Round 1 Babtha 1

Question 1

Three of the following points lie on the same straight line. Which point does **not** lie on this line?

(-2, 14) (-1, 8) (1, -1) (2, -6)

Question 2

Express as a single logarithm

$$2\log_a 6 - \log_a 3$$

Round 2 Babtha 2

Question 1

The sum of the first nine terms of a linear progression is 117. The sum of the next four terms is 91. Find the first term of the progression.

Question 2

The equation of a curve is $y = (3 - 2x)^3 + 24x$.

Find $\frac{d^2y}{dx^2}$.

Round 3 Babtha 3

Question 1

The coordinates of the points A and B are $(-1,-2)$ and $(7,4)$ respectively. Find the equation of the circle, c, for which AB is a diameter.

Answer in form $(x-h)^2+(y-k)^2=r^2$

Question 2

Find the number of different arrangements that can be made from the 9 letters of the word JEWELLERY in which the three Es are together and the two Ls are together.

Round 4 Babtha 4

Question 1

Find the area of the rectangle $ABCD$ with vertices A(-7, -7), B (8, -1), C (4, 9) and D (-11, 3).

Question 2

At the start of this month, Mathilde and Salah each had 100 coins. For Mathilde, this was 25% more coins than she had at the start of last month. For Salah, this was 20% fewer coins than he had at the start of last month. What was the total number of coins that they had at the start of last month?

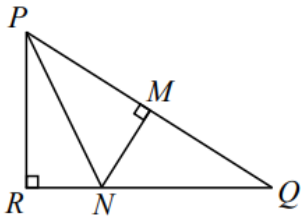
Round 5 Babtha 5

Question 1

A rectangle has length 8 cm and width π cm. A semi-circle has the same area as the rectangle. What is its radius?

Question 2

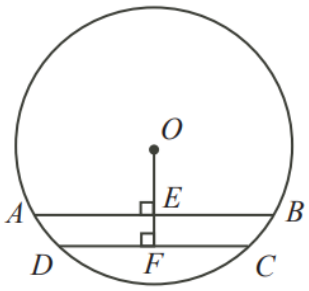
In the diagram, $\triangle PQR$ is right-angled at R, $|PR| = 12$, and $|QR| = 16$. Also, M is the midpoint of PQ and N is the point on QR so that MN is perpendicular to PQ.



What is the area of $\triangle PRN$?

Round 6 Babtha 6

Question 1



In the diagram, the circle has centre O.

OF is perpendicular to DC at F and OF is perpendicular to AB at E. If $|AB| = 8$, $|DC| = 6$ and $|EF| = 1$, determine the radius of the circle.

Question 2

A line with slope 3 and another line with slope −1 intersect at P(3, 6). What is the distance between the x-intercepts of the two lines?

Round 7 Babtha 7

Question 1

Two fair dice each have four sides painted blue and two sides painted red. If the two dice are tossed, what is the probability that exactly seven of the blue faces will be visible (that is, not face down on the table)?

Question 2

The area of the triangle RQU is 28 units squared where $R(-1, -5)$, $Q(3, -1)$ and $U(-2k, 3k)$. Find the value of k, where $k \in R$ and $k > 0$.

Question 3

The mean of t^2, $2t$ and 3 is 9. If $t < 0$, determine the value of t.

Question 4

If Q(5, 3) is the midpoint of the line segment with endpoints P(1, p) and R(r, 5), what are the values of p and r?

Round 8 Babtha 8

Question 1

Fionn wrote 4 consecutive integers on a whiteboard. Lexi came along and erased one of the integers. Fionn noticed that the sum of the remaining integers was 847. What integer did Lexi erase?

Question 2

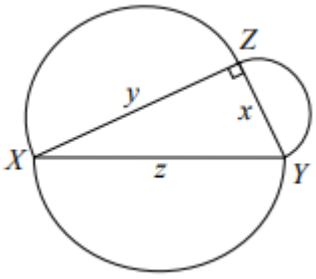
Suppose that $f(x) = x^2 + (2n-1)x + (n^2 - 22)$ for some integer n. What is the smallest positive integer n for which $f(x)$ has no real roots?

Question 3

Determine all real numbers x for which $\frac{1}{\sqrt{x^2+7}} = \frac{1}{4}$.

Question 4

In the diagram, semi-circles are drawn on the sides of right-angled ΔXYZ, as shown. If the area of the semi-circle with diameter YZ is 50π and the area of the semi-circle with diameter XZ is 288π, determine the area of the semi-circle with diameter XY.



Tie Break

1 Solve for x, y, z

$$3x - y + 3z = 1$$
$$x + 2y - 2z = -1$$
$$4x - y + 5z = 4$$

2 Simplify $\frac{-2+3i}{3+2i}$

3 The first three terms of a geometric sequence are $2x - 4$, $x + 1$ and $x - 3$. Find the two possible values of x.

4 Solve for x

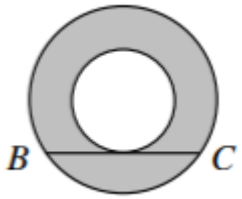
$$2\log_9 x = \frac{1}{2} + \log_9(5x + 18), x > 0.$$

5 The equation of a circle is $x^2 + y^2 = 130$. Find the slope of the tangent to the circle at the point (– 7, 9).

6 The area of a sector of a circle is $27\ cm^2$. The length of the radius of the circle is 6 cm. Find, in radians, the measure of the angle in the sector.

7 Find all the solutions of the equation $15sin^2x - 4\ cosx - 11 = 0$ in the domain $0° \leq x \leq 360°$.
Give your answers correct to the nearest degree.

8 The diagram shows two concentric circles. A tangent to the inner circle cuts the outer circle at B and C, where $|BC| = 2x$. Express the area of the shaded region in terms of x.



9 Five cards are drawn together at random from a standard pack of 52 playing cards.
Find, in decimal form, correct to two significant figures, the probability that the five cards include the four aces.

10 Find the two complex numbers $a + bi$ such that

$$(a + bi)^2 = -3 + 4i.$$

Answers

R1	Q1	$(-1,8)$
	Q2	$\log_a 12$
R2	Q1	7
	Q2	$72 - 48x$
R3	Q1	$(x-3)^2 + (y-1)^2 = 5^2$
	Q2	720
R4	Q1	174 sq units
	Q2	205
R5	Q1	$4\ cm$
	Q2	21
R6	Q1	$r = 5$
	Q2	8
R7	Q1	$\frac{4}{9}$
	Q2	$k = 2$
	Q3	$t = -6$
	Q4	p = 1 and r = 9.
R8	Q1	283
	Q2	$n = 23$
	Q3	$x = \pm 3$ (both needed for point)
	Q4	338π
Tiebreak	1	$x = -1, y = 2, z = 2$
	2	i
	3	1 & 11
	4	$x = 18$
	5	$m = \frac{7}{9}$
	6	$1.5\ Rads$
	7	$66°, 132°, 228°, 294°$ (degrees symbol not needed but can be used to separate teams if still tied after tiebreak).
	8	πx^2
	9	$0 \cdot 000018$
	10	$\pm(1 + 2i)$

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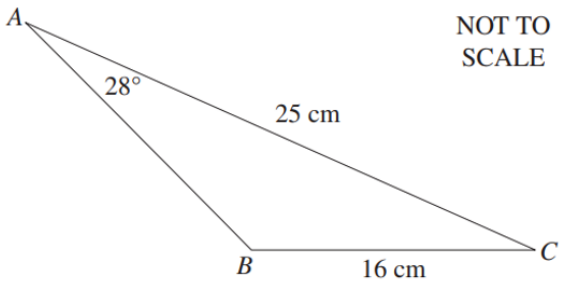
Babhta 1 **Round 1**

Q1

The diagram shows a triangle ABC where $AC = 25$ cm, $BC = 16$ cm, $\angle BAC = 28°$ and ABC is obtuse.



Find the size of the obtuse angle ABC correct to the nearest degree.

Q2

a, b, c and *d* are four integers written in order of size, starting with the smallest. The sum of *a, b* and *c* is 70 and the mean of *a, b, c*, and *d* is 25. The range of the four integers is 14.

Work out the median of *a, b, c,* and *d*.

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Babhta 2 **Round 2**

Q1

Sheldon has a large number of cards. On the back of each card there is either a honeycomb pattern (hexagons) or a brick pattern (rectangles). On the front of each card there is either a 1 or a 2. As Sheldon went through all the cards, he found that 30% of the cards have a honeycomb pattern on the back. Of the cards with a brick pattern on the back, 80% have a 1 on the front.
Determine the percentage of all the cards that have a brick pattern on the back **and** a 2 on the front.

Q2

If $z_1 = a - 6i$, $z_2 = 1 + bi$ and $z_1 z_2 = -17 - 9i$, find the integer values of a **and** b**.**

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Babhta 3 **Round 3**

Q1

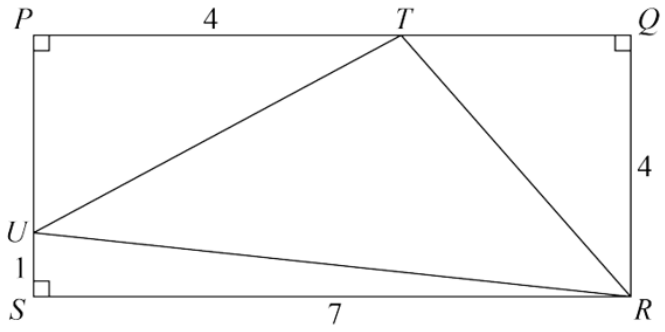
Arya has never travelled to another country, but has a collection of foreign coins given to him by friends and family who have. In his collection he has 10 coins from Africa, 6 coins from Asia, 7 coins from South America, and 8 coins from Europe. One day Arya's grandfather added some Australian coins to the collection.

After he did that, he told Arya that if he took a coin at random from the collection, the probability of it being from either Africa or Asia was $\frac{4}{9}$.

How many Australian coins did Arya's grandfather add to the collection?

Q2

Rectangle $PQRS$ has $QR = 4$ and $RS = 7$. ΔTRU is inscribed in rectangle $PQRS$ with T on PQ such that $PT = 4$ and U on PS such that $SU = 1$. Determine the **exact** value of $\angle RUS + \angle PUT$.



Babhta 4 **Round 4**

Q1

The Bord Gais Energy Theatre has four levels of seating: gold, silver, red, and black. One night, the manager of the theatre was asked how many patrons are in the theatre. The manager replied that $\frac{1}{6}$ of the patrons in the theatre that night are in the gold seating, $\frac{1}{4}$ of the patrons are in either the red seating or the black seating, there are three times as many patrons in the silver seating as in the red seating, and there are 138 patrons in the black seating. How many patrons were in the theatre that night?

Q2

The line l_1 has equation $3x - 2y = 10$ and crosses the x-axis at point A.
The line l_2 is perpendicular to l_1 and crosses the x-axis at (9, 0). l_2 crosses the y-axis at B**.**

Find the area of the triangle OAB where O is the origin.

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Babhta 5 **Round 5**

Q1

Simeon has a rope that is 108 cm long and is asked to cut the rope once so that one of the pieces can be arranged, with its two ends touching, to form a square, and the other piece can be arranged, with its two ends touching, to form a rectangle with one side length of 6 cm. Furthermore, the area of the square will be equal to the area of the rectangle. Where should Simeon make the cut to the original piece of rope?

Q2

A curve has the equation $y = x\sqrt{x} + \frac{48}{\sqrt{x}}$, $x > 0$.

Find the coordinates of the point on the curve where the gradient is 0.

Babhta 6 **Round 6**

Q1

A restaurant owner was experimenting the taste of her homestyle lemon drink. She started with 60 litres of water. She removed 15 litres of water and replaced it with 15 litres of pure lemon juice.

After thoroughly stirring the new mixture, she discovered that it was too 'lemony' So, she removed 10 litres of the new mixture and replaced it with 10 litres of water. She thoroughly stirred the mixture and concluded that the new mixture was just right.

Determine the ratio of pure lemon juice to water in the final 60 litre mixture.

Q2

In the expansion of $(3 + px)^3$ the coefficient of x^2 is twice the coefficient of x^3.

Find the non-zero value of p.

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Babhta 7 **Round 7**

Q1
A particle is shot vertically upwards from a point 100 metres above ground level. The position of the particle, y metres above the ground after t seconds, is given by $y(t) = -5t^2 + 70t + 100$.
Find the maximum height above ground level reached by the particle.

Q2
Calculate the sum of the arithmetic series 4 + 10 + 16 + · · · + 1354

Q3
History and Geography are two of the subjects students may decide to study. For a group of 40 students, the following is known.
• 7 students study neither History nor Geography
• 20 students study History
• 18 students study Geography
A student is chosen at random. Find the probability that the student studies **both** History and Geography (answer as fraction in simplest form).

Q4
In a particular country, the hourly rate of pay for adults who work is normally distributed with a mean of €25 and a standard deviation of €5. Two adults who both work are chosen at random. Find the probability that at least one of them earns between €15 and €30 per hour. (answer as a decimal correct to 4 decimal places)

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Babhta 8 **Round 8**

Q1

A population, P, which is initially 5000, varies according to the formula

$$P = 5000b^{\frac{-t}{10}},$$

where b is a positive constant and t is time in years, $t \geq 0$.

The population is 1250 after 20 years.

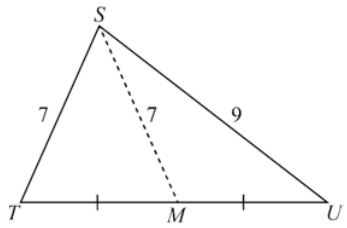
Find the value of t, correct to one decimal place, for which the instantaneous rate of decrease is 30 people per year.

Q2

Write $\frac{(1-\sqrt{3}i)^4}{1+\sqrt{3}i}$ in the form $a + bi$, where a and b are real constants.

Q3

In ΔSTU, a median is drawn from vertex S, meeting side TU at point M. The length of side ST is 7 cm, the length of side SU is 9 cm and the length of the median SM is 7 cm.



Calculate the length of TU.

Q4

For what values of x, in the interval $0 \leq x \leq \frac{\pi}{4}$, does the line $y = 1$ intersect the graph of $y = 2\sin 4x$?

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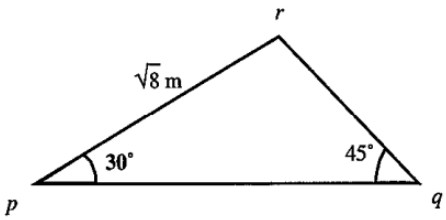
Tie-break

1. Express $\frac{1-\sqrt{2}}{1+\sqrt{2}}$ in the form $a\sqrt{2}-b$, where $a, b \in N$
2. Solve for x, y, z

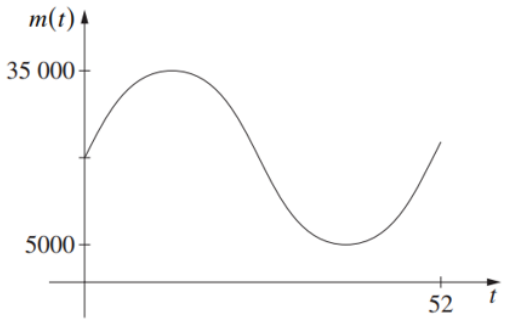
$$x + y - z = 0$$
$$x - y + z = 4$$
$$x - y - z = -8$$

3. Differentiate $\frac{2x-3}{x+1}$ with respect to x, answer in simplest form.
4. Find $\frac{dy}{dx}$ if $y = ln\sqrt{x^2+1}$, answer in simplest form.
5. Points $(1,-1)$, $(-6,-2)$ and $(3,-5)$ are on a circle C.
   Find the equation of C.
6. Find the area of the triangle shown in the diagram,
   correct to 1 decimal place.



7. Four numbers have a mean of p. Five numbers have a mean x. The nine numbers have a mean of q. Express x in terms of p and q.

8. The population of mice on an isolated island can be modelled by the function $m(t) = a\sin\left(\frac{\pi}{26}t\right) + b$, where t is the time in weeks and $0 \leq t \leq 52$. The population of mice reaches a maximum of $35{,}000$ when $t = 13$ and a minimum of $5{,}000$ when $t = 39$. The graph of $m(t)$ is shown.
   What are the values of a **and** b?



