

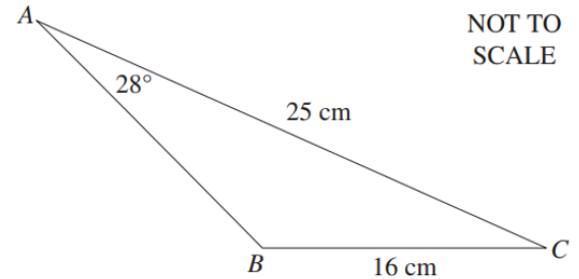


**Babhta 1**

**Round 1**

Q1

The diagram shows a triangle  $ABC$  where  $AC = 25$  cm,  $BC = 16$  cm,  $\angle BAC = 28^\circ$  and  $ABC$  is obtuse.



Find the size of the obtuse angle  $ABC$  correct to the nearest degree.

Q2

$a$ ,  $b$ ,  $c$  and  $d$  are four integers written in order of size, starting with the smallest. The sum of  $a$ ,  $b$  and  $c$  is 70 and the mean of  $a$ ,  $b$ ,  $c$ , and  $d$  is 25. The range of the four integers is 14.

Work out the median of  $a$ ,  $b$ ,  $c$ , and  $d$ .



**Babhta 2**

**Round 2**

Q1

Sheldon has a large number of cards. On the back of each card there is either a honeycomb pattern (hexagons) or a brick pattern (rectangles). On the front of each card there is either a 1 or a 2. As Sheldon went through all the cards, he found that 30% of the cards have a honeycomb pattern on the back. Of the cards with a brick pattern on the back, 80% have a 1 on the front.

Determine the percentage of all the cards that have a brick pattern on the back **and** a 2 on the front.

Q2

If  $z_1 = a - 6i$ ,  $z_2 = 1 + bi$  and  $z_1 z_2 = -17 - 9i$ , find the integer values of **a** and **b**.



**Babhta 3**

**Round 3**

Q1

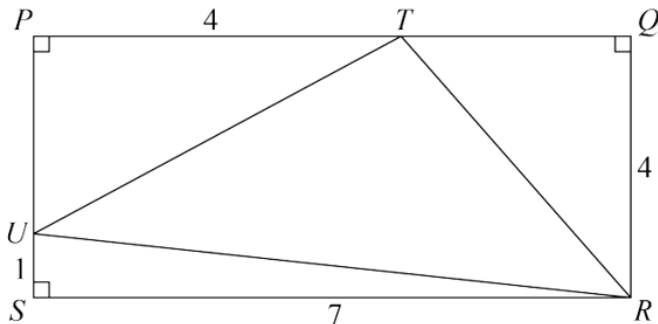
Arya has never travelled to another country, but has a collection of foreign coins given to him by friends and family who have. In his collection he has 10 coins from Africa, 6 coins from Asia, 7 coins from South America, and 8 coins from Europe. One day Arya's grandfather added some Australian coins to the collection.

After he did that, he told Arya that if he took a coin at random from the collection, the probability of it being from either Africa or Asia was  $\frac{4}{9}$ .

How many Australian coins did Arya's grandfather add to the collection?

Q2

Rectangle  $PQRS$  has  $QR = 4$  and  $RS = 7$ .  $\triangle TRU$  is inscribed in rectangle  $PQRS$  with  $T$  on  $PQ$  such that  $PT = 4$  and  $U$  on  $PS$  such that  $SU = 1$ . Determine the **exact** value of  $\angle RUS + \angle PUT$ .





**Babhta 4**

**Round 4**

Q1

The Bord Gais Energy Theatre has four levels of seating: gold, silver, red, and black. One night, the manager of the theatre was asked how many patrons are in the theatre. The manager replied that  $\frac{1}{6}$  of the patrons in the theatre that night are in the gold seating,  $\frac{1}{4}$  of the patrons are in either the red seating or the black seating, there are three times as many patrons in the silver seating as in the red seating, and there are 138 patrons in the black seating. How many patrons were in the theatre that night?

