Peter's Problem 2022 -Suggested Solutions

Problem No. 1

There are 52 pages in a tabloid newspaper.
What other 3 pages are on the same sheet as page 10?

Solution to No. 1

Five sheets are required to give the first 10 pages with odd numbered pages on the right and even numbered on the left. Thus page 10 will be on the left backing page 9.
These five sheets also give the last 10 pages from pages 43 to 52. The page numbers on the same sheet as page 10 are pages 43 and 44.
Answer: The page numbers are 9, 43, 44

Problem No. 2

The MetroSur underground railway in Madrid runs in a circle loop in both directions – *Sentido Andén 1 (To Platform 1)* or *Sentido Andén 2 (To Platform 2)*. It takes one hour to complete the loop. On certain days of the week the trains in the direction of *Andén 1 (Platform 1)* leave Puerta del Sur station every 10 minutes starting on the hour while the trains in the direction of *Andén 2 (Platform 2)* leave Puerta del Sur station every 10 minutes starting 5 minutes after the hour. On occasion, the Metro has run on a 24-hour basis.

On one of these occasions if I take a complete round trip from Puerta del Sur station in one direction, how many trains will I pass which are going in the opposite direction?
Show all calculations on the Long Answers sheet.

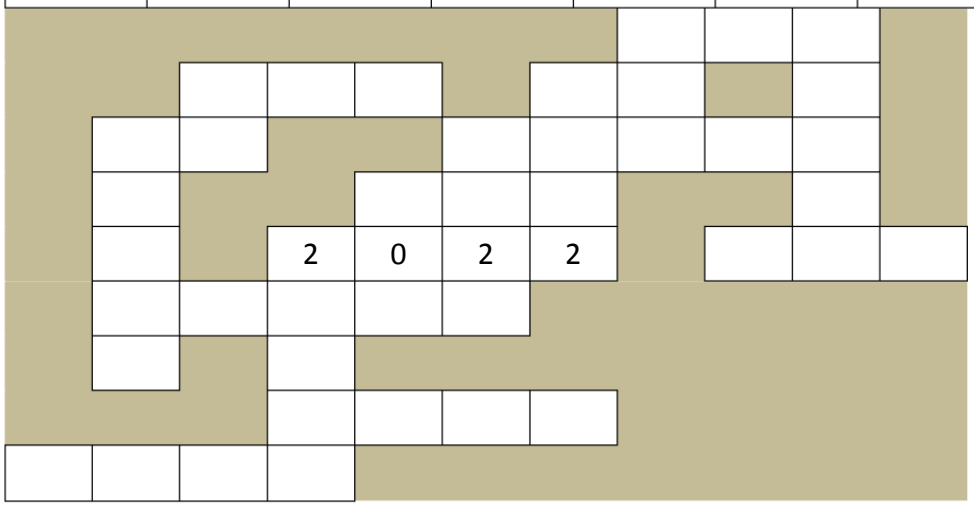
Solution to No. 2

If I leave Puerta del Sur at 12:00 in the direction of Platform 1 the first train that I meet is the 11:05 from Puerta del Sur in the direction of Platform 2 (which gets to Puerta del Sur at 12:05), followed next by the 11:15, 11:25, 11:35, 11:45, 11:55, 12:05, 12:15, 12:25, 12:35, 12:45 and the 12:55 which is the last as I will arrive in Puerta del Sur again at 13:00. That is 12 trains in total.
Answer: 12 trains.

