Find the value of p for which:

$$f''(x) = pf(x)$$

where $p \in \mathbb{Z}$, and $f''(x)$ is the second derivative of $f(x)$.

Round 7 Babtha 7

Question 1

Sand is pouring out of a tube at 1 cubic meter per second. It forms a pile which has the shape of a right cone. The height of the cone is equal to the radius. How fast is the sandpile rising when it is 2 meters high? Give your answer in terms of π.

Question 2

$u_1, u_2, u_3, u_4, u_5, u_6, \ldots\ldots$ is a sequence where $u_1 = 4$ and $u_{n+1} = (-1)^n u_n + 20$. Find the value of u_{2024}.

Question 3

A is an obtuse angle such that:

$$\sin\left(A + \frac{\pi}{3}\right) + \sin\left(A - \frac{\pi}{3}\right) = \frac{4}{5}$$

Find the value of $\tan A$.

Question 4

A maths teacher has 7 pencils to give to four students and must give out all 7 pencils. The maths teacher can give none, or a mix of equal amounts and unequal amounts to the four students. In how many ways can the teacher do this?

Round 8 Babtha 8

Question 1

Let $y = \frac{1}{x} + \cos 2x$. Find the value of $n \in \mathbb{Z}$, such that:

$$\frac{d^5y}{dx^5} = -\frac{n!}{x^6} - 2^n \sin 2x.$$

Question 2

The National Lottery Daily Million is a draw game in which you can pick any six numbers from 1 to 39. At each draw six random numbers are drawn. If you match all 6 numbers, you win €1 million. It costs €1 to enter. You play the game once. What is your overall expected profit/loss, taking the entry fee into account? Give your answer correct to 4 decimal places.

Question 3

Let $f(x) = x^2 - 7x + 12$. Find the range of values of $x \in \mathbb{R}, x \neq 2, x \neq 3$, for which

$$\frac{f(x)}{f(x+1)} \geq 3$$

Question 4

What is the remainder when 2024^{2024} is divided by 9?

Answers

R1	Q1	22
	Q2	10
R2	Q1	$\frac{53}{160}$
	Q2	$-\frac{3}{2}$
R3	Q1	23
	Q2	$\frac{3r}{5}$
R4	Q1	$A = z_2, \quad B = z_3, \quad C = z_1, \quad D = z_5, \quad E = z_4$
	Q2	2179
R5	Q1	$53\pi\ m^2$
	Q2	8
R6	Q1	$5;\ -\frac{1}{3}$
	Q2	9
R7	Q1	$\frac{1}{4\pi}\ m/s$
	Q2	-16
	Q3	$-\frac{4}{3}$
	Q4	120
R8	Q1	5
	Q2	− €0.6935 or €0.6935 loss
	Q3	$1 \leq x < 2$
	Q4	1