Team Maths Regional 2023 Possible Worked Solutions

Babhta 1 **Round 1**

Q1

$$\frac{\sin B}{25} = \frac{\sin 28}{16}$$

$$=> \ B = \sin^{-1}\left(\frac{25}{16}(\sin 28\,)\right)$$

$B = 47.184777^o$ $(acute)$ or

$B = 180 - \ 47.184777 = 132.815223^o$ $(obtuse) \approx 133^o$

Q2

In ascending order: a, b, c, d

Mean of a,b,c,d = 25 => sum of all equals 100

$$a + \ b + c \ + d \ = 100$$

$$-a - b - c \qquad = \ -70$$

$$=> d = 30$$

Range: 14

$$=> 30 - a = 14$$

$$=> a = 16$$

$$a + b + c = 70$$

$$=> b + c = 54$$

Median of b,c = median of a,b,c,d due to ascending order

$$\frac{a+b}{2} = \frac{54}{2} = 27$$

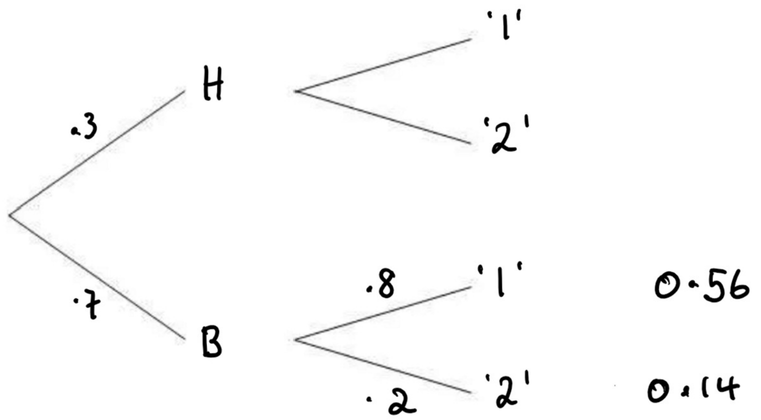
Babhta 2 **Round 2**

Q1

Possible Cards: H1 H2 B1 B2

$$P(H) = 0.3; => P(B) = 0.7; \quad P('1'|B) = 0.8; => P('2'|B) = 0.2$$

$$=> P(B \cap '2') = P(B) \times P('2'|B) = 0.7 \times 0.2 = 0.14$$



Q2

$$z_1 = a - 6i; \quad z_2 = 1 + bi; \qquad z_1 z_2 = -17 - 9i$$

$$z_1 z_2 = (a - 6i)(1 + bi) = a + abi - 6i - 6bi^2 = a + 6b + (ab - 6)i$$

$=> a + 6b = -17; \qquad ab - 6 = -9;$

$=> a = -17 - 6b;$

$=> b(-17 - 6b) = -9$

$=> 6b^2 + 17b - 3 = 0$

$=> b = \frac{1}{6}\ ; \ b = -3$

Take integer value $b = -3$

$=> a = -17 - 6(-3) = 1$

$a = 1; b = -3;$

Babhta 3 **Round 3**

Q1

Coins

African	Asian	South American	European	Australian
10	6	7	8	x

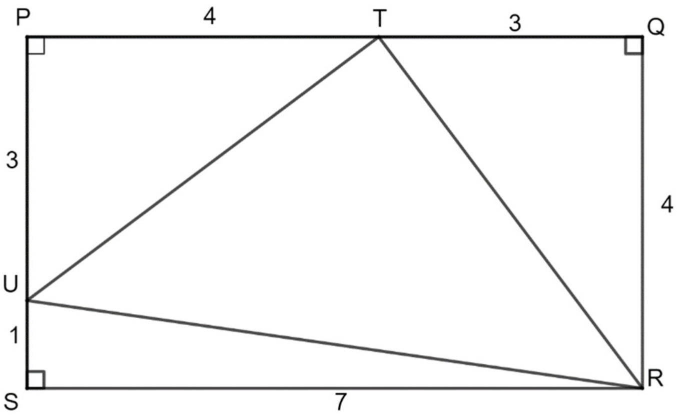
$$\frac{4}{9} = \frac{16}{31 + x}$$

$$4(31 + x) = 144$$

$$31 + x = 36$$

$$x = 5$$

Q2



$$|UT| = \sqrt{3^2 + 4^2} = 5 = |TR|$$

$$|UR| = \sqrt{1^2 + 7^2} = \sqrt{50}$$

Cosine Rule for ΔTRQ

$$5^2 = 5^2 + \left(\sqrt{50}\right)^2 - 2(5)\left(\sqrt{50}\right)\cos(|\angle TUR|)$$

