Round 1 Babtha 1

Question 1

2024 is a tetrahedral number and, as such, can be written in the form:

$$2024 = \frac{n(n+1)(n+2)}{6}$$

Solve for $n \in \mathbb{N}$.

Question 2

Given that $z = 3 + i$, where $i^2 = -1$, find the value of $k \in \mathbb{R}$ such that:

$$z + \frac{k}{z}$$

is real.

Round 2 Babtha 2

Question 1

James is the penalty taker for his soccer team. The probability James scores his first penalty is $\frac{7}{10}$. If subsequent penalties in a match are awarded, the probability of James scoring depends on his previous success or failure. If James scored the previous penalty, the probability of scoring again is $\frac{3}{4}$. If he missed, the probability is $\frac{1}{2}$.

On a particular day three penalties are awarded.

What is the probability James scores exactly two penalties? Give your answer as a fraction written in its simplest form.

Question 2

Let $f(x) = -\frac{5}{2}x + 1$. Solve for $k \in \mathbb{Q}$, for which:

$$f\big(x + f(x)\big) = kf(x)$$

Round 3 Babtha 3

Question 1

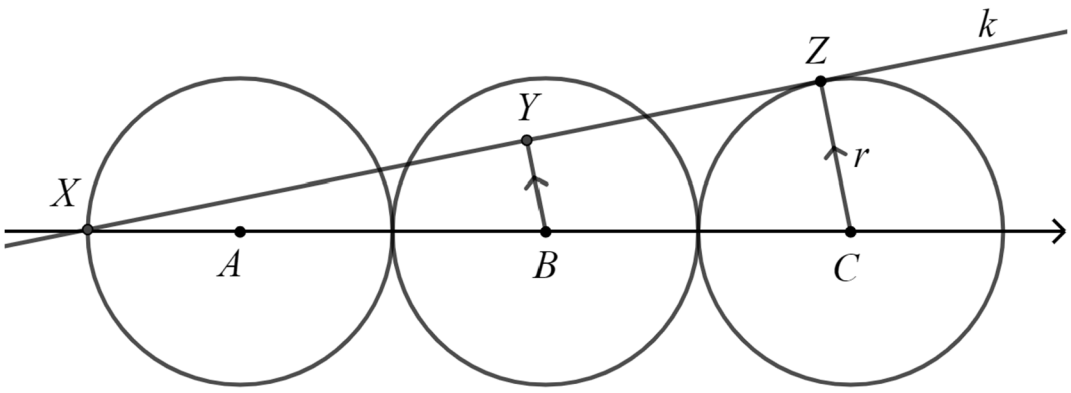
A box contains discs of equal size, coloured red, white or black. There are 11 red discs, 7 white discs and x black discs. Three discs are selected together at random from the box, without replacement.

If the probability that all three discs are all different in colour is equal to the probability that they are all black, find the value of x.

Question 2

3 circles of equal radius r, have centres A, B, and C respectively, on the x-axis. They touch externally as shown in the diagram. The line k is a tangent to the third circle at the point Z. The line k intersects the first circle on the x-axis at the point X. BY is drawn parallel to CZ.

Express $|BY|$ in terms of r. Your answer must be given in its simplest form.



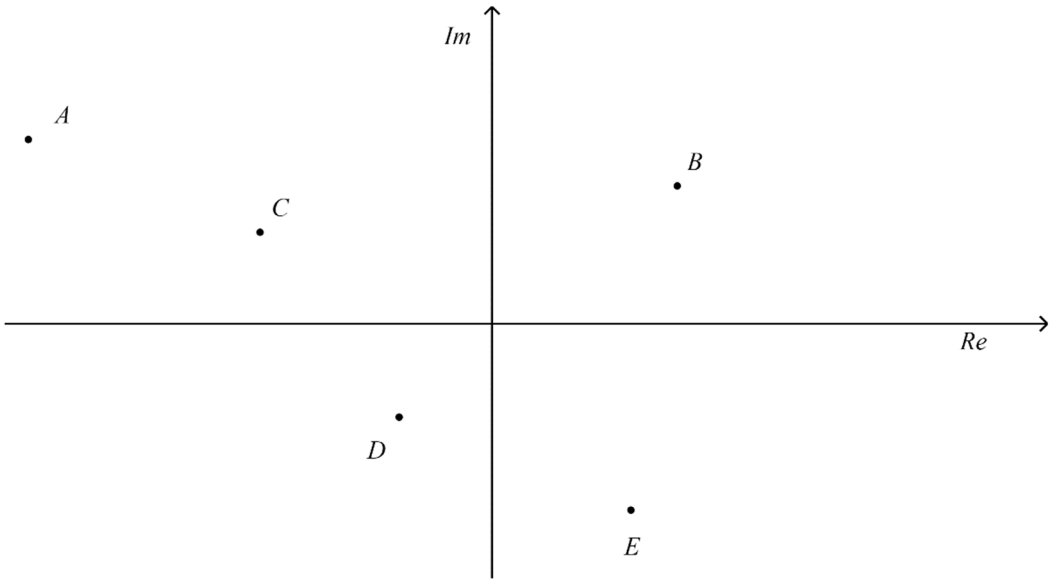
Round 4 Babtha 4

Question 1

5 complex numbers are shown with the same scale on each axis. They satisfy the following conditions:

$z_2 = 2z_1$; $z_5 = z_1 + z_4$; $z_4 = -iz_3$

Identify each point A, B, C, D, E correctly as to their z_n equivalent.



Question 2

A battery manufacturer claims a mean battery life of 2190 minutes and standard deviation 76 minutes for a particular battery. They conducted a random sample of 200 batteries, found the mean battery life of the sample and calculated a p-value of 0.037.

The sample mean battery life was lower than the previously established mean. Calculate the sample mean battery life from their sample. Give your answer in minutes, correct to 4 significant figures.

Round 5 Babtha 5

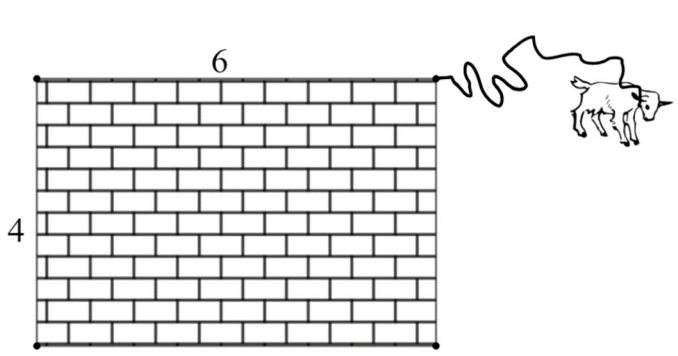
Question 1

Gertie, the mathematical goat, is tied by a rope to the corner of a 4x6 metre solid brick shed that is surrounded entirely by grass. The length of the rope is 8 metres. The goat can move freely in all directions that the rope and shed do not impede.

Calculate the area of grass that the goat can eat.

Give your answer in terms of π.

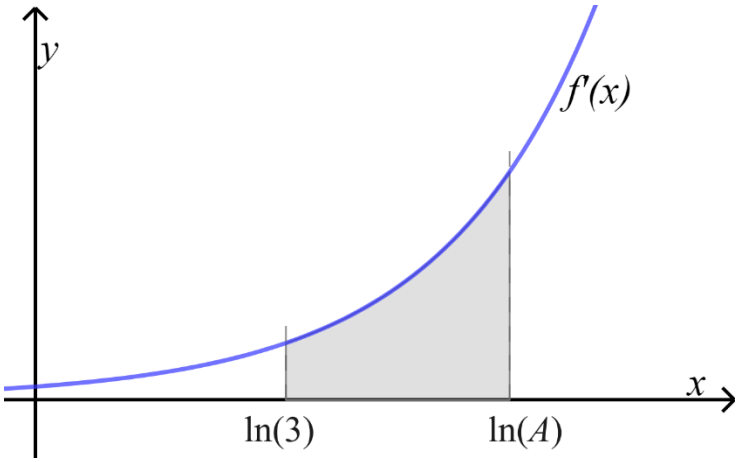
Diagram NOT to scale.



Question 2

Let $f(x) = \frac{2}{3}(1 + e^x)^{\frac{3}{2}} + C$, where C is a constant.

The diagram shows part of the graph of the function $f'(x)$. The shaded region in the diagram is between the graph and the x-axis, from $x = \ln(3)$ to $x = \ln(A)$.



This shaded region has an area of $\frac{38}{3}$ square units.

Find the value of A.

Round 6 Babtha 6

Question 1

Find the values of k for which the four numbers below have standard deviation $\sigma = \sqrt{20}$

$$1, \qquad k, \qquad 3k-2, \qquad 9$$

Question 2

Let $f(x) = \frac{e^{3x}+e^{-3x}}{3}$. Find the value of p for which:

$$f''(x) = pf(x)$$

where $p \in \mathbb{Z}$, and $f''(x)$ is the second derivative of $f(x)$.

Round 7 Babtha 7

Question 1

Sand is pouring out of a tube at 1 cubic meter per second. It forms a pile which has the shape of a right cone. The height of the cone is equal to the radius. How fast is the sandpile rising when it is 2 meters high? Give your answer in terms of π.

Question 2

$u_1, u_2, u_3, u_4, u_5, u_6, \ldots\ldots$ is a sequence where $u_1 = 4$ and $u_{n+1} = (-1)^n u_n + 20$. Find the value of u_{2024}.

Question 3

A is an obtuse angle such that:

$$\sin\left(A + \frac{\pi}{3}\right) + \sin\left(A - \frac{\pi}{3}\right) = \frac{4}{5}$$

Find the value of $\tan A$.

Question 4

A maths teacher has 7 pencils to give to four students and must give out all 7 pencils. The maths teacher can give none, or a mix of equal amounts and unequal amounts to the four students. In how many ways can the teacher do this?

Round 8 Babtha 8

Question 1

Let $y = \frac{1}{x} + \cos 2x$. Find the value of $n \in \mathbb{Z}$, such that:

$$\frac{d^5y}{dx^5} = -\frac{n!}{x^6} - 2^n \sin 2x.$$

Question 2

The National Lottery Daily Million is a draw game in which you can pick any six numbers from 1 to 39. At each draw six random numbers are drawn. If you match all 6 numbers, you win €1 million. It costs €1 to enter. You play the game once. What is your overall expected profit/loss, taking the entry fee into account? Give your answer correct to 4 decimal places.

Question 3

Let $f(x) = x^2 - 7x + 12$. Find the range of values of $x \in \mathbb{R}, x \neq 2, x \neq 3$, for which

$$\frac{f(x)}{f(x+1)} \geq 3$$

Question 4

What is the remainder when 2024^{2024} is divided by 9?

Answers

R1	Q1	22
	Q2	10
R2	Q1	$\frac{53}{160}$
	Q2	$-\frac{3}{2}$
R3	Q1	23
	Q2	$\frac{3r}{5}$
R4	Q1	$A = z_2, \quad B = z_3, \quad C = z_1, \quad D = z_5, \quad E = z_4$
	Q2	2179
R5	Q1	$53\pi\ m^2$
	Q2	8
R6	Q1	$5;\ -\frac{1}{3}$
	Q2	9
R7	Q1	$\frac{1}{4\pi}\ m/s$
	Q2	-16
	Q3	$-\frac{4}{3}$
	Q4	120
R8	Q1	5
	Q2	− €0.6935 or €0.6935 loss
	Q3	$1 \leq x < 2$
	Q4	1

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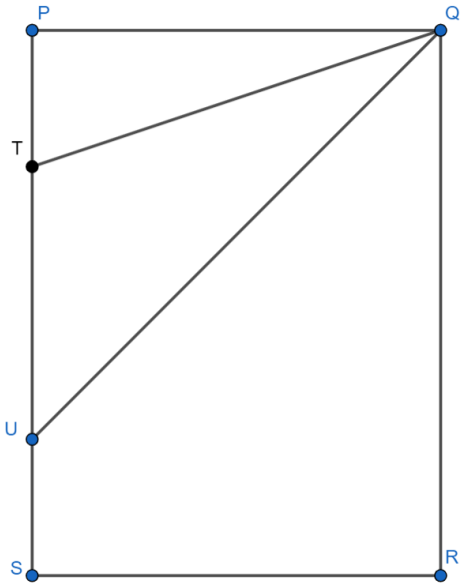
Round 1

Question 1

In rectangle $PQRS$, $|PQ| = 3$ and $|QR| = 4$. Points T and U are on side PS such that $|PT| = |US| = 1$.



Calculate the measure of $|\angle TQU|$, in degrees correct to 1 decimal place.

Question 2

Solve the equation,

$$\log_5(x-2) = 1 - \log_5(x-6), x \in \mathbb{R}, x > 6.$$

Round 2

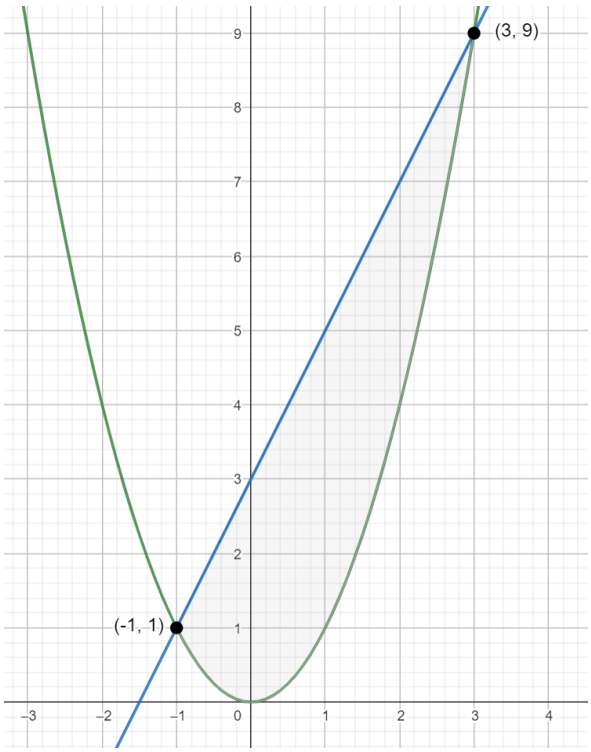
Question 1

The equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ has five solutions.

Determine all five values of x that satisfy the equation.

Question 2

The parabola $y = x^2$ meets the line $y = 2x + 3$ at the points $(-1,1)$ and $(3,9)$ as shown in the diagram



Find the area enclosed by the parabola and the line.

Answer in the form $\frac{a}{b}$, where $a, b \in N$.

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Round 3

Question 1

If $f(x) = e^{2x}(2x + 1)$, find $f'(0)$

Question 2

A family has four children, each with a different age. The product of their ages is 17,280. The sum of the ages of the three oldest children is 40 and the sum of the ages of the three youngest children is 32. Determine all possibilities for the ages of the four children.

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Round 4

Question 1

Find the real number a such that for all $x \neq 9$,

$$\frac{x-9}{\sqrt{x}-3} = \sqrt{x} + a.$$

Question 2

The curve C has equation $y = x\sqrt{lnx}$, $x > 0$.

The equation of the tangent to C at the point where $x = a$ is

$4y = bx - a$, where a and b are non zero constants.

Find the exact value of a.