9. Find the value of $\frac{x}{y}$ when $\frac{2x+3y}{x+6y} = \frac{4}{5}$
10. Three consecutive terms of an arithmetic series are $4x + 11, 2x + 11$ and $3x + 17$.
    Find the value of x.

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Answer Key 2023

Round 1	
1	$133°$
2	27
Round 2	
1	14%
2	$a = 1, b = -3$ *(both required for mark)*
Round 3	
1	5
2	$135°$
Round 4	
1	2484
2	10 *(ignore any reference to units)*
Round 5	
1	$48cm$ **OR** $60cm$
2	$(4,32)$
Round 6	
1	$5:19$
2	$\frac{9}{2}$ **OR** 4.5
Round 7	
1	$345\ m$
2	$153{,}454$
3	$\frac{1}{8}$
4	0.9671
Round 8	
1	$t = 35.3$
2	$4 + 4\sqrt{3}i$
3	$8\ cm$
4	$x = \frac{\pi}{24}$ **and** $x = \frac{5\pi}{24}$ *(both required for mark)*

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Tiebreak Answers

1	$2\sqrt{2}-3$
2	$x=2, y=4, z=6$
3	$\frac{5}{(x+1)^2}$
4	$\frac{x}{x^2+1}$
5	$(x+2)^2+(y+5)^2=25$ **OR** $x^2+y^2+4x+10y+4=0$
6	2.7
7	$\frac{9q-4p}{5}$
8	$a=15{,}000$ and $b=20{,}000$
9	1.5 or $\frac{3}{2}$
10	-2

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Round 1 Babtha 1

Question 1

A drawer contains 5 red markers and n blue markers. One marker is drawn at random and not replaced. A second marker is then selected at random. If the probability that both markers are blue is $\frac{1}{6}$, find the total number of markers in the box at the start.

Question 2

Determine the real numbers p and q such that $(p + iq)^2 = 5 + 12i$, where $i = \sqrt{-1}$.

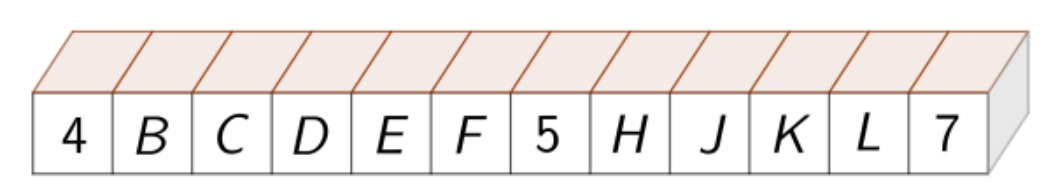
Round 2 Babtha 2

Question 1

A fair, six-sided die is thrown seven times. What is the probability that a '6' occurs on exactly 2 of the 7 throws? Give your answer correct to 3 decimal places.

Question 2

Twelve blocks are arranged as illustrated in the diagram.



Each letter shown on the front of a block represents a number. The sum of the numbers on any four consecutive blocks is 25.

Determine the value of B + F + K.

Round 3 Babtha 3

Question 1

The positive integers can be arranged as follows.

Row 1	1					
Row 2	2	3				
Row 3	4	5	6			
Row 4	7	8	9	10		
Row 5	11	12	13	14	15	
Row 6	16	17	18	19	20	21
⋮	⋮	⋮	⋮	⋮	⋮	⋮

More rows and columns continue to list the positive integers in order, with each new row containing one more integer than the previous row. How many integers, less than 2022, are in the column that contains the number 2022?

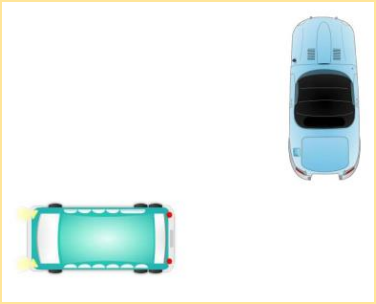
Question 2

XYZ is a triangle where $|XY| = 8$ cm and $|YZ| = 6$ cm. Given that the area of triangle XYZ is $12cm^2$, find the two possible values of $|\angle XYZ|$.

Round 4 Babtha 4

Question 1

At 7:00 a.m., Sahil drives north at 48 km/h. At the same time from the same intersection, Brenda drives west at 64 km/h. At what time will they be 260 km apart?



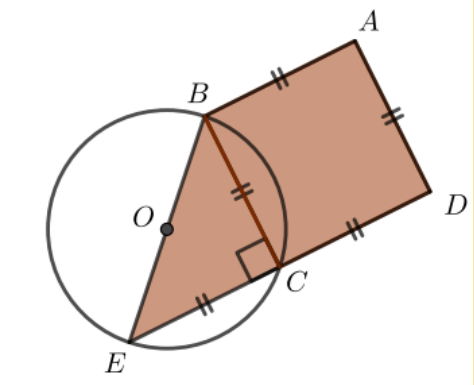
Question 2

Georgina enters a 12 km race. She wants to finish the race in one hour and twenty minutes. She starts off jogging at a speed of 7 km/h. After 30 minutes, she realizes that she needs to increase her speed to finish the race in her desired time. For the remaining time, what speed must she run at to finish the race in exactly one hour and twenty minutes?

Round 5 Babtha 5

Question 1

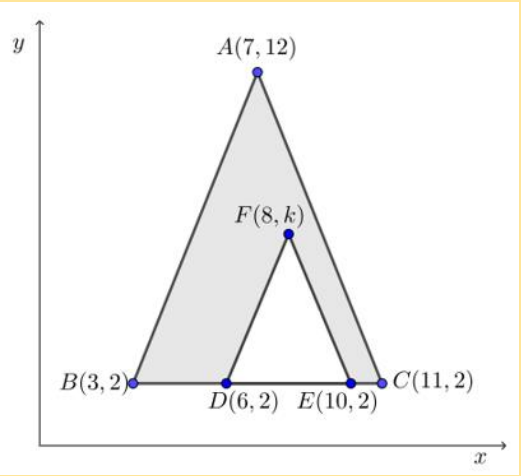
Quadrilateral ABED is made up of square ABCD and right isosceles triangle BCE. BE is a diameter of the circle with centre O. Point C is also on the circle.



If the area of ABED is $24cm^2$, what is the length of BE, correct to 1 decimal place?

Question 2

Points $A(7,12)$, $B(3,2)$, $C(11,2)$, $D(6,2)$ and $E(10,2)$ are placed on the Cartesian plane, as shown. The point F is placed inside ΔABC so that the area of the shaded region is $32\ units^2$. If the point F is $(8,k)$, find the value of k.

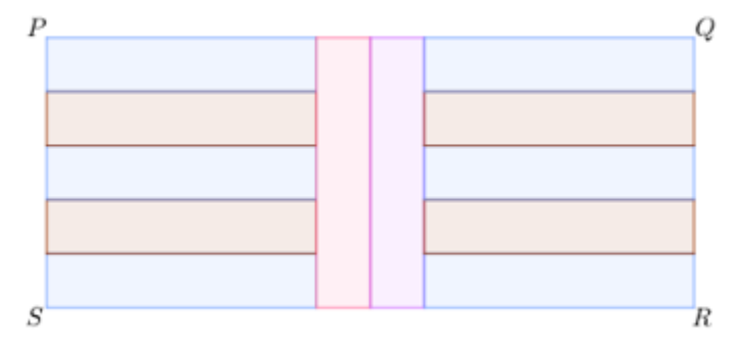


Round 6 Babtha 6

Question 1

Twelve identical smaller rectangles are arranged as shown in the diagram to form a large rectangle $PQRS$.

If the area of rectangle $PQRS$ is $540\ cm^2$, determine the integer dimensions of the smaller rectangles.



Question 2

By solving the following simultaneous equations, determine the value of $x + y + z$

$$3x - y + 3z = 1$$

$$x + 2y - 2z = -1$$

$$4x - y + 5z = 4.$$

Round 7 Babtha 7

Question 1

The first three terms of a geometric sequence are $2x - 4$, $x + 1$, and $x - 3$. Find the two possible values of x.

Question 2

Bill and Ben win some money in their local lottery. They share the money in the ratio $3: 4$. Ben decides to give €40 to his sister. The amount that Bill and Ben have now is in the ratio $6: 7$. Calculate the total amount of money won by Bill and Ben.

Question 3

The area of a sector of a circle is 27 cm^2. The length of the radius of the circle is 6 cm. Find, in radians, the measure of the angle in the sector.

Question 4

By writing $cos2x$ in terms of $sinx$, or otherwise, find all the solutions of the equation $cos2x - sinx = 1$, in the domain $0° \leq x \leq 360°$.

Round 8 Babtha 8

Question 1

The junior and senior students at Mathville High School are going to present an exciting musical entitled, "Math, What is it Good For?". A large group of students came out to an information meeting. After a brief introduction to the musical, 15 senior students decided that it was not for them, and they left. At that point, twice as many junior students as senior students remained. Later in the meeting, after the 15 senior students had left, $\frac{3}{4}$ of the junior students and $\frac{1}{3}$ of the remaining senior students also left. This left 8 more senior students than junior students. All the remaining students stuck it out and went on to produce an amazing show. How many students remained to perform in the school musical, "Math, What is it Good For?"

Question 2

α and β are real numbers such that $\alpha + \beta = -7$ and $\alpha\beta = 11$.

Find the value of $\alpha^2 + \beta^2$.

Question 3

Solve $\log_6(x+5) = 2 - \log_6 x$, for $x > 0$.

Question 4

Given $y = 2x - sin2x$, $\frac{dy}{dx}$ can be written in the form $ksin^2x$, where $k \in Z$. Find the value of k.

Tiebreak Questions

1	What is the sum of the positive integers from 1 to 1,000,000?
2	Find the acute angle between the lines $x - 2y + 1 = 0$ and $y = 3x - 4$. Answer in degrees (no unit needed).
3	Nine cards are numbered from 1 to 9. Three cards are drawn at random from the nine cards. The probability that the card numbered 8 is not drawn is written in its simplest form as $\frac{a}{b}$. Find the value of $a + b$.
4	The polynomial $Q(x)$that satisfies $x^3 + 2x^2 - 3x - 7 = (x - 2)Q(x) + 3$ is written in its simplest form, $ax^2 + bx + c$. Find the value of $a + b + c$.
5	The point *P* divides the interval from $A(-4, -4)$ to $B(1,6)$ internally in the ratio $2:3$. Find the x-coordinate of *P*.
6	Find the term independent of x in the binomial expansion of $\left(x^2 - \frac{1}{x}\right)^9$.
7	A is an acute angle such that $\tan A = \frac{8}{15}$. $\sin 2A$ can be written in its simplest form as $\frac{p}{q}$. Evaluate $p + q$.
8	The equation of the tangent to the curve $y = \sqrt{x + 2}$ at the point (7, 3) can be written in its simplest form as $ax + by + c = 0$. What is the value of $a + b + c$?
9	An infinite geometric series has a first term of 8 and a limiting sum of 12. The common ratio can be written as $\frac{p}{q}$, in simplest form. Calculate $p + q$.
10	If the recurring decimal $1.\dot{2}$ is written in its simplest form $\frac{a}{b}$ where $a, b \in \mathbb{N}$, find the value of $a + b$.

BABHTA 1 ROUND 1

Q1.1) In a given triangle ABC, using the usual notation, side a = 3 cm, side b = 4 cm and side c = 5 cm
Find the numerical value of Sin(A)+ Sin(2B) + Sin(3C).

Answer in form $\frac{a}{b}$,where a and b $\in$ N, in simplest form.

Q1.2) Find, in terms of n, the value of ${}^{n}C_{1} + 6({}^{n}C_{2}) + 6({}^{n}C_{3})$

Answer in simplest form

BABHTA 2 ROUND 2

Q2.1) Four integers are added to the set {3, 4, 5, 5, 8} increasing the mean, median and mode each by 1.

Write down the elements of the new set **in ascending order.**

Q2.2) What is the value of the base **b**, in surd form, when

$$\log_b(10) + \log_b(10)^2 + \log_b(10)^3 + \ldots + \log_b(10)^{10} = 110$$

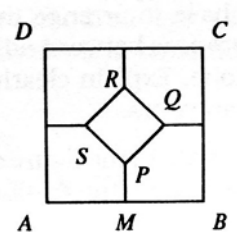
BABHTA 3 ROUND 3

Q3.1) Find the value of the integral $\int(1+\mathrm{Sin}(3x))dx$

Q3.2) Find the 5th term of the expansion $(x-3y)^{12}$.
Answer in simplest form ax^py^q, where a, p, q $\in$ N.

BABHTA 4 ROUND 4

Q4.1) ABCD and PQRS are concentric squares. The diagonals of PQRS are parallel to the sides of ABCD.



If |AB| is 1 cm and |PQ| = $\frac{1}{2}$ cm, calculate the length of [PM],
where M is the midpoint of the line segment [AB].

Answer in form $\frac{a - \sqrt{b}}{c}$, where a , b and c $\in$ N,
in simplest form.

Q4.2) Find all the cube roots of $4 + 4\sqrt{3}i$.

Answers in the form r(Cos(A) + iSin (A)), where A is in degrees.

BABHTA 5 ROUND 5

Q5.1) Given that a and b $\in$ R, o < a < b and $a^2 + b^2 = 6ab$ find the numerical value of $\frac{a+b}{a-b}$.

Answer in simplest surd form.

Q5.2) Solve for x and y

$x(x + y) = 108$

$y(x + y) = \frac{297}{4}$

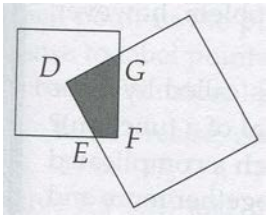
Answers in the form (x, y) where x and y $\in$ Q

BABHTA 6 ROUND 6

Q6.1) A jar has 499 fair coins and one similar coin with heads on both sides.
A coin is chosen from the jar at random and flipped 9 times.
Given that it comes up heads every time, what is the probability that the coin is the two headed coin?

Answer in form $\frac{a}{b}$, where a and b $\in$ N.

Q6.2) A square of side 3 m and a square of side 4 m overlap as shown in the diagram.
D is the centre of the 3 m square.
Find the area of the shaded region DGFE.



Answer in the simplest form $\frac{a}{b}$, where a and b $\in$ N.

BABHTA 7 ROUND 7

Q7.1) Solve for x : $|x^2 - 4x + 5| \leq 6$, where $x \in R$.

Answer in the form $a \leq x \leq b$ in simplest surd form.

Q7.2) If $\frac{5(10^{27}-1)}{9}$ is written as an integer how many times does 5 appear in the answer?

Q7.3) Two vertices of an equilateral triangle lie on the line $y = 2x - 2$, and the third vertex lies on the line $y = 2x + 2$.
Find the area of the triangle.

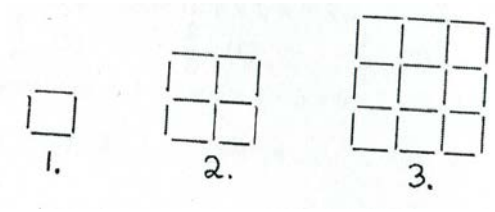
Answer in form $\frac{a\sqrt{b}}{c}$, where a, b and $c \in N$.

Q7.4) Three fair 6-sided dice are thrown.
What is the probability that the three numbers rolled are three consecutive numbers, in some order?

Answer in the simplest form $\frac{a}{b}$, where a and $b \in N$.

BABHTA 8 ROUND 8

Q8.1) Square grid patterns of matches are shown, namely a 1-square, a 2-square and a 3-square.



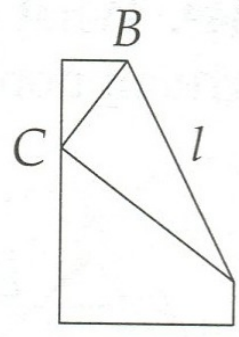
How many matches must be added to a 500-square to make a 501-square?

