Round 1

Question 1

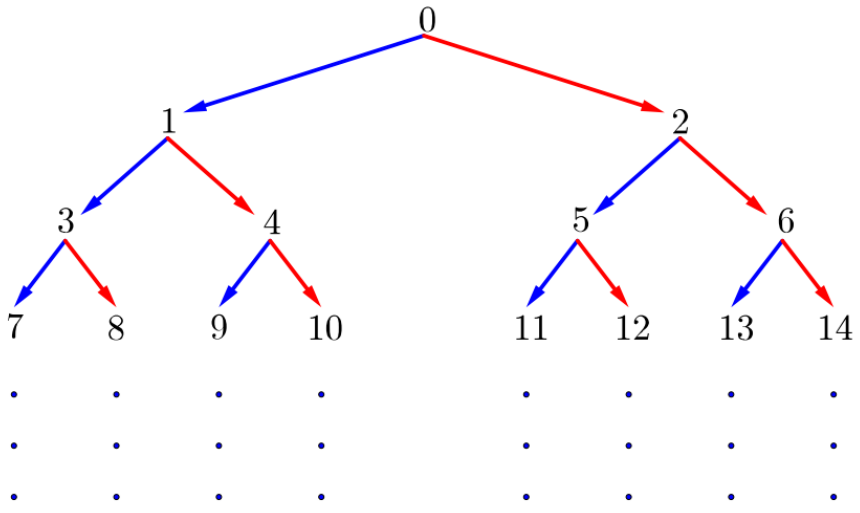
The number $A8$ is a two-digit number with tens digit A and units (ones) digit 8.
Similarly, $3B$ is a two-digit number with tens digit 3 and units digit B.
When $A8$ is multiplied by $3B$, the result is the four-digit number $C730$. That is,

$$\begin{array}{r} A8 \\ \times\ 3B \\ \hline C730 \end{array}$$

If A, B, and C are each different digits from 0 to 9, determine the values of A, B, and C.

Question 2

Consider the following number tree.



In this number tree, the integers greater than or equal to 0 are written out in increasing order, with the top row containing one integer and every row after containing twice as many integers as the row above it.
If row 1 contains the integer 0, what is the fifth number in row 10?

Round 2

Question 1

Alexia, Benito, and Carmen won a team art competition at their local park. As part of their prize, they can paint the 5 km (5000 m) paved path through the park any way they like. They decided Alexia will paint the first 70 m, then Benito will paint the next 15 m, then Carmen will paint the next 35 m. They will keep repeating this pattern until they reach the end of the 5 km path. What percentage of the path will each person paint?

Question 2

Greta currently works 45 hours per week and earns a weekly salary of $729. She will soon be starting a new job where her salary will be increased by 10% and her hours reduced by 10%. How much more will she be earning per hour at her new job?

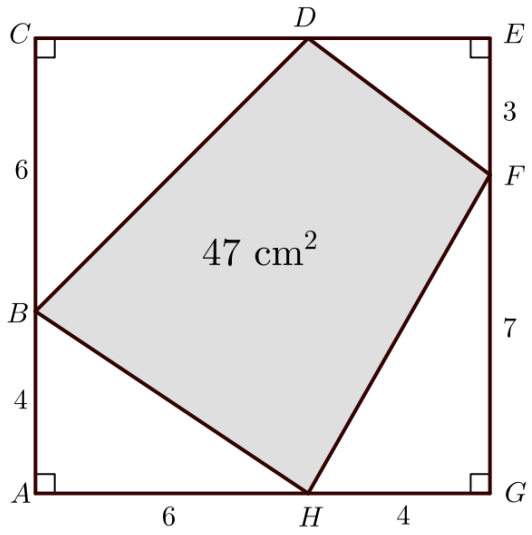
Round 3

Question 1

Quadrilateral $BDFH$ is constructed so that each vertex is on a different side of square $ACEG$. Vertex B is on side AC so that $AB = 4$ cm and $BC = 6$ cm. Vertex F is on EG so that $EF = 3$ cm and $FG = 7$ cm. Vertex H is on GA so that $GH = 4$ cm and $HA = 6$ cm. The area of quadrilateral $BDFH$ is 47 cm^2.