Q2

The line  $l_1$  has equation  $3x - 2y = 10$  and crosses the  $x$ -axis at point A. The line  $l_2$  is perpendicular to  $l_1$  and crosses the  $x$ -axis at  $(9, 0)$ .  $l_2$  crosses the  $y$ -axis at B.

Find the area of the triangle OAB where O is the origin.



**Babhta 5**

**Round 5**

Q1

Simeon has a rope that is 108 cm long and is asked to cut the rope once so that one of the pieces can be arranged, with its two ends touching, to form a square, and the other piece can be arranged, with its two ends touching, to form a rectangle with one side length of 6 cm. Furthermore, the area of the square will be equal to the area of the rectangle. Where should Simeon make the cut to the original piece of rope?

Q2

A curve has the equation  $y = x\sqrt{x} + \frac{48}{\sqrt{x}}$ ,  $x > 0$ .

Find the coordinates of the point on the curve where the gradient is 0.



**Babhta 6**

**Round 6**

Q1

A restaurant owner was experimenting the taste of her homestyle lemon drink. She started with 60 litres of water. She removed 15 litres of water and replaced it with 15 litres of pure lemon juice.

After thoroughly stirring the new mixture, she discovered that it was too 'lemony' So, she removed 10 litres of the new mixture and replaced it with 10 litres of water. She thoroughly stirred the mixture and concluded that the new mixture was just right.

Determine the ratio of pure lemon juice to water in the final 60 litre mixture.

Q2

In the expansion of  $(3 + px)^3$  the coefficient of  $x^2$  is twice the coefficient of  $x^3$ .

Find the non-zero value of  $p$ .



**Babhta 7**

**Round 7**

Q1

A particle is shot vertically upwards from a point 100 metres above ground level. The position of the particle,  $y$  metres above the ground after  $t$  seconds, is given by  $y(t) = -5t^2 + 70t + 100$ .

Find the maximum height above ground level reached by the particle.

Q2

Calculate the sum of the arithmetic series  $4 + 10 + 16 + \dots + 1354$

Q3

History and Geography are two of the subjects students may decide to study. For a group of 40 students, the following is known.

- 7 students study neither History nor Geography
- 20 students study History
- 18 students study Geography

A student is chosen at random. Find the probability that the student studies **both** History and Geography (answer as fraction in simplest form).

Q4

In a particular country, the hourly rate of pay for adults who work is normally distributed with a mean of €25 and a standard deviation of €5. Two adults who both work are chosen at random. Find the probability that at least one of them earns between €15 and €30 per hour. (answer as a decimal correct to 4 decimal places)



**Babhta 8**

**Round 8**

Q1

A population,  $P$ , which is initially 5000, varies according to the formula

$$P = 5000b^{\frac{-t}{10}},$$

where  $b$  is a positive constant and  $t$  is time in years,  $t \geq 0$ .

The population is 1250 after 20 years.

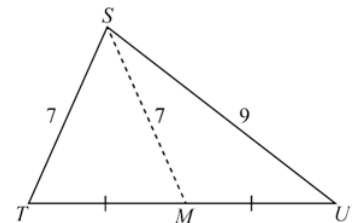
Find the value of  $t$ , correct to one decimal place, for which the instantaneous rate of decrease is 30 people per year.

Q2

Write  $\frac{(1-\sqrt{3}i)^4}{1+\sqrt{3}i}$  in the form  $a + bi$ , where  $a$  and  $b$  are real constants.

Q3

In  $\Delta STU$ , a median is drawn from vertex  $S$ , meeting side  $TU$  at point  $M$ . The length of side  $ST$  is 7 cm, the length of side  $SU$  is 9 cm and the length of the median  $SM$  is 7 cm.



Calculate the length of  $TU$ .

Q4

For what values of  $x$ , in the interval  $0 \leq x \leq \frac{\pi}{4}$ , does the line  $y = 1$  intersect the graph of  $y = 2 \sin 4x$ ?





