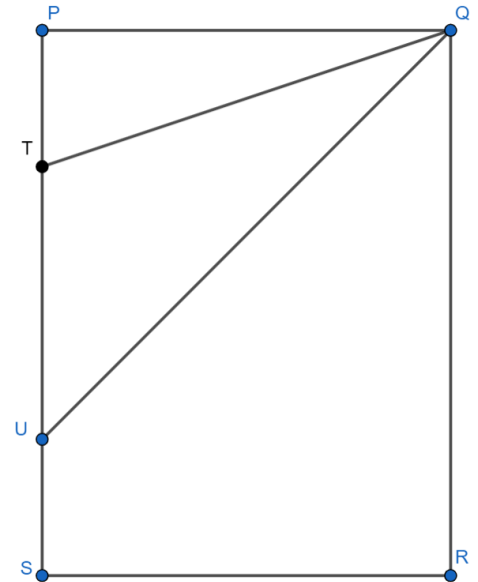


Round 1

Question 1

In rectangle $PQRS$, $|PQ| = 3$ and $|QR| = 4$.
Points T and U are on side PS such that
 $|PT| = |US| = 1$.

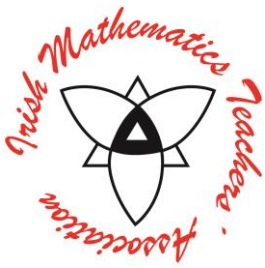
Calculate the measure of $|\angle TQU|$, in degrees
correct to 1 decimal place.



Question 2

Solve the equation,

$$\log_5(x - 2) = 1 - \log_5(x - 6), x \in \mathbb{R}, x > 6.$$



Round 2

Question 1

The equation $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$ has five solutions.

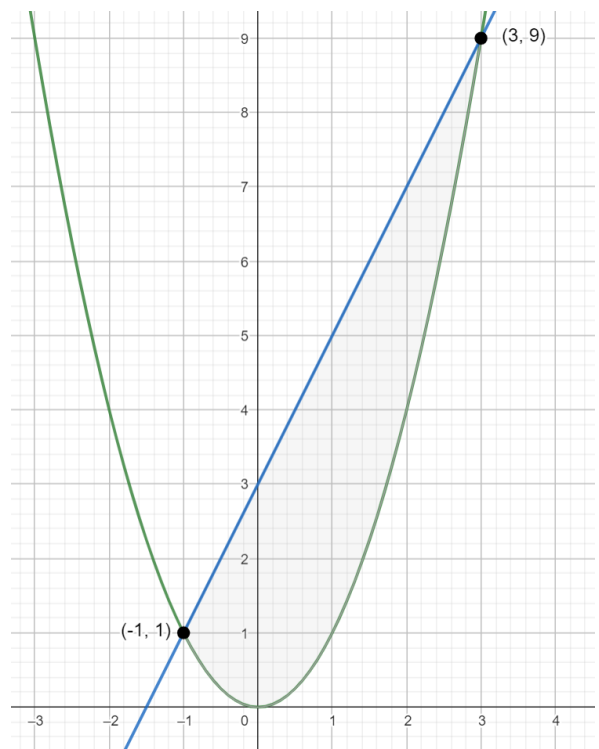
Determine all five values of x that satisfy the equation.

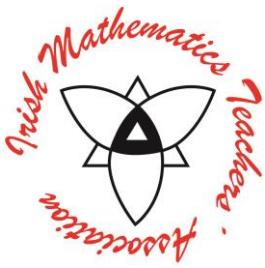
Question 2

The parabola $y = x^2$ meets the line $y = 2x + 3$ at the points $(-1, 1)$ and $(3, 9)$ as shown in the diagram

Find the area enclosed by the parabola and the line.

Answer in the form $\frac{a}{b}$, where $a, b \in \mathbb{N}$.





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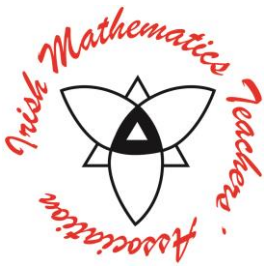
Round 3

Question 1

If $f(x) = e^{2x}(2x + 1)$, find $f'(0)$

Question 2

A family has four children, each with a different age. The product of their ages is 17,280. The sum of the ages of the three oldest children is 40 and the sum of the ages of the three youngest children is 32. Determine all possibilities for the ages of the four children.