The fourth vertex of quadrilateral $BDFH$, labelled D, is located on side CE so that the lengths of CD and DE are both positive integers.

Determine the lengths of CD and DE.

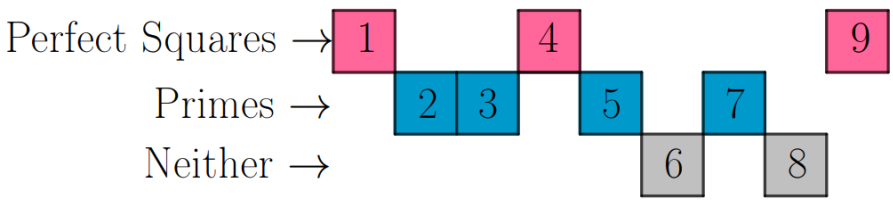


Question 2

The number 7 has only two positive factors, 1 and itself. A positive integer greater than 1 whose only positive factors are 1 and itself is said to be *prime*.

A *perfect square* is an integer created by multiplying an integer by itself. The number 25 is a perfect square since it is 5×5 or 5^2.

Determine the smallest perfect square that has three different prime numbers as factors.

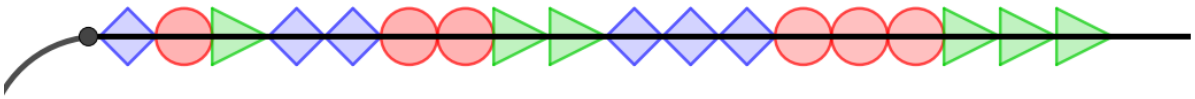


Round 4

Question 1

A necklace is to be created that contains only square shapes, circular shapes, and triangular shapes. A total of 180 of these shapes will be strung on the necklace in the following sequence: 1 square, 1 circle, 1 triangle, 2 squares, 2 circles, 2 triangles, 3 squares, 3 circles, 3 triangles, with the number of each shape type increasing by one every time a new group of shapes is placed. The diagram illustrates how the first 18 shapes would be strung.

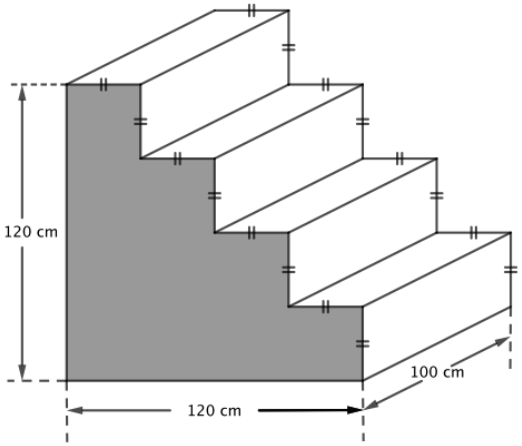
Once the necklace is completed, how many of each shape would the necklace contain?



Question 2

The stairs are to be painted gold and the two sides are to be painted black. One of the two sides that is to be painted black is shaded in the diagram. The back and bottom of the structure will not be seen and will not be painted.

Determine the total area to be painted gold and the total area to be painted black.



Round 5

Question 1

A circular spinner is divided into three sections. An arrow is attached to the centre of the spinner. The arrow is spun once. The probability that the arrow stops on the largest section is 50%. The probability it stops on the next largest section is 1 in 3. What is the probability it stops on the smallest section?

Question 2

A train is 1000 metres long. It is travelling at a constant speed, and approaches a tunnel that is 3000 metres long. From the time that the last car on the train has completely entered the tunnel until the time when the front of the train emerges from the other end, 30 seconds pass. Determine the speed of the train, in kilometres per hour.