$$25 = 25 + 50 - 2(5)\left(\sqrt{50}\right)\cos(|\angle TUR|)$$

$$|\angle TUR| = \cos^{-1}\frac{\sqrt{2}}{2} = 45^o$$

$$=> |\angle RUS| + |\angle PUT| = 180^o - 45^o = 135^O$$

Or:

$$=> |\angle RUS| + |\angle PUT| = 180^o - \tan^{-1}\frac{4}{3} - \tan^{-1}\frac{7}{1} = 45^o$$

However, finding $\tan^{-1}\frac{4}{3}$ and $\tan^{-1}\frac{7}{1}$ separately leads to rounding errors!!

Q1

Let $G + S + R + B = T$

G	S	R	B
$\frac{1}{6}T$	$3R$	R	138

$\frac{1}{4}T = R + 138$

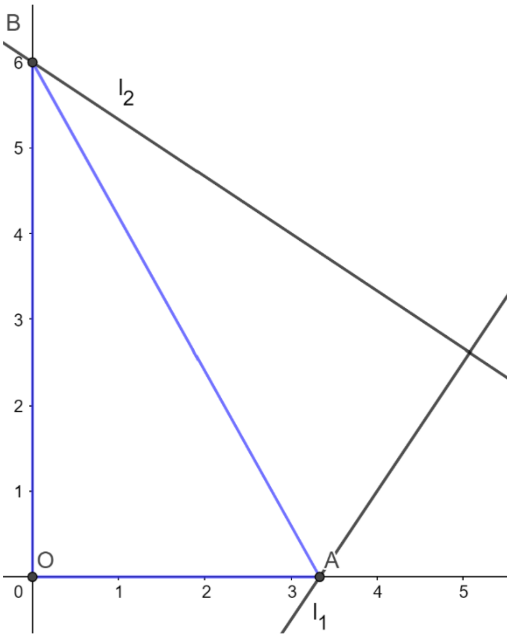
$=> \; 6T = 24R + 3312$

$T = \frac{1}{6}T + \; 4R + 138$

$=> \; 5T = 24R + 828$

$=> T = 2484$

Q2



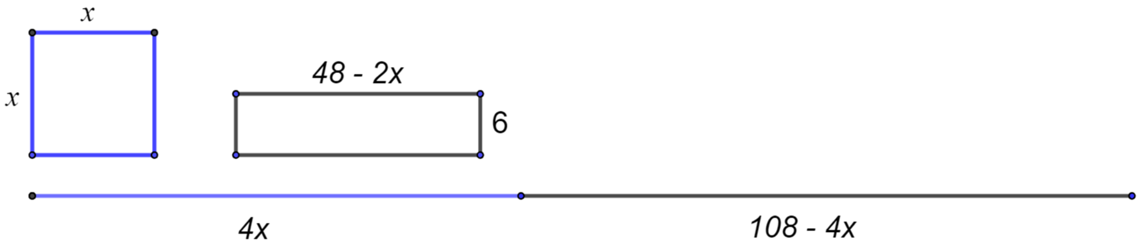
l_1 slope: $\frac{3}{2}$ x-intercept: $(\frac{10}{3}, 0)$ = A

l_2 slope: $-\frac{2}{3}$ equation: $y - 0 \; = \; -\frac{2}{3}(x - 9)$ y-intercept: (0,6) =B

Area ΔOAB: $\frac{1}{2}\left(\frac{10}{3}\right)(6) = 10$

Babhta 5 **Round 5**

Q1



$x^2 = 6(48 - 2x)$

$=> x^2 + 12x - 288 = 0$

$=> x = -24, \quad x = 12$

$=> x = 12; \quad 4x = 48$

Cut at 48cm or equivalently, 108 - 48 = 60cm.

Q2

$y = x\sqrt{x} + \frac{48}{\sqrt{x}} = x^{\frac{3}{2}} + 48x^{-\frac{1}{2}}$

$=> \frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + 48\left(-\frac{1}{2}\right)x^{-\frac{3}{2}} = \frac{3\sqrt{x}}{2} - \frac{24}{x\sqrt{x}}$

Point where gradient =0 iff turning point i.e. $\frac{dy}{dx} = 0$

$=> \frac{3\sqrt{x}}{2} - \frac{24}{x\sqrt{x}} = 0$

$=> \frac{3\sqrt{x}}{2} = \frac{24}{x\sqrt{x}}$

$=> 3x(\sqrt{x})\sqrt{x} = 48$

$=> x^2 = 16$

$=> x = \pm 4$ but $x > 0 \quad => \quad x = 4$

$y = 4\sqrt{4} + \frac{48}{\sqrt{4}} = 32$

$(4, 32)$

Babhta 6 **Round 6**

Q1

$$W : L = 3 : 1$$

Thus, $\frac{3}{4}$ of solution is W and ¼ is L.

