



Foireann Mata 2022 : an Chraobh Team Maths Final 2022



Babhtha 1

Round 1

Q1.1 The first three terms of a geometric sequence are

$$2x - 4, x + 1, x - 3.$$

Find the two possible values of x .

Q1.2 Given $y = mx + c$ is a tangent to the circle $x^2 + y^2 = a^2$, find an expression for c in terms of a and m .



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Babhta 2

Round 2

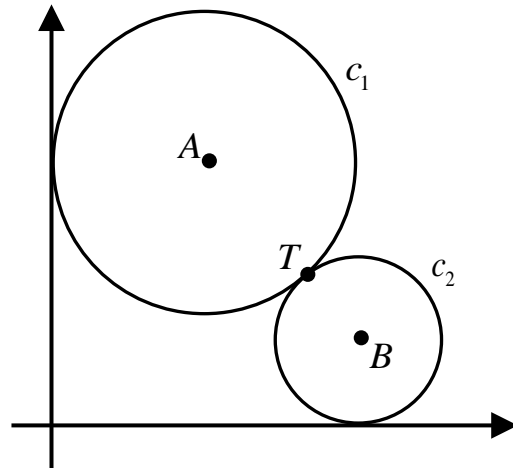
Q2.1 Solve for x , $2^{2x+1} - 15(2^x) - 8 = 0$.

Q2.2 Two circles c_1 and c_2 touch each other at the point $T(9, 3.2)$ as shown in the diagram.

c_1 has radius 5 and c_2 has radius 2.

Find the coordinates of A and B , the centres of the circles.

Note that both c_1 and c_2 are on or above the x -axis.





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Babhta 3

Round 3

- Q3.1 A person coughs when a foreign object is in their windpipe. The velocity of the cough depends on the size of the object. A certain person has a windpipe of radius 20 mm. If a foreign object has a radius of r mm, then the velocity, V mm/s, needed to remove the object by a cough is given by :

$$V(r) = k(20r^2 - r^3),$$

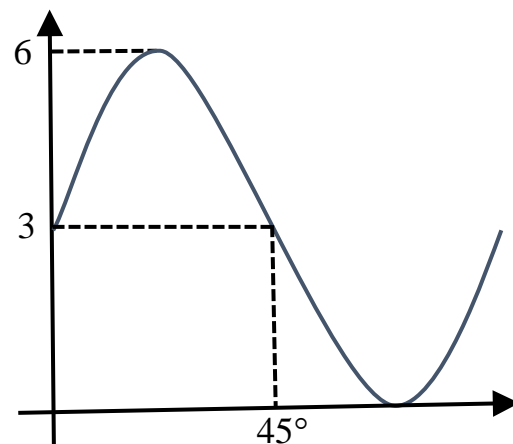
where $0 \leq r \leq 20$, and k is a constant, where $k > 0$.

Find the radius of the foreign object that gives the maximum velocity needed to remove the object?

- Q3.2 The graph shows the function

$$f(x) = c + a \sin bx.$$

Find the values of a , b and c .





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Babhta 4

Round 4

- Q4.1 A circle intersects a line at the points $A(-3, 0)$ and $B(5, -4)$.
The midpoint of $[AB]$ is m .
The distance from the centre of the circle to m is $\sqrt{5}$.
Find the equations of the two circles that satisfy these conditions.
- Q4.2 The speeds of 150 randomly selected cars were recorded as they passed a check-point on a motorway. The mean speed of the cars was 115 km/h and the standard deviation was 24 km/h.
Find the 95% confidence interval for the mean speed of cars passing the check-point correct to one place of decimals.



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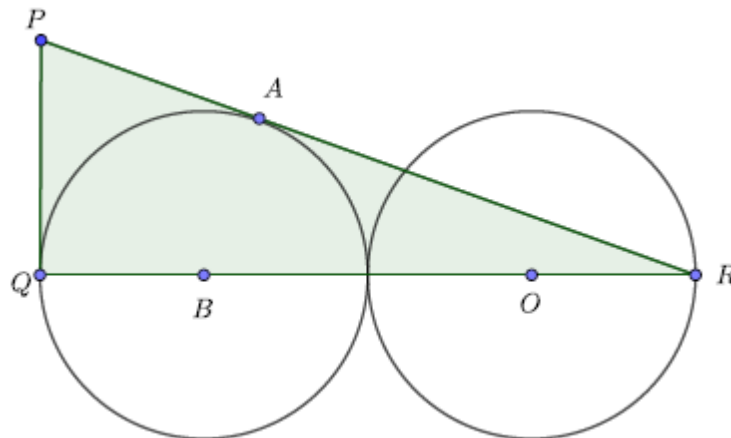
Babhta 5

Round 5

Q5.1 Find the number of ways in which 10 different books can be given to Andrew, Breda, Colin and Deirdre, if Andrew is to receive 4 books, Breda 3 books, Colin 2 books and Deirdre 1 book.

Q5.2 Two circles, with centres O and B and each with a radius of 2, are tangent to each other. A straight line is drawn through O and B meeting the circles at Q and R . Two other sides of $\triangle PQR$ are drawn such that side $[PR]$ is tangent to the circle with centre B at A and side $[PQ]$ is tangent to the circle with centre O at Q .

Determine the length of $[PQ]$. Give your answer in the form $a\sqrt{2}$.





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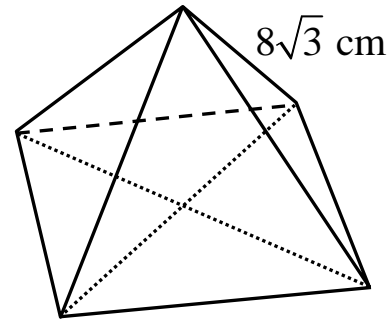
Babhta 6

Round 6

Q6.1 A group of Transition Year students was given the task of making a paper model of a pyramid.