Q8.2) Find all the complex numbers such that $|z - 1| = |z + 3| = |z - i|$, where $z = x + iy$.

Answers in the form $a + ib$ where a and $b \in R$.

Q8.3) Find the values of x and y for which $x^2 + 6x + y^2 = 4$.
Answers in the form (x , y), where x and y $\in$ Z.

Q8.4) A corner of a rectangular piece of paper of width 8 cm is folded over so that it coincides with the point C on the opposite side.
Given that |BC| = 5 cm, calculate, in cms, the length of the fold *l*.



Answer in form $a\sqrt{b}$, where a and b $\in$ N.

Cumann Oidí Matamaitice na hÉireann : Irish Mathematics Teachers Association
MATA FÓIRNE 2020,Babhta Réigiúnach: TEAM MATHS 2020, Regional Round

SCOILT TIEBREAK

T1) Find all the numerical values of x for which $7^{2x} = 45(7^x) + 4(7^2)$

T2) What is the relationship between a and b $\in$R for which the line x + y = a is a tangent to the circle $x^2 + y^2 = b$?

T3) What is the remainder when 8^{2020} is divided by 9?

T4) John will need €7000 in three years time to purchase a car.
How much should he deposit at the end of each month into an account that pays 6% EAR to achieve his goal?
Answer to the nearest euro.

T5) Find the greatest integer, x, for which $3^{20} > 32^x$?

T6) Find the acute angle between the lines 2x – y = 3 and x – 3y = 4.

T7) Find the area enclosed in the graph $x^2 + y^2 = 16x + 32y$.
Answer in terms of π.

T8) Given $1.2 \leq a \leq 5.1$ and $3 \leq b \leq 6$ find the highest possible value for the quotient $\frac{a}{b}$.

.

T9) Find all the ordered pairs of real numbers (x , y), such that

$x^3 - y^3 = 19$ and

$x^2y - xy^2 = 6$

T10) Find the area of the triangle with vertices (5,4), (3,6) and (2,1)

Answer Key Regional Round

Round 1

Q1.1 $\frac{14}{25}$ Q1.2 n^3

Round 2

Q2.1 {3,4,5,5,6,6,6,8, 11} Q2.2 b = $\sqrt{10}$

Round 3

Q3.1 $x - \frac{Cos(3x)}{3} + C$ Q3.2 40095 x^8y^4

Round 4

Q4.1 $\frac{2-\sqrt{2}}{4}$

Q4.2 $2(Cos(20^0) + iSin(20^0)), 2(Cos(140^0) + iSin(140^0)), 2(Cos(260^0) + iSin(260^0))$

Round 5

Q5.1 $-\sqrt{2}$ (only) Q5.2 $(8, \frac{11}{2}), (-8, -\frac{11}{2})$

Round 6

Q6.1 $\frac{512}{1011}$ Q6.2 $\frac{9}{4}$

Round 7

Q7.1 $2 - \sqrt{5} \leq x \leq 2 + \sqrt{5}$ Q7.2 27

Q7.3 $\frac{16\sqrt{3}}{15}$ Q7.4 $\frac{1}{9}$

Cumann Oidí Matamaitice na hÉireann : Irish Mathematics Teachers Association
MATA FÓIRNE 2020,Babhta Réigiúnach: TEAM MATHS 2020, Regional Round

Round 8

Q8.1 2004

Q8.2 - 1 – i

Q8.3 $(0, \pm 2),(-6, \pm 2),(-1, \pm 3),(-5, \pm 3)$

Q8.4 $5\sqrt{5}$

Tiebreak

T1 2

T2 $b = \frac{a^2}{2}$ or $a = \pm\sqrt{2b}$

T3 1

T4 €178

T5 6

T6 45^0

T7 320π

T8 1.7

T9 (3, 2) and (-2, - 3)

T10 6

Babhta 1 Round 1

Q1.1 Find the equation of the line passing through (- 3, 2) which is perpendicular to the line $5x - 2y + 7 = 0$.

Answer in the form $ax + by + c = 0$, where a , b and $c \in Z$.

Q1.2 How many triangles can be formed using the vertices of a regular dodecagon (12 sided polygon).

Answer in simplest form.

Babhta 2 Round 2

Q2.1 Given that $0^\circ \leq A \leq 360^\circ$ find, in degrees, the values of A for which Tan(A) = 2Sin(A).

Q2.2 Given z = x + iy find all the solutions for which $(z - 1)^3 = 1$
Answers in form a + ib, a, b $\in$ R in non decimal form, where appropriate where i = $\sqrt{-1}$.

Babhta 3 Round 3

Q3.1 Find the numerical value of the integral $\int_1^2 (4x + 4 - \frac{1}{x^2})\,dx$

Answer in form $\frac{a}{b}$ where a and b $\in$ N.

Q3.2 Given $\log_{15}(5) =$ **a** write $\log_{15}(9)$ in terms of **a.**

Babhta 4 Round 4

Q4.1 Find the points of intersection of the circle
$x^2 + y^2 - 4x - 6y + 9 = 0$
and the line $x + y - 7 = 0$

Answers in form (x,y) , where x and y $\in$ Z.

Q4.2 What is the probability that the ace of hearts and the ace of spades in a standard pack of 52 cards will occur next to one another in a thoroughly shuffled standard deck of 52 cards?

Answer in simplest form $\frac{a}{b}$, where a and b $\in$ N

Babhta 5 Round 5

Q5.1 Solve for real x : $\frac{x^2+x-1}{x^2+1} < \frac{1}{2}$.

Q5.2 Divide $x^6+x^4+8x^3-12x^2-3x+15$ by x^3-3x+3.

Babhta 6 Round 6

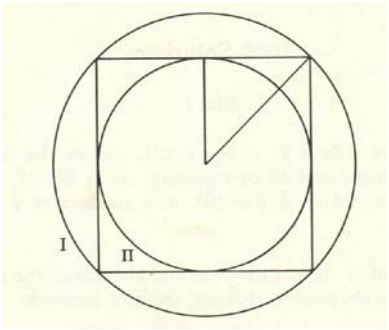
Q6.1 15 balls are arranged in a straight line at intervals of 2 metres and a bucket is placed in the same straight line, 5 metres from the first ball to the left of the leftmost ball. The competitor has to start from the bucket, bring the balls one at a time and place them in the bucket.

What is the total distance he has to travel?

Q6.2 Two congruent 30° - 60° - 90° triangles are placed so that their hypotenuses coincide.
If the hypotenuse of each triangle is 12 cm, calculate the area common to both triangles.

Answer in simplest form $a\sqrt{b}$, where a and b $\in$ N.

Babhta 7 Round 7



Q7.1 Circle I is circumscribed about a given square and circle II is inscribed in the same given square.
Find the ratio of the area of circle I to circle II.

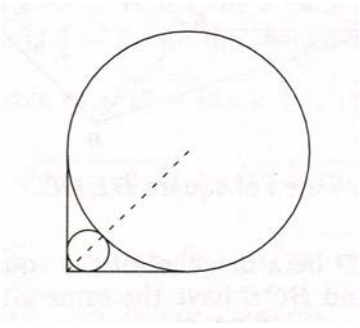
Answer in simplest form a : b, where a and b $\in$ N.

Q7.2 Find the values of x for which $3^{2+x} + 3^{2-x} = 82$.

Q7.3 Given that $\frac{1}{p+q} = \frac{1}{p} + \frac{1}{q}$, find the numerical value of $(\frac{p}{q})^3$

.

Q7.4 Two line segments are perpendicular and are tangents to the large circle.
It is required to place a small circle in the space left by the large circle as shown.
Given that the radius of the large circle is **r** and the radius of the small circle is **s** find the ratio **r : s.**



Answer in form a + $b\sqrt{c}$: 1 , where a, b and c $\in$ N.

Babhta 8 Round 8

Q8.1 Given that a, b and c are real numbers such that

$$a^2+2b=7,\ b^2+4c=-7 \text{ and } c^2+6a=-14$$

Find the numerical value of $a^2+b^2+c^2$.

Q8.2 The equation $x^2+px+q=0$, where p and q are different numbers, has solutions $x = p$ and $x = q$ where p and $q \neq 0$

Determine all such ordered pairs (p , q), p and $q \in Z$.

Answer in simplest form.

Q8.3 Find the possible values of $p \in R$ so that the two lines

$x - y = 2$ and $px + y = 3$ intersect strictly in the first quadrant.

Q8.4 At the beginning of each of 20 consecutive years €250 is invested at 4% APR.
Find the total amount at the end of the 20 years.

Answer to nearest euro.

Cumann Oidí Matamaitice na hÉireann : Irish Mathematics Teachers Association

FOIREANN MATA 2019 , Babhta Réigiúnach TEAM MATHS 2019 , Regional Round

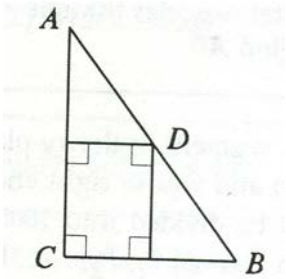
Tiebreak Round 1

T1) How many digits are there in the product $2^{2019} \times 5^{2009}$?

T2) The square of a positive integer is called a perfect square.
If x is a perfect square find the next larger perfect square.

Answer in simplest form.

T3) Three vertices of the rectangle shown are the midpoints of the sides of the triangle ABC. Given that the area of ABC is 24 units, what is the area of the rectangle?



T4) The weights of a large group of people are normally distributed with a mean of 68 kg and a standard deviation 10 kg. If a person, chosen at random has a weight x kg, find the probability that this weight lies between 60 kg and 70 kg.

T5) Find all the values of x and y for which $x^6 = y^2 + 53$.

Answer in the form (x, y) where x and y $\in$ N.

T6) Given a + b =1 and $a^2 + b^2 = 2$, find the value of $a^4 + b^4$.

Answer in simplest form $\frac{a}{b}$, where a and b $\in$ N

T7) Solve the pair of simultaneous equations :

$$xy + 7x + 2y + 14 = 0$$
$$xy - 4x - 5y + 20 = 0$$

Answer in form (x, y) , where x and y $\in$ Z

T8) Given $x = \frac{1}{2 - \sqrt{3}}$ and $y = \frac{\sqrt{3}}{3}$, find the value of $\frac{x - y}{1 + xy}$.

Answer in simplest form.

Cumann Oidí Matamaitice na hÉireann : Irish Mathematics Teachers Association

FOIREANN MATA 2019 , Babhta Réigiúnach TEAM MATHS 2019 , Regional Round

Answer Key Regional Round

Round 1

Q 1.1 $2x + 5y - 4 = 0$

Q1.2 220

Round 2

Q 2.1 $0^0, 180^0, 360^0, 60^0, 300^0$

Q2.2 2, $\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

Round 3

Q3.1 $\frac{19}{2}$

Q3.2 2(1 – a) or 2 – 2a

Round 4

Q4.1 (2 , 5) , (4 , 3)

Q4.2 $\frac{1}{26}$

Round 5

Q5.1 - 3 < X < 1

Q5.2 $x^3 + 4x + 5$

Round 6

Q6.1 570

Q6.2 $12\sqrt{3}$

Round 7

Q7.1 2 : 1

Q7.2 - 2 , 2

Q7.3 1

Q7.4 $3 + 2\sqrt{2}$: 1

Round 8

Q8.1 14

Q8.2 (1, -2)

Q8.3 $-1 < p < \frac{3}{2}$

Q8.4 €7742

Cumann Oidí Matamaitice na hÉireann : Irish Mathematics Teachers Association

FOIREANN MATA 2019 , Babhta Réigiúnach TEAM MATHS 2019 , Regional Round

Tiebreak

T1	2013	T2	$x + 2\sqrt{x} + 1$	T3	12
T4	0.3674	T5	(3 , 26)	T6	$\frac{7}{2}$
T7	(5 , -7) , (-2 , 4)	T8	1		

TEAM MATHS 18 , REGIONAL ROUND , January 26, 2018

Round 1

Q1.1) Find both square roots of – 7 – 24i , where i = $\sqrt{-1}$.
Answers in form a + ib, where a and b $\in$ Z.

Q1.2) What is numerical value of the units digit of the following sum ?

$$13^{841} + 17^{508} + 24^{617}$$

Round 2

Q2.1) When $(1 - \frac{1}{a})^6$ is expanded , find the sum of the coefficients of the three highest powers of $\frac{1}{a}$.

Q2.2) A square and an equilateral triangle have equal perimeters. The area of the triangle is $9\sqrt{3}$ cm^2.Find , in cms, the length of the diagonal of the square.
Answer in form $\frac{a\sqrt{b}}{c}$, where a , b and c $\in$ N.

Round 3

Q3.1) Find the range of real values of x for which $(x + 8)(x – 3) < 3x$

Q3.2) For what values of p and q is the expression $x^4 + 6x^3 + 13x^2 + px + q$ a perfect square?

Round 4

Q4.1) In a given triangle ABC , M is the midpoint of [BC] .
Write $|AB|^2 + |AC|^2$ in terms of |AM| and |MB|.

Q4.2) Let f be a function such that $f(x-1) = 2x^2-10x+3$.
Find the value of f(x) in terms of x

Round 5

Q5.1) Two circles, radii 7 cm and 9 cm, cut each other.
Given that their centres are 11 cm apart find the length of the common chord.
Answer correct to 1 decimal place.

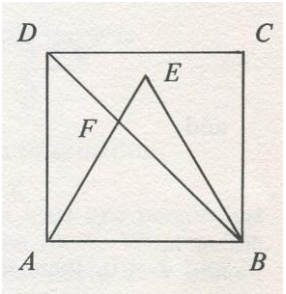
Q5.2) Find the equation of the line which passes through the point (- 5, 4) such that the portion of the line between the axes is divided by the point, **from x-axis to y-axis** in the ratio 1 : 2.

Answer in form ax + by + c = 0, where a , b and c $\in$ Z.

Round 6

Q6.1) Given that $(\sqrt{3}+\sqrt{2})^2 = 5+2\sqrt{6}$ and $(\sqrt{3}-\sqrt{2})^2 = 5-2\sqrt{6}$ find the values of x for which $(5+2\sqrt{6})^{\sin(x)} + (5-2\sqrt{6})^{\sin(x)} = 2\sqrt{3}$, where $0^0 \le x \le 360^0$

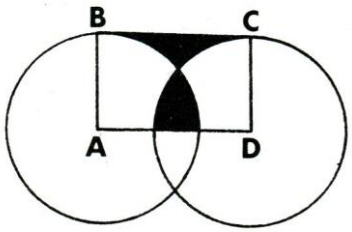
Q6.2) Vertex E of an equilateral triangle ABE is in the interior of a square ABCD, and F is the point of intersection of the diagonal [BD] and the line segment [AE].
Given that $|AB| = \sqrt{1+\sqrt{3}}$ calculate the area of the triangle ABF.
Answer in form $\frac{\sqrt{a}}{b}$, where a and b $\in$ N.



Round 7

Q7.1) The lengths of the medians of a right-angled triangle which are drawn from the vertices of the acute angles are 5 cm and $\sqrt{40}$ cm.
Calculate the length of the hypotenuse of the triangle.
Answer in form $a\sqrt{b}$, where a and b $\in$ N.

Q7.2) In the diagram are two congruent circles,



with centres A and D. Radii [AB] and [CD] intersect the tangent [BC].
Given that the shaded regions have equal areas and |AB| = 1, find the area of the quadrilateral ABCD.

Q7.3) Simplify $\dfrac{12}{3+\sqrt{5}+2\sqrt{2}}$

Answer in simplest surd form.

Q7.4) For how many values of **a** do the equations

$$x^2 + ax + 1 = 0 \text{ and } x^2 - x - a = 0$$

have a common real solution?

Round 8

Q8.1) Solve simultaneously for x and y :

$$x^2 - xy + y^2 = 7$$
$$x - xy + y = -1$$

Answer in form (x , y) , where x and y Ï N

Q8.2) For what positive values of m is the line $x + y = \sqrt{2m}$ a tangent to the circle $x^2 + y^2 = m$?

Q8.3) Given that q is an acute angle and $\text{Sin}(\frac{q}{2}) = \sqrt{\frac{x - 1}{2x}}$, find the value of $\text{Tan}(q)$ in terms of x.