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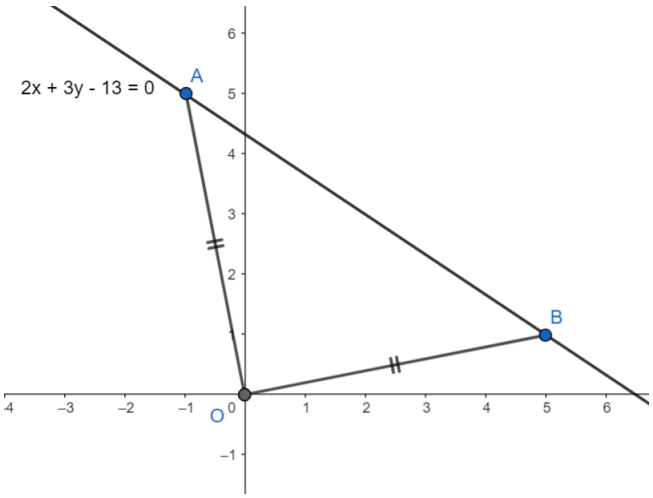
Round 5

Question 1

ΔOAB is an isosceles right-angled triangle with

- vertex O located at the origin; and
- vertices A and B located on the line $2x + 3y - 13 = 0$ such that $\angle AOB = 90°$ and $|OA| = |OB|$.



Calculate the area of ΔOAB.

Question 2

Solve the following trigonometric equation

$$\frac{\cos 2x}{1 + \cos 2x} = 1 - 2\tan x, \qquad 0 \leq x \leq 2\pi.$$

Write your answers in terms of π.

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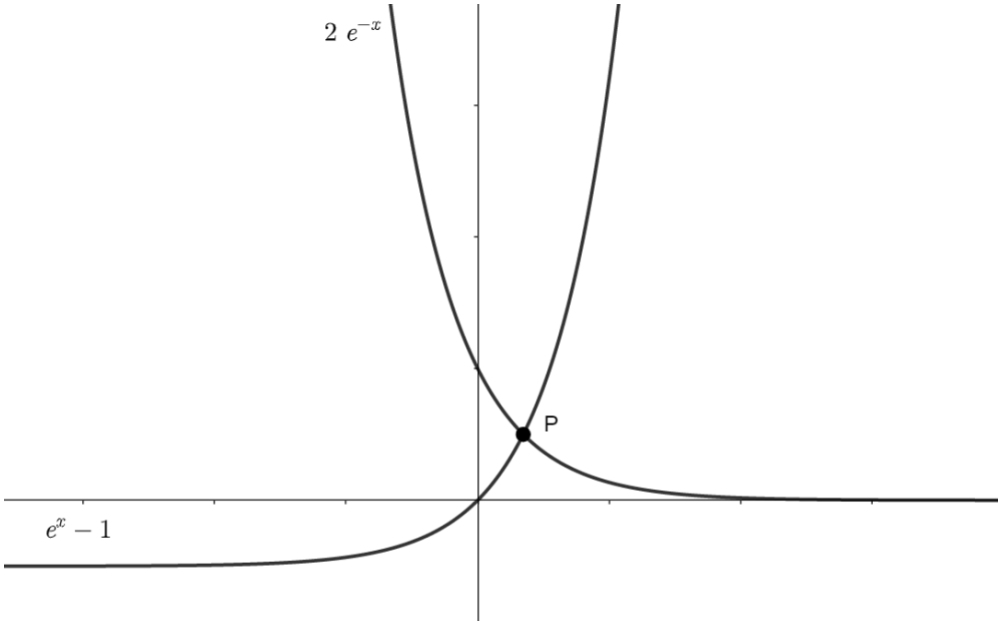
Round 6

Question 1

Given $y = ln\left(\frac{3+x}{\sqrt{9-x^2}}\right)$, find $\frac{dy}{dx}$ and express it in the form $\frac{a}{b-x^n}$

Question 2



The figure above shows the graphs of the curves with equations $y = 2e^{-x}$ and $y = e^x - 1$.

The two graphs intersect at the point P.

Calculate the exact coordinates of P.

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Round 7

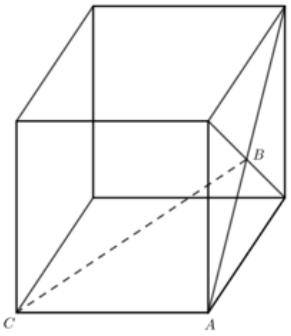
Question 1

A cylinder has a radius of $\left(\frac{1}{\sqrt{2}-1}\right)$cm and a height of $(\sqrt{2}+1)$cm.

Express the volume in the form $\pi(a+b\sqrt{c})$.

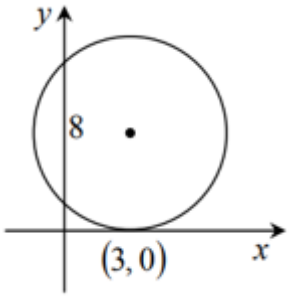
Question 2

Points A and C are vertices of a cube with side length 2 cm and B is the point of intersection of the diagonals of one face of the cube, as shown.



Find the length of $|CB|$.

Question 3



A circle has its centre in the first quadrant.

The $x-axis$ is a tangent to the circle at the point $(3,0)$. The circle cuts the $y-axis$ at points that are 8 units apart.

Find the equation of the circle in the form $(x-h)^2+(y-k)^2=j$, where $h,j,k \in N$.

Question 4

Five unbiased coins are tossed eight times. Find the probability of getting three heads and two tails exactly four times, correct to three decimal places.

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Round 8

Question 1

Find the value of the term which is independent of x in the expansion $\left(x^2 - \frac{1}{x}\right)^9$

Question 2

Solve the simultaneous equations

$$2^{x+2} + 3^y = 19$$

$$4^x - 2(3^y) = 10, \quad \text{for } x, y \in R.$$

Question 3

Find the exact value of $\tan 2(\sin^{-1}\frac{3}{5})$

Question 4

A group consists of 7 boys and 5 girls. If three of the group are picked at random, what is the probability that more girls than boys are picked? Answer as a fraction in simplest form.

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Tiebreak Round

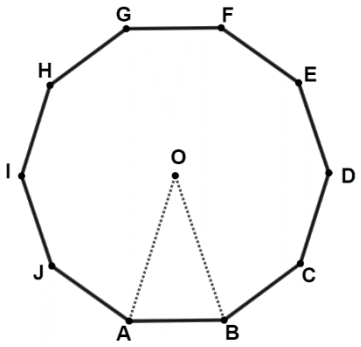
1 Find the range of value sof $t \in R$ for which the quadratic equation $(2t-1)x^2 + 5tx + 2t = 0$ has real roots.

2 Find all the solutions of the equation $\sin 4x + \sin 2x = 0, \quad 0° \leq x \leq 360°$. Answer in degrees.

3 If $y = \sqrt{\frac{x^2-1}{x^2+1}}$, find the value of $\frac{dy}{dx}$ when $x = \sqrt{3}$ and write your answer in the form $\sqrt{\frac{a}{b}}$, where $a, b \in N$.

4 The amounts due on monthly mobile phone bills in Ireland are normally distributed with mean €53 and standard deviation €15.
If a monthly phone bill is chosen at random, find the probability that the amount due is between €47 and €74, as a decimal to 3 decimal places.

5 The diagram shows a regular decagon. The decagon has centre O. The perimeter of the shape is $80\ cm$. By considering triangle OAB, calculate the area of the 10-sided shape, in square centimetres correct to two significant figures.



6 Calculate the distance from the point (– 3, 2) to the line 5x – 12y – 6 = 0, correct to 3 decimal places.

7 Two numbers have a sum of 8 and a product of 15. What is the sum of the reciprocals of these numbers?

8 Write $-\sqrt{3} - i$ in general polar form.

9 An unbiased six sided die is tossed three times. Given that the sum of the first two tosses equals the third, what is the probability that at least one 2 is tossed?

10 Find the altitude of the equilateral triangle whose area and perimeter have the same numerical value.

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Answers

Round 1	
1	$26.6°$
2	$x = 7$
Round 2	
1	$x = -10, 1, 2, 4, 6$
2	$\frac{32}{3}$ units sq
Round 3	
1	4
2	8, 9, 15, and 16
Round 4	
1	$x = 3$
2	$a = e^4$
Round 5	
1	13
2	$\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$
Round 6	
1	$\frac{3}{9 - x^2}$
2	$P(\ln 2, 1)$
Round 7	
1	$\pi(7 + 5\sqrt{2})$
2	$\sqrt{6}$
3	$(x-3)^2 + (y-5)^2 = 25$
4	0.149
Round 8	
1	84
2	$x = 2, y = 1$
3	$\frac{24}{7}$
4	$\frac{4}{11}$

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Tiebreak Answers	
1	$t \leq \frac{-8}{9}$ or $t \geq 0$
2	$0°, 60°, 90°, 120°, 180°, 240°, 270°, 300°, 360°$
3	$\sqrt{\frac{3}{32}}$
4	0.575
5	490
6	3.462
7	$\frac{8}{15}$
8	$2[\cos\left(\frac{7\pi}{6} + 2n\pi\right) + i\sin\left(\frac{7\pi}{6} + 2n\pi\right)$ or $2[\cos(210° + n360°) + i\sin(210° + n360°)$ **Accept**: $2[\cos(-150 + 360n) + i\sin(-150 + 360n)]$ or $2[\cos\left(\frac{-5\pi}{6} + 2n\pi\right) + i\sin\left(\frac{-5\pi}{6} + 2n\pi\right)]$
9	$\frac{8}{15}$
10	6

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Babhta 1 — Round 1

Q1.1 The first three terms of a geometric sequence are

$$2x - 4\ ,\ x + 1,\ x - 3.$$

Find the two possible values of x.

Q1.2 Given $y = mx + c$ is a tangent to the circle $x^2 + y^2 = a^2$, find an expression for c in terms of a and m.

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Babhta 2 — Round 2

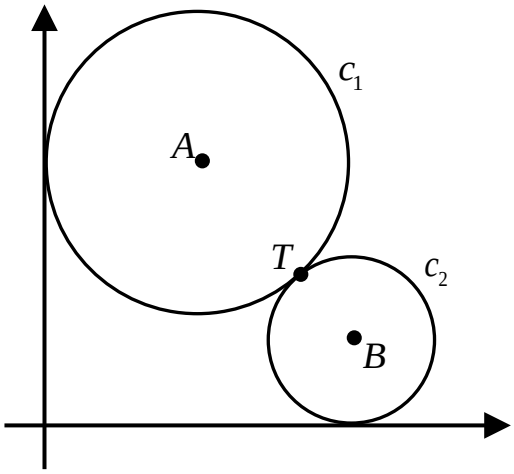
Q2.1 Solve for x, $2^{2x+1} - 15(2^x) - 8 = 0$.

Q2.2 Two circles c_1 and c_2 touch each other at the point T (9, 3·2) as shown in the diagram.

c_1 has radius 5 and c_2 has radius 2.

Find the coordinates of A and B, the centres of the circles.

Note that both c_1 and c_2 are on or above the x-axis.



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Babhta 3 Round 3

Q3.1 A person coughs when a foreign object is in their windpipe. The velocity of the cough depends on the size of the object. A certain person has a windpipe of radius 20 mm. If a foreign object has a radius of r mm, then the velocity, V mm/s, needed to remove the object by a cough is given by :

$$V(r) = k(20r^2 - r^3),$$

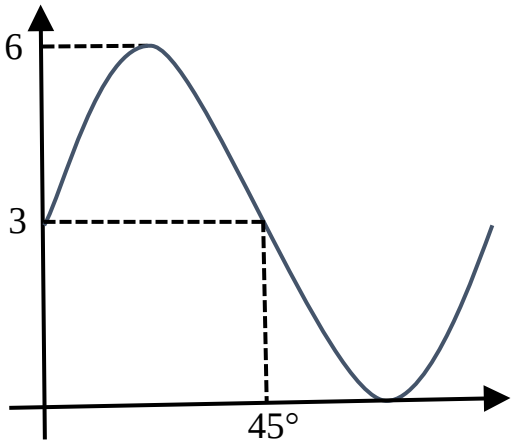
where $0 \le r \le 20,$ and k is a constant, where $k > 0$.

Find the radius of the foreign object that gives the maximum velocity needed to remove the object?

Q3.2 The graph shows the function

$$f(x) = c + a\sin bx.$$

Find the values of a, b and c.



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Babhta 4 — Round 4

Q4.1 A circle intersects a line at the points $A(-3, 0)$ and $B(5, -4)$.

The midpoint of $[AB]$ is m.

The distance from the centre of the circle to m is $\sqrt{5}$.

Find the equations of the two circles that satisfy these conditions.

Q4.2 The speeds of 150 randomly selected cars were recorded as they passed a check-point on a motorway. The mean speed of the cars was 115 km/h and the standard deviation was 24 km/h.

Find the 95% confidence interval for the mean speed of cars passing the check-point correct to one place of decimals.

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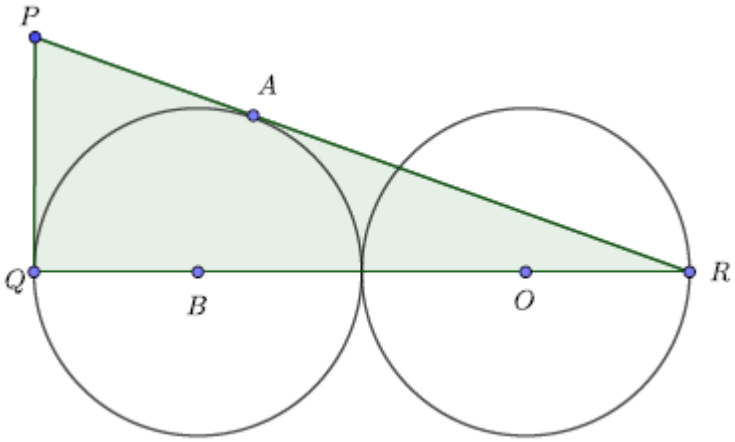
Babhta 5 Round 5

Q5.1 Find the number of ways in which 10 different books can be given to Andrew, Breda, Colin and Deirdre, if Andrew is to receive 4 books, Breda 3 books, Colin 2 books and Deirdre 1 book.

Q5.2 Two circles, with centres O and B and each with a radius of 2, are tangent to each other. A straight line is drawn through O and B meeting the circles at Q and R. Two other sides of ΔPQR are drawn such that side $[PR]$ is tangent to the circle with centre B at A and side $[PQ]$ is tangent to the circle with centre B at Q.

Determine the length of $[PQ]$. Give your answer in the form $a\sqrt{2}$.



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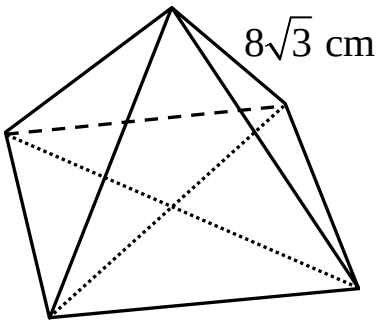
Babhta 6 — Round 6

Q6.1 A group of Transition Year students was given the task of making a paper model of a pyramid.

Each of the slant edges has a length of $8\sqrt{3}$ cm and the length of the square base is twice the height of the pyramid.

Find the height of the model.



Q6.2 $h(x) = 3x - 5$ and $k(x) = x - 2$ are two functions where $x \in \mathbb{R}$.
Solve for x, $h^{-1} \circ k(x) = k^{-1} \circ h(x)$.

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Babhta 7 — Round 7

Q7.1 Find $\frac{dy}{dx}$, given $y = xe^x - e^x$.

Q7.2 Solve for k: $\frac{k!}{(k-2)!} = 30$, where $k \in \mathbb{N}$.

Q7.3 Find the equation of the tangent to the curve $y = e^{\frac{1-x}{1+x}}$ at the point (1, 1). Give your answer in the form $ax + by + c = 0$.

Q7.4 At a meeting there are 7 students, 3 teachers and 11 parents. Two people are chosen at random. Find the probability that the first person chosen was a teacher, given that the second person chosen was a teacher.