Each of the slant edges has a length of $8\sqrt{3}$ cm and the length of the square base is twice the height of the pyramid.

Find the height of the model.



Q6.2 $h(x) = 3x - 5$ and $k(x) = x - 2$ are two functions where $x \in \mathbb{R}$.
Solve for x , $h^{-1} \circ k(x) = k^{-1} \circ h(x)$.



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Babhta 7

Round 7

Q7.1 Find $\frac{dy}{dx}$, given $y = xe^x - e^x$.

Q7.2 Solve for k : $\frac{k!}{(k-2)!} = 30$, where $k \in \mathbb{N}$.

Q7.3 Find the equation of the tangent to the curve $y = e^{\frac{1-x}{1+x}}$ at the point $(1, 1)$. Give your answer in the form $ax + by + c = 0$.

Q7.4 At a meeting there are 7 students, 3 teachers and 11 parents. Two people are chosen at random. Find the probability that the first person chosen was a teacher, given that the second person chosen was a teacher.



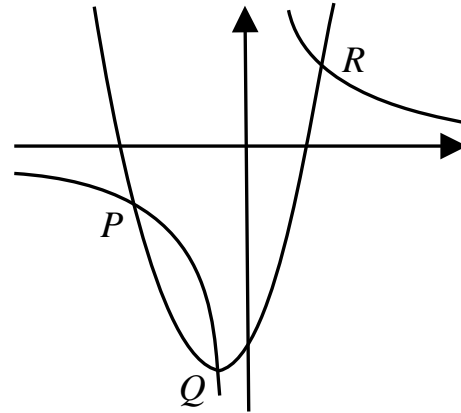
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Babhta 8

Round 8

- Q8.1 The diagram shows the graphs of the functions $f(x) = x^2 + 2x - 11$ and $g(x) = \frac{12}{x}$, $x \neq 0$ and $x \in \mathbb{R}$.
Find the coordinates of P , Q and R , the points of intersection of f and g .



- Q8.2 Suppose that $\omega^3 = 1$, $\omega \neq 1$, and k is a positive integer. Find the two possible values of $1 + \omega^k + \omega^{2k}$ which belong to \mathbb{Z} .
- Q8.3 Solve $5\sin^2 \theta - 3\sin \theta \cos \theta - 2\cos^2 \theta = 0$, for $0 < \theta < 360^\circ$.
Give your answers to the nearest degree.
- Q8.4 Find the value of $k > 0$ for which $\int_1^k (2x + 3)dx = 6$.



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Tie-break Final 2022

- T1 Find the value of k if $k(x^2 + 2y^2) + (y - 2x + 1)(y + 2x + 3) = 0$ represents a circle.
- T2 The third term in the expansion of $(x + k)^8$ is $252x^6$.
Find the possible values of k .
- T3 Solve for x and y , $3x + 2y = 1$ and $x^2 + 2xy + 15 = 0$.
- T4 Given that $2^{x+1} - 5^y = 131$ and $2^{x-4} + 5^{y-2} = 13$, find the value of x and the value of y , where $x, y \in \mathbb{Z}$.
- T5 Mary takes two tests in general knowledge. The first test, test A, has a pass rate of 60%. The second test is Test B. The probability that a candidate passes both tests is 0.4.
Find the probability that Mary fails both tests.
- T6 If $y = \sqrt{x^2 - 2x - 3}$, where $x \in \mathbb{R}$, find the range of values for x for which $y \in \mathbb{R}$.
- T7 Solve for x , $\log_2 8 + \log_5 \left(\frac{1}{25} \right) + \log_9 3 = \log_{16} x$.
- T8 The lengths of the sides of a triangle are 2 cm, 3 cm, and 4 cm, respectively. Find the measure of the largest angle correct to the nearest degree.



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Answer Key Final 2022

Q1.1 $x = 1, 11$

Q1.2 $c = \pm a\sqrt{1+m^2}$

Q2.1 $x = 3$

Q2.2 $A(5, 6 \cdot 2) \quad B(10 \cdot 6, 2)$

Q3.1 $\frac{40}{3} \text{ mm}$

Q3.2 $a = 3, b = 4, c = 3$

Q4.1 $x^2 + (y + 4)^2 = 25$ or $x^2 + y^2 + 16y - 9 = 0$
 $(x - 2)^2 + y^2 = 25$ or $x^2 + y^2 - 4x - 21 = 0$

Q4.2 $[111 \cdot 2, 118 \cdot 8]$

Q5.1 12,600

Q5.2 $2\sqrt{2}$

Q6.1 8 cm

Q6.2 $x = 1 \cdot 5$



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Q7.1 $\frac{dy}{dx} = xe^x$

Q7.2 6

Q7.3 $x + 2y - 3 = 0$

Q7.4 $\frac{1}{10}$

Q8.1 $P(-4, -3), Q(-1, -12), R(3, 4)$

Q8.2 0 and 3

Q8.3 $45^\circ, 158^\circ, 225^\circ, 338^\circ$

Q8.4 2



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Tie-break

T1 $k = -5$

T2 $k = \pm 3$

T3 $(3, -4)$ and $\left(-\frac{5}{2}, \frac{17}{4}\right)$

T4 $x = 7, y = 3$

T5 $\frac{2}{15}$

T6 $x \leq -1$ or $x \geq 3$

T7 64

T8 104°