Q8.4) Two equal parallel chords are drawn 8 cm apart in a circle of radius 8 cm. Calculate the area of the part of the circle that lies between the chords.

Answer in form $a\sqrt{b} + \frac{c\pi}{d}$, where a , b , c and d $\in$ N.

TIEBREAK

T1) Calculate the value of $\frac{e^x - e^{-x}}{e^x + e^{-x}}$, where e = 2.718 and x = 0.378.

Answer correct to four significant figures.

T2) L(- 1, 0) , M(3, 7) and N(5, -2) are the midpoints of the sides [BC], [CA] and [AB] respectively of the triangle ABC.
Find the equation of [AB].

Answer in form ax + by + c = o, where a, b and c $\in$ Z.

T3) For $0^O \leq A \leq 360^O$, find the values of A for which 3Cos(2A) + Sin(A) = 1
Answers to nearest degree.

T4) Given xy = b and $\frac{1}{x^2} + \frac{1}{y^2} = a$, calculate the value of $(x + y)^2$ in terms of a and b.

T5) In how many ways can the letters of the word **ORANGE** be arranged so that the three vowels must not come together?

T6) The line **y = mx + c** a tangent to the circle $x^2 + y^2 = a^2$.
Write **c** in terms of **a** and **m, where a , m** and **c** Î **R**

T7) If $x - y : x + y = 5 : 9$, find the numerical value of the ratio $x^2 - xy + y^2 : x^2 + 4y^2$.
Answer in form a : b

where a and b Î N

T8) Determine x such that $(2.4)^x = (5.3)^{x-1}$.

Answer correct to 3 decimal places.

T9) A circle of radius r is inscribed in a right angled isosceles triangle . and a circle of radius R is circumscribed about the triangle . Calculate the ratio R : r .

Answer in form $a + \sqrt{b} : 1$ where a and b Î N.

T10) Find , in degrees, the least positive angle x for which

$$(2^{\sin^2(x)})(2^{\cos^2(x)})(2^{\tan^2(x)}) = 2^2$$

T11) Find the value of A + B , where A and B Î Z for which

$$\frac{5x - 11}{2x^2 + x - 6} = \frac{A}{x + 2} + \frac{B}{2x - 3}.$$

Answer Key Team Maths 2018 Regional Round.

Round 1 Q1.1 3 – 4i , -3 + 4i Q1.2 8

TEAM MATHS 18 , REGIONAL ROUND , January 26, 2018

Round 2 Q2.1 10 Q2.2 $\frac{9\sqrt{2}}{2}$

Round 3 Q3.1 $-6 < X < 4$ Q3.2 p =12, q = 4

Round 4 Q4.1 $2|AM|^2 + 2|MB|^2$ Q4.2 $2x^2 - 6x - 5$

Round 5 Q5.1 11.4 Q5.2 $8x - 5y + 60 = 0$

Round 6 Q6.1 $30^0, 150^0, 210^0, 330^0$ Q6.2 $\frac{\sqrt{3}}{2}$

Round 7 Q7.1 $2\sqrt{13}$ Q7.2 $\frac{\pi}{2}$

Q7.3 $1+\sqrt{2}+\sqrt{5}-\sqrt{10}$ Q7.4 1

Round 8 Q8.1 (2,3), (3,2), Q8.2 ALL positive REALS

Q8.3 $\sqrt{x^2-1}$ Q8.4 $32\sqrt{3} + \frac{64\pi}{3}$

Tiebreak

T1 0.3609 T2 $7x - 4y - 43 = 0$

T3 $42^0, 138^0, 210^0, 330^0$ T4 $ab^2 + 2b$

T5 576 T6 $\pm a\sqrt{1+m^2}$

T7 3 : 5 T8 2.105

T9 $1 + \sqrt{2} : 1$ T10 45^0

T11 2

BABHTA 1 ROUND 1

Q1.1) z = a + ib , where a and b $\in \mathbb{R}$ and i = $\sqrt{-1}$.
If f(z) = z^2 , calculate f(3 + 4i)

Answer in form a + ib, where a and b $\in$ Z

Q1.2) ABCD is a square of side a and [BC] is produced to E so that $|CE| = \frac{1}{2}|BC|$.
Calculate $|AE|^2 - |DE|^2$ in terms of a.

BABHTA 2 ROUND 2

Q2.1) In how many ways can three of the letters A, B , C, a , b , c, d be arranged in a row, using one capital and two small letters, so that the capital letter is always first, if each letter is used only once?

Q2.2) The positive integers are arranged in a triangular grid as shown.

1						
2	3	4				
5	6	7	8	9		
10	11	12	13	14	15	16
.	.	.	.	.	.	.
.	.	.	.	.	.	. and so on

What is the fourth number in the 60^{th} row?

BABHTA 3 ROUND 3

Q3.1) Find the equation of the line through (1,4) which is parallel to the line $2x - y - 3 = 0$.
Answer in the form $ax + by + c = 0$,
where a , b and c $\in Z$.

Q3.2) If $x^2 + y^2 = 10$ and xy = 3 find all possible values of x + y.

BABHTA 4 ROUND 4

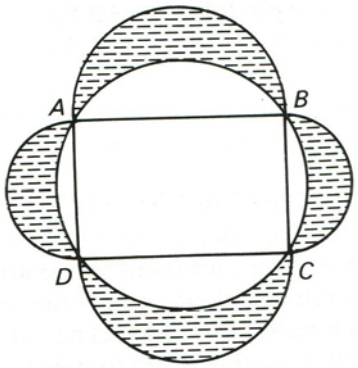
Q4.1) $x^4 - 48x + 28 = (x^2 + ax + 2)(x^2 + 4x + b)$.
Find the values of a and b, where a and b $\in$ R.

Q4.2) A triangle ABC has vertices A(18 , - 4), B(13 , 6), and C(x , 12).
The area of the ΔABC is 50 square units.
Find all possible points C in form (x , y).

BABHTA 5 ROUND 5

Q5.1) A club identifies each of its members with a unique three-digit number, the sum of whose digits is 24. How many such numbers exist?

Q5.2)



ABCD is a rectangle such that |AB| = 8 cm and |BC| = 6 cm. Semicircles are drawn on the four sides of the rectangle ABCD, each semicircle lying outside the rectangle. The circle passing through A, B , C and D determines four crescent shaped regions, one inside each semicircle as shown in the diagram.
Find the total area of the four crescent shaped regions in simplest form.

BABHTA 6 ROUND 6

Q6.1) Solve for a and b :

$\log(8x^3+4x^2-2x-1) = \log(2x-1) + 2\log(ax+b)$.

Q6.2) A dice rests on a table. Sam can see three faces and a total of 9 spots. Pat, who is on the opposite side of the table, can see three faces and 15 spots.
How many spots are on the top face of the dice?

BABHTA 7 ROUND 7

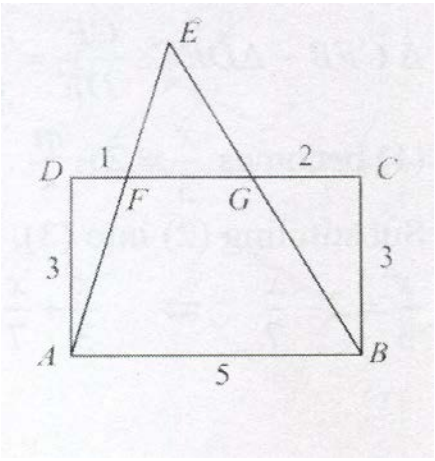
Q7.1) f and g are functions such that f(x) = ax + 5 and g(x) = cx – 3, where a and c $\in$ N.
If f(g(x)) = g(f(x)) calculate the values of a and c.

Q7. 2) Given two different two-digit positive integers whose product is 2016.

Find the greatest possible sum of the four digits.

Q7.3) In the expansion $(x + 3y + \frac{z}{4})^3$ find the coefficient of yz^2.
Answer in form $\frac{a}{b}$, where a and b $\in$ N.

Q7. 4) In the rectangle ABCD. |AB| = 5 and |BC| = 3.
Points F and G are on [CD] so that |DF| = 1 and |GC| = 2.
AF and BG intersect at E.
Calculate the area of the triangle AEB.



Answer in form $\frac{a}{b}$, where a and b $\in$ N.

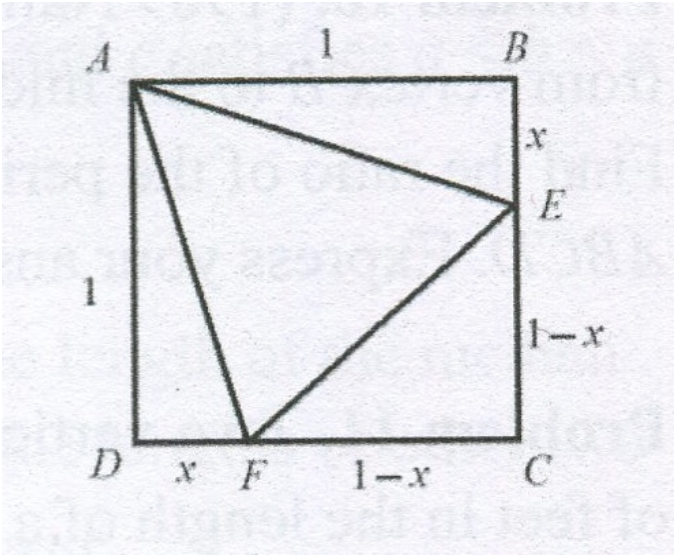
BABHTA 8 ROUND 8

Q8.1) If $0^0 \le x \le 180^0$ and $0^0 \le y \le 180^0$ find the pairs of angles, in the form (x,y),which satisfy the equations :

$$\text{Sin}(x + y) = +\frac{1}{\sqrt{2}}$$

$$\text{Cos}(2x) = -\frac{1}{2}$$

Q8.2) In the diagram an equilateral triangle is inscribed in a square of side 1 unit. From the diagram use the value of x to find the area of the triangle in form $a\sqrt{b} - c$, where a , b and c $\in$ N.



Q8.3) The coordinates of the vertices of a cyclic quadrilateral are A(2 , 5) , B(- 4 , 0) and D(7 , 0).
If the angles at B and D are right angles calculate the coordinates of vertex C.

Q8.4) Solve for x : $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$.

TIEBREAK

T1) The price of coat was reduced by 40% and the new price was then reduced by a further 40% during a sale.
By what percentage was the original price reduced?

T2) The sides of a triangle ABC are 10, 24 and 26 cm long.
A rectangle, with width 3cm, has an area equal to that of the triangle.
Find the perimeter of the rectangle, in cms.

T3) If $0^o \leq x \leq 360^0$ find the values of x for which
$3\text{Sin}^2(x) - \text{Sin}(x)\text{Cos}(x) - 4\text{Cos}^2(x) = 0$.
Answers to nearest degree.

T 4) Find the common ratio of the geometric sequence where the first term exceeds the third by 60 and the sum of the first two terms is 48.

Answer in form $\frac{a}{b}$, where a and b $\in$ Z.

T5) Find the points of intersection of the circles:

$x^2 + y^2 + 4x - 2y - 5 = 0$ and $x^2 + y^2 + 2x - 7 = 0$

Answers in coordinate form (x , y).

T6) A train travelling at 60 km/h takes 15 seconds to pass a certain point.
How long is the train?

T7) A complex number in the form a + ib , when multiplied by its conjugate a – ib yields 13, where a and b $\in$ R.
Find the numerical value of $a^4 + 2a^2b^2 + b^4$.

T8) Write in simplest form:

$$((x + 1)^2(x - 1)^2)((x^2 + 1)^2)$$

T9) The members of a Team Maths 2017 team are chosen at random from a class of twelve boys and fifteen girls.
Find the probability that the team consists of two boys and two girls.
Answer in simplest form $\frac{a}{b}$, where a and b $\in$**N.**

T10) Simplify $\dfrac{\dfrac{x}{1+x}+\dfrac{1-x}{x}}{\dfrac{x}{1+x}-\dfrac{1-x}{x}}$

Answer in simplest form as a fraction in terms of x.

T11) Find the perimeter of a right-angled triangle whose hypotenuse is 2 units and whose area is 1 square unit.

Answer in form $a+b\sqrt{c}$, where a , b and c $\in$ N.

T12) (m , 3) and (1 , m) are two points on the line l. The slope of l is m.
Given that m > 0 , find the value of m in surd form.

T13) The point with coordinates (pCos(A) , pSin(A)) lies on the circle $x^2+y^2=16$.
Find the value of p, where p > 0.

Answer key , Round 1 2017

Round 1	Q1.1)	-7 + 24i	Q1.2)	$2a^2$
Round 2	Q2.1)	36	Q2.2)	3485
Round 3	Q3.1)	2x – y + 2 = 0	Q3.2)	± 4
Round 4	Q4.1)	a = -4 , b = 14	Q4.2)	(20, 12) and (0,12)
Round 5	Q5.1)	10	Q5.2)	48 cm^2
Round 6	Q6.1)	a=2 , b =1	Q6.2)	5
Round 7	Q7.1)	a = 1, c =1	Q7.2)	20
	Q7.3)	$\frac{9}{16}$	Q7.4)	$\frac{25}{2}$
Round 8	Q8.1)	$(60^0,75^0)$ and $(120^0,15^0)$	Q8.2)	$2\sqrt{3}-3$
	Q8.3)	(1, -6)	Q8.4)	2

Tiebreak

T1) 64% T2) 86 T3) $53^0, 233^0, 135^0, 315^0$

T4) $-\frac{1}{4}$ T5) (-3,-2) and (1,2) T6) 250m

T7) 169 T8) $(x^4-1)^2$ **or** x^8-2x^4+1

T9) $\frac{77}{195}$ T10) $\frac{1}{2x^2-1}$

T11) $2+2\sqrt{2}$ T12) $\sqrt{3}$ T13) 4

TEAM MATHS 2017 Solutions

Round 1

Q1.1 f(z) = $z^2 = (3+4i)^2 = 9 + 24i - 16 = -7 + 24i$.

Q1.2 $|AE|^2 = a^2 + (\frac{3}{2}a)^2 = \frac{13a^2}{4}$

$|DE|^2 = a^2 + (\frac{1}{2}a)^2 = \frac{5a^2}{4}$

$|AE|^2 - |DE|^2 = 2a^2$

Round 2

Q2.1 Number of ways = 3 x 4 x 3 = 36

Q2.2 Last number in nth row = n^2.

Last number in 59th row = 59^2 = 3481.
First number in 60th row is 3482.
Therefore 4th number in 60th row is 3485.

Round 3

Q3.1 Any line parallel to 2x – y – 3 =0 has form l:2x = y – k = 0.
(1,4) is on $l \Rightarrow$2- 4 +k = 0. So k = 2.

l: 2x – y + 2 = 0.

Q3.2 $x^2 + y^2 = 10$ and x y = 3.
So 2xy = 6.
$x^2 + 2xy + y^2 = 16$. i.e $(x + y)^2 = 16 \Rightarrow x + y = \pm 4$.

Round 4

Q4.1 $x^4 - 48x + 28 = (x^2 + ax + 2)(x^2 + 4x + b)$
$= x^4 + (4 + a)x^3 + (4a + b + 2)x^2 + (ab + 8)x + 2b$

4 + a = 0 , a = - 4, 2b = 28, b = 14.