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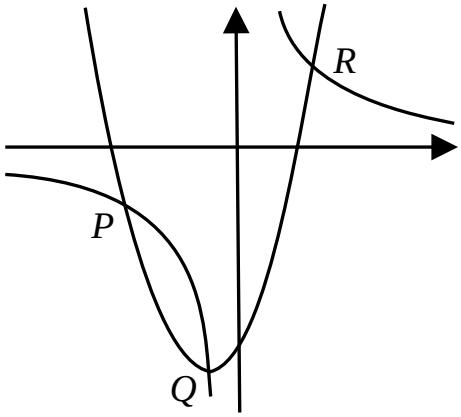
Babhta 8 — Round 8

Q8.1 The diagram shows the graphs of the functions $f(x) = x^2 + 2x - 11$ and $g(x) = \frac{12}{x}, x \neq 0$ and $x \in \mathbb{R}$.
Find the coordinates of *P*, *Q* and *R*, the points of intersection of *f* and *g*.



Q8.2 Suppose that $\omega^3 = 1, \omega \neq 1,$ and k is a positive integer. Find the two possible values of $1 + \omega^k + \omega^{2k}$ which belong to $\mathbb{Z}$.

Q8.3 Solve $5\sin^2\theta - 3\sin\theta\cos\theta - 2\cos^2\theta = 0,$ for $0 < \theta < 360°$.
Give your answers to the nearest degree.

Q8.4 Find the value of $k > 0$ for which $\int_1^k (2x+3)dx = 6$.

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Tie-break Final 2022

T1 Find the value of k if $k(x^2+2y^2)+(y-2x+1)(y+2x+3)=0$ represents a circle.

T2 The third term in the expansion of $(x+k)^8$ is $252x^6$.
Find the possible values of k.

T3 Solve for x and y, $3x+2y=1$ and $x^2+2xy+15=0$.

T4 Given that $2^{x+1}-5^y=131$ and $2^{x-4}+5^{y-2}=13$, find the value of x and the value of y, where $x, y \in \mathbb{Z}$.

T5 Mary takes two tests in general knowledge. The first test, test A, has a pass rate of 60%. The second test is Test B. The probability that a candidate passes both tests is 0·4.
Find the probability that Mary fails both tests.

T6 If $y=\sqrt{x^2-2x-3}$, where $x \in \mathbb{R}$, find the range of values for x for which $y \in \mathbb{R}$.

T7 Solve for x, $\log_2 8+\log_5\left(\frac{1}{25}\right)+\log_9 3=\log_{16} x$.

T8 The lengths of the sides of a triangle are 2 cm, 3 cm, and 4 cm, respectively. Find the measure of the largest angle correct to the nearest degree.

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Team Maths Final 2022

Answer Key Final 2022

Q1.1 $x = 1, 11$

Q1.2 $c = \pm a\sqrt{1+m^2}$

Q2.1 $x = 3$

Q2.2 $A(5,6\cdot 2)\quad B\ (10\cdot 6, 2)$

Q3.1 $\frac{40}{3}$mm

Q3.2 $a = 3, b = 4, c = 3$

Q4.1 $x^2 + (y+4)^2 = 25$ or $x^2 + y^2 + 16y - 9 = 0$
$(x-2)^2 + y^2 = 25$ or $x^2 + y^2 - 4x - 21 = 0$

Q4.2 [111·2, 118·8]

Q5.1 12,600

Q5.2 $2\sqrt{2}$

Q6.1 8 cm

Q6.2 $x = 1\cdot 5$

Foireann Mata 2022 : an Chraobh
Team Maths Final 2022

Q7.1 $\frac{dy}{dx} = xe^x$

Q7.2 6

Q7.3 $x + 2y - 3 = 0$

Q7.4 $\frac{1}{10}$

Q8.1 $P(-4, -3)$, $Q(-1, -12)$, $R(3, 4)$

Q8.2 0 and 3

Q8.3 $45°, 158°, 225°, 338°$

Q8.4 2

Foireann Mata 2022 : an Chraobh
Team Maths Final 2022

Tie-break

T1 $k = -5$

T2 $k = \pm 3$

T3 $(3,-4)$ and $\left(-\frac{5}{2},\frac{17}{4}\right)$

T4 $x = 7, y = 3$

T5 $\frac{2}{15}$

T6 $x \leq -1$ or $x \geq 3$

T7 64

T8 104°

Foireann Mata 2019 : an Chraobh
(Team Maths Final 2019)

Babhta 1 Round 1

Q1.1 The line $12x + 5y = 60$ forms a triangle with the x and y axes. Calculate the sum of the lengths of the altitudes of the triangle?

Answer in the form $\frac{a}{b}$, where a and b $\in$ N in lowest terms.

Q1.2 Given that $\text{Tan}(A) + \text{Tan}(B) = a$
$\text{Cot}(A) + \text{Cot}(B) = b$

and $\text{Tan}(A + B) = c$

Write c in terms of a and b.

Answer in simplest fractional form in terms of a and b.

Foireann Mata 2019 : an Chraobh
(Team Maths Final 2019)

Babhta 2 Round 2

Q2.1 P, Q and R are the points (3, 4) , (7, -2) and (- 2 , -1) respectively. Find the equation of the median through R of the triangle PQR.

Answer in form ax + by + c = 0 , where a , b and c $\in$ Z.

Q2.2 Find the slope of the tangent to the curve $y = x(x - 2)^2 + 5$ at the point (3,8).

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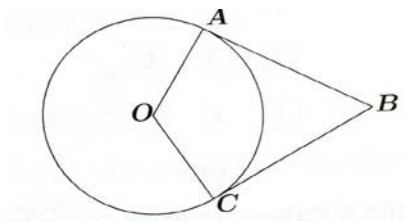
Babhta 3 Round 3

Q3.1 The triangle ABC is equilateral with sides tangent to the circle with centre O and radius $\sqrt{3}$.
Find the area of the quadrilateral ABCO.
Answer in simplest surd form



Q3.2 f is a function such that f(f(x)) = f (x + 2) – 3
for all integers x.
Given that f(1) = 4 and f(4) = 3 calculate the value of f(5) ?

Foireann Mata 2019 : an Chraobh
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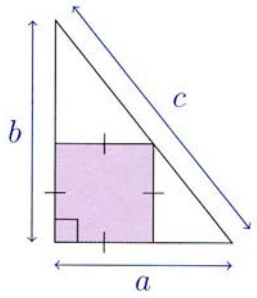
Babhta 4 Round 4

Q4.1 A square is drawn in the corner of a right angled triangle with sides of lengths a ,b and c as shown.

Write the ratio of the unshaded area to the shaded area as a fraction, in simplest form, in terms of a,b and c.



Q4.2 A sequence of fractions obeys the following rule : given any two successive terms **p** and **q** of the sequence, the next term is obtained by dividing their product **p.q** by their sum **p + q**.

Given that the first two terms are $\frac{1}{2}$ and $\frac{1}{3}$, find the value of the tenth term.

Answer in form $\frac{a}{b}$, where a and b $\in$ N in lowest terms.

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Babhta 5 Round 5

Q5.1 A fair die is rolled six times. Find the probability of rolling at least a five at least five times.

Answer in the form $\frac{a}{b}$, where a and b $\in$ N in lowest terms.

Q5.2 Given that

$x^3 + y^3 = 5$ and

$x^2 + y^2 = 3$, find possible values of x + y.

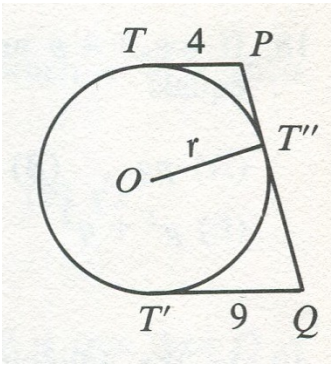
Answers in non-decimal form.

Foireann Mata 2019 : an Chraobh
(Team Maths Final 2019)

Babhta 6 Round 6

Q6.1 A certain number consists of three digits, which are in arithmetical progression.
The number divided by the sum of its digits is equal to 26, but if 198 is added to the number the digits are reversed.
What is the number?

Q6.2 In the diagram TP and $T'Q$ are parallel tangents to a circle of radius r, where T and T' are the points of tangency.
$PT''Q$ is a third tangent with T''as point of tangency.



If |TP| = 4 cm and |T'Q| = 9cm , find the length of the radius, r.

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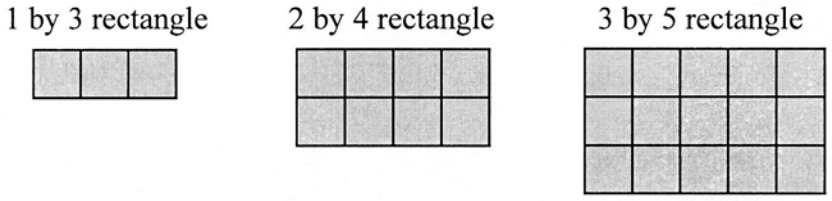
Babhta 7 Round 7

Q7.1 A unit square is a square whose sides are 1 unit long.
Rectangles can be formed by joining unit squares.
For the three examples shown below, their perimeters are, respectively, 8 units, 12 units, and 16 units.
The number of line segments of unit length that are **NOT** part of their perimeters are, respectively 2, 10 , and 22.



If a 50 by 100 rectangle is formed with unit squares, how many line segments would **NOT** be part of the perimeter?

Q7.2 The Binomial $(x+y)^9$ is expanded in decreasing powers of x.
The second and third terms have equal values when evaluated at x = p and y = q, where p and q are positive numbers whose sum is one.
Find the value of p.

Answer in simplest form $\frac{a}{b}$, where a and b $\in$ N.

Q7.3 At the start of the year Joan borrowed €25 000 at an APR of 8%. She wishes to repay in twelve equal instalments over twelve years, with the first repayment one year after she takes out the loan.
Calculate, to the nearest euro, the amount of each repayment.

Q7.4 Given that (2 + 3i) is a root of $3z^3 - 13z^2 + 43z - 13 = 0$, find the other two roots.

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Babhta 8 Round 8

Q8.1 Simplify

$$\frac{1}{2\sqrt{1}+1\sqrt{2}}+\frac{1}{3\sqrt{2}+2\sqrt{3}}+\frac{1}{4\sqrt{3}+3\sqrt{4}}+.....+\frac{1}{100\sqrt{99}+99\sqrt{100}}$$

Answer in the form $\frac{a}{b}$, where a and b $\in$ N in lowest terms.

Q8.2) In a popular TV game if a contestant spins €1 on the Big Wheel, she wins €1000 and gets 2 bonus spins in which she has a $\frac{1}{20}$ chance of winning an additional €10 000 and a $\frac{1}{10}$ chance of winning an additional €5 000. Given that the contestant has a $\frac{1}{10}$ chance of spinning €1, what is the expected value of the contestant's winnings?

Q8.3 Two positive integers p and q satisfy the equation $p^2q = 2016$.

How many different values of p are possible?

Q8.4 Solve for x and y : $2x + 3y = 2$ and $2x^2 + 3xy - 3y^2 = \frac{2}{3}$

Answer in the form (x,y), where x and y $\in$ Q in lowest terms.

Foireann Mata 2019 : an Chraobh
(Team Maths Final 2019)

Tiebreak Final 19

T1) Solve the equation $2019x^2 - 2019^{2019} = 0$.

Answers in form a^b, a $\in$ Z and b $\in$ N.

T2) What is the probability that a five card poker hand contains exactly two aces?

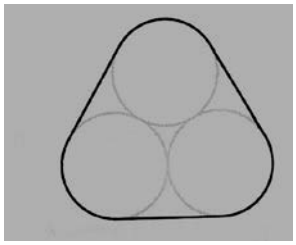
Answer correct to 5 decimal places

T3) Write in the form ax + by + c = 0 when x = $\frac{3 - t}{1 + t}$ and y = $\frac{5t}{1+ t}$,
where a, b and c $\in$ Z.

T4) Find the values of x $\in$ R for which $|x^2 - 3x| = 4$.

T5) Three cylindrical pipes of radius 1 meter are held together by a tight metal band as shown.
The length of the metal band is a + $b\pi$, where a and b $\in$ N.

Find the value of a + b.



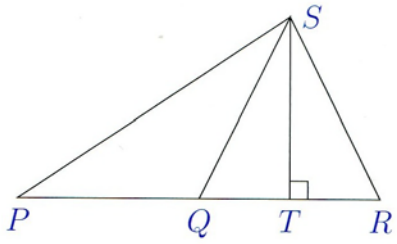
T6) Calculate the sum of the units digits of the sum $26^{26} + 33^{33} + 45^{45}$.

T7) Find the value of $\frac{1}{\log_2(36)} + \frac{1}{\log_3(36)}$ in simplest form.

T8) In the triangle |PQ| = |SQ| = |SR| = |QR|
and |<STR| = 90^0.

Calculate the ratio |PS| : |TR| .

Answer in form $a\sqrt{b}$: 1, a, b $\in$ N



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ANSWER KEY FINAL 19

Q1.1	$\frac{281}{13}$	Q1.2	$\frac{ab}{b-a}$
Q2.1	$2x - 7y - 3 = 0$	Q2.2	7
Q3.1	$3\sqrt{3}$	Q3.2	12
Q4.1	$\frac{c^2}{2ab}$	Q4.2	$\frac{1}{144}$
Q5.1	$\frac{13}{729}$	Q5.2	2 and $-1 \pm \sqrt{6}$
Q6.1	234	Q6.2	6
Q7.1	9850	Q7.2	$\frac{4}{5}$
Q7.3	€3317	Q7.4	$\frac{1}{3},\ 2 \pm 3i$
Q8.1	$\frac{9}{10}$	Q8.2	€200
Q8.3	6	Q8.4	$(\frac{1}{2},\frac{1}{3}), (3,-\frac{4}{3})$

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TIEBREAK

T1	$\pm 2019^{1009}$	T2	0.03993
T3	5x +4y - 15 = 0	T4	4 AND – 1
T5	8	T6	14
T7	$\frac{1}{2}$	T8	$2\sqrt{3} : 1$

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Babhta 1 Round 1

Q1.1) A triangle has angles of 30^0 and 45^0. If the side opposite the 45^0 angle has length 8 cm . Calculate the length of the side opposite the 30^0 angle.

Answer in form $a\sqrt{b}$, where a and b ∈ N

Q1.2) Find the equations of the tangents to the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ which are parallel to the line $4x + 3y + 5 = 0$.

Answers in form $ax + by + c = 0$, where a, b and c ∈ Z

Babhta 2 Round 2

Q2.1) For x ĩ R , find the range of values for which
$1 \leq |x - 2| \leq 7$

Q2.2) An equilateral triangle is drawn with side length **a**. A new equilateral triangle is formed by joining the midpoints of the sides of the first one. Then a third equilateral triangle is formed by joining the midpoints of the sides of the second one; and so on forever. In terms of **a** what is the limit of the sum of the perimeters of all the triangles drawn?

Babhta 3 Round 3

Q3.1) Solve for x and y ∈ N : $2^{2x} - 3^{2y} = 55$.

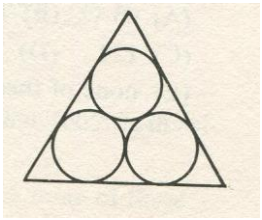
Answer in form (x, y) .

Q3.2) If $\frac{x}{a-b} = \frac{y}{b-c} = \frac{z}{c-a}$, find the numerical value of $x + y + z$, where a , b and c are distinct real numbers.

Babhta 4 Round 4

Q4.1) An open box with volume $24\,cm^3$ is to be made from a square piece of tin by cutting 2 cm from each corner and turning up the sides.
Calculate the length of the side of the square piece required.
Answer correct to 3 decimal places.

Q4.2) Each of the three circles in the adjoining figure is externally tangent to the other two , and each side of the triangle is tangent to two of the circles.
If each circle has a radius 3 cm, calculate the perimeter of the triangle.



Answer in simplest surd form.