Question 3

Find the point of intersection of the lines:

$$x = \frac{1}{2}y + 3 \qquad \& \qquad y = \frac{1}{2}x - 3.$$

Question 4

If $q^2x = p + 2q^2$ and $y = q(x - 4)$, express y in terms of p and q in the form $\frac{ap+bq^n}{cq}$, $a, b, c, n \in Z$

Round 6

Question 1

U is the universal set and P and Q are two subsets of U.

$$\#U = 20$$

$$\#(P \cap Q) = x$$

$$\#(P \backslash Q) = 2x$$

$$\#((P \cup Q)') = 4$$

$$\#Q = 2(\#P).$$

Find $\#Q$.

Question 2

A square sheet of cardboard measures 6 cm by 6 cm. A square of side x cm is removed from each corner. The remaining piece of cardboard is folded to form an open box. Find an expression, in x, for the area of one of the four equal sides of the box, in cm^2, in its simplest form.

Question 3

Simplify:

$$\frac{2^{2^{0^2}} + 2^{2^0} + 2^2 + 2}{2^{0^{2^2}} + 2^{0^2} + 2^0 + 2}.$$

Question 4

Every day, a sprinkler first turns on at 9 AM. To save water, Alice turns it off every 50 minutes, at 9:50 AM, 10:40 AM, and so on. To water the plants, Bob turns it on every 60 minutes, at 10 AM, 11 AM, and so on. They continue doing this until 2 PM. At 2 PM, the sprinkler turns off for the rest of the day. What fraction of the day is the sprinkler on? (grammar issue – is it ok to end a sentence with 'on'?)

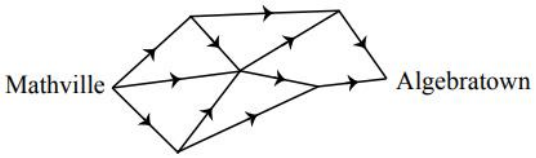
Round 7

Question 1

A square and a regular hexagon have equal perimeters. If the area of the hexagon is $24\sqrt{3}$, what is the area of the square?

Question 2

On the map shown, there are a number of routes from Mathville to Algebratown.



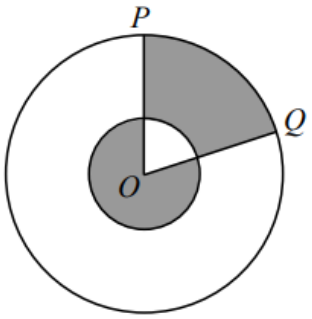
Each route must travel along the roads in the direction marked by the arrows. What is the total number of routes from Mathville to Algebratown?

Question 3

Abdullah and Mary are 90 metres apart. Starting at the same time, they run towards each other. Abdullah runs twice as fast as Mary. How far has Abdullah run when they meet?

Question 4

In the diagram, two circles are centred at O. The smaller circle has a radius of 1 and the larger circle has a radius of 3. Points P and Q are placed on the larger circle so that the areas of the two shaded regions are equal. If $\angle POQ = x°$, what is the value of x?



Round 8

Question 1

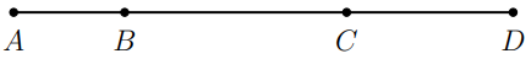
If $\frac{a}{b} = \frac{2}{3}$ and $\frac{c}{b} = \frac{1}{5}$ and $\frac{c}{d} = \frac{7}{15}$, find the value of $\frac{ab}{cd}$.

Question 2

What is the largest integer that x can be, so that $\frac{x}{11} < \frac{2}{3}$?

Question 3

Line segment AD is divided into three segments by points B and C, so that $AB : BC = 1 : 2$ and $BC : CD = 6 : 5$. The length of AD is 56 units. What is the length of AB?