Q4.2 A(18 , -4) , B(13 , 6) , C(x, 12)

Area of triangle ABC = $\frac{1}{2}|18(6 - 12) + 13(12 + 4) + x(-4 - 6)| = 50$

$$\Rightarrow \frac{1}{2}|100 - 10x| = 50$$

$$\Rightarrow 100 - 10x = \pm 100, \text{ x = 0 or 20.}$$

So C(20 , 12) or C(0 , 12)

Round 5

Q5.1 24 = 9 + 9 + 6 , giving 699, 969, 996
24 = 9 + 8 +7 , giving 987, 978, 897, 879, 798, 789
24 = 8 + 8 +8 , giving 888

TOTAL 3 + 6 +1 = 10

Q5.2 Area of rectangle = 8 x 6 = 48 cm^2

Area of circle = $\pi(5)^2 = 25\pi \text{ cm}^2$

Area of semicircles = $2(\frac{1}{2})\pi(4)^2 + 2(\frac{1}{2})\pi(3)^2 = 25\pi \text{ cm}^2$

Area of crescents = $25\pi - (25\pi - 48) = 48 \text{ cm}^2$

Round 6

Q6.1 $\log(8x^3 + 4x^2 - 2x - 1) = \log(2x - 1) + 2\log(ax + b)$

$$= \log(2x - 1)(ax + b)^2$$

$$= \log(2a^2x^3 + (4ab - a^2)x^2 + (2b^2 - 2ab)x - b^2)$$

$2a^2 = 8, a = 2$ since $2x - 1 > 0 \Rightarrow x > \frac{1}{2}$

$4ab - a^2 = 4 \Rightarrow 8b - 4 = 4$, b = 1

So a = 2 and b = 1

Q6.2 Let there be x spots on top face. Let the sides Sam sees have x , y and z spots.
So x + y +z = 9
The opposite sides of a dice ad up to 7.

So Pat see x , 7 – y and 7 – z spots.
Therefore x + 7 - y + 7 – z = 15
x – y – z = 1 and x + y + z = 9
2x = 10, x = 5 , top face has 5 spots.

Round 7

Q7.1 f(x) = ax + 5 , g(x) = cx - 3 , where a and c $\in$ N.

f(g(x)) = f (cx – 3 = a(cx – 3) + 5 = acx – 3a +5

g(f(x)) = g(ax +5) = c(ax +5) - 3 = acx +5c -3

f(g(x)) = g(f(x)) $\Rightarrow$ acx – 3a + 5 = acx + 5c -3 $\Rightarrow -3a + 5 = 5c - 3$

$$\Rightarrow c = \frac{-3a + 8}{5} > 0 \text{ since c } \in \text{ N. } \Rightarrow -3a + 8 > 0$$

$$\Rightarrow a < \frac{8}{3} = 2\frac{2}{3}$$

Possible values for a are 1 and 2.

When a = 1 , c = 1 and when a = 2, c = $\frac{2}{5} \notin$ N.

Answers a = 1 , c = 1.

Q7.2 2016 = $2^5 x\ 3^2 x\ 7$
= 21 x 96, sum of digits = 18
= 63 x 32 , sum of digits = 14
= 42 x 48, sum of digits = 18
= 56 x 36, sum of digits = 20

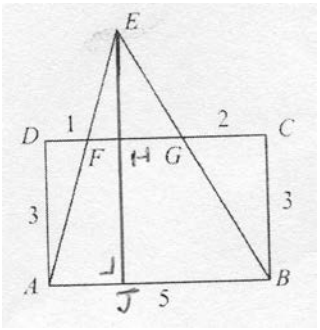
Greatest sum of digits is 20.

Q7.3 $((x + 3y) + \frac{z}{4}) = (x + 3y)^3 + 3(x + 3y)^2(\frac{z}{4}) + 3(x + 3y)(\frac{z^2}{16}) + \frac{z^3}{16}$

Taking relevant term $3(x + 3y)(\frac{z^2}{16}) = \frac{3xz^2}{16} + \frac{9yz^2}{16}$

Co-efficient of yz^2 is $\frac{9}{16}$.

Q7.4 Drop [EJ] perpendicular to [AB]. Let [EJ] cut [CD] at H.



Δ's EFG and AEB similar $\Rightarrow \frac{|FG|}{|AB|} = \frac{|EF|}{|EA|} = \frac{2}{5}$.

Δ's EFH and EAJ similar $\Rightarrow \frac{|EF|}{|EA|} = \frac{2}{5} = \frac{|EH|}{|EJ|} = \frac{|EH|}{|EH+3|}$

$\Rightarrow 5|EH| = 2|EH| + 6,\ 3|EH| = 6,\ |EH| = 2$

$\Rightarrow |EJ| = 5$

Area of triangle AEB = $\frac{1}{2}(5)(5) = \frac{25}{2}$ sq. units.

Round 8

Q8.1 Sin (x + y) = $\frac{1}{\sqrt{2}} \Rightarrow x + y = 45^0 \text{or } 135^0$

Cos (2x) = $-\frac{1}{2} \Rightarrow 2x = 120^0 \text{or } 240^0 \Rightarrow x = 60^0 \text{or } 120^0$.

So y = $15^0 \text{or } 75^0$.

Q8.2 $side^2 = 1 + x^2 = 2(1 - x)^2$

$1 + x^2 = 2 - 4x + 2x^2 \Rightarrow x^2 - 4x + 1 = 0$

$\Rightarrow x = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$

$\Rightarrow x = 2 - \sqrt{3}$, since $x<1$

$side^2 = 1 + (2 - \sqrt{3})^2 = 8 - 4\sqrt{3}$

Area of triangle = $\frac{1}{2}(8 - 4\sqrt{3})\text{Sin}\,(60^{0)} = 2\sqrt{3} - 3$ sq.units

Q8.3 Slope of AB = $\frac{5 - 0}{2 + 4} = \frac{5}{6} \Rightarrow$ Slope of BC = $-\frac{6}{5}$

Eqn of BC is $y - 0 = -\frac{6}{5}(x + 4)$

i.e $5y = -6x - 24$ i.e $6x + 5y = -24$ (i)

Slope of AD = $\frac{5-0}{2-7} = -1 \Rightarrow$ slope of CD = 1

Eqn of CD is $y = x - 7$ (ii)

Substituting (i) $\Rightarrow 6x + 5(x - 7) = -24$, $6x + 5x - 35 = -24$

$11x = 11$, $x = 1$, $y = -6$.

So C(1 , -6)

Q8.4 $\sqrt{x + 7} + \sqrt{x + 2} = \sqrt{6x + 13}$

Squaring $x = 7 + 2\sqrt{(x + 7)(x + 2)} + x + 2 = 6x + 13$

$2\sqrt{(x + 7)(x + 2)} = 4x + 4$, $x^2 + 9x + 14 = 4x^2 + 8x + 4$,

$3x^2 - x - 10 = 0$, $(3x + 5)(x - 2) = 0$, $x = \frac{5}{3}$ or 2.

Checking $x = \frac{5}{3}$ invalid. So $x = 2$ only.

Babhta 1 Round 1

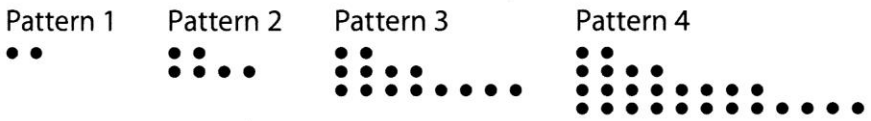
1) Let the smallest of four consecutive whole numbers be $2n-1$. Calculate, in terms of n, the difference between the product of the two even numbers and the product of the two odd numbers.

2) Find the derivative of $\text{Sin}^2(3x)$.
Answer in the form $a\text{Sin}(bx)$, where a and b $\in$ N.

Babhta 2 Round 2

1) Find the equation of the circle which passes through the corners of the square with vertices (2, -1) , (5, 3) , (9, 0) and (6, -4). Answer in the form $x^2 + y^2 + 2gx + 2fy + c = 0$, where g, f and c ∈ Z.

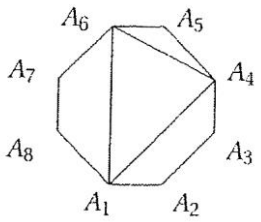
2) Dots are arranged in the following pattern:



If this pattern continues how many dots will be in the 30th pattern?

Babhta 3 — Round 3

1) How many different triangles can you draw as in the figure, if three vertices of the triangle have to be among the points shown $A_1,..., A_8$?

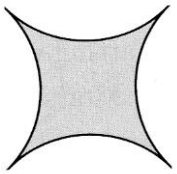


2) A two-digit number, greater than 10, is written at random. What is the probability that the sum of its digits is 5? Answer in simplest form $\frac{a}{b}$, where a and b Í N.

Babhta 4 Round 4

1) ABCD is a rhombus in which the measure of the angle ABC is 26°.
An equilateral triangle BCX is drawn on the side [BC], and a square DCYZ is drawn on the side [DC], both outside the rhombus.
Calculate the measure of the angle CXY.
Answer in degrees.

2) The figure shown is formed from four identical arcs, each a quarter of the circumference of a circle of radius 5 centimetres. What is the area of the figure in square centimetres?



Answer in the form $a - b\pi$, where a and $b \in \mathbb{Z}$.

Babhta 5 Round 5

1) Let x and y have opposite signs and satisfy the two equations

$$x + y + xy = -2 \text{ and } x^2 + y^2 + x^2y^2 = 64.$$

Find the numerical value of xy.

2) If $f(\text{Sin}(x)) = \text{Sin}(3x)$ calculate the numerical value of $f(\text{Cos}(30^\circ))$

Babhta 6 Round 6

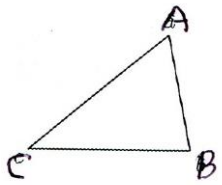
1) The geometric mean of a set of n positive numbers is defined as the n^{th} root of the product of those numbers. The geometric mean of a set of three numbers is 3 and the geometric mean of another set of three numbers is 12. Calculate the numerical value of the geometric mean of the combined set of the six numbers.

2) A rhombus has half the area of a square with the same side length.
Calculate the ratio of the longer diagonal to the shorter diagonal of the rhombus.
Answer in the form $a+\sqrt{b}:1$, where a and $b\hat{I}$ N.

Babhta 7 Round 7

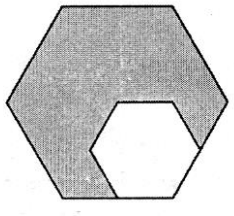
1) Find the equation of the tangent to the curve $y = x^3 - x^2 - 5x + 4$ at the point where the curve crosses the y-axis.
Answer in form $ax + by + c = 0$, where a , b and c $\in Z$.

2) In the diagram $|AC| = 2x$, $|BC| = 2x + 1$ and $|\angle ACB| = 30^\circ$.
Given that the area of the triangle ABC is 18, calculate the value of x.



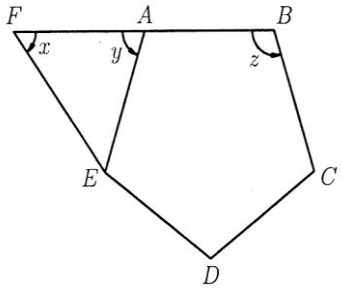
3) x and y are positive integers and $\sqrt{x} + \sqrt{y} = \sqrt{333}$.
Find the numerical value of $x + y$.

4) The length of an edge of the large regular hexagon is twice the length of an edge of the small regular hexagon.
The small hexagon has an area of 4 cm^2.
Calculate the area of the large hexagon.



Babhta 8 Round 8

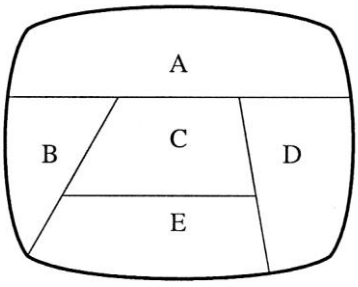
1) In the diagram ABCDE is a regular pentagon, and FAB is a straight line. $|FA|=|AB|$.



Find the ratio of the angles x, y and z in simplest form $x : y : z$., where x , y and $z \in N$.

2) The average of two positive numbers is 30% less than one of them and x% more than the second number. Calculate the value of x.

3) In the figure below four different colours (red, yellow, blue and green) are available to colour the regions.
The only restrictions are that each region must be entirely one colour and no adjacent regions are allowed to have the same colour.
If at least three of the four colours are used how many different colouring schemes are possible?



4) Pat purchases a car for €17 800 on January 1st. She makes a down payment of €1500 and then takes out a 5 year loan on the balance at an APR of 6%.
She agrees to repay the loan in equal monthly instalments beginning on the following February 1st and at the beginning of each subsequent month.
Calculate her monthly repayment to the nearest cent.

Team Maths 2016

Scoilt **Tiebreak**

1) A triangle ABC has sides $a = 6$ cm, $b = 8$ cm and c =10 cm.
Calculate the length of the perpendicular from C to [AB].
Answer to 1 decimal place.

2) The transformation $(x , y) \rightarrow (2 - y, x - 2)$ is applied three successive times .
Starting with (1 , 1) find the coordinates of the resultant point.

3) P is a point on the xy-plane whose coordinates are both integers.
For how many values of P is it exactly 10 units from the origin?

4) Tan(A) + Tan(B) = 25 and Cot(A) + Cot(B) = 30.
Calculate the value of Tan(A + B).

5) How many solutions does the equation $2^{2x} = 4^{x+1}$ have?

6) A computer password consists of a letter followed by 3 or 4 digits.
Find the total number of passwords in which no digit is repeated.

7) What is the greatest common factor (divisor) of 19! and 19! + 17 ?

8) The average of six distinct real numbers is 275. The average of the four smallest numbers is 200. The average of the four greatest numbers is 340.
What is the average of the two middle numbers?

9) Given that $1 - r + r^2 - r^3 + ... = s$ and $1 + r^2 + r^4 + r^6 + ... = 4s$, find the value of s.

Answer in form $\frac{a}{b}$, where a and b $\in N$.

10) The line segment [AB] is divided internally in the ratio 7 : 5 by the point P with coordinates (3, 2.75).
A has coordinates (- 4 , 1) .
Find the coordinates of B in simplest form (*x*,y).

11) Find in simplest form the range of values of x for which $|3 - 4x| < 7$, $x \in R$.

12) Solve for x : $e^{2x} - 3e^x - 4 = 0, \quad x \in R$.
Answer correct to 2 decimal places..

13) The expression $\frac{2x^2 + 1}{x^2 - 3}$ may be written in the form $2 + \frac{A}{x^2 - 3}$.
Find the value of A $\in N$.

ANSWER KEY Team Maths Round 1

Round 1	q1	4n +1	q2	3Sin6x
Round 2	q1	$x^2 + y^2 - 11x + y + 18 = 0$	q2	1742
Round 3	q1	56	q2	$\frac{1}{18}$
Round 4	q1	62°	q2	100 - 25p
Round 5	q1	-6	q2	0
Round 6	q1	6	q2	$2+\sqrt{3} : 1$
Round 7	q1	5x + y – 4 = 0	q2	4
	q3	185	q4	16
Round 8	q1	3 : 4 : 6	q2	75
	q3	72	q4	€313.92

Tiebreak

q1	4.8	q2	(3, 1)
q3	12	q4	150
q4	0	q6	149760
q7	17	q8	255
q9	$\frac{4}{7}$	q10	(8,4)
q11	$-1 < x < 2.5$	q12	1.39
q13	7		

Cumann Oidí Matamaitice na hÉireann **Irish Mathematics Teachers Association**

Foireann Mata 2015 **Team Maths 2015**

Babhta Réigiúnach **Regional Round**

BABHTA 1 ROUND 1

1) Write $2^n - 2^{n-1}$ in form 2^x

2) 120 people are having a meeting at which they are to be separated into equal-sized groups having at least three but no more than 12 to a group.
How many different group sizes are possible?

BABHTA 2 ROUND 2

1) Find the sum of the distances from one vertex of a square of side 2 cm to the midpoints of each of the sides of the square.

Answer in the form $a + b\sqrt{c}$, where a, b and $c \in N$.

2) A circle, which passes through the origin, cuts off intercepts of lengths 4 and 6 units on the positive x- and y- axes respectively. Find the equation of the circle.

Answer in form $x^2 + y^2 + 2gx + 2fy + c = 0$

BABHTA 3 ROUND 3

1) If a and b $\in$ R find the numerical value of a + b when

$$\frac{4\cos^2(\theta)-3}{1-2\sin(\theta)} = a + b\sin(\theta) \text{ , where } \text{Sin}(\theta) \neq \frac{1}{2}$$

2) For what positive value of k does the line x + y = k intersect the circle $x^2 + y^2 = 3$ at one point only?
Answer in the form $\sqrt{a}$, where a $\in$ N

BABHTA 4 ROUND 4

1) A rectangle with sides in the ratio 3 : 5 is inscribed in a circle. The four vertices of the rectangle are on the circle.
Calculate the ratio of the area of the rectangle to the area of the circle.
Answer in simplest form $\frac{a}{b\pi}$, where a and b $\in$ N

2) The following array of integers is called a ladder; each horizontal row is called a rung. Find the sum of the pair of integers on the 7th row.