Remove 10l, 50l remaining. Amounts in Litres:

$W: \quad \frac{3}{4}(50) = 37.5 \qquad\qquad L: \quad \frac{1}{4}(50) = 12.5$

Add 10l of water.

$W: \quad 37.5 + 10 = 47.5 \qquad\qquad L: \quad 12.5$

Ratio of Lemon to Water

$$L : W = 12.5 : 47.5 = 5 : 19$$

Q2

$$(3 + px)^3 = \binom{3}{0}(3)^3(px)^0 + \binom{3}{1}(3)^2(px)^1 + \binom{3}{2}(3)^1(px)^2 + \binom{3}{3}(3)^0(px)^3$$

The coefficient of x^2 is $9p^2$. The coefficient of x^3 is p^3.

Given that the coefficient of x^2 is twice the coefficient of x^3:

$$=> \ 9p^2 = 2p^3$$

$$=> \frac{9}{2} = p$$

Babhta 7 **Round 7**

Q1

$y = -5t^2 + 70t + 100;$ $\quad\quad => \dfrac{dy}{dx} = -10t + 70$

Maximum height reached when $\frac{dy}{dx} = 0$:

$$=> -10t + 70 = 0$$

$$=> t = 7$$

$$=> y = -5(7)^2 + 70(7) + 100 = 345m$$

Q2

$$4 + 10 + 16 + \ ... \ + 1354$$

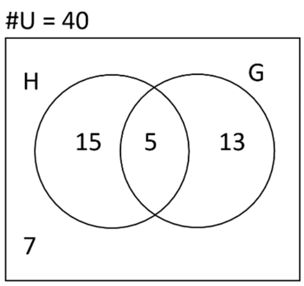
$a = 4; d = 6;$ $\quad\quad T_n = 4 + (n-1)(6) = 6n - 2$

$=> 6n - 2 = 1356$

$=> n = 226$

$S_{226} = \dfrac{226}{2}\{8 + (225)(6)\} = 153{,}454$

Q3



$P(H \cap G) = \dfrac{5}{40} = \dfrac{1}{8}$

Q4

$P(15 \le X \le 30) = P\left(\dfrac{15-25}{5} \le Z \le \dfrac{30-25}{5}\right) = P(-2 \le Z \le 1)$

$= P(Z \le 1) - \big(1 - P(Z \le 2)\big) = \ 0.8413 - (1 - .9772) = 0.8185$

Probability of at least one earning between €15 and €30 is equal to (1 – Probability neither earn that).

$1 - \binom{2}{0}(0.8185)^0(0.1815)^2 = 0.96705775 \ \approx 0.9671$

Babhta 8 **Round 8**

Q1

$P = 5000b^{-\frac{t}{10}}$ $\quad => 1250 = 5000b^{-\frac{20}{10}}$

$$=> \frac{1250}{5000} = \frac{1}{4} = b^{-2}$$

$$=> b = \pm 2 \qquad b > 0 \quad => \quad b = 2$$

$P = 5000(2^{-\frac{t}{10}})$ $\quad => \frac{dP}{dt} = -\frac{5000}{10}(\ln(2))\left(2^{-\frac{t}{10}}\right) = -(500\ln 2)\left(2^{-\frac{t}{10}}\right)$

Instantaneous rate of decrease: 30 per year

$$-(500\ln 2)\left(2^{-\frac{t}{10}}\right) = -30$$

$$=> \left(2^{-\frac{t}{10}}\right) = \frac{3}{(50\ln 2)}$$

$$=> -\frac{t}{10} = \log_2 \frac{3}{(50\ln 2)}$$

$$=> t = (-10)\log_2 \frac{3}{(50\, ln\, 2)} = 35.301 \approx 35.3 \text{ years}$$

Q2

$$\frac{(1-\sqrt{3}i)^4}{1+\sqrt{3}i} \times \frac{(1-\sqrt{3}i)}{(1-\sqrt{3}i)} = \frac{(1-\sqrt{3}i)^5}{1+3} = \frac{(1-\sqrt{3}i)^5}{4} = \frac{1}{4}(1-\sqrt{3}i)^5$$