Question 4

If

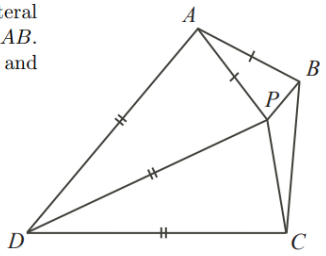
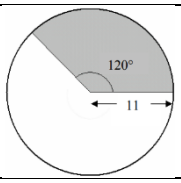
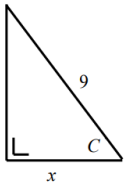
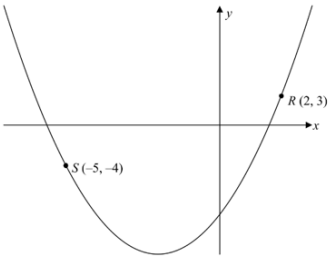
$$x = \left(1 - \tfrac{1}{12}\right)\left(1 - \tfrac{1}{11}\right)\left(1 - \tfrac{1}{10}\right)\left(1 - \tfrac{1}{9}\right)\left(1 - \tfrac{1}{8}\right)\left(1 - \tfrac{1}{7}\right)\left(1 - \tfrac{1}{6}\right)\left(1 - \tfrac{1}{5}\right)\left(1 - \tfrac{1}{4}\right)\left(1 - \tfrac{1}{3}\right)\left(1 - \tfrac{1}{2}\right)$$

what is the value of x?

Answers

Round 1	
1	A = 7, B = 5 and C = 2
2	515
Round 2	
1	Alexia will paint 58.8% of the path, Benito will paint 12.5% of the path, and Carmen will paint 28.7% of the path.
2	$3.60 more per hour
Round 3	
1	8 and 2
2	900
Round 4	
1	66 square shapes, 59 circular shapes, and 55 triangular shapes
2	the total area to paint gold is 24 000 cm2 and the total area to paint black is 18 000 cm2 .
Round 5	
1	$\frac{1}{6}$
2	$240\ km/h$
3	$(2, -2)$
4	$y = \frac{p - 2q^2}{q}$
Round 6	
1	12
2	$6x - 2x^2$
3	2
4	$\frac{5}{48}$
Round 7	
1	36
2	8
3	$60m$ (units required)
4	$40°$ (units required)
Round 8	
1	$\frac{70}{9}$
2	7
3	12
4	$\frac{1}{12}$

Tie Break Round

1	If a + b = 9 − c and a + b = 5 + c, what is the value of c?
2	Aoife is paid €51 for the first week of her part-time summer job. For each week after the first, she is paid €100. How many weeks in total does she have to work for her average weekly pay to be €93?
3	In the diagram, point P is inside quadrilateral $ABCD$. Also, $DA = DP = DC$ and $AP = AB$. If $\angle ADP = \angle CDP = 2x°$, $\angle BAP = (x + 5)°$, and $\angle BPC = (10x - 5)°$, what is the value of x? 
4	Find, correct to the nearest cm^2 , the area of the shaded region in the diagram. 
5	l is the line $3x - 4y + 7 = 0$ and contains the point $P(-1, h)$. m is the line $4x + 3y - 24 = 0$ and contains the point $Q(k, 0)$. l and m intersect at the point R. Find the coordinates of R.
6	Given that $\cos C = \frac{2}{3}$, find the value of x. 
7	Divide $x^3 + 5x^2 - 29x - 105$ by $x + 3$.
8	Part of the graph of the function $y = x^2 + ax + b$, where $a, b \in Z$, is shown. The points $R(2,3)$ and $S(-5, -4)$ are on the curve. Find the values of a & b. 
9	In a table quiz, 100 questions were asked. Team M answered 72 questions correctly. Team Nanswered 38 questions correctly. Find the minimum number of questions answered correctly by both teams.
10	A box contains red and blue pens. The probability that a pen picked at random from the box is red is $\frac{2}{7}$. Some green pens are then added to the box, so that 25% of the pens in the box are now green. One pen is then picked at random from the box. Find the probability that this pen is blue (as a fraction in simplest form).

Tiebreak Answers

1	2
2	7
3	$13°$ (unit required)
4	127 (no units needed)
5	$(3,4)$
6	6
7	$x^2 + 2x - 35$
8	$a = 4, b = -9$
9	10
10	$\frac{15}{28}$