1	1
2	3
5	7
12	17
.	.
.	.
.	.

BABHTA 5 ROUND 5

1) Find the sum of the roots of the equation

$$9^{2x+1} - 28.3^{2x} + 3 = 0$$

Answer in simplest form $\frac{a}{b}$, where a and b $\in$ Z

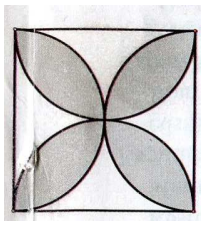
2) A box contains three coins; one coin is fair, one coin is two-headed, and one coin is weighted so that the probability of heads appearing is $\frac{1}{3}$. A coin is selected at random and tossed. Find the probability that a head will appear?

Answer in simplest form $\frac{a}{b}$, where a and b $\in$ N

BABHTA 6 ROUND 6

1) Two sides of a parallelogram have length 5 and the other two sides have length 7.
The length of one diagonal is 11.
Calculate the length of the other diagonal.
Answer correct to one decimal place.

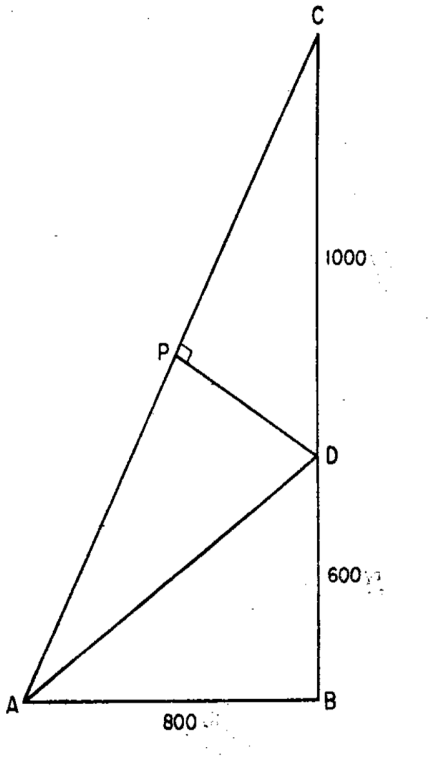
2) Four semicircles are drawn in the interior of a square using each side of the square as a diameter.
The area of the square is 64 square units.
Find the area of the shaded region.



Answer in form $a\pi$ - b, where a and b $\in$ N

BABHTA 7 ROUND 7

1) A and B are two points on a straight road and B is 800 m east of A.
D and C are two landmarks which are due north of B. It is known that $|BD| = 600$m and $|DC| = 1000$m.
Calculate the shortest distance, $|DP|$, from D to a straight road which joins AC.
Answer in simplest form $a\sqrt{b}$, where a and $b \in N$



2) If x is a positive real number and $\left(x+\frac{1}{x}\right)^2 = 7$ find the value of $x^3+\frac{1}{x^3}$
Answer in simplest form $a\sqrt{b}$ where a and $b \in N$

3) The complex number z satisfies $z + |z| = 2 + 8i$.

Find the value of $|z|$.

4) Find the value of $x + y + z$ if $\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}=12$ and $xyz=\frac{1}{18}$
Answer in simplest form $\frac{a}{b}$, where a and $b \in N$.

BABHTA 8 ROUND 8

1) Find all the solutions to the equation Cos(A) + Sin(A) = $\sqrt{\frac{3}{2}}$ in the domain O < A < π

Answers in terms of π.

2) Find the value of $x_2 + x_4 + x_6 + \ldots\ldots + x_{98}$ if $x_1, x_2, x_3 \ldots\ldots$ is an arithmetic progression with common difference 1, given that $x_1 + x_2 + x_3 \ldots\ldots + x_{98} = 137$.

3) The lengths of the sides of a triangle are 10, 17 and 21. What is the length of the shortest altitude of the triangle?

4) Find all the real positive values of p and r which satisfy the following equations:

$$p + pr + pr^2 = 26$$
$$p^2r + p^2r^2 + p^2r^3 = 156$$

Answers in the form (p , r).

Cumann Oidí Matamaitice na hÉireann **Irish Mathematics Teachers Association**

Foireann Mata 2015 **Team Maths 2015**

Babhta Réigiúnach **Regional Round**

SCOILT TIEBREAK

1) The number 13 is prime. If you reverse the digits you also get a prime number, 31. Find the largest prime number that satisfies this condition if the sum of the two primes is 110.

2) If Alex stands on a table and Brian stands on the floor then Alex is 80 cm taller than Brian.
If Brian stands on the same table and Alex stands on the floor then Brian is 1 m taller than Alex.
How high is the table?

3) An equilateral triangle is cut into four equilateral triangles, each with a perimeter of 12 cm.
What is the perimeter, in cm, of the original equilateral triangle?

4) If the product $15^6 \times 28^5 \times 55^7$ was evaluated, it would end with a string of consecutive zeros.
How many zeros are in this string?

5) The numbers 1 to 10 are placed around a circle. Sue crosses out 1, then 4, and then 7. Continuing in a clockwise direction she crosses out every third number of those remaining, until only two numbers are left.
What is the sum of the two remaining numbers?

6) If $P = 3^x + 3^{-x}$ and $Q = 3^x - 3^{-x}$ what is the numerical value of $P^2 - Q^2$?

7) The graph of $5x - 3y - 7 = 0$ is translated 3 units up and 2 units to the right.
What is the equation of the new graph in the form
$ax + by + c = 0$, where a, b and $c \in Z$.

8) What is the 2015^{th} digit when $\frac{7}{13}$ is written as a decimal?

9) $a = \log_8(225)$ and $b = \log_2(15)$. Write a in terms of b.

10) a , b and c are real numbers which satisfy the following equations:

$$a - b + c = 2$$
$$b - c + a = -3$$
$$c - a + b = 5$$

Find the numerical value of $a + b + c$.

11) Find the altitude of the equilateral triangle whose area and perimeter have the same numerical value.

12) If $16^{x+1} = 3$, what is the value of 2^{4x+2}?

13) If $2^x = 3$, $3^y = 5$ and $5^z = 8$ what is the numerical value of the product xyz?

14) In the triangle ABC, $|AB| = 11$, $|AC| = 9$ and the length of the altitude from A to $[BC] = 7$. Calculate the length of the side $[BC]$.

Answer in simplest form $a\sqrt{b}$, where a and b $\in$ N.

Answers Team Maths Regional Round 2015

Round 1 **Q1** 2^{n-1} **Q2** 7

Round 2 **Q1** $2+2\sqrt{5}$ **Q2** $x^2+y^2-4x-6y=0$

Round 3 **Q1** **3** **Q2** $\sqrt{6}$

Round 4 **Q1** $\frac{30}{17\pi}$ **Q2** 408

Round 5 **Q1** $-\frac{1}{2}$ **Q2** $\frac{11}{18}$

Round 6 **Q1** 5.2 **Q2** $32\pi - 64$

Round 7 **Q1** $200\sqrt{5}$ **Q2** $4\sqrt{7}$
Q3 17 **Q4** $\frac{2}{3}$

Round 8 **Q1** $\frac{\pi}{12}, \frac{5\pi}{12}$ **Q2** 93
Q3 8 **Q4** $(18, \frac{1}{3})$ **and** $(2, 3)$

Cumann Oidí Matamaitice na hÉireann **Irish Mathematics Teachers Association**

Foireann Mata 2015 **Team Maths 2015**

Babhta Réigiúnach **Regional Round**

Tiebreak

Q1 73 **Q2** 90 **Q3** 24 **Q4** 10 **Q5** 10

Q6 4 **Q7** $5x - 3y - 8 = 0$ **Q8** 6 **Q9** $a = \frac{2}{3}b$

Q10 4 **Q11** 6 **Q12** $\frac{3}{4}$ **Q13** 3 **Q14** $10\sqrt{2}$

Irish Maths Teachers Association

Foireann Mata 2014 **Comórtas Réigiúnach**
(Team Maths 2014 **Regional Round)**

BABHTA 1 ROUND 1

1) Calculate, in its simplest form, the value of i^{2014}, where $i = \sqrt{-1}$.

2) Find the numerical value of the derivative of Sin(3x) when x = $\frac{\pi}{3}$.

Irish Maths Teachers Association

Foireann Mata 2014 **Comórtas Réigiúnach**
(Team Maths 2014 **Regional Round)**

BABHTA 2 ROUND 2

1) What is the numerical value $\text{Cos}(15°)\text{Sin}(15°)$?
Answer in the form $\frac{a}{b}$, where a and b $\in$ N

2) Find the range of values of x for which $3 > 5x + 2x^2, x \in R$

Irish Maths Teachers Association

Foireann Mata 2014 **Comórtas Réigiúnach**
(Team Maths 2014 **Regional Round)**

BABHTA 3 ROUND 3

1) Find the length of the tangents from $(-9, 3)$ to the circle

$$x^2 + y^2 - 4x - 2y - 20 = 0$$

2) If $3^x = 12$, calculate the value of 9^{x-1}.

Irish Maths Teachers Association

Foireann Mata 2014 **Comórtas Réigiúnach**
(Team Maths 2014 **Regional Round)**

BABHTA 4 ROUND 4

1) Find the fifth term in the arithmetic sequence whose first term is 1, whose common difference is non-zero and whose second, tenth and thirty fourth term are the first three terms of a geometric sequence.
Answer in the form $\frac{a}{b}$, where a and $b \in$ N and in its lowest terms.

2) Three vertices of a parallelogram are (1, 1), (3, 5) and $(-1, 4)$.
Find all possible ordered pairs that could be the coordinates of the fourth vertex.

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Foireann Mata 2014 (Team Maths 2014 **Comórtas Réigiúnach Regional Round)**

BABHTA 5 ROUND 5

1) Solve for x : $2.3^{2x+1} - 5.3^{x} - 6 = 0$.

Answer correct to 2 decimal places.

2) Find the value of x If $\log_2(\log_2(\log_2(x))) = 2$

Answer in the form a^b, where a and b $\in$ N, and b is the smallest integer greater than 1

Irish Maths Teachers Association

Foireann Mata 2014 (Team Maths 2014 **Comórtas Réigiúnach Regional Round)**

BABHTA 6 ROUND 6

1) Two identical bottles are filled with weak milk solutions. In one bottle the ratio of the volume of milk to the volume of water is 1 : 25. In the other bottle the ratio of the volume of milk to the volume of water is 1 : 77.
If the entire contents of the two bottles are mixed together, the ratio of the volume of milk to the volume of water in the mixture is 1 : N.
Find the value of N.

2) Each of the first six prime numbers is written on a separate card.
The cards are shuffled and two cards are selected without replacement. What is the probability that the sum of the numbers selected is prime and unique?
Answer in the form $\frac{a}{b}$, where a and b $\in$ N and in its lowest terms.

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Foireann Mata 2014 **Comórtas Réigiúnach**
(Team Maths 2014 **Regional Round)**

BABHTA 7 ROUND 7

1) If $x_1, x_2, x_3, \ldots\ldots, x_n$ are real numbers and $x_n = \frac{n}{x_{n-1}}$ for $n \geq 1$, where $n \in N$, calculate the product $x_1x_2x_3\ldots\ldots x_8$.

2) Find the numerical value of $25^{\frac{1}{2} - \log_5(\sqrt{2})}$

Answer in the form $\frac{a}{b}$, where a and $b \in N$ and in its lowest terms.

3) Calculate the value of the positive integer n where

$$2014^{2013} < 2013^n < 2014^{2014}$$

4) Three cubes of volume 1, 8 and 27 are glued together at their face.
Calculate the smallest possible surface area of the resulting configuration.

Irish Maths Teachers Association

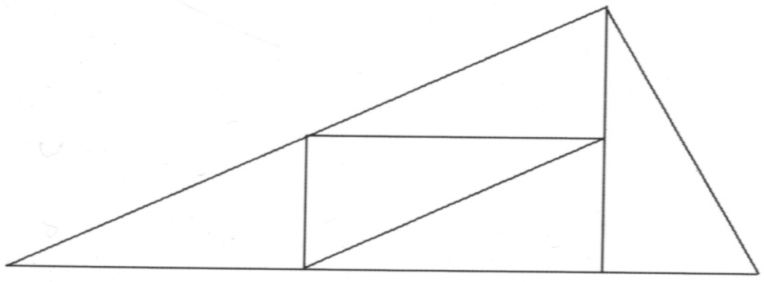
Foireann Mata 2014 **Comórtas Réigiúnach**
(Team Maths 2014 **Regional Round)**

BABHTA 8 ROUND 8

1) A tennis player computes her "win ratio" by dividing the number of matches she has won by the total number of matches played.
At the start of a weekend, her win ratio was exactly 0·5.
During the weekend she plays four matches, winning three and losing one. At the end of the weekend her "win ratio" is greater than 0·503.
What is the largest number of matches that she could have won before the weekend began?

2) The numbers 1447, 1005 and 1231 have something in common: each is a four digit number beginning with 1 that has exactly two identical digits.
How many such numbers are there?

3) The architecture of a sculpture in a certain city is based on frames as shown in which a large triangle is subdivided into 5identical triangles, each similar to the large triangle.



If the shortest side of one of the smallest triangles is 1metre, how many metres of framing are required to construct the whole shape?
Answer in the form $a + b\sqrt{c}$, where a, b and $c \in \mathrm{N}$

4) The product of a two-digit number and the same number with its digits reversed is 3154. Find the sum of the two numbers.

Irish Maths Teachers Association

Foireann Mata 2014 **Comórtas Réigiúnach**
(Team Maths 2014 **Regional Round)**

Tiebreak

1) If $3^m = 7$ and $7^n = 81$ what is the value of the product mn?

2) What is the shortest distance between the curves $y = 4x^2 + 2$ and $y = -3x^2 - 4$?

3) What is the sum of all three-digit integers whose units digit is 7?

4) An unbiased six sided die is tossed three times.
Given that the sum of the first two tosses equals the third, what is the probability that at least one "2" is tossed?

5) Suppose that the temperature during June is normally distributed with a mean of 20^0C and a standard deviation of 3·33 degrees. Find the probability that the temperature is between $21 \cdot 11^0$ and $26 \cdot 66^0$.
Answer correct to 2 decimal places.

6) If $0^0 < A < 90^0$ and Sin(A) = 0.1, find the value of $\log_{10}(\text{Tan}(A)) + \log_{10}(\text{Cos}(A))$

7) Find the numerical value of $x^3 + y^3$, where

$$x + y = 1$$
and
$$x^2 + y^2 = 2$$

Answer in the form $\frac{a}{b}$, where a and b $\in$ N

8) What is the sum of the digits of the product $2^{2012} \times 5^{2014}$?

9) The addition below is incorrect. What is the largest digit that can be changed in order to make the left hand side equal to the right hand side.

$$641 + 852 + 973 = 2456$$

10) Find the value of k such that the product $(3 + 2i)(3 + ki)$ is a real number.

11) The line y = $\frac{1}{2}$ intersects the graph of y = Sin(x) at A and B.
Find the length of the line segment [AB] in the domain $[0, \pi]$

Answer in terms of π, in simplest form.

12) Each of the numbers 2, 5, 11 and 13 are assigned in some order to p, q, r and s.
What is the largest possible value of the pq + pr + ps ?

13) The base of a triangle is tripled and the altitude is halved. Calculate the ratio of the area of the new triangle to the area of the original triangle.

Answer in the form $a : b$ where a and $b \in$ N.

14) Simplify the following expression $1-\dfrac{1}{1-\dfrac{1}{1-\dfrac{1}{2014}}}$.

15) Take any two -digit number **ab**. If we reverse the digits we get the two-digit number **ba**.
If we now make a four-digit number of the form **abba,** what is the smallest positive two-digit integer, that always divides **abba** for all a and b $\in$ N?