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Round 4

Question 1

Find the real number a such that for all $x \neq 9$,

$$\frac{x - 9}{\sqrt{x} - 3} = \sqrt{x} + a.$$

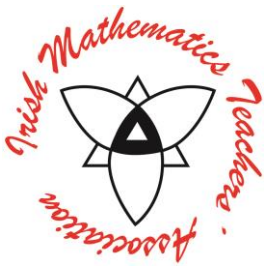
Question 2

The curve C has equation $y = x\sqrt{\ln x}$, $x > 0$.

The equation of the tangent to C at the point where $x = a$ is

$$4y = bx - a, \text{ where } a \text{ and } b \text{ are non zero constants.}$$

Find the exact value of a .



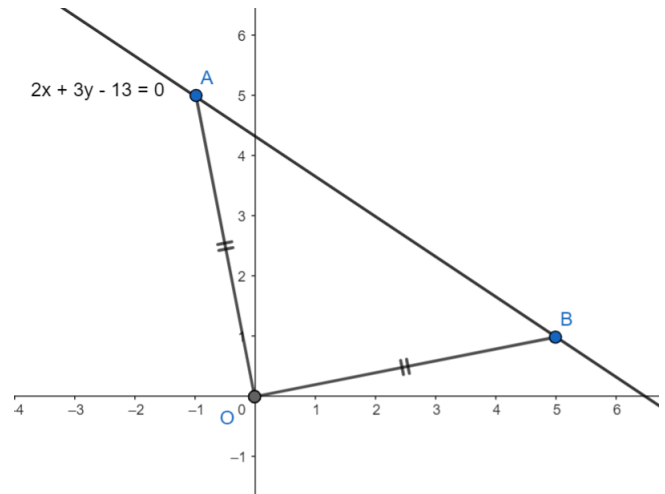
Round 5

Question 1

$\triangle OAB$ is an isosceles right-angled triangle with

- vertex O located at the origin; and
- vertices A and B located on the line $2x + 3y - 13 = 0$ such that $\angle AOB = 90^\circ$ and $|OA| = |OB|$.

Calculate the area of $\triangle OAB$.

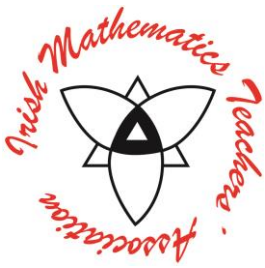


Question 2

Solve the following trigonometric equation

$$\frac{\cos 2x}{1 + \cos 2x} = 1 - 2 \tan x, \quad 0 \leq x \leq 2\pi.$$

Write your answers in terms of π .