Babhta 5 Round 5

Q5.1) Given $g(x) = 1 - x^2$ and $f(g(x)) = \frac{1-x^2}{x^2}$ when $x \neq 0$, find the numerical value of $f(\frac{1}{2})$

Q5.2) A set of consecutive positive integers beginning with 1 is written on a blackboard.
One number is erased.
The average of the remaining numbers is $35\frac{7}{17}$.
How many numbers were in the set and what number was erased?

Babhta 6 Round 6

Q6.1) Find **ALL** the values of x , y and z which satisfy the following equations:

$$y + z = yz - 1$$
$$z + x = zx - 2$$
$$x + y = xy - 5$$

Q6.2) The lengths of the sides of a triangle are consecutive integers, and the largest angle is twice the smallest angle.
Find the cosine of the smallest angle.

Answer in the form $\frac{a}{b}$, where a , b ∈ N.

Babhta 7 Round 7

Q7.1) Given that $\frac{3x^2 - 10x - 2}{(x-1)(x-2)(2x+1)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{2x+1}$

find the numerical value A + B + C.

Q7.2) Solve for x : $\sqrt{x^2 - 7x + 10} - \sqrt{x^2 + x - 6} = x - 2$

Q7.3) Given $0^0 \leq x \leq 90^0$ find all the values of x for which
Sin(7x)Cos(2x) =Sin(2x)Cos(3x)

Q7.4) Find the values of m for which the quadratic equation

$12(m + 2)x^2 - 12(2m - 1)x - 38m - 11 = 0$

will have equal roots.

Babhta 8 Round 8

Q8.1) Two identical jars are filled with alcohol solutions, the volume of alcohol to the volume of water being p : 1 in one jar and q : 1 in the other jar. If the entire contents of the two jars are mixed together, calculate the ratio of the volume of alcohol to the volume of water in the mixture.
Answer as a fraction in terms of p and q.

Q8.2) Given that $i^2 = -1$, for how many integers **n** is $(n+i)^4$ an integer?

Q8.3) Calculate, in terms of n, the sum of the first n terms of the series

$$1 + (1 + 2) + (1 + 2 + 4) + (1 + 2 + 4 + 8) + \ldots\ldots + (1 + 2 + 4 \ldots\ldots + 2^{n-1})$$

Answer in terms of n in simplest form, without brackets.

Q8.4) A box contains two 1 cent coins, four 5 cent coins and six 10 cent coins.
Six coins are drawn without replacement, each coin having an equal probability of being chosen.
What is the probability that the value of the coins drawn is at least 50 cents?

Answer in simplest form $\frac{a}{b}$, where a and b $\in$ N.

Craobhchomórtas Foireann Mata 2018
Team Maths Final 2018

Tiebreak

T1 Find ALL solutions to the equations $\text{Cos}^2(x) = \frac{3}{4}$, in the range $-90^0 < x < 180^0$.
Answers in degrees.

T2 Find the range of values of $x \in \mathbf{R}$ for which

$$\frac{3x}{2x - 3} > 6.$$

T 3 If p leaves a remainder of 3 when divided by 5 and q leaves a remainder of 2 when divided by 5, what is the remainder when the product pq is divided by 5? p and q are positive integers.

T4 Given that a circle and a square are of equal area calculate the ratio of the circumference of the circle to the perimeter of the square.
Answer, as a percentage, correct to one decimal place .

T5 Toy cars A, B , C, D and E are arranged in order of increasing height with A being the smallest and E being the largest. The heights are all consecutive even integers and, when added their sum is 450.

Find the value of $\frac{2E - 3C + A}{4B - 3D}$.

Answer in simplest form $\frac{a}{b}$, where a and $b \in N$.

T6 Find all the values of $x \in R$ where $(2x - 5)^3 + 6(2x - 5) = 5(2x - 5)^2$.

Answers in simplest form $\frac{a}{b}$, where a and $b \in N$.

T7 Two angles in a triangle measure 25^0 and 85^0. The longest side measures 10 cm. Find the length of the shortest side correct to two decimal places.

T8 Express $\frac{5}{2 - \sqrt{3}}$ in the form $a(b + \sqrt{c})$ where a , b and $c \in \mathbf{N}$.

T9 The points (1 , -1) , (a , 2) and (b , 1) are collinear .
Express a in terms of b.

T10 Solve for x : $27^{2x-1} = 81^{x+2}$.

Answer in the form $\frac{a}{b}$, where a and b $\in$ N.

T11 A three-digit integer contains one of each of the digits 2 , 6 and 5.
What is the probability that the integer is divisible by 5?

Answer in simplest form $\frac{a}{b}$, where a and b $\in$ N.

T12 How many integer values of k are there such that the graphs of $y = x^2$ and $y = kx - 1$ do not intersect?

T13 If the graph of the equation $y = (x + 2)^2$ is reflected in the y-axis, what is the equation of the resulting graph?
Answer in form $y = ax^2 + bx + c$, where a , b , and c $\in$ Z.

T14 Find the perpendicular distance from the point $(4, -3)$ to the line $y = -2x - 4$.

Answer in form $\frac{a\sqrt{b}}{b}$,where a and b $\in$ N.

ANSWER KEY : TEAM MATHS FINAL 2018

Round 1

Q1.1 $4\sqrt{2}$ Q1.2 $4x + 3y + 19 = 0$ and $4x + 3y - 31 = 0$

Round 2

Q2.1 $-5 \le x \le 1$ and $3 \le x £ 9$ Q2.2 6a

Round 3

Q3.1 (3 , 1) Q3.2 0

Round 4

Q4.1 7.464 Q4.2 $18\sqrt{3}+18$

Round 5

Q5.1 1 Q5.2 69 Numbers , 7 erased.

Round 6

Q6.1 x = 4 or -2 , y = 3 or -1, z = 2 or 0 Q6.2 $\frac{3}{4}$

Round 7

Q7.1 2 Q.2 2

Q7.3 $0^0, 36^0, 72^0, 22.5^0, 67.5^0.$ Q7.4 $-\frac{1}{2}$ OR -1

Round 8

Q8.1 $\frac{p + q + 2pq}{p + q + 2}$ Q8.2 3 Q8.3 $2^{n+1} - n - 2$ Q8.4 $\frac{127}{924}$

Babhta 1 Round 1

Q1.1 In how many ways can the letters of the word **COURAGE** be arranged, if the vowels are always to occupy the odd positions.
Answer in non-factorial form.

Q1.2 The inscribed circle of a triangle ABC touches [BC], [CA] and [AB] at X , Y and Z respectively.
|BC| = a cm, |CA| = b cm , and |AB| = c cm,
|AY| = x cm, |BZ| = y cm and |CX| = z cm.
Write x in terms of a , b and c.

Babhta 2 Round 2

Q2.1 Find the equations of the lines which pass through the point (- 2, 1) and make an angle of 45^0 with the line 3x + y + 5 =0.
Answers in form ax + by + c = o ,where a , b and c $\in Z$

Q2.2 x and y are integers such that $x^2+y^2=29$.
Calculate the maximum value of |x - y|

Babhta 3 Round 3

Q3.1 Find the coordinates of the image of the point (6, 0) after a 30^0 rotation anticlockwise about the origin.

Answer as a coordinate in simplest surd form.

Q3.2 f is a function and $f(x) = \frac{6x + 1}{3x - 2}$

Find the inverse function, $f^{-1}(x)$ in terms of x.

Answer in simplest form.

Babhta 4 Round 4

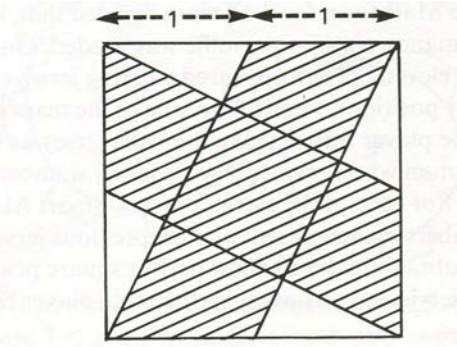
Q4.1 A man borrowed €20000 at 5% compound interest. The principal and interest are to be repaid in 20 equal annual instalments, with the first repayment one year after he takes out the loan. Calculate the amount of each instalment.
Answer to nearest euro.

Q4.2 If $z = \text{Cos}(\theta)+i\text{Sin}(\theta)$, write $\dfrac{1-\dfrac{1}{z^2}}{1+\dfrac{1}{z^2}}$

in simplest trig. form

Babhta 5 Round 5

Q5.1



In the diagram the square has a side 2 cm long, Line segments are drawn joining its vertices to the midpoints of the sides as shown in the diagram to form a cross. Find the area of the shaded cross.

Answer in form $\frac{\mathbf{a}}{\mathbf{b}}$, where a and b $\in N$.

Q5.2 Jane and Sarah each throw a dice.
What is the probability that Jane's throw is higher than Sarah's?

Answer in simplest form $\frac{\mathbf{a}}{\mathbf{b}}$, where a and b $\in N$.

Babhta 6 Round 6

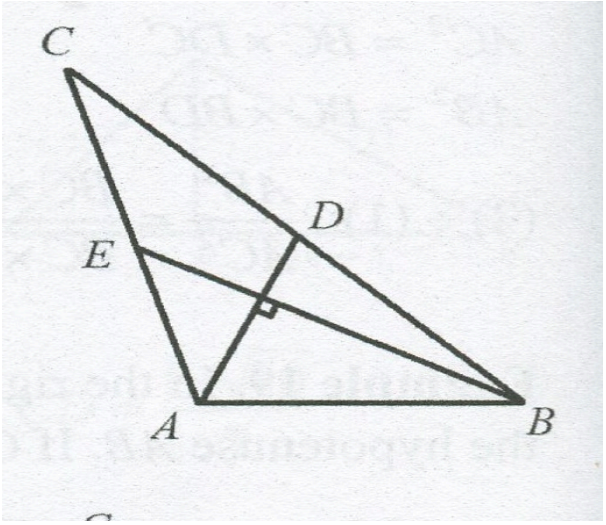
Q6.1 When a two-digit number is added to another two-digit number which has the same digits in reverse, the sum is a perfect square.
Find the sum of all such two-digit numbers less than 50.

Q6.2 Let f(x) = $\log_b(x)$ and let g(x) = $x^2 - 4x + 4$.
Given that f(g(x)) = g(f(x)) = 0 has exactly one solution and b > 1, find the value of b.

Answer in form $\sqrt{a}$, where a $\in N$.

Babhta 7 Round 7

Q7.1 In the triangle ABC , |BC| = 30 , |AC| = 20. [AD] and [BE] are two medians as shown in diagram and [AD] is perpendicular to [BE].
Find the length of [AB].
Answer in form $a\sqrt{b}$, where a and b $\in N$.



Q7.2 The numbers 49, 29, 9 , 40 , 22 , 15 , 53 , 33, 13 , 47 are grouped in pairs so that the sum of each pair is the same. Which number is paired with 15?

Q7.3 Five lamps A , B , C , D , E, are arranged in a row, each with its own switch. Initially, all five lamps are off. Jill starts pressing switches in the following order :
A, B, C, D, E, A, B, C, D, E, A, ……until she has pressed 2017 switches, counting repetitions. Assuming that each press turns a lamp off if it is on, or on if it is off , which lamps are finally on?

Q7.4 Calculate the value of the constant a if the coefficient of x^3 in the expansion $(a+2x)^4$ is 160.

Babhta 8 Round 8

Q8.1 The points A(-3 , - 9) and B(9 , -4) are the endpoints of a diameter of a circle.
This circle touches the x-axis at T.
Find the ratio of the length of [TA] to the length of [TB].

Answer in simplest form a : b, where a and b $\in$ N.

Q8.2 What is the product of the positive integers m and n such that m > n and

$$\frac{1}{m} + \frac{1}{n} + \frac{1}{mn} = 1?$$

Q8.3 An express train, travelling at uniform speed, leaves station A at 3 pm and reaches station B at 6 pm. A slower train, travelling at uniform speed, leaves station B at 1.30 pm and arrives at station A at 6pm.
At what time do they meet?

Q8.4 Solve for x and y : $x^3 + y^3 = 91$

$$x^2y + xy^2 = 84$$

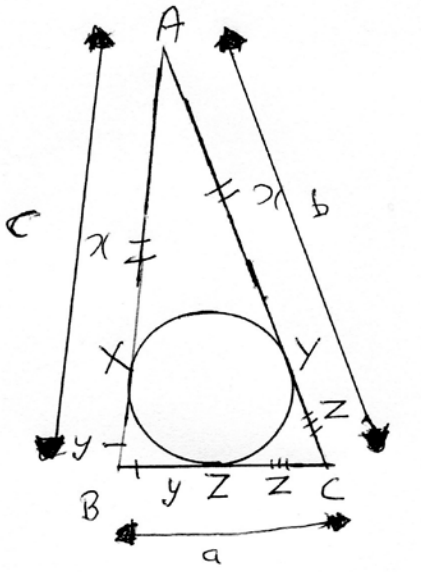
Answer in coordinate form (x , y).

TEAM Maths : Answer key final17

Round 1	Q1.1	144	Q1.2	x= $\frac{1}{2}(b + c - a)$
Round 2	Q2.1	x + 2y =0 and 2x – y + 5 =0	Q2.2	7
Round 3	Q3.1	$(3\sqrt{3}, 3)$	Q3.2	$\frac{2x + 1}{3x - 6}$
Round 4	Q4.1	€1605	Q4.2	iTan(θ)
Round 5	Q5.1	$\frac{16}{5}$ cm^2	Q5.2	$\frac{5}{12}$
Round 6	Q6.1	114	Q6.2	$\sqrt{3}$
Round 7	Q7.1	$2\sqrt{65}$	Q7.2	47
	Q7.3	C, D and E	Q7.4	5
Round 8	Q8.1	3 : 2	Q8.2	mn =6
	Q8.3	4 : 12 PM	Q8.4	(3 , 4) and (4 , 3)

Round 1

Q1.1 COURAGE has 7 letters: 4 vowels and 3 consonants
Vowels can occupy the odd positions in 4! ways.
Consonants can occupy the even positions in 3! ways.
Number of arrangements = 4! x 3! = 144 .



Q1.2 |AX| = |AY| = x, |BX| = |BZ| = y, |CY| = |CZ| = z

2x + 2y + 2z = a + b +c, a = y + z,
2x + 2(y + z) = a + b + c ,
2x + 2 a = a + b + c , x = $\frac{1}{2}(b + c - a)$

Round 2

Q2.1 P(- 2 , 1) , slope = tan (45 ^{0}) = 1

Any line through (- 2 , 1) has form , $\mathbf{l}$: y – 1 =m(x + 2).

3x + y + 5 = 0 has slope -3.

Tan (45 ^{0}) = $\pm\left(\frac{m+3}{1-3m}\right) = 1$, m + 3 = 1 – 3m, m = $-\frac{1}{2}$.

Or m +3 = -1 +3m , m = 2

Lines are x + 2y = 0 and 2x – y + 5 = 0.

Q2.2 $x^2 + y^2 = 29 \Rightarrow$ eg $x = \pm 5, y = \pm 2$.

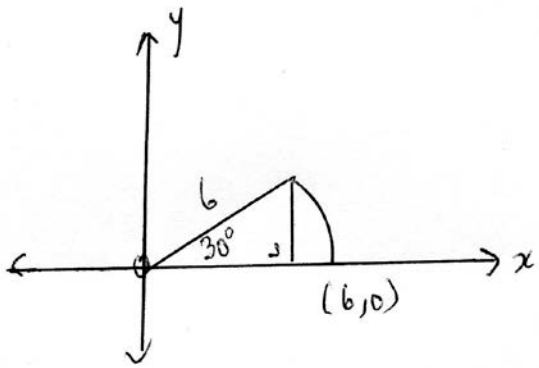
For max value take opposite signs. Max = |5 – (-2)| = |-5 -2| = 7

Round 3

Q3.1 $\mathrm{Sin}(30^0) = \frac{1}{2} = \frac{y}{6}$, y = 3

$\mathrm{Cos}(30^0) = \frac{\sqrt{3}}{2} = \frac{x}{6}$, x = $3\sqrt{3}$.