Irish Maths Teachers Association

Foireann Mata 2014 (Team Maths 2014 **Comórtas Réigiúnach Regional Round)**

ANSWER KEY

ROUND 1	Q 1 –1	Q2 – 3		
ROUND 2	Q1 ¼	Q2 $-3 < x < \frac{1}{2}$		
ROUND 3	Q1 10	Q2 16		
ROUND 4	Q1 $\frac{7}{3}$	Q2 (–3, 0), (1, 8), (5, 2)		
ROUND 5	Q1 0·37	Q2 256^2		
ROUND 6	Q1 N=38	Q2 $\frac{1}{5}$		
ROUND 7	Q1 384	Q2 $\frac{5}{2}$	Q3 2014	Q4 72
ROUND 8	Q1 164	Q2 432	Q3 $10+4\sqrt{5}$	Q4 121

TIEBREAK

Q1 4
Q2 6
Q3 49680
Q4 8/15
Q5 0·35
Q6 –1
Q7 5/2
Q8 7
Q9 7
Q10 –2
Q11 2π/3
Q12 234
Q13 3 : 2
Q14 2014
Q15 11

Foireann Mata 2013 **Comórtas Réigiúnach**
(Team Maths 2013 **Regional Round)**

Babhta 1 Round 1

1) Expand $(1 - x^2)(1 + x^3)$

2) The point (4, 3) is reflected in the x-axis to a point P. Then P is reflected in the y-axis to a point Q. What is the sum of the coordinates of Q?

Foireann Mata 2013 **Comórtas Réigiúnach**
(Team Maths 2013 **Regional Round)**

Babhta 2 Round 2

1) Find the radius of the circle which cuts the x-axis at $x = 2$ and $x = 4$ and the y-axis is a tangent to the circle.

2) Find the values of z^3 when $z^6 + 2z^3 + 2 = 0$
Answer in the form $x + iy$ where x and $y \in \mathbb{R}$

Foireann Mata 2013 Comórtas Réigiúnach
(Team Maths 2013 Regional Round)

Babhta 3 Round 3

1) A bag has 3 red and k white marbles. Two marbles are chosen at random from the bag. If the probability that the two marbles are the same colour is $\frac{1}{2}$, find the value of k, if $k > 1$.

2) In the subtraction below some of the digits have been replaced by letters.

$$\begin{array}{r} A4B7C \\ 5D8E6 \\ \hline 28499 \end{array}$$

Find the numerical value of A + B + C + D + E.

Foireann Mata 2013 Comórtas Réigiúnach
(Team Maths 2013 Regional Round)

Babhta 4 Round 4

1) If $f(x) = ax^2 + bx + c$ and the points (4, -9), (-1, - 4) and (0,-1) lie on the curve $y = f(x)$, calculate the product **abc.**

2) If $\tan(2\theta) = \frac{5}{12}$, calculate all possible values of $\sin(\theta)$.

Answers in the form $\frac{a}{\sqrt{b}}$, where a and b $\in \mathbb{Z}$

Babhta 5 Round 5

1) Using the usual notation find the area of the triangle ABC where $a = 3{\cdot}34$, $b = 5{\cdot}62$ and $c = 3{\cdot}84$.
Answer correct to 2 decimal places.

2) A piece of wire 12 m long is cut into two pieces, one of which is bent to form a square and the other to form a circle. Calculate, in terms of π, the length of the side of the square when the combined areas of the two figures is a minimum. Answer as a fraction.

Foireann Mata 2013 **Comórtas Réigiúnach**
(Team Maths 2013 **Regional Round)**

Babhta 6 Round 6

1) Two players compete in a tennis match of up to 3 sets. The match ends when a player wins two sets.
If player *A* has a probability of 60% in winning a set, what is this player's probability of winning the match?

Answer correct to 3 decimal places.

2) Find the accurate value of $\int_0^{\frac{\pi}{2}} \frac{Sin(x)dx}{1+4Cos(x)}$

Babhta 7 Round 7

1) In how many ways can a pigeon fancier separate his 10 pigeons into a group of 4 and a group of 6 if he has to keep a certain two of the pigeons in separate groups?

2) Components made by a certain process have a thickness which is normally distributed about a mean of 3 cm with a standard deviation 0·03 cm. A component is considered defective if its thickness is greater than 3·05 cm or less than 2·95 cm. What percentage of the components are defective? Answer correct to 1 decimal place.

3) If $\sin(x) + \sin^2(x) = 1$, find the value of $\cos^2(x) + \cos^4(x)$.

4) Find the derivative of $\ln\left(\frac{4x^2 - 1}{3x + 1}\right)$ when $x = 1$.
Answer in simplest form $\frac{a}{b}$, where a and $b \in \mathbb{Z}$

Babhta 8 Round 8

1) If $(3x + 2 – 3y)(2x – 3 + 2y) = 0$ and $x + y \geq 2$ find the value of $x – y$.
Answer in the simplest form $\frac{a}{b}$ where a and b $\in \mathbb{Z}$.

2) How many 6-digit numbers start and / or end with an even digit? (A six digit number cannot begin with 0)

3) Three fair dice are tossed once. What is the probability that the faces show three consecutive integers?
Answer in simplest form $\frac{a}{b}$ where a and b $\in \mathbb{N}$.

4) Two candles of equal length are lit at 10 pm. Each candle burns at a different constant rate.
One candle takes 6 hours to burn out; the other takes 3 hours.
At what time is one candle exactly twice as long as the other?

Foireann Mata 2013 **Comórtas Réigiúnach**
(Team Maths 2013 **Regional Round)**

Tiebreak

1) A total of twenty eight handshakes were exchanged at the end of a party.
Assuming that everyone shook hands with everyone else at the party, how many people attended the party?

2) Find the coordinates of the point where the tangent to the curve $y= x^3 - 2x + 1$ at (2, 5) meets the curve for the second time.

3) Find $\int e^{\frac{\pi}{2}-t} dt$

4) Find the value of $(4^{-1} - 3^{-1})^{-1}$ in its simplest form.

5) In a special sort of lottery a player may buy a ticket on which he selects 10 different integers from the range 1 to 100 (inclusive). 10 numbers are then drawn at random. The player wins if his ticket contains none of the numbers drawn.
What is the probability that his ticket wins? Answer correct to 4 decimal places.

6) If $\frac{a}{a+2b} = -3$ calculate the value of $\frac{a}{b}$, giving the answer as a fraction in simplest form.

7) A large watermelon weighs 20kg, with 98% of its weight being water. It is left to stand in the sun, and some of the water evaporates so that now only 95% of its weight is water. What does it now weigh, if only the water content changes?

8) The roots of $x^3 + 4x^2 - 7x - 10 = 0$ are -5 , -1 and 2.
What are the roots of $(x-3)^3 + 4(x-3)^2 - 7(x-3) - 10 = 0$?

9) p, q and r are consecutive odd integers. If $p < q < r$ find the value of

$$p^2 - 2q^2 + r^2$$

10) Suppose five days before the day after tomorrow was Wednesday. What day of the week was yesterday?

11) Solve for x : $\frac{e^{2x}}{2} - e^x - 12 = 0$. Answer correct to 3 decimal places.

12) My house number is the lowest number on the street that, when divided by 2, 3 , 4 , 5 or 6 , will always leave a remainder of 1.
However when divided by 11 there is no remainder.
What is my house number?

13) I bought two watches, an expensive one and a cheap one. The expensive one cost €200 more than the cheap one. I spent €220 for both. How much did I pay for the cheap watch?

14) In a knock-out tennis tournament all players who entered took part. (there were no walkovers). In all there were 39 matches played before the outright winner emerged.
How many players entered the competition?

Answers:

Round 1	Q1	$1+x^3-x^2-x^5$	Round 2	Q1	3
	Q2	-7		Q2	-1 + i, -1 – i
Round 3	Q1	6	Round 4	Q1	2
	Q2	28		Q2	$\pm\frac{5}{\sqrt{26}}, \pm\frac{1}{\sqrt{26}}$
Round 5	Q1	6.25	Round 6	Q1	.648
	Q2	$\frac{12}{4+\pi}$		Q2	$\frac{1}{4}\ln(5)$
Round 7	Q1	112	Round 8	Q1	$-\frac{2}{3}$
	Q2	9.5%		Q2	650 000
	Q3	1		Q3	$\frac{1}{9}$
	Q4	$\frac{23}{12}$		Q4	12 midnight

Tiebreak	Q1	8
	Q2	(-4,-55)
	Q3	$-e^{\frac{\pi}{2}-t}+c$
	Q4	-12
	Q5	0.3305
	Q6	$-\frac{3}{2}$
	Q7	8kg
	Q8	-2, 2 , 5
	Q9	8
	Q10	Friday
	Q11	1.792
	Q12	121
	Q13	€10
	Q14	40

BABHTA 1 - ROUND 1

1) Express $\frac{-5}{i-\sqrt{3}}$ in the form $r(\cos\theta + i\sin\theta)$

2) Find the coordinates of the point P which divides the line segment joining

A(-9, 14) and B(5·4, -5·2) internally
in the ratio 5 : 1

BABHTA 2 - ROUND 2

1) If α and β are roots of the equation $2x^2 + 5x + 1 = 0$ form an equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

Put answer in the form $ax^2 + bx + c = 0$,
where **a**, **b**, **c** $\in$ Z

2) If a committee of 5 is to be chosen from 9 people at random, what is the probability that a particular person will be on the committee?
Leave your answer in the form $\frac{a}{b}$,
where a and b $\in$ Z

BABHTA 3 - ROUND 3

1) The point (3, a) is a solution of the equations

$$x + 4y + 5 = 0$$

$$bx^2 + 9xy + 25 = 4y^2$$

Find the value of a and the value of b, where **a** and **b** $\in Z$

2) The points A(1,1), B(0,0) and C(2,-1) form a triangle.

Find the measure of the angle $\angle ABC$ to the nearest degree.

BABHTA 4 - ROUND 4

1) In how many ways can a careless secretary place four letters in four envelopes so that nobody gets the right letter?

2) Solve the equation $\tan(x) + \tan(2x) = 0$, where $0 \leq x \leq 2\pi$.
Answers in terms of π.

BABHTA 5 - ROUND 5

1) Find the coordinates of the foot of the perpendicular from the point (2, -6) to the line $3y - x + 2 = 0$

2) The probability that John will beat Joe in a game of chess is $\frac{2}{3}$. In a series of 6 games, what is the probability that John will win 4 or more games? Answer in form $\frac{a}{b}$ where **a** and **b** $\in$ Z

BABHTA 6 - ROUND 6

1) Find $\int \frac{\cos^5(x)}{\text{Sin}^2(x)}\,dx$

2) Find the equations of the tangents from (-1, 5) to the circle $x^2 + y^2 - 4x + 2y - 31 = 0$

Answers in the form ax+by+c=0, where **a**, **b** and **c** $\in Z$

BABHTA 7 - ROUND 7

1) In a private sweep, 100 tickets are sold. A man buys one ticket. A prize is awarded on each draw of a ticket. Assuming replacement of a ticket after each draw, find the least number of prizes which must be awarded so that the probability of the man winning at least one prize is at least $\frac{1}{4}$.

2) Find the numerical value of the greatest term of the expansion $(1+2x)^9$ when x = $\frac{1}{3}$. Leave your answer as a fraction.

3) Find , in radians, correct to 3 significant figures, the acute angle between the lines

$3x + 2y = 5$ and $2x - y = 6$.

4) Find the value of $\dfrac{Cos(\frac{\pi}{8}) + iSin(\frac{\pi}{8})}{Cos(\frac{3\pi}{8}) - iSin(\frac{3\pi}{8})}$

BABHTA 8 - ROUND 8

1) Differentiate Cos (x) , when x is in degrees.

2) Find the value of $\text{Sin}^2(x) + \text{Sin}^2(60^0 - x) + \text{Sin}^2(60^0 + x)$ in the form $\frac{a}{b}$, where **a** and **b** $\in$ Z.

3) Julie devises a game in which a player can choose to pay at random from a 1 cent coin, a 5 cent coin, a 20 cent coin and a 50 cent coin. How much should she have charged and still hope to break as even as possible?

4) If two litres of a 20% acid solution are mixed with 8 litres of a 50% acid solution, what is the percentage concentration of the resulting solution?

SCOILT – TIEBREAK

1) Find the value of x if $x - 2(1 - 3x) = 6 + 3(4 - x)$

2) Find the slope of the line passing through A(-3,2) and B(4,-1).
Answer as a fraction.

3) How many pairs of parallel faces on a regular cube?

4) What is the maximum number of points of intersection of four distinct straight lines?

5) Find the values of x if $x^2 = 2x + 1$. Answer in surd form.

6) Find the range of values of x for which $x^2 \geq 4x + 5$, $x \in R$

7) Solve the following equations for x and y:

$$2y^2 - 3x = 0$$
$$4y - x = 6$$

Write your answer in coordinate form.

8) Find the value of $Cos(15^0)$ in simplest surd form.

9) Find the length of the tangent from A(8, -3) to the circle $x^2 + y^2 - 2x + 8y - 23 = 0$.
Answer in surd form.

10) Using only 5c , 10c and 20c coins, in how many ways can you make up 35 cents?

11) One bag contains 4 white and 4 black balls, a second bag contains 3 white and 6 black balls, and a third contains 1 white and 5 black balls.
If one ball is drawn from each bag find the probability that all are white.

12) Using the letters of the word EQUATIONS how many 5 letter "words" contain all the consonants?

13) Find the value of $\left(\sqrt{3} - i\right)^{10}$ in the form x + iy.

14) Find the number of digits that are required to number the pages of a book from page 1 to page 250.

Answer Key

Round 1	**Q1**	$\frac{5}{2}(\text{Cos}(\frac{\pi}{6})+\text{iSin}(\frac{\pi}{6}))$ **or 2.5(Cos (30) +iSin(30))**
	Q2	**P(3, -2)**
Round 2	**Q1**	$2x^2-21x+2=0$
	Q2	$\frac{5}{9}$
Round 3	**Q1**	**a = -2 , b = 5**
	Q2	72^0
Round 4	**Q1**	**9**
	Q2	$0,\pi,2\pi,\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}$
Round 5	**Q1**	$(\frac{1}{5},-\frac{3}{5})$ **or (0·2, - 0·6)**
	Q2	$\frac{496}{729}$
Round 6	**Q1**	$-\frac{1}{\text{Sin}(x)}-2\text{Sin}(x)+\frac{\text{Sin}^3(x)}{3}+C$
	Q2	**y-5=0 and 4x - 3y + 19 = 0**

Round 7	**Q1**	**29**
	Q2	$\frac{224}{9}$
	Q3	**1.05 radians**
	Q4	***i***
Round 8	**Q1**	$-\frac{\pi}{180}\text{Sin}(\frac{\pi x}{180})$ **or equivalent**
	Q2	$\frac{3}{2}$
	Q3	**19 cent**
	Q4	**44%**

Tiebreak answers:

1)	$x = 2$
2)	$-\frac{3}{7}$
3)	3
4)	6
5)	$1 \pm \sqrt{2}$
6)	$x \leq -1$ or $x \geq 5$
7)	(6,3)
8)	$\frac{\sqrt{6}+\sqrt{2}}{4}$ OR $\frac{\sqrt{3}+1}{2\sqrt{2}}$
9)	$\sqrt{10}$
10)	6
11)	$\frac{1}{36}$
12)	600
13)	$512 + 512\sqrt{3}i$
14)	642

ROUND ONE

Q. 1 Solve the simultaneous equations:

$$3x + y = 5$$
$$x^2 + y^2 = 25$$

Q. 2 Find the equation of the circle which has [*AB*] as a diameter, where *A* and *B* are the points (–2, 1) and (6, –3) respectively.

Write your answer in the form $x^2 + y^2 + 2gx + 2fy + c = 0, \quad g, f, c \in Z.$

ROUND TWO

Q. 1 Given that α and β are the roots of the equation $2x^2 - 6x + 1 = 0$,
find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ in its simplest form.

Q. 2 Find the measure of the acute angle between the line l: $2x + y - 3 = 0$ and the line k: $x - 3y + 2 = 0$.

Give your answer correct to the nearest degree.