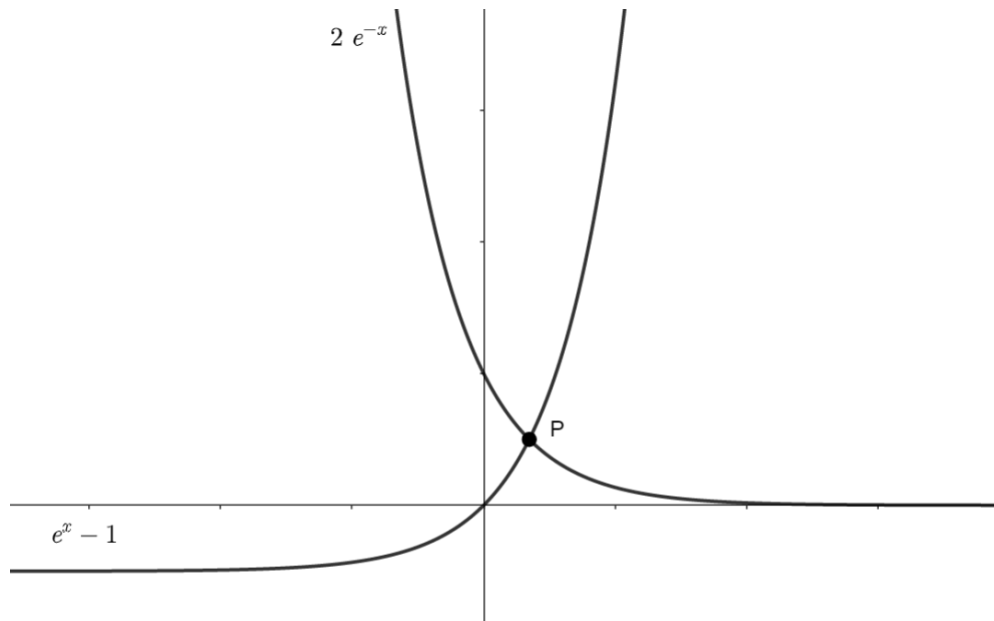


Round 6

Question 1

Given $y = \ln\left(\frac{3+x}{\sqrt{9-x^2}}\right)$, find $\frac{dy}{dx}$ and express it in the form $\frac{a}{b-x^n}$

Question 2

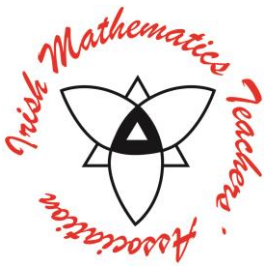


The figure above shows the graphs of the curves with equations

$$y = 2e^{-x} \text{ and } y = e^x - 1.$$

The two graphs intersect at the point P .

Calculate the exact coordinates of P .



Round 7

Question 1

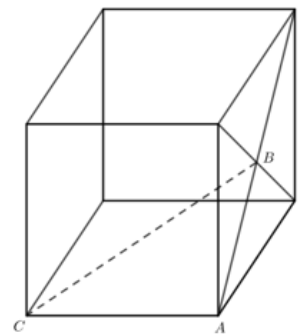
A cylinder has a radius of $\left(\frac{1}{\sqrt{2}-1}\right)$ cm and a height of $(\sqrt{2} + 1)$ cm.

Express the volume in the form $\pi(a + b\sqrt{c})$.

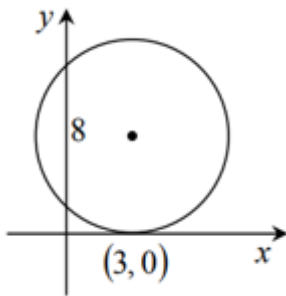
Question 2

Points A and C are vertices of a cube with side length 2 cm and B is the point of intersection of the diagonals of one face of the cube, as shown.

Find the length of $|CB|$.



Question 3



A circle has its centre in the first quadrant.

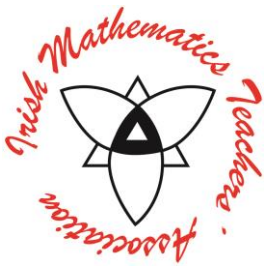
The x - axis is a tangent to the circle at the point $(3,0)$. The circle cuts the y - axis at points that are 8 units apart.

Find the equation of the circle in the form

$$(x - h)^2 + (y - k)^2 = j, \text{ where } h, j, k \in N.$$

Question 4

Five unbiased coins are tossed eight times. Find the probability of getting three heads and two tails exactly four times, correct to three decimal places.



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Round 8

Question 1

Find the value of the term which is independent of x in the expansion $\left(x^2 - \frac{1}{x}\right)^9$

Question 2

Solve the simultaneous equations

$$2^{x+2} + 3^y = 19$$

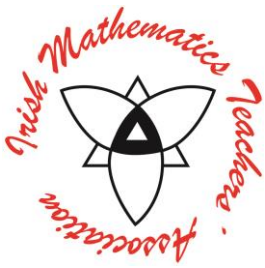
$$4^x - 2(3^y) = 10, \text{ for } x, y \in R.$$

Question 3

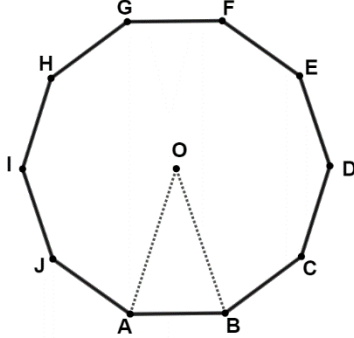
Find the exact value of $\tan 2\left(\sin^{-1} \frac{3}{5}\right)$

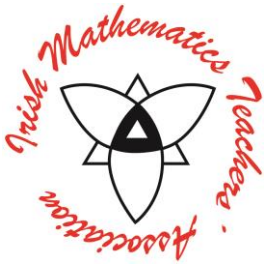
Question 4

A group consists of 7 boys and 5 girls. If three of the group are picked at random, what is the probability that more girls than boys are picked? Answer as a fraction in simplest form.