Re-write in polar form: $(1-\sqrt{3}i) = 2\,(\cos 300 + i\sin 300)$

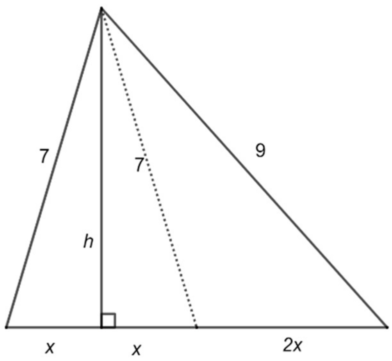
$$(1-\sqrt{3}i)^5 = (2(\cos 300 + i\ \sin 300))^5$$

$$= 2^5(\cos 5(300) + i\sin 1500) = 32\,(\frac{1}{2} + \frac{\sqrt{3}}{2}i) = 16(1+\sqrt{3}\,i)$$

$$=> \frac{1}{4}(1-\sqrt{3}i)^5 = \frac{1}{4}(16)(1+\sqrt{3}\,i) = 4 + 4\sqrt{3}\,i$$

Babhta 8 **Round 8**

Q3



$9^2 = h^2 + (3x)^2$

$81 = h^2 + 9x^2$

$=> \ 81 = 49 - x^2 + 9x^2$

$=> \ 32 = 8x^2$

$=> \ 4 = x^2$

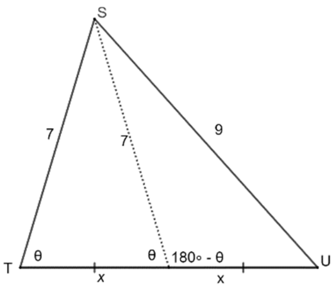
$=> \ x = \ \pm 2 \quad => x = 2$

$=> 4x = 8$cm

$7^2 = h^2 + x^2$

$49 = h^2 + x^2$

OR



$7^2 = 7^2 + x^2 - 2(7)(x)\cos\theta$ and $9^2 = 7^2 + x^2 - 2(7)(x)\cos(180 - \theta)$

$=> x^2 - 14x\cos\theta = 0$

$=> x^2 - 32 - 14x\cos(180 - \ \theta) = 0$

But $\cos(180 - \theta) = \ -\cos\theta$

$=> \ \ x^2 - 32 + 14x\cos\theta = 0$

$=> x^2 \qquad - 14x\cos\theta = 0$

$\underline{=> x^2 - 32 + 14x\cos\theta = 0}$

$=> \ 2x^2 - 32 = \ 0 \qquad => x^2 = 16 \quad => x = 4 \quad => 2x = 8$ = length

Q4

$2\sin 4x = 1 \qquad => \ \sin 4x = \frac{1}{2}$

$=> 4x = \ \sin^{-1}\frac{1}{2} = \frac{\pi}{6} + 2n\pi$ or $4x = \frac{5\pi}{6} + 2n\pi$

$=> x = \frac{\pi}{24} + \frac{n\pi}{2}$ or $x = \frac{5\pi}{24} + \frac{n\pi}{2}$

However: $0 \le x \le \frac{\pi}{4}$

$=> x = \frac{\pi}{24}$ or $x = \frac{5\pi}{24}$

Cesit Réitigh **Tie-break**

Q1. $\frac{(1-\sqrt{2})}{(1+\sqrt{2})} \times \frac{(1-\sqrt{2})}{(1-\sqrt{2})} = \frac{1+2-2\sqrt{2}}{1-2} = 2\sqrt{2} - 3$

Q2. $x + y - z = 0$ $\qquad$ $x - y + z = 4$

$\underline{x - y + z = 4}$ $\qquad$ $\underline{x - y - z = -8}$

$2x = 4$ $\qquad$ $2x - 2y = -4$

$=> x = 2$ $\qquad$ $x - y = -2$

$=> y = 4$

$x + y - z = 0$

$2 + 4 = 6 = z$

$(x, y, z) = (2,4,6)$

Q3.

$y = \frac{2x-3}{x+1}$ $\qquad$ $=> \frac{dy}{dx} = \frac{(x+1)(2)-(1)(2x-3)}{(x+1)^2} = \frac{5}{(x+1)^2}$

Q4.