Point is $(3\sqrt{3}, 3)$



Q3.2 $f(x) = y = \dfrac{6x+1}{3x-2}$, $6x + 1 = 3xy - 2y$, $x = \dfrac{-2y-1}{6-3y} = \dfrac{2y+1}{3y-6}$.

$$f^{-1}(x) = \frac{2x+1}{3x-6}.$$

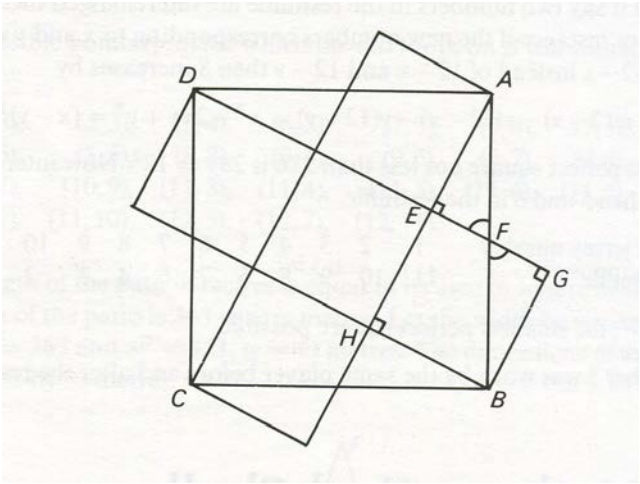
Round 4

Q4.1 A $= €\dfrac{20000(1.05)^{20}(.05)}{(1.05)^{20}-1}$ = €1605 to nearest euro.

Q4.2 $\dfrac{1-\frac{1}{z^2}}{1+\frac{1}{z^2}} = \dfrac{\frac{1}{z}(z-\frac{1}{z})}{\frac{1}{z}(z+\frac{1}{z})} = \dfrac{z-\frac{1}{z}}{z+\frac{1}{z}} = \dfrac{2iSin\theta}{2Cos\theta} = i\tan\theta$

Round 5

Q5.1



Extend the cross as shown above. The triangles AEF and BGF are congruent. So BG = AE = BH and EGBH is a square. Also EF = GF = $\frac{1}{2}$EG.

Since the areas of triangles AEF and BGF are equal, we see that the area of the five small squares forming a cross equals the area of square ABCD, which is 4, and so each square has area $\frac{4}{5}$. Now triangle AEF has area $\frac{1}{4}$ of the area of a small square and so the four unshaded triangles in the original figure have total area $\frac{4}{5}$. Thus the area of the shaded cross is $4-\frac{4}{5} = \frac{16}{5}$.

Q5.2 Total number of outcomes = 6 x 6 = 36

	Jane	Sarah
Throw	2, 3,4, 5, 6	1
Throw	3,4,5,6	1,2
Throw	4,5,6	1,2,3
Throw	5,6	1,2,3,4
Throw	6	1,2,3,4,5

Number of outcomes where Jane's score is greater than Sarah's
= 5 + 4 + 3 + 2 +1 = 15

p(Jane > Sarah) = $\frac{15}{36} = \frac{5}{12}$

Round 6

Q6.1 Let the number 10a + b.

So 10a + b + 10b + a = $\mathbf{n^2}$, for some n. , 11(a + b) = $\mathbf{n^2}$

For a perfect square a + b = 11.

There are 8 possible pairs (2,9) , (9, 2). (8,3). (3,8), (4,7), (7,4), (6,5), (5,6).

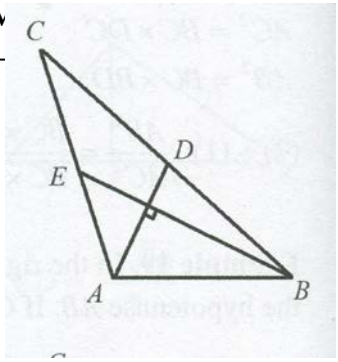
Sum of numbers < 50 is 29 + 38 + 47 = 114

Q6.2 f(x) = $\log_b(x)$, g(x) = $x^2 - 4x + 4 = (x - 2)^2$

f(g(x)) = 0. So $f(x - 2)^2 = \log_b(x - 2)^2 = 2\log_b(x - 2) = 0$
x -2 = $b^0 = 1$, x = 3

g(f(x)) = 0. So g($\log_b(x)$) = $(\log_b(x) - 2)^2 = 0$

$\log_b(x) = 2$, $x = b^2 = 3$, $b = +\sqrt{3}$



Round 7

Q7.1

By Pythagoras theorem

$4m^2+n^2=15^2,\ 4n^2+m^2=10^2=100,$ $5m^2+5n^2=325,\ m^2+n^2=65,$

$|AB|^2=4m^2+4n^2=4x65,$ $|AB| = 2\sqrt{65}.$

Q7.2 Arrange the numbers in ascending order : 9, 13, 15, 22, 29, 33, 40, 47, 49, 53

9 + 53 = 62, 13 + 49 = 62 etc
So 15 + 47 = 62. ANSWER 47.

Q7.3 At start lamps A , B , C , D and E are off
Then 1st to 5^{th} time all 5 lamps are on
Then 6^{th} to 10^{th} time all 5 lamps are off
Then 11th to 15th times all 5 lamps are on

Continuing the pattern 2011th to 2015^{th} time all lamps are on.
So tuning the switches 2016 and 2017 times lamps **A and B** are **off** and so **C , D** and **E** are **on.**

Q7.4 $(a+2x)^4=a^4+4a^3(2x)+6a^2(4x^2)+4a(8x^3)+16x^4$

32a = 160 , a = 5

Round 8

Q8.1 A(-3 , -9) , B(9, - 4) , Centre C $(3,-\frac{13}{2})$

Radius $= \sqrt{(3+3)^2+(-\frac{13}{2}+9)^2} = \frac{13}{2}$

Equation of Circle : $(x-3)^2+(y+\frac{13}{2})^2=\frac{169}{4}$

ie $x^2+y^2-6x+13y+9=0$

Circle cuts x-axis (y =0) , giving $x^2-6x+9=0 \Rightarrow (x-3)^2=0$

So x = 3 , y = 0. So T(3, 0) , A(-3 , -9) , B(9, - 4)

|AT| = $\sqrt{36+81}=\sqrt{117}=3\sqrt{13}$
|TB| = $\sqrt{36+16}=\sqrt{52}=2\sqrt{13}$

So ratio |TA| : |TB| = 3 : 2

Q8.2 $\frac{1}{m}+\frac{1}{n}+\frac{1}{mn}=1 \Rightarrow$ n + m + 1 = 0 $\Rightarrow$ mn – m – 1 = 0

Adding 2 to each side

mn – m – n +1 = 2 , (m – 1)(n – 1) = 2 x 1

m – 1 = 2 , m = 1 and n – 1 = 1 , n = 2, since m > n.
So mn = 6

Q8.3 Train 1 (from A to B) takes 3 hours
Train 2 (from B to A) takes 4.5 hours
Let d be distance between the stations

Train 1 speed = $\frac{d}{3}$ km/h

Train 2 speed = $\frac{d}{4.5}$ km/h = $\frac{2d}{9}$km/h

Let trains meet x hours after 3 pm

In x hours train 1 has $\frac{xd}{3}$travelled km and train 2 has travelled $\frac{(x+\frac{3}{2})(2d)}{9}$ km.

When they meet $\frac{xd}{3}+\frac{(x+\frac{3}{2})(2d)}{9}=d$

$\Rightarrow \frac{x}{3}+\frac{(x+\frac{3}{2})(2)}{9}=1 \Rightarrow$3x +2x+ 3 = 9, 5x = 6 , x = 6.2 hours after 3 pm

i.e 1 hour 12 minutes after 3 pm at 4.12 pm

Q8.4

$x^3+y^3=91$

$x^2y+xy^2=84$ and $\Rightarrow 3x^2y+3xy^2=252$

Adding $x^3+3x^2y+3xy^2+y^3=343$,
$(x+y)^3=343$, x + y = 7 , y = 7 – x .

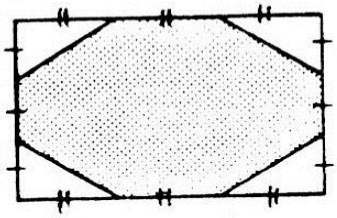
Substituting $x^2y+xy^2=84 \Rightarrow$xy(x + y) = 84, xy(7) = 84, xy =12.

So x(7 – x) = 12 , $x^2-7x+12=0$, x = 3 or 4 and y = 4 or 3.

Answers (4, 3) and (3, 4)

Babhta 1 **Round 1**

1) The sides of a 12 x 9 rectangle are trisected.



The points are joined as shown in the diagram.
Calculate the perimeter of the octagon formed.

2) Given the following pattern

$$1 + 2 = 3$$
$$4 + 5 + 6 = 7 + 8$$
$$9 + 10 + 11 + 12 = 13 + 14 + 15$$
$$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

If the pattern continues, find the value of the first term immediately after the "=" in the 20^{th} row.

Babhta 2 **Round 2**

1) How many 5-digit positive integers contain the digit 6 exactly once?
A 5-digit number is ³ 10 000.

2) What is the smallest positive integer n such that $\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)^n = 1$?

Babhta 3 **Round 3**

1) Find the value of the integral $\int_2^4 e^2 dx$ in terms of e.

2) Solve for x : $\sqrt{16^{x+1}} = 8^{1-x}$, where $x \in Q$.

Answer in the form $\frac{a}{b}$, where a and b $\in$ N.

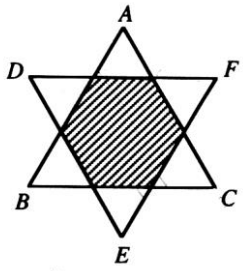
Babhta 4 **Round 4**

1) Two sides of a parallelogram lie along the lines $x+y=5$ and $3x+y=9$ respectively. The diagonals of the parallelogram intersect at the point (3, -1). Calculate the coordinates of all the vertices of the parallelogram.

2) ABC is a right angled triangle. The right angle is at B. The line segment [AD] is the bisector of the acute angle A , where D Ï [BC] .
[AD] divides the opposite side [BC] into segments of lengths 1 and 2, where |BD| =1 and |DC| = 2.
Calculate the length of the bisector [AD].

Babhta 5 **Round 5**

1) ABC and DEF are equilateral triangles, each of area 12.
They overlap to form a regular hexagon.
Calculate the area of the hexagon.



2) The function f(x) = ax + b satisfies the equalities :

f(f(f(1))) = 29 and f(f(f(0))) = 2.

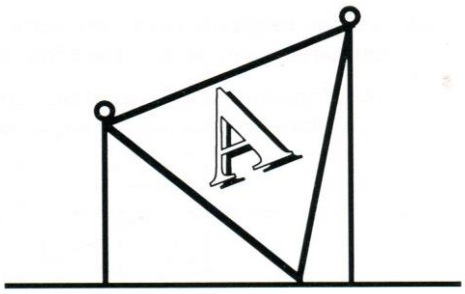
That is $f_0 f_0 f(1)=29$ and $f_0 f_0 f(0)=2$

Calculate the value of a.

Babhta 6 **Round 6**

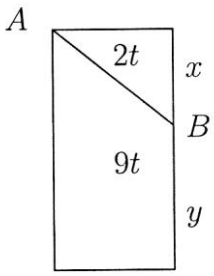
1) Three six-sided dice are rolled at once.
What is the probability that at least two of the dice show the same number?
Answer in simplest form $\frac{a}{b}$, where a and b Ì N.

2) A large flag in the shape of an equilateral triangle is suspended by two of its corners from the tops of two vertical poles, one 4 m high and the other 3 m high.
The third corner of the flag just touches the ground.
Find the length of a side of the flag.
Answer in the form $\sqrt{\frac{a}{b}}$, where a and b Î N.



Babhta 7 **Round 7**

1) The line segment [AB] divides the rectangle into two parts.
The ratio of the areas of these parts is 2 : 9.
Calculate the ratio of the lengths x and y in simplest form.
Answer in form simplest x : y, where x and y Ì N.



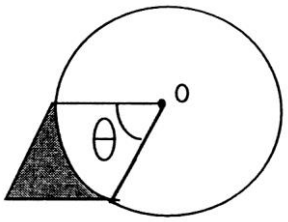
2) Two machines move at constant speeds around a circle of circumference 600 cm, starting together at the same point.
If they travel in the same direction then they next meet after 20 seconds.
If they travel in opposite directions then they next meet after 5 seconds.
At what speed, in cms, per second ,is the slower machine travelling?

3) The Fibonacci sequence is 1 ,1, 2 , 3 , 5, 8,…….
Every term after the first two terms is the sum of the two previous terms.
Let **a, b , c** and **d** be four successive terms of the Fibonacci sequence.
If **a** + **b** + **c** = **3194** and **b** + **c** + **d** = **5168,** find the value of **a.**

4) John and Mary bought a house on March 1st, 1996. They obtained a 30 year loan from the bank at an annual interest rate of 7.5% APR. Their monthly repayment at the end of each month was €1100. The first repayment was made at the end of March 1996. In February 2016 they had a substantial win in the Lotto. They paid the February 2016 instalment as usual and later decided to pay off the loan from March 2016.
How much did they have to pay to clear the loan?
Answer to the nearest euro.

Babhta 8 **Round 8**

1) The diagram shows a disc of area 1 cm^2,
centre o, and a parallelogram whose sides are all of equal length.
q is an angle in radian measure.
Find an expression for the shaded area in terms of π and q.



2) The lines y = mx + 1 , where m is positive integer, and 13x + 9y = 183 intersect at a point P. For how many values of m are the coordinates of P integers?

3) In a series of 5 games to be played between two equally matched teams, the first team to win 3 games becomes the champion. Team X has won the first game.
If there are no draws find the probability that team X will be the champion.

4) John wants to make a secret code of five letter "words".
He makes the following rules:

(a) No more than two vowels or two consonants in succession
(b) No "word" to start or end with two consonants

He rejects the letter A as too obvious, so he has 21 consonants and 4 vowels to choose from.
How many possible code "words" are possible?

Note : a letter may be repeated, for example **BECCU**

Answer Key for TEAM MATHS Final 16

Round 1	Q1	34	Q2	421
Round 2	Q1	29889	Q2	8
Round 3	Q1	$2\,e^2$	Q2	$\frac{1}{5}$
Round 4	Q1	(2.3) , (4, -5) , (5, -6) , (1,4)	Q2	2
Round 5	Q1	8	Q2	3
Round 6	Q1	$\frac{4}{9}$	Q2	$\sqrt{\frac{52}{3}}$
Round 7	Q1	$\frac{4}{7}$	Q2	45
	Q3	610	Q4	€93680
Round 8	Q1	$\frac{\sin(q)}{p} - \frac{q}{2p}$ or $\frac{2\sin(q) - q}{2p}$	Q2	1
	Q3	$\frac{11}{16}$	Q4	647472

Answer Key Tiebreak 2016

TIEBREAK

q1	0	q 2	$\frac{x}{x - 1}$	q3	4y	q4	$\frac{1}{3}$
q5	11	q6	165 km	q7	$\frac{1}{3}(\log_{10}(x)+5)$	q8	1 + i
q9	k= $\frac{80}{19}$	q10	6	q11	$\frac{4}{3}$	q12	126

q13 2 q14 $\frac{4\sqrt{3}}{7}$

Foireann Mata 2016 (An Chraobh) Team Maths 2016 (Final)

Scoilt Tiebreak

1) In how many different ways can fourteen 10-cent coins be divided into three piles with an odd number of 10-cent coins in each pile?