## Tie-break

1. Express  $\frac{1-\sqrt{2}}{1+\sqrt{2}}$  in the form  $a\sqrt{2} - b$ , where  $a, b \in \mathbb{N}$
2. Solve for  $x, y, z$

$$x + y - z = 0$$

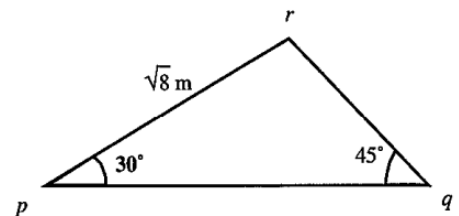
$$x - y + z = 4$$

$$x - y - z = -8$$

3. Differentiate  $\frac{2x-3}{x+1}$  with respect to  $x$ , answer in simplest form.
4. Find  $\frac{dy}{dx}$  if  $y = \ln\sqrt{x^2 + 1}$ , answer in simplest form.
5. Points  $(1, -1)$ ,  $(-6, -2)$  and  $(3, -5)$  are on a circle C.

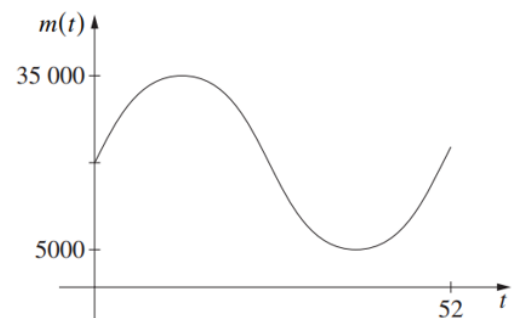
Find the equation of C.

6. Find the area of the triangle shown in the diagram, correct to 1 decimal place.



7. Four numbers have a mean of  $p$ . Five numbers have a mean  $x$ . The nine numbers have a mean of  $q$ . Express  $x$  in terms of  $p$  and  $q$ .

8. The population of mice on an isolated island can be modelled by the function  $m(t) = a \sin\left(\frac{\pi}{26}t\right) + b$ , where  $t$  is the time in weeks and  $0 \leq t \leq 52$ . The population of mice reaches a maximum of 35,000 when  $t = 13$  and a minimum of 5,000 when  $t = 39$ . The graph of  $m(t)$  is shown.



What are the values of  $a$  and  $b$ ?

9. Find the value of  $\frac{x}{y}$  when  $\frac{2x+3y}{x+6y} = \frac{4}{5}$
10. Three consecutive terms of an arithmetic series are  $4x + 11$ ,  $2x + 11$  and  $3x + 17$ .  
Find the value of  $x$ .



Foireann Mata 2023  
Team Maths 2023

Answer Key 2023

Round 1	
1	133°
2	27
Round 2	
1	14%
2	$a = 1, b = -3$ (both required for mark)
Round 3	
1	5
2	135°
Round 4	
1	2484
2	10 (ignore any reference to units)
Round 5	
1	48cm <b>OR</b> 60cm
2	(4,32)
Round 6	
1	5:19
2	$\frac{9}{2}$ <b>OR</b> 4.5
Round 7	
1	345 m
2	153,454
3	$\frac{1}{8}$
4	0.9671
Round 8	
1	$t = 35.3$
2	$4 + 4\sqrt{3}i$
3	8 cm
4	$x = \frac{\pi}{24}$ <b>and</b> $x = \frac{5\pi}{24}$ (both required for mark)



### Tiebreak Answers

1	$2\sqrt{2} - 3$
2	$x = 2, y = 4, z = 6$
3	$\frac{5}{(x+1)^2}$
4	$\frac{x}{x^2 + 1}$
5	$(x+2)^2 + (y+5)^2 = 25$ <b>OR</b> $x^2 + y^2 + 4x + 10y + 4 = 0$
6	2.7
7	$\frac{9q - 4p}{5}$
8	$a = 15,000$ and $b = 20,000$
9	1.5 or $\frac{3}{2}$
10	-2