Tiebreak Round

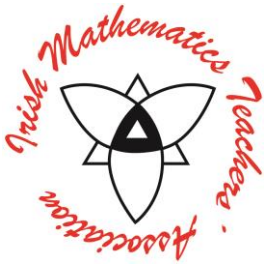
- 1 Find the range of values of $t \in \mathbb{R}$ for which the quadratic equation $(2t - 1)x^2 + 5tx + 2t = 0$ has real roots.
- 2 Find all the solutions of the equation $\sin 4x + \sin 2x = 0$, $0^\circ \leq x \leq 360^\circ$. Answer in degrees.
- 3 If $y = \sqrt{\frac{x^2-1}{x^2+1}}$, find the value of $\frac{dy}{dx}$ when $x = \sqrt{3}$ and write your answer in the form $\sqrt{\frac{a}{b}}$, where $a, b \in \mathbb{N}$.
- 4 The amounts due on monthly mobile phone bills in Ireland are normally distributed with mean €53 and standard deviation €15. If a monthly phone bill is chosen at random, find the probability that the amount due is between €47 and €74, as a decimal to 3 decimal places.
- 5 The diagram shows a regular decagon. The decagon has centre O . The perimeter of the shape is 80 cm. By considering triangle OAB , calculate the area of the 10-sided shape, in square centimetres correct to two significant figures.
- 6 Calculate the distance from the point $(-3, 2)$ to the line $5x - 12y - 6 = 0$, correct to 3 decimal places.
- 7 Two numbers have a sum of 8 and a product of 15. What is the sum of the reciprocals of these numbers?
- 8 Write $-\sqrt{3} - i$ in general polar form.
- 9 An unbiased six-sided die is tossed three times. Given that the sum of the first two tosses equals the third, what is the probability that at least one 2 is tossed?
- 10 Find the altitude of the equilateral triangle whose area and perimeter have the same numerical value.



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Answers

Round 1	
1	26.6°
2	$x = 7$
Round 2	
1	$x = -10, 1, 2, 4, 6$
2	$\frac{32}{3}$ units sq
Round 3	
1	4
2	8, 9, 15, and 16
Round 4	
1	$x = 3$
2	$a = e^4$
Round 5	
1	13
2	$\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$
Round 6	
1	$\frac{3}{9 - x^2}$
2	$P(\ln 2, 1)$
Round 7	
1	$\pi(7 + 5\sqrt{2})$
2	$\sqrt{6}$
3	$(x - 3)^2 + (y - 5)^2 = 25$
4	0.149
Round 8	
1	84
2	$x = 2, y = 1$
3	$\frac{24}{7}$
4	$\frac{4}{11}$



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Tiebreak Answers	
1	$t \leq \frac{-8}{9}$ or $t \geq 0$
2	$0^\circ, 60^\circ, 90^\circ, 120^\circ, 180^\circ, 240^\circ, 270^\circ, 300^\circ, 360^\circ$
3	$\sqrt{\frac{3}{32}}$
4	0.575
5	490
6	3.462
7	$\frac{8}{15}$
8	$2[\cos(\frac{7\pi}{6} + 2n\pi) + i\sin(\frac{7\pi}{6} + 2n\pi)]$ or $2[\cos(210^\circ + n360^\circ) + i\sin(210^\circ + n360^\circ)]$ Accept: $2[\cos(-150 + 360n) + i\sin(-150 + 360n)]$ or $2[\cos(\frac{-5\pi}{6} + 2n\pi) + i\sin(\frac{-5\pi}{6} + 2n\pi)]$
9	$\frac{8}{15}$
10	6