2 $x = 1 + 2^{p}$ and $y = 1 + 2^{-p}$. Write y in terms of x.

3) $y^2 = y + 4$.
Calculate the value of $y^4 - y^3 - 16$ in terms of y^k, where k =1.

4) x , y and z form a geometric sequence with a common ratio r and $x \neq y$.
If x , 2y and 3z form an arithmetic sequence , find the numerical value of r.
Answer in simplest form $\frac{a}{b}$, where a and b $\in Z$.

5) $f(x) = px^4 + qx^2 + x + 5$, where p and q are constants.
Find the value of f(4) if f(- 4) = 3.

6) If John drove at 110 km/h instead of 100 km/h he would save 9 minutes when travelling between two towns.
Calculate the distance, in km, between the towns.

7) Calculate the inverse of the function $f(x) = 10^{3x - 5}$.

8) Write $1 - i^{-1}$ in the form a + ib, where a and b $\in Z$

9) If $\frac{\log_b(a)}{\log_c(a)} = \frac{19}{99}$, then $\frac{b}{c} = c^k$.

Find the value of k in simplest form $\frac{p}{q}$, where p and q Î N.

10) There are 10 digits when using the denary system in mathematics
0, 1, 2, 3, 4,5, 6,7, 8, and 9 .

The Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21,……. starts with two 1's and each term after that is the sum of the two previous terms. Which one of the ten digit listed above is the last to appear in the units position of a number in the Fibonacci sequence ?

11) If 4(9a – 13b) = 6(a – 2b) , what is the ratio of a to b in simplest form $\frac{a}{b}$, where a and b Î N ?

12) Each letter below represents a positive integer. A = 31 and F = 7.

31 B C D E 7 G H I

The sum of any three adjacent numbers is 164. Find the value of H

13) Solve for x: ln(x) + ln(x+2) = ln(x+6).

14) When Sin(A) = $\frac{1}{7}$ and Tan (A) >0 ,calculate the numerical value of Sin($A + \frac{\rho}{2}$).

Answer form $\frac{a\sqrt{b}}{c}$ where a , b and c Î N.

Foireann Mata 2015 (An Chraobh) Team Maths 2015 (Final)

Babhta 1 Round 1

1) Given $c^x = 3$, what is the numerical value of $c^{4x} - 5$?

2) Given $2x^2 - 9x + 14 = a(x-1)(x-2) + b(x-1) + c$,

find the numerical value of $a + b + c$.

07 – March – 2015

Foireann Mata 2015 (An Chraobh) Team Maths 2015 (Final)

Babhta 2 Round 2

1) Let x be a positive real number.
Find the value of x when $x - 1$ is the reciprocal of $x + ½$.
Answer in the form $\frac{a}{b}$, where a and $b \in \mathbb{Z}$.

2) The coordinates of three vertices of a parallelogram ABCD are:
A$(-5, 1)$, B$(2, 4)$ and C$(1, 1)$.

Calculate the area of the parallelogram.

07 – March – 2015

Foireann Mata 2015 (An Chraobh) Team Maths 2015 (Final)

Babhta 3 Round 3

1) For $0 < \theta \leq 2\pi$, find all the solutions to the equation $\sin^2\theta = 1 - \cos\theta$

Answer in terms of π.

2) A line divides a circle of radius length 2 into two arcs.
If the length of the smaller arc is $\frac{\pi}{3}$, what is the area of the region bounded by the smaller arc and the line?

Answer in the form $\frac{\pi}{a} - b$, where a and $b \in \mathbb{Z}$.

07 – March – 2015

Foireann Mata 2015 (An Chraobh) Team Maths 2015 (Final)

Babhta 4 Round 4

1) The quadratic expression $4x^2 + 11x + 3$ can be written in the form $a(x + b)^2 + c$.

Calculate, in its simplest form, the numerical value of $ab^2 + c$.

2) A coin is weighted so that P(head) = $\frac{2}{3}$ and P(tail) = $\frac{1}{3}$.
The coin is tossed once.

If a head appears, then a number is selected at random from the whole numbers 1 to 9.
If a tail appears, a number is selected at random from the whole numbers 1 to 5.

Find the probability that an even number appears.
Answer in its simplest form $\frac{a}{b}$, where a and $b \in \mathbb{N}$.

07 – March – 2015

Foireann Mata 2015 (An Chraobh) Team Maths 2015 (Final)

Babhta 5 Round 5

1) Let a be a positive real number.
The curve $y = 1 - ax^2$ intersects the x and y axes at three points which form the vertices of an equilateral triangle.

Find the numerical value of a in its simplest form.

2) John enters a tall building. He has a decision to make: should he take the stairs or the lift.

If he decides to take the stairs, it would take him 20 seconds to walk up each flight of stairs.
If he decides to take the lift, he would have to wait 3 minutes for the lift to arrive, after which it would take 3 seconds to move up each floor.

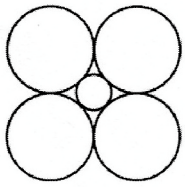
Calculate the minimum number of floors for which taking the lift takes less time than taking the stairs.

07 – March – 2015

Foireann Mata 2015 (An Chraobh) Team Maths 2015 (Final)

Babhta 6 Round 6

1) Four circles of radius length 1 are touching each other and a smaller circle, as seen in the diagram.
What is the radius length of the smaller circle?



Answer in its simplest form $a\sqrt{b}+c$, where a, b and $c \in \mathbb{Z}$.

2) For a given arithmetic sequence, the ratio of the sum of the first m terms to the sum of the first n terms is $m^2 : n^2$. Find, in simplest form, the ratio of the m^{th} term to the n^{th} term in terms of m and n, where $m \neq n$.

07 – March – 2015

Foireann Mata 2015 (An Chraobh) Team Maths 2015 (Final)

Babhta 7 Round 7

1) In the triangle ABC, using the usual notation, write the expression

$$\frac{4 \times (\text{Area } ABC)}{b^2+c^2-a^2}$$

as a single trigonometrical function in its simplest form.

2) An equilateral triangle is circumscribed about a circle of radius length 9.
Find the area of the triangle.
Answer in its simplest form $a\sqrt{b}$, where a and $b \in \mathbb{N}$.

3) Given $\text{Sin } x - \text{Cos } x = \frac{3}{5}$, find the numerical value of

$$\text{Sin}^3 x - \text{Cos}^3 x$$

Answer in its simplest form $\frac{a}{b}$, where a and $b \in \mathbb{N}$..

4) I have a bag with 12 slips of paper in it. Some of the slips have a "2" written on them and the rest have a "7" written on them.
If the expected value of the number randomly drawn from the bag is 3·25, how many slips have a "2" written on them?

07 – March – 2015

Foireann Mata 2015 (An Chraobh) Team Maths 2015 (Final)

Babhta 8 Round 8

1) Find the complex number z such that $\frac{2z-3i}{z+2} = -5 + i$

Answer in the form $a + bi$ where a and $b \in \mathbb{Q}$..

2) Given that $(a + c)(b + c)(a + b) \neq 0$ and $b^2 - a^2 = c^2 - b^2$
find the numerical value of

$$\frac{2}{a+c} - \frac{1}{b+c} - \frac{1}{a+b}$$

3) The lines $y = mx + 4$ and $y = 2mx + 3$ intersect at an angle $\tan^{-1}(\frac{1}{4})$
Find all possible values for m.

Answers in the form $a + b\sqrt{c}$ where a, b and $c \in \mathbb{Q}$..

4) Calculate the range of values of x for which $3^{2x+1} + 3 < 3^{x+2} + 3^x$

TM 15 Final answers

Round 1	q1	76	q2	6
Round 2	q1	$\frac{3}{2}$	q2	18
Round 3	q1	$\frac{1}{2}\pi,\ \frac{3}{2}\pi,\ 2\pi$	q2	$\frac{\pi}{3}-1$
Round 4	q1	3	q2	$\frac{58}{135}$
Round 5	q1	3	q2	11
Round 6	q1	$\sqrt{2}-1$	q2	$2m-1 : 2n\text{-}1$

Round 7	q1	tan A	q2	$243\sqrt{3}$
	q3	$\frac{99}{125}$	q4	9
Round 8	q1	$-\frac{3}{2}+\frac{1}{2}i$	q2	0
	q3	$\pm 1 \pm \frac{\sqrt{2}}{2}$	q4	$-1 < x < 1$

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BABHTA 1 **ROUND 1**

1) The curve $y = 17(x + 3)(x - 4)$ intersects the x-axis at A and B. Find the length of the line segment $[AB]$.

2) Let $(1 + i)^3 = x + iy$, where x and $y \in \mathbb{R}$ and where $i = \sqrt{-1}$. Find the value of $x + y$.

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BABHTA 2 **ROUND 2**

1) Find all the values of B, where $0 < B < \pi$, such that
$6\text{Cos}^2(B) - \text{Sin}(B) = 4$

Answers in terms of π.

2) Find the distance from (5, 8) to the nearest point on the circle

$$2x^2 + 2y^2 + 4x - 8y = 6$$

Answer in the form $a\sqrt{b}$ where a and $b \in \mathbb{N}$.

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BABHTA 3 **ROUND 3**

1) Find two numbers a and b such that

$$(a-b) : (a+b) : ab \;=\; 1 : 5 : 24\,, \quad b \neq 0$$

2) In the equation below, calculate the value of $x + y$.

$3^{2x} - 2^{2y} = 17$, where x and $y \in \mathbb{N}$.

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BABHTA 4 **ROUND 4**

1) The lengths of the sides of a right- angled triangle are in geometric progression and the shortest side has a length of 2 cm.
What is the length of the hypotenuse in simplest surd form?

Answer in the form $a + \sqrt{b}$, where a and $b \in \mathbb{N}$.

2) Three straight lines l, m and n have slope $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, respectively.
All three lines have the same y-intercept.
If the sum of the x-intercepts of the three lines is 36, calculate the value of the y-intercept.

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BABHTA 5 **ROUND 5**

1) Find the smallest positive integer, a, such that 2014 is a term of the following arithmetic sequence:

$a,\ a+7,\ a+14,\ a+21,\ \ldots\ldots\ldots$

2) We are given three boxes as follows:

Box A has 10 light bulbs, of which 4 are defective.
Box B has 6 light bulbs, of which 1 is defective.
Box C has 8 light bulbs, of which 3 are defective.

If we draw a box at random and then draw a bulb at random, what is the probability that the bulb is defective?

Answer in the form $\frac{a}{b}$, where a and $b \in \mathbb{N}$.

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BABHTA 6 **ROUND 6**

1) Solve the following system of equations for x , y and z:

$$xy + xz = 8 - x^2$$
$$xy + yz = 12 - y^2$$
$$yz + zx = -4 - z^2$$

2) In a triangle, two of its angles, A and B are acute.
If $\tan(A) = \frac{3}{4}$ and $\sin(B) = \frac{5}{13}$, find the cosine of the third angle, i.e. $\cos(C)$, in the form $\frac{a}{b}$, where a and $b \in \mathbb{Z}$.

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BABHTA 7 **ROUND 7**

1) A square and an equilateral triangle have the same perimeter.
Find the ratio of the area of the triangle to the area of the square.
Answer in the form $a\sqrt{b} : c$, where a, b and $c \in \mathbb{N}$.

2) $f(x) = (1+x) + (1+x)^2 + (1+x)^3 + \cdots + (1+x)^{100}$
What is the coefficient of x^{98}?

3)

"1"

"10" "2"

"9" "3"

"8" "4"

"7" "5"

"6"

Ten people form a circle. Each person calls out a whole number. Then each person computes the average of the numbers of the two people *adjacent to him / her in the circle* and writes down that average. The figure above shows the average of the two neighbouring numbers for each person (not the original number the person picked).
What number was called by the person who wrote down the average 6?

4) The student lockers at a certain college are numbered consecutively, beginning with locker number 1. The plastic digits used to number the lockers cost eight cents each.
For example, it costs eight cents to label locker number 8 and sixteen cents to label locker number 14. If it costs €555·92 to label all lockers, how many student lockers are there at the college?

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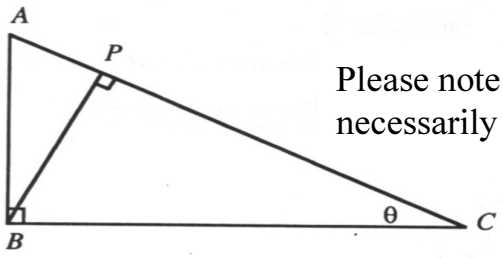
BABHTA 8 ROUND 8

1) In the right-angled triangle ABC, $|AC| = 4$, $|BP| = 1$ and $BP \perp AC$.

Find all possible values of the angle θ, in degrees.



Please note: This diagram is not necessarily drawn to scale.

2) The curve $y = ax^2 + bx + 1$ passes through the point (1, 2).
For what values of a does the curve intersect the x-axis at two distinct points?
Answer in simplest surd form.

3) If $x^2 + x = 1$, calculate the value of the expression

$$x^3 + 2x^2 + 2014.$$

4) On Wednesday, Jackie drove from home to work at an average speed of 70 km/h and arrived 1 minute late. On Thursday, she left home at the same time as on Wednesday and took the same route. This time she drove at an average speed 75 km/h and arrived 1 minute early.
What distance did she have to travel to work?

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ANSWER KEY

ROUND 1	Q 1 $\boxed{7}$	Q2 $\boxed{0}$		
ROUND 2	Q1 $\boxed{\frac{\pi}{6}, \frac{5\pi}{6}}$	Q2 $\boxed{4\sqrt{2}}$		
ROUND 3	Q1 $\boxed{a = 12 \\ b = 8}$	Q2 $\boxed{5}$		
ROUND 4	Q1 $\boxed{1 + \sqrt{5}}$	Q2 $\boxed{-4}$		
ROUND 5	Q1 $\boxed{5}$	Q2 $\boxed{\frac{113}{360}}$		
ROUND 6	Q1 $\boxed{2, 3, -1 \text{ and } -2, -3, 1}$	Q2 $\boxed{-\frac{33}{65}}$		
ROUND 7	Q1 $\boxed{4\sqrt{3} : 9}$	Q2 $\boxed{5050}$	Q3 $\boxed{1}$	Q4 $\boxed{2014}$
ROUND 8	Q1 $\boxed{15°, 75°}$	Q2 $\boxed{a > 3 + 2\sqrt{2} \text{ and } a < 3 - 2\sqrt{2}}$	Q3 $\boxed{2015}$	Q4 $\boxed{35 \text{ km}}$

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Scoilt **Tiebreak**

1) Susie has 138 identical cubes. She uses these to build a solid cube, using as many of the 138 cubes as possible. How many cubes will remain unused?

2) For how many values of $n \in \mathbb{Z}$, would right-angled triangles with sides of length $\sqrt{n+1}$, $\sqrt{n+2}$ and $\sqrt{n+3}$ exist?

3) What is the largest possible area of a rectangle whose perimeter is 36 cm?

4) If p% of q is k, what is q% of p?

5) The year 2013 contained four digits whose values are consecutive integers.
How many years after 2013 must we wait for this event to occur again?

6) Two different digits are selected at random from the digits 1 through 9. If the sum of the digits is even, find the probability that both digits are odd.
Answer in simplest form $\frac{a}{b}$, where a and $b \in \mathbb{N}$.

7) Suppose the waist measurements of 800 young people are normally distributed with a mean of 66 cm and standard deviation 5 cm. Find the number of young people with waists greater than or equal to 72 cm.

Please **turn over** to continue ➔

8) Let $\cos(2A) = -\frac{7}{9}$ and $0 < A < \frac{\pi}{2}$. Find the value of $\cos(A)$

9) Find the numerical value of

$$\sin\left(\frac{\pi}{6}\right) + \sin^2\left(\frac{\pi}{6}\right) + \sin^3\left(\frac{\pi}{6}\right) + \ \ldots\ldots\ldots$$

10) The solutions to the equation $\sqrt[4]{x} = \frac{12}{7 - \sqrt[4]{x}}$ are A and B.
Calculate the value of $A + B$.