$y = \ln\sqrt{x^2+1} = \frac{1}{2}\ln(x^2+1)$ $\qquad$ $=> \frac{dy}{dx} = \frac{1}{2}\frac{(2x)}{(x^2+1)} = \frac{x}{x^2+1}$

Q5.

$(1,-1), \quad (-6,-2), \quad (3,-5) \in C$

$1 + 1 + 2g - 2f + c = 0$ $\qquad$ $=> 2g - 2f + c = -2$

$36 + 4 - 12g - 4f + c = 0$ $\qquad$ $=> 12g + 4f - c = 40$

$9 + 25 + 6g - 10f + c = 0$ $\qquad$ $=> -6g + 10f - c = 34$

$2g - 2f + c = -2$ $\qquad$ $2g - 2f + c = -2$ $\qquad$ $g - 2f = -8$

$\underline{12g + 4f - c = 40}$ $\qquad$ $\underline{-6g + 10f - c = 34}$ $\qquad$ $\underline{14g + 2f = 38}$

$14g + 2f = 38$ $\qquad$ $-4g + 8f = 32$ $\qquad$ $15g = 30$

$=> 7g + f = 19$ $\qquad$ $=> g - 2f = -8$ $\qquad$ $g = 2$

Cesit Réitigh **Tie-break**

Q6

$g - 2f = -8$

$2 - 2f = -8$

$=> f = 5$

$2g - 2f + c = -2$

$2(2) - 2(5) + c = -2$

$=> c = 4$

$=> C: x^2 + y^2 + 4x + 10y + 4 = 0$

OR

$(1, -1), \quad (-6, -2), \quad (3, -5) \in C$

$(1, -1), \ (-6, -2)$

Slope: $\frac{-2+1}{-6-1} = +\frac{1}{7} => \perp slope: -7$

Mid Point: $\left(-\frac{5}{2}, -\frac{3}{2}\right)$

$\perp$ Line:

$y + \frac{3}{2} = -7\left(x + \frac{5}{2}\right)$

$14x + 2y - 38$

$(1, -1), \ (3, -5)$

Slope: $\frac{-5+1}{3-1} = -2 => \perp slope: +\frac{1}{2}$

Mid Point: $(2, -3)$

$\perp$ Line:

$y + 3 = \frac{1}{2}(x - 2)$

$x - 2y = 8$

$x - 2y = 8$

$\underline{14x + 2y = -38}$

$15x = -30$

$=> x = -2$

$-2 - 2y = 8$

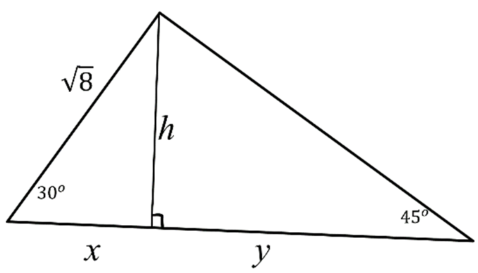
$=> y = -5$

Centre: $(-2, -5)$

Radius: $\sqrt{(1+2)^2 + (-1+5)^2} = 5$

Equation: $(x + 2)^2 + (y + 5)^2 = 25$

Q6



$x = (\sqrt{8}) \cos 30 = \sqrt{6}$

$h = (\sqrt{8}) \sin 30 = \sqrt{2}$

$=> y = h = \sqrt{2}$ isosceles triangle

Area: $\frac{1}{2}(\sqrt{6} + \sqrt{2})(\sqrt{2}) = 1 + \sqrt{3} \approx 2.7$ units squared.

Cesit Réitigh **Tie-break**

Q7

Mean of 4 numbers p => sum of those 4 is $4p$

Mean of 5 numbers x => sum of those 5 is $5x$

Mean of 9 numbers q => sum of those 9 is $9q$

Sum of 9 take the sum of 5 is the sum of 4:

$9q - 5x = 4p$

$=> \ x = \dfrac{9q - 4p}{5}$

Q8

b: Mid-way Line: halfway between 5000 and 35,000 => $b = 20{,}000$

a: Amplitude: height above/below mid-way line => $a = 15{,}000$

Q9

$\dfrac{2x + 3y}{x + 6y} = \dfrac{4}{5}$

$=> 10x + 15y = 4x + 24y$

$=> 6x = 9y$

$=> \dfrac{x}{y} = \dfrac{9}{6} = \dfrac{3}{2}$

Q10

$3x + 17 - (2x + 11) = 2x + 11 - (4x + 11)$

$=> -2x = x + 6$

$=> \ -6 = 3x$

$=> \ x = \ -2$