ROUND THREE

Q. 1 Write $-\sqrt{3}-i$ in general polar form.

Q. 2 The perpendicular distance from the point $(k, 3)$ to the line $3x + 4y - 2 = 0$ is 1 unit, where $k \in Q$.

Find the values of k.

ROUND FOUR

Q. 1 Find the values of A for which

$$\sqrt{3}\tan A = -1, \text{ where } 0^o \le A \le 360^o.$$

Q. 2 Find the range of values of x that satisfies the inequality

$$\frac{x-2}{x+3} \ge \frac{1}{2}, \quad x \ne -3.$$

Team Number ➔ ☐

ROUND FIVE

NB. For this Round 5, answer the questions on **this page** and hand it up to the markers.

Q. 1 Write the equation of the tangent to the curve

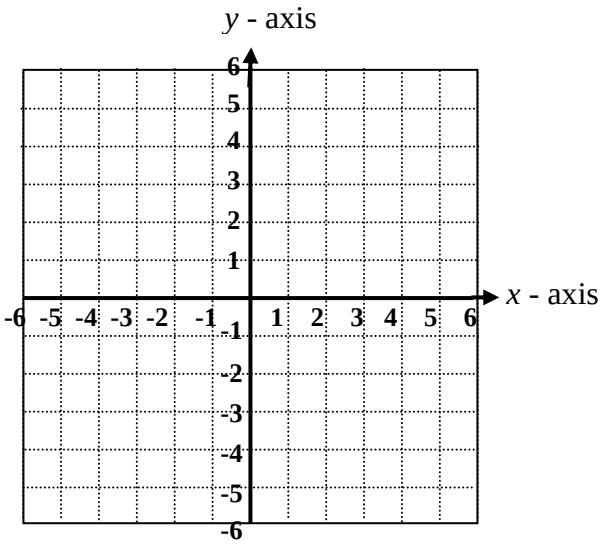
$$x^2 + y^2 + x = 1 \text{ at the point } (0, 1)$$

in the form $ax + by + c = 0$, where $a, b, c \in Z$.

Answer to Q. 1 _____

Q. 2 Within the limits of the given diagram, clearly indicate the set of points that satisfy the inequality

$$2x + y \leq 4.$$



ROUND SIX

Q. 1 In the National Lottery, forty five balls numbered 1 to 45 are placed in a drum. Six balls are taken at random from the drum as the winning combination. If you select six numbers from 1 to 45, what is the probability that, at most, only one of them matches with the winning combination? Give your answer in decimal form, correct to two decimal places.

Q. 2 Find the value of x and the value of y for which

$$\begin{pmatrix} 3 & -5 \\ 5 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 18 \end{pmatrix}.$$

ROUND SEVEN

Q. 1 Find all the values of x that satisfy the equation

$$\sin 4x + \cos 2x = 0, \text{ where } 0^o \le x \le 180^o.$$

Q. 2 A, B and C are points on the same level ground.
$|\,AB\,| = 12$ m and $|\,\angle CAB\,| = 90^o$.
A vertical pole is erected at C.
The angles of elevation of the top of the pole from A and B are 60^o and 30^o respectively.

Calculate $|\,AC\,|$ in metres, correct to one decimal place.

Q. 3 Find the equations of the asymptotes of the graph of

$$f(x) = \frac{x-3}{2x+5}.$$

Q. 4 Evaluate $\int_1^9 \frac{x-1}{x+\sqrt{x}}\,dx$.

ROUND EIGHT

Q. 1 Solve the equation

$$2\log_3 x - \log_3(x - 2) = 2.$$

Q. 2 Mary is playing cards with Liam and Sam. She has nine clubs in her hand. The other four clubs, the Ace, 2, 3 and 4 are randomly dealt to Liam and Sam. Liam holds at least two of the four cards and may have all four.

If Liam shows Mary that he has the 2 of clubs, what is the probability that Sam has the Ace of clubs?

Q. 3 Simplify

$$\lim_{h \to 0} \left(\frac{\tan(x + h) - \tan x}{h} \right).$$

Q. 4 P is the set of lines $3x + y - t = 0$ for all real values of t.
Q is the set of lines $kx - 2y - 8 = 0$ for all real values of k.

Write the equation of the line that is common to the sets P and Q in the form $ax + by + c = 0$, where $a, b, c \in Z$.

Team Number _____

TIE BREAK 1

Write the answers on this page and hand it up

Q. 1 Evaluate $\int_0^{\pi} \frac{\sin^2 x}{1+\cos x}\,dx$.

Answer Q 1 _____

Q. 2 Solve the equation

$$3^{2x} + 8(3^x) - 9 = 0.$$

Answer Q 2 _____

Q. 3 For what range of values is $\tan^{-1}(x)$ defined for all real values of x?

Answer Q 3 _____

Q. 4 Using only digits from the set {2, 3, 4, 5}, how many even numbers can be made, if no two digits in any number are the same?

Answer Q 4 _____

Team Number _____

TIE BREAK 2

Write the answer on this page and hand it up

Find $\dfrac{d\cos^2 x}{dx} + \dfrac{d\sin^2 x}{dx}$

Answer _____

Answers

Round 1: Q. 1 $(0, 5), (3, -4)$

Q. 2 $x^2 + y^2 - 4x + 2y - 15 = 0$

Round 2: Q. 1 6

Q. 2 82^o

Round 3: Q. 1 $2[\cos({}^{7\pi}/_{6} + 2n\pi) + i\sin({}^{7\pi}/_{6} + 2n\pi)]$ or
$2[\cos(210° + n360°) + i\sin(210^o + n360°)]$

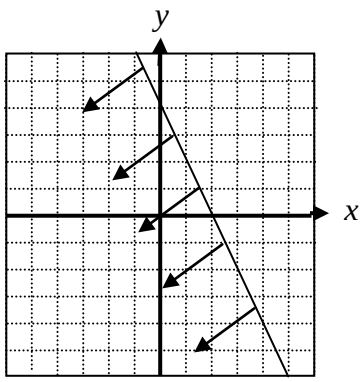
Q. 2 $-5,\ -{}^{5}/_{3}$

Round 4: Q. 1 $150^o, 330^o$ OR ${}^{5\pi}/_{6}, {}^{11\pi}/_{6}$

Q. 2 $\{x|\ x < -3\} \cup \{x|\ x \geq 7\}$

Round 5: Q. 1 $x + 2y - 2 = 0$

Q. 2



Round 6: Q. 1 0·82

Q. 2 $(2, -1)$

Round 7: Q. 1 $45^o, 105^o, 135^o, 165^o$

Q. 2 4·2 m

Q. 3 $x = -{}^{5}/_{2}\ ,\ y = ½$

Q. 4 4

Round 8: Q. 1 3, 6

Q. 2 ${}^{3}/_{7}$

Q. 3 $\sec^2 x$ or $\dfrac{1}{\cos^2 x}$

Q. 4 $3x + y + 4 = 0$

Tie Break 1: Q. 1 π

Q. 2 0

Q. 3 $-90^o < \tan^{-1}(x) < 90^o$ or $-{}^{\pi}/_{2} < \tan^{-1}(x) < {}^{\pi}/_{2}$

Q. 4 32

Tie Break 2: 0

ROUND ONE

Q. 1 Express $\frac{m}{m-n}+\frac{n}{n-m}$ in its simplest form.

Q. 2 The roots of $3x^2+5x+4=0$ are α and β.

Find the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$.

ROUND TWO

Q. 1 Write $\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^{20}$

in the form $\frac{x}{2} + i\frac{y}{2}$, where $x, y \in \boldsymbol{R}$, using surd values where relevant.

Q. 2 Find the values of x which satisfy the equation

$\sin x = \sqrt{3}\cos x$, where $0° \le x \le 360°$.

ROUND THREE

Q. 1 Find the range of values of $\boldsymbol{x}$ that satisfy the inequality

$$\frac{x+5}{x-2} > \frac{1}{2}, \qquad x \neq 2.$$

Q. 2 Find the equations of the two tangents to the circle

$$x^2 + y^2 - 6y - 8 = 0$$

which are parallel to the line $4x - y = 0$.

Write your answers in the form $ax + by + c = 0$, where $a, b, c \in \boldsymbol{Z}$

ROUND FOUR

Q. 1 $z = -1 + 3i$ satisfies the equation

$$z^2 - z - 4iz + 5i - 5 = 0$$

Find the other solution of the equation.

Q. 2 $y = \sin\left(\log_e x^3\right)$.

What is the value of $\frac{dy}{dx}$ when $\boldsymbol{x} = 1$?

ROUND FIVE

Q. 1 The parametric equations

$$x = \frac{8t}{1+4t^2}, \qquad y = \frac{2-8t^2}{1+4t^2}$$

define the circle

$$x^2 + y^2 = r^2.$$

Find the value of r.

Q. 2 Find the 2×2 matrix M such that

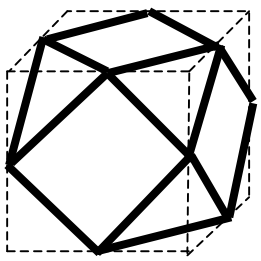
$$\begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix} M \begin{pmatrix} 4 & -3 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}.$$

ROUND SIX

Q. 1 A die has six square faces and eight triangular faces. When thrown, it always lands on one of its faces and is twice as likely to land on any square face as any triangular face.

When the die is thrown, what is the probability that the face it lands on is square ?

Write the answer in the form $\frac{a}{b}$, where $a, b \in \boldsymbol{N}$.



Q. 2 The die in question 1 is made by cutting away the eight corners through the midpoints of the edges of a regular 6-faced die.
Each edge of the 6-faced die was 2 cm in length.

Find the sum of the edges of the resulting 14-faced die in the form $a\sqrt{b}$, where $a, b \in \boldsymbol{N}$.

ROUND SEVEN

Q. 1 Find the shortest distance between the lines

$$3x + 4y - 10 = 0 \quad \text{and} \quad 3x + 4y = 0.$$

Q. 2 Evaluate $\int_0^1 \frac{1}{e^{\sqrt{x}} . \sqrt{x}}\, dx$

Give your answer correct to two decimal places.

Q. 3 From a boat which is due east of a lighthouse, the angle of elevation of the top of the lighthouse is $45°$. From another boat which is due north of the lighthouse, the angle of elevation of the top of the lighthouse is $30°$.
The top of the lighthouse is 200 metres above the level of each boat.

Calculate the distance between the two boats.

Q. 4 Find a value of x that satisfies the equation

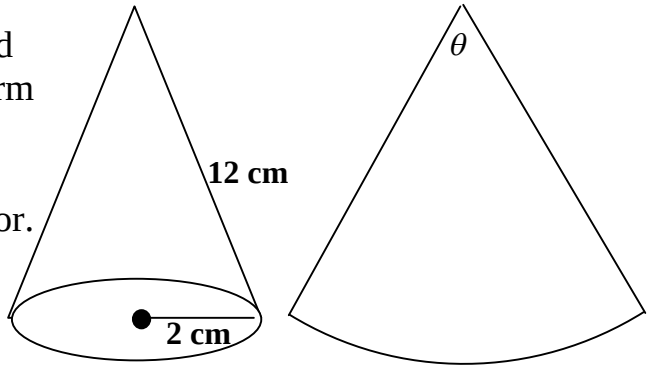
$$\log_3(4 - x) + \log_3(6 - x) = 1.$$

ROUND EIGHT

Q. 1 A hollow cone of base radius 2 cm and slant height 12 cm is opened out to form a sector of a circle.

θ is the angle at the centre of the sector.

Find, in degrees, the value of θ.



Q. 2 Write, in the form $ax + by + c = 0$ where $a, b, c \in \mathbf{Z}$, the equation of the tangent to the curve

$$x^2y^2 - x^2 + y^2 = 7 \text{ at the point } (1, 2)$$

Q. 3 Solve the equation

$$6^x = 300,$$

giving your answer correct to two decimal places.

Q. 4 In a game show, each of five contestants chooses a set of boxes of a particular colour.
No two contestants choose the same colour.
There are six boxes in each colour.
One of the thirty boxes contains a prize of a car and all of the other boxes have small cash prizes.
All of the boxes are equally likely to contain the prize of the car.

Each contestant chooses a colour and then chooses boxes, one at a time, in that colour.

What is the probability that the **third** contestant to choose a colour will win the car when she chooses her **second** box?

TIE BREAK 1

Write the answers on this page and hand it up

Q. 1 $y = \sin 5$.

Find $\frac{dy}{dx}$. **Answer** _____

Q. 2 Find $\int \frac{1}{\sqrt{x^3}} dx$.

Write your answer in surd form. **Answer** _____

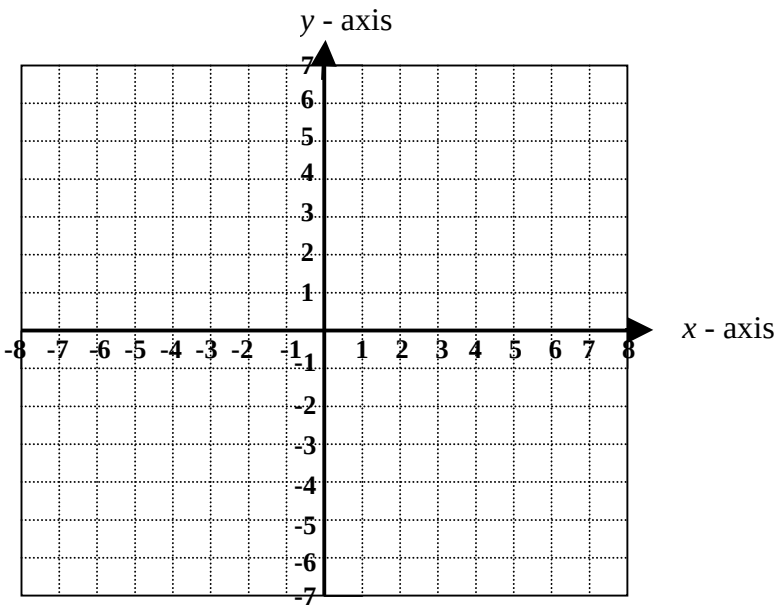
Q. 3 Find the values of $\boldsymbol{x}$ which satisfy the equation

$\cos^2 2\boldsymbol{x} - \sin 2\boldsymbol{x} = 1$, where $0^o \leq \boldsymbol{x} \leq 180^o$.

Answer _____

Q. 4 Within the limits of the given diagram, plot the graph of

$y = |x|$, where $x \in \boldsymbol{R}$.



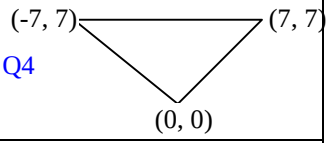
TIE BREAK 2

Write the answer on this page and hand it up

Using only digits from the set {3, 5, 6, 8}, how many numbers greater than 600 can be made, if no two digits in any number are the same?

Answer _____

ANSWERS - Round 1, 2010.

ROUND	Question 1.	Question 2.	Question 3.	Question 4.
1	Q1. 1	Q2. $\frac{1}{12}$		
2	Q1 $-\frac{1}{2}-i\frac{\sqrt{3}}{2}$	Q2 60^o and 240^o		
3	Q1. $\{x \mid x < -12\} \cup \{x \mid x > 2\}$	Q2. $4x - 1y + 20 = 0$ and $4x - 1y - 14 = 0$		
4	Q1. $2 + i$	Q2. 3		
5	Q1 $r = 2$	Q2 $\begin{pmatrix} -1 & -3 \\ -1 & -4 \end{pmatrix}$		
6	Q1. $\frac{3}{5}$ or equivalent	Q2. $24\sqrt{2}$ cm		
7	Q1 2	Q2 1·26	Q3. 400 metres	Q4. $x = 3$
8	Q1 60^o	Q2. $3x + 4y - 11 = 0$	Q3. $x = 3{\cdot}18$	Q4 $\frac{1}{30}$ (*or* $0{\cdot}0\dot{3}$ *or* equivalent)
TIE 1	Q1 0	Q2 $-\frac{2}{\sqrt{x}} + c$	Q3 0°, 90°, 135°, 180°	Q4 
TIE 2	Q1 36			