11) If $a + b = 1$ and $a^2 + b^2 = 2$, calculate the value of $a^3 + b^3$.

Answer in the form $\frac{a}{b}$, where a and $b \in \mathbb{N}$.

12) Calculate the value of the term independent of x in the expansion of

$$\left(x^2 - \frac{2}{x^2}\right)^8.$$

13) Find the value of x when $4^{20} + 4^{20} = 2^x$.

14) Find the equation of the line through $(-1, 5)$ which is perpendicular to the line $y + 5x - 1 = 0$.
Answer in the form $ax + by + c = 0$, where a, b and $c \in \mathbb{N}$.

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Tiebreak Answers

Q1	13
Q2	1
Q3	81
Q4	k
Q5	18
Q6	$\frac{5}{8}$
Q7	92
Q8	$\frac{1}{3}$
Q9	1
Q10	337
Q11	$\frac{5}{2}$
Q12	1120
Q13	41
Q14	Any correct solution

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BABHTA 1 – ROUND 1

1) When $(x + 3)(3x – t)(2x – 1)$ is multiplied out, the coefficient of x^2 is 5.

What is the value of t ?

2) Find the value of $\dfrac{2^1 + 2^0 + 2^{-1}}{2^{-2} + 2^{-3} + 2^{-4}}$ in its simplest form.

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BABHTA 2 – ROUND 2

1) What is the value of $\sqrt[3]{2\sqrt{2}}$?

2) At an airport, there is a moving walkway 500metres long, which moves at a speed of 4km/h.
Alice and Bob step on the walkway at the same time. Alice walks at a speed of 6 km/h on the walkway while Bob stands still. When Alice comes to the end of the walkway, how far behind her is Bob?

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BABHTA 3 – ROUND 3

1) Find all the roots of $8x^3 - 44x^2 + 86x - 65 = 0$, given that one root is $\frac{1}{2}(3 - 2i)$.

2) A fair coin is tossed five times.
What is the probability of obtaining at least three consecutive heads?
Answer in the form $\frac{a}{b}$, where a and $b \in \mathrm{N}$.

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BABHTA 4 – ROUND 4

1) Using the usual notation, find the radius length of the circumcircle of the triangle ABC, where $a = 5$ cm, $b = 6$ cm and $c = 8$ cm.
Answer to nearest cm.

2) If $xy = 2$, $yz = 3$ and $zx = 5$, what is the value of $x^2 + y^2 + z^2$?
Answer in the form $\frac{a}{b}$, where a and $b \in$ N.

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BABHTA 5 – ROUND 5

1) If a, b and c are positive integers such that

$$a + \frac{1}{b + \frac{1}{c}} = \frac{37}{16},$$ calculate the value of $a + b + c$.

2) Evaluate, correct to 3 significant figures,

$$\int_1^2 \frac{\left(e^{2x} - e^{-2x}\right)}{2} dx$$

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BABHTA 6 – ROUND 6

1) If $2000^2 - 1996^2 = 111ak^2$, where a and k are integers, what is the maximum of $k - a$?

2) The first digit of a six-digit number is 1.This digit, 1, is now moved from the first digit position to the end so that it becomes the last digit. The new six-digit number is now 3 times larger than the original number. What was the original number?

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BABHTA 7 – ROUND 7

1) If x and y are real numbers such that $0 < x < y$ and $x^2 + y^2 = 6xy$, find the value of $\dfrac{x+y}{x-y}$

2) Arrange in ascending order of magnitude:

$$2^{800},\quad 3^{600},\quad 5^{400},\quad 6^{300}.$$

3) A solid circular cylinder is to be made from 10 cm³ of material. Calculate, correct to 2 decimal places, the height of the cylinder when the surface area is a minimum. (Use the most accurate value of π).

4) When the digits of a two-digit number are reversed, the result is another two-digit number that is 62·5% smaller than the original number. What is the original number?

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BABHTA 8 – ROUND 8

1) Find the value of x, in the form $\frac{a}{b}$ where a and $b \in \mathrm{N}$,
when $8^{3x+1} - 8^{3x} = 448$

2) When $A + B + C = 180^\circ$,
$\mathrm{Sin}(2A) - \mathrm{Sin}(2B) + \mathrm{Sin}(2C) = k\mathrm{Cos}(A)\mathrm{Sin}(B)\mathrm{Cos}(C)$.
Find the value of k.

3) Find all the positive solutions to the equation
$(\log_{10}(x^2))^2 = \log_{10}(x^4)$

4) A hotel cleaner has 8 master keys at home to open all rooms in the hotel. Each room can be opened by just one of these keys.
If 40% of the rooms are left unlocked, what is the probability that the cleaner can get into a specific room if she selects three keys at random before leaving home to go to work?

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SCOILT – TIEBREAK

1) What is the value of $\dfrac{-2x^4}{(-2x)^4}$?

2) The lengths of the altitudes of a parallelogram are 3 cm and 5 cm and its perimeter is 32 cm. What is the area of the parallelogram?

3) The ratio of the volumes of two cubes is 1 : 8. What is the ratio of their surface areas?

4) Find the value of $\dfrac{\frac{1}{5}+\frac{1}{7}}{\frac{2}{5}+\frac{2}{7}}$

5) The point $(a, -2)$ is on the line joining the points $(0, 6)$ and $(-3, 0)$.
Find the value of a.

6) What is the numerical value when the sum of the first 80 odd natural numbers is subtracted from the sum of the first 80 even natural numbers? An even natural number begins with 2.

7) The odd numbers are arranged in a triangle as shown:

1
3 5
7 9 11
13 15 17 19
……..

If the pattern continues in this form what is the sum of the numbers in the 12^{th} row?

Turn over for 7 more questions ➔

8) What is the numerical value of $\text{Sin}(70^0)\text{Cos}(20^0)+\text{Cos}(70^0)\text{Sin}(20^0)$?

9) A fair die is tossed 7 times. What is the probability that a 5 or a 6 occurs exactly 3 times? Answer correct to 2 decimal places.

10) Let X be a random variable with standard normal distribution.
Find $p(-1{\cdot}37 < X < 2{\cdot}01)$

11) Find the equations of the tangents to the circle $x^2 + y^2 - 2x + 8y - 23 = 0$ which have a slope of 3. Answer in the form $ax + by + c = 0$, where a, b and $c \in \text{N}$.

12) In a list of five numbers, the first number is 2 and the last number is 12. The product of the first three numbers is 30, the product of the three numbers in the middle is 90 and the product of the last three numbers is 360.
What is the value of the middle number?

13) What is the last non-zero digit of the product $2^{59} \times 3^4 \times 5^{53}$?

14) The sum of the digits of a nine-digit integer is 8. What is the product of these digits?

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Answers – Final 2013

Round 1	Round 2
(1) 5	**(1)** $\sqrt{2}$
(2) 8	**(2)** 300 metres
Round 3	**Round 4**
(1) $\frac{1}{2}(3\pm 2i)$, 2·5 (or equivalent)	**(1)** 4
(2) $\frac{1}{4}$	**(2)** $\frac{361}{30}$
Round 5	**Round 6**
(1) 10	**(1)** 11
(2) 11·8	**(2)** 142857
Round 7	**Round 8**
(1) $-\sqrt{2}$	**(1)** $\frac{2}{3}$
(2) 6^{300}, 2^{800}, 5^{400}, 3^{600}	**(2)** 4
(3) 2·34 cm.	**(3)** 1 and 10
(4) 72	**(4)** $\frac{5}{8}$ or equivalent

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Answers – Tiebreak, Final 2013

1) $-\frac{1}{8}$	8) 1
2) 30	9) 0·26
3) 1 : 4	10) 0·8925
4) $\frac{1}{2}$ or equivalent	11) $3x - y + 13 = 0$ and $3x - y - 27 = 0$
5) -4	12) 5
6) 80	13) 4
7) $1728 = 12^3$	14) 0

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Foireann Mata 2012 (An Chraobh) : Team Maths 2012 (Final)

BABHTA 1 - ROUND 1

1) Find the shortest distance between the circles

$x^2 + y^2 = 9$ and $x^2 + y^2 - 12x + 6y + 41 = 0$.

Give your answer in surd form.

2) Find the numerical value of $2^{\log_3 3^5}$.

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BABHTA 2 - ROUND 2

1) The houses along the street in which John lives are numbered using consecutive odd numbers, starting with 1 on one side of the street, and by using even numbers on the other side.

John's house number is 137. If the numbering had commenced at the other end of the street John's house would have been number 85.

How many houses are there on John's side of the street?

2) Find the range of values of x for which

$$\frac{x+1}{x-1} \geq 2$$

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BABHTA 3 - ROUND 3

1) If 3 unbiased coins are tossed 8 times, find the probability of getting 2 heads and 1 tail exactly 3 times.

Answer correct to 3 significant figures.

2) Find the equations of the lines which pass through the point (2,1) and are inclined at an angle of 45^0 to the line $y = 3x + 3$.

Answers in the form $ax+by+c = 0$,where a,b and $c \in Z$.

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BABHTA 4 - ROUND 4

1) Solve $\frac{x}{4}+\frac{y}{3}=\frac{9}{2}$

$\frac{4}{x}+\frac{3}{y}=1$

Write your answers in coordinate form (x, y).

2) There are 18 points in a plane. Five of the points are in a straight line. Apart from these, no three of the points are collinear.
How many triangles can be formed by joining the 18 points?

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BABHTA 5 - ROUND 5

1) A six-seater carriage has two rows of three seats opposite each other.

Four people enter this carriage. If one couple must sit beside each other, how many arrangements are possible?

2) If x is a positive number and $x \neq 1$ express

$$\frac{1}{\log_3(x)} + \frac{1}{\log_4(x)} + \frac{1}{\log_5(x)}$$ as a single log.

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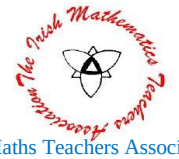
BABHTA 6 - ROUND 6

1) If $\text{Sin}(\theta+\alpha) - \text{Cos}(\theta+\alpha) = \text{Cos}(\theta)\text{Sin}(\alpha)$, calculate the value of $\text{Tan}(\theta)$, given that $\text{Tan}(\alpha) = \frac{7}{24}$.

2) A teacher bought 100 puzzles for €82·90 to give out to members of her maths team. She bought three different types of puzzle.

Type A cost 40c each, type B cost 70c each and type C cost €1 each.

How many more type C puzzles than type A puzzles did she buy?



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Foireann Mata 2012 (An Chraobh) : Team Maths 2012 (Final)

BABHTA 7 - ROUND 7

1) When Mary and Julia ran a 100 metre race, Mary won by 10 metres. When Julia ran against Anne, Julia won that race by 20 metres. Assuming that Mary and Anne race at exactly the same speeds as each ran in her previous race, by how much would Mary beat Anne?

2) If crocodiles consider that:
one dog is worth three rats, one pig is worth as much as a dog and a rat, and five cats are worth three dogs, then how many cats would ninety seven rats and thirty two pigs be worth?

3) The expression $ax^2 + bx + c$ takes the value 2 when $x = -2$ and when $x = 4$ and its minimum value is -7.
Find the value of

$$a + b + c.$$

4) A triangle has an area of 9 square units. It is formed by the x-axis and the lines $x = 1$ and $y = \boldsymbol{mx - 4}$

If $m < 0$, find all possible values of m.

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BABHTA 8 - ROUND 8

1) Find the coordinates of P, the point of intersection of the curves
$y = e^x$ and $y = 2 + 3e^{-x}$

Answer in non-decimal format.

2) Find the value of $\int_0^{\frac{\pi}{2}} \sin^2(x)\cos^2(x)\mathrm{d}x$

Answer in terms of π.

3) Write down the set of all the positive integer solutions to the equation

$$a^2 - 7a + b^2 - 7b + 2ab = 0$$

Answers in coordinate form (a, b).

4) A computer is programmed to scan the digits of the counting numbers, starting at 1.

For example, if it scans

1 2 3 4 5 6 7 8 9 10 11 12, it has scanned 15 digits.

The computer begins its task and scans the first 1788 digits.
What was the last counting number scanned?

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Answers (Final 2012)

Round 1	**Round 2**
(1) $3\sqrt{5}-5$	(1) 111
(2) 32 or 2^5	(2) $1 < x \leq 3$
Round 3	**Round 4**
(1) ·282 or 0·282 to 3 significant figs	(1) (6, 9) and (12, 4·5)
(2) $2x + y - 5 = 0$ and $x - 2y = 0$	(2) 806
Round 5	**Round 6**
(1) 96	(1) $\frac{24}{31}$
(2) $\log_x(60)$	(2) 43
Round 7	**Round 8**
(1) 28 m	(1) (ln(3), 3)
(2) 125 cats	(2) $\frac{\pi}{16}$
(3) – 7	(3) (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)
(4) m = – 2 or – 8	(4) 632

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Scoilt : Tiebreak – Final 2012

Irish Maths Teachers Association

1) If $f(x) = x^2 - 7x + k$ and $f(k) = -9$, find the value of $f(-1)$.

2) Find the value of x if $\log_3(\log_2(x)) = 2$.

3) If Cos (A) = $\frac{3}{5}$ and $\pi \leq \mathbf{A} \leq 2\pi$, find the value of tan (A).

Answer in the form $\frac{a}{b}$, where a and $b \in$ Z.

4) Find the length of the longest possible chord of the circle $x^2 + y^2 = 16$ that passes through $(\mathbf{1}, \mathbf{-2})$.

5) A palindrome is a number that remains the same when its digits are written in reverse order. For example 252 is a palindrome.
A car's clock-distance (in kilometres) reads 15951. Find the least number of kilometers required for the next palindrome to appear.

6) An unbiased coin is tossed four times. On the first three occasions it turns up tails. What is the probability that on the fourth occasion it will again turn up tails?

7) The roots of the equation $x^2 + x + 2 = 0$ are α and β.
Find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$. Answer in the form $\frac{a}{b}$, where a and $b \in$ Z.

8) A committee consists of 5 elected members. The members are to be elected from 5 women and 3 men. Find the number of ways the committee may be elected when not more than 2 men may be on the committee.

9) Find the 9th term of the expansion $\left(x - \frac{1}{\sqrt{x}}\right)^{12}$ in simplest form.

10) Find the area of the triangle with vertices (0, 0), $(-1, 3)$ and $(7, -1)$

11) If repetitions are not allowed, how many three-digit odd numbers can be formed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9? A three-digit number cannot begin with 0.

12) Find the numerical value of $\int_3^7 \frac{x\,dx}{1+x^2}$ as a single term.

13) Solve for x : $3x^2 + 2x + 2 \; < \; 2x^2 + x + 4$.

14) Two numbers have a sum of 8 and a product of 15.
What is the sum of the reciprocals of these numbers?

Irish Maths Teachers Association

Tiebreak answers (Final 2012)

1) 11	8) 46
2) 512	9) 495
3) $-\frac{4}{3}$	10) 10
4) 8	11) 320
5) 110	12) $\ln\sqrt{5}$ or $\frac{1}{2}\ln(5)$
6) $\frac{1}{2}$	13) $-2 < x < 1$
7) $-\frac{1}{2}$	14) $\frac{8}{15}$

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TEAM MATH FINAL 2011

ROUND 1

Time allowed: **Six** minutes

Q. 1 Solve the simultaneous equations

$$2x + y + z = 0$$
$$3x - 2y + 6z = -6$$
$$3y + 2z = 11.$$

Q. 2 Calculate the distance from the point $(-2, 3)$ to the line

$$5x - 12y - 6 = 0.$$

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TEAM MATH FINAL 2011

ROUND 2

Time allowed: **Six** minutes

Q. 1 Write, in the form $a + bi$, the values of z that satisfy the equation $z^2 + i = 0$, where $a, b \in \boldsymbol{R}$.

Q. 2 Find the centre and radius of the circle

$$2x^2 + 2y^2 + 6x - 8y = 0.$$

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TEAM MATH FINAL 2011

ROUND 3

Time allowed: **Six** minutes

Q. 1 Find the equations of the two circles that contain the points (–2, 3) and (4, 1) and have radius 10.
Write your answers in the form $x^2 + y^2 + 2gx + 2fy + c = 0$.

Q. 2 Find the range of values of x for which

$$\frac{x-}{x} > \quad , \; x \neq 0.$$

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TEAM MATH FINAL 2011

ROUND 4

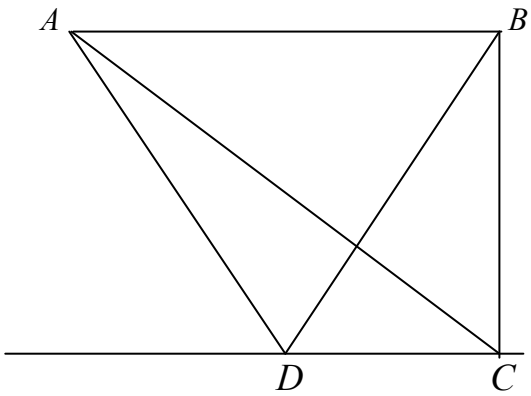
Time allowed: **Six** minutes

Q. 1 Solve the equation

$$\log_e(2x - 5) + \log_e(x - 3) = 2\log_e(2x - 6) - \log_e 2.$$

Q. 2 In the diagram,
$|\angle CBA| = 90^o$, $|\angle ACB| = 20^o$,
$|BC| = 20$
$DC \,//\, AB$ and $|AD| = |BD|$.

Calculate $|\angle ADB|$ in degrees, correct to one decimal place.



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TEAM MATH FINAL 2011

ROUND 5

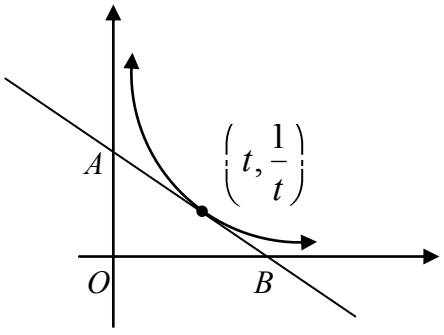
Time allowed: **Six** minutes

Q. 1 Evaluate

$$\int_0^1 \frac{2x+1}{1+x^2}\,dx.$$

Write your answer in terms of π and log.

Q. 2 A tangent to the curve $y = \frac{1}{x}$ at the point $\left(t, \frac{1}{t}\right)$ intersects the y-axis at A and the x–axis at B.



Find the co-ordinates of A and B in terms of t.
Find the area of triangle AOB, where O is the origin.

Team Number _____

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TEAM MATH FINAL 2011

ROUND 6

Time allowed: **Six** minutes

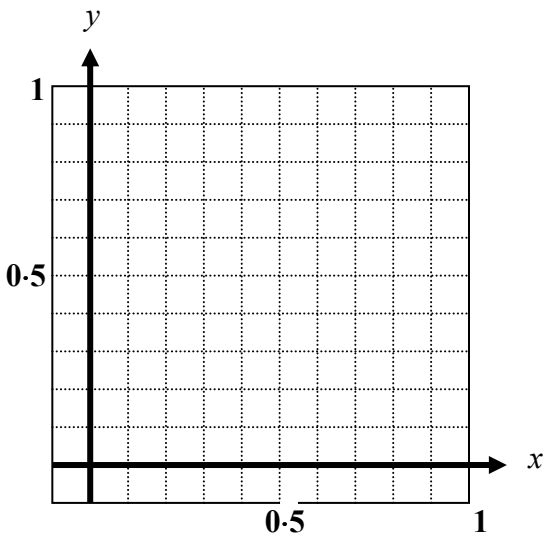
Please answer the questions on this question paper and hand it up.

Q. 1 Within the limits of the given diagram, clearly indicate the set of points that satisfy

$$x \geq 2y,$$

where $x, y \in \boldsymbol{R}$,

$x \geq 0$ and $y \geq 0$.



Q. 2 Express tan $22{\cdot}5^{\circ}$ in its simplest surd form.

Answer to question 2 _____

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TEAM MATH FINAL 2011

ROUND 7

Time allowed: **Six** minutes

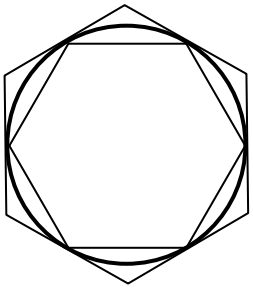
Q. 1 $x = \cos^2 t$, $y = \cos 2t$.

Find $\dfrac{dy}{dx}$ in its simplest form.

Q. 2 Points A and B are on a given plane and are 6 units apart.
How many lines can be drawn on that plane so that A is 2 units from each of the lines and B is 4 units from each of the lines?

Q. 3 Find the total number of ways in which six people can be divided into two groups with at least one person in each group and where all six people are involved each time.

Q. 4 A regular hexagon is inscribed in a circle (radius r) and another regular hexagon is circumscribed about the circle.



Find the ratio of the area of the inscribed hexagon to that of the circumscribed hexagon.

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TEAM MATH FINAL 2011

ROUND 8

Time allowed: **Six** minutes

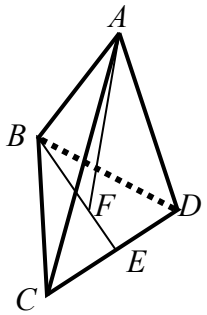
Q. 1 Find the values of x that satisfy the equation

$$\cos x + \cos 3x + 2\cos 2x = 0, \text{ where } 0 \le x \le 180^{o}.$$

Q.2 For what value of x is

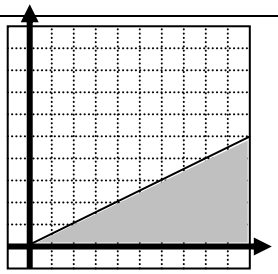
$$9^{x}.4^{x} = 216?$$

Q. 3 $ABCD$ is a regular tetrahedron whose edges are of length 6.
$|CE| = |ED|$
$AF \perp BE$ and $|BF| : |FE| = 2 : 1$.
Calculate $|AF|$.
Write your answer in surd form.



Q. 4 In a talent show, five judges, including Dana and John are randomly seated in a row.
What is the probability that Dana is seated to the left of John but not next to him?

FINAL 2011
ANSWERS

Round	Question 1	Question 2	Question 3	Question 4
1	$x = -2, y = 3, z = 1$	Distance = 4		
2	$z = \mp\frac{1}{\sqrt{2}} \pm \frac{1}{\sqrt{2}}i$	$(-1{\cdot}5, 2)$, r = 2·5		
3	$x^2 + y^2 + 4x + 14y - 47 = 0$ $x^2 + y^2 - 8x - 22y + 37 = 0$	$x < 0$		
4		$\lvert \angle ADB \rvert = 20{\cdot}6^\circ$		
5	$\frac{\pi}{4} + \log_e 2$	$A(0, \frac{2}{t}), B(2t, 0)$ Area = 2		
6		$\tan 22{\cdot}5^\circ = \sqrt{2} - 1$		
7	$\frac{dy}{dx} = 2$	3 lines	31 ways	3 : 4
8	$x = 45^\circ, 135^\circ, 180^\circ$	$x = 1\dot{5}$	$\lvert AF \rvert = \sqrt{24}$ or $2\sqrt{6}$	$\frac{3}{10}$

OLLSCOIL NA hÉIREANN, CORCAIGH
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UNIVERSITY COLLEGE, CORK

TEAM MATH FINAL 2010

ROUND 1

Time allowed: **Six** minutes

Q.1 Express $x = \dfrac{2t+1}{3t-1}$, $y = \dfrac{t+2}{3t-1}$ in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{R}$.

Q.2 Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 5x - \sin x}{3x}$

ROUND 2

Time allowed: **Six** minutes

Q.1 Find the value of x that satisfies the equation

$$\log_3 x + \log_9 16 = 0, \text{ where } x \in \mathbb{Q}.$$

Q.2 The line $x - y + 3 = 0$ passes through the centre of a circle.
The points $A(-1, 2)$ and $B(2, -4)$ are on the circle.
Write the equation of the circle in the form $x^2 + y^2 + 2gx + 2fy + c = 0$, where $g, f, c \in \mathbb{R}$.

ROUND 3

Time allowed: **Six** minutes

Q.1 Solve the equation $\sqrt{x} + \dfrac{24}{\sqrt{x}} = 10$, where $x > 0$.

Q.2 Write $\cos^4 x - \sin^4 x$ in the form $a \cos bx$, where $a, b \in \mathbb{Z}$.

ROUND 4

Time allowed: **Six** minutes

Q.1 Find the values of θ that satisfy the equation

$$|1 + \cos 2\theta + i \sin 2\theta| = \sin 2\theta,$$

where $0° \leqslant \theta \leqslant 360°$.

Q.2 Find the equations of the two lines that pass through the point $(2, -1)$ and make an angle of $60°$ with the x-axis.

Write your answers in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{R}$, using surd values where relevant.

ROUND 5

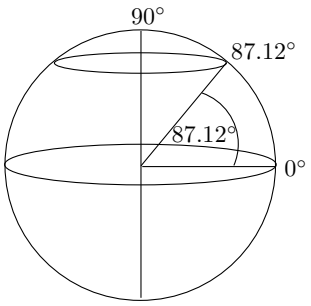
Time allowed: **Six** minutes

Q.1 A committee of *five* is to be formed from a group of *nine* people which includes Jack and Jill. Jack will **not** serve on the committee without Jill, but Jill will serve without Jack.

In how many ways can the committee be formed?

Q.2 Taking the earth as a sphere, the diagram shows the circular parallels of latitude $0°$ and $87.12°$.

Taking the length of latitude $0°$ to be $40\,000$ km, calculate the length of latitude $87.12°$, correct to the nearest km.



ROUND 6

Time allowed: **Six** minutes

Q.1 Writing x^3 as $x^2 \cdot x$ or otherwise, evaluate

$$\int_{\sqrt{2}}^{\sqrt{7}} \frac{x^3}{\sqrt{2 + x^2}}\, dx.$$

Write your answer in the form $\dfrac{a}{b}$, where $a, b \in \mathbb{Z}$.

Q.2 An ATM machine contains thirty notes.
There is at least one €10, one €50 and one €100 note in the machine.
These are the only notes in the machine and the total value of the thirty notes is €2010.

How many of each of the notes are in the machine?

Write, clearly, the two sets of possible answers.

ROUND 7

Time allowed: **Six** minutes

Q.1 Find, correct to two decimal places, the value of x that satisfies

$$2^{2x} - 2^{x+3} - 20 = 0, \text{ where } x \in \mathbb{R}.$$

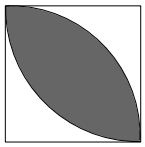
Q.2 The volume of a spherical balloon is increasing at the rate of $20\,\text{cm}^3$ per second, i.e. $\dfrac{dv}{dt} = 20$.

Find $\dfrac{dr}{dt}$ when $r = 1\,\text{cm}$.

Write your answer in the form $\dfrac{a}{\pi}$, where $a \in \mathbb{Z}$.

Q.3 The diagram shows a square of side $4\,\text{cm}$ in length and two arcs, each of radius $4\,\text{cm}$ and with centres at opposite vertices of the square.

Calculate, in terms of π, the area of the shaded region between the two arcs.



Q.4 Simplify $\begin{pmatrix} \cos 60^\circ & -\sin 60^\circ \\ \sin 60^\circ & \cos 60^\circ \end{pmatrix}^{11}$.

Write your answer in the form $\dfrac{1}{2}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where $a, b, c, d \in \mathbb{R}$, using surd values where relevant.

ROUND 8

Time allowed: **Six** minutes

Q.1 The result, either ***win***, ***lose*** or ***draw***, of each of four matches is forecast.

Each of the possible results is equally likely for each match.

Find the probability that the forecast is correct for exactly two of the four matches.

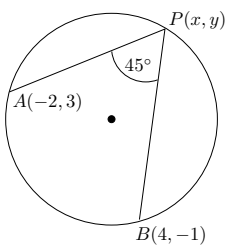
Q.2 Evaluate $\displaystyle\int_3^8 \frac{1}{\sqrt{16 + 6x - x^2}}\,dx$.

Write your answer in terms of π.

Q.3 $A(-2, 3)$, $B(4, -1)$ and $P(x, y)$ are points on a circle such that $|\angle APB| = 45^\circ$.

Using the formula $\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$, or otherwise, find the equations of two circles that satisfy these conditions.

Write your answers in the form $x^2 + y^2 + 2gx + 2fy + c = 0$, where $g, f, c \in \mathbb{Z}$.



Q.4 α and β are the roots of the equation $x^2 - 3x + 1 = 0$.

Calculate the value of $\alpha - \beta$, where $\alpha > \beta$.

Write your answer in the form $\sqrt{a}$, where $a \in \mathbb{Z}$.

TIE BREAK 1

Maximum time allowed: **Six** minutes

The team may benefit if the answers are handed up before those of other teams.

Q.1 Evaluate

$$\int_1^4 \cos^2 x\, dx + \int_1^4 \sin^2 x\, dx.$$

Q.2 $z = 2 - 3i$ is a root of

$$z^2 + az + b = 0, \text{ where } a, b \in \mathbb{R}.$$

Find the value of a and the value of b.

Q.3 l is the line $2x + 3y - 1 = 0$ and k is the line $x - 2y - 5 = 0$.

Find the equation of the line that passes through the origin and through the point $l \cap k$.

Write your answer in the form $ax + by = 0$, where $a, b \in \mathbb{Z}$.

Q.4 Suppose $y = \log_{e^2} x$. Find $\dfrac{dy}{dx}$.

TIE BREAK 2

Maximum time allowed: **Six** minutes

The team may benefit if the answers are handed up before those of other teams.

Write

$$\left(\sin\frac{\pi}{3} + i\cos\frac{\pi}{3}\right)^{20}$$

in the form $\dfrac{x}{2} + i\dfrac{y}{2}$, where $x, y \in \mathbb{R}$, using surd values where relevant.

FINAL 2010
ANSWERS

Round	Question 1	Question 2	Question 3	Question 4
1	$7x - 5y - 3 = 0$ or equivalent	$\frac{4}{3}$		
2	$x = \frac{1}{4}$	$x^2 + y^2 + 17x + 11y - 10 = 0$		
3	$x = 16$ and $x = 36$	$1 \cos 2x$		
4	$\theta = 90°$ and $\theta = 270°$	$\sqrt{3}x - 1y - 1 - 2\sqrt{3} = 0$ and $\sqrt{3}x + 1y + 1 - 2\sqrt{3} = 0$ or equivalent		
5	91	2010 km		
6	$\frac{13}{3}$	$1 \times$ €10, $18 \times$ €50, $11 \times$ €100 and $6 \times$ €10, $9 \times$ €50, $15 \times$ €100		
7	$x = 3.32$	$\frac{5}{\pi}$ cm/sec	$8\pi - 16\,\text{cm}^2$	$\frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$
8	$\frac{8}{27}$	$\frac{\pi}{2}$	$x^2 + y^2 + 2x + 4y - 21 = 0$ $x^2 + y^2 - 6x - 8y - 1 = 0$	$\alpha - \beta = \sqrt{5}$
TIE 1	3	$a = -4$, $b = 13$	$9x + 17y = 0$ or equivalent	
TIE 2	$-\frac{1}{2} - i\frac{\sqrt{3}}{2}$			$\frac{dy}{dx} = \frac{1}{2x}$