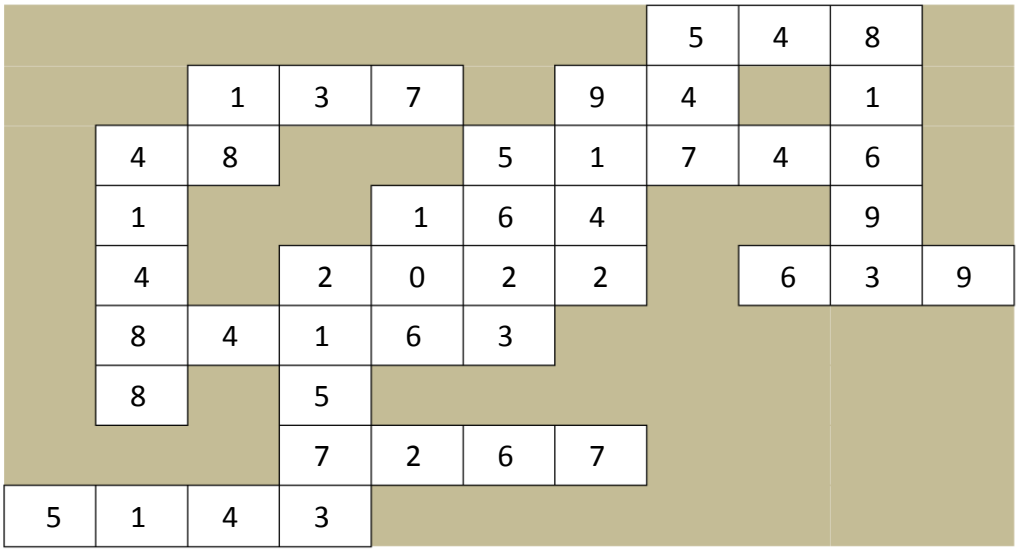
Problem No. 3

There are 22 numbers in the list below. 19 of the numbers may be inserted in the grid. Write down the sum of the 3 numbers from the list which cannot be included on the grid

18		106		2022		21573
48		137		5143		41488
94		164		5623		51746
98		547		7267		52486
		548		9142		81693
		639		9442		84163



Solution to No. 3



98 + 9442+ 52486 = 62026

Answer:62026

Problem No. 4

What is the next number in the sequence 3, 3, 4, 6, 9, 13, 18, 24, 31, …

Solution to No. 4

The differences between the successive terms of the sequence are:3 – 3 = 0; 4 – 3 = 1; etc,
The sequence of difference is 0, 1, 2, 3, 4, 5, 6, 7, …The next difference is 8.
Therefore, the next number on the original sequence is 31 + 8 = 39.
Answer: 39

Problem No. 5

In the summer of 2021 Laura got a job in the local supermarket, working every third day. John also got a job there, but working Saturdays only. The shop is open 7 days a week. If both started work on June 5th, what was the next date on which both of them were working on the same day? Give the answer in the format DD/MM/YY

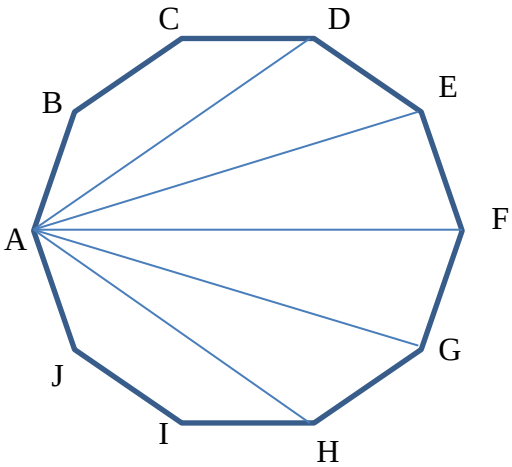
Solution to No. 5

Since June 5th 2021 was a Saturday we can set out the calendar as follows:
John works on Saturday's only: June 5th, 12th, 19th,26th, …
Laura works on Saturday June 5th and on each third day following: June 5th, 8th, 11th, 14th, 17th, 20th, 23rd, 26th, …
They both work again on the same day on Saturday June 26th 2021.
Answer: 26/06/21

Problem No. 6

The sides of a regular decagon are labelled consecutively from A to J. How many diagonals, each longer than |AC|, can be drawn in the decagon?

Solution to No. 6



Label the vertices A, B, C, D, E, F, G, H, I, J as illustrated.
|AC| = |AI| so two of the seven diagonals from A cannot be counted. Similarly, we can count, at most, five diagonals from each of the other vertices. After the third vertex we find that some or all of the diagonals have been counted already.
We have : From A: 5 diagonals; From B: 5 diagonals; From C: 5 diagonals; From D: 4 diagonals since one is a repeat (|DA|); From E: 3 diagonals (2 repeats); From F: 2 diagonals (3 repeats); From G: 1 diagonal (4 repeats).
All diagonals from H, I and J are already counted.
Thus the total is 5 + 5 + 5 + 4 + 3 + 2 + 1 = 25 diagonals.
Answer: 25.

Problem No. 7

How many three-digit multiples of 9 only contain odd digits?
Note: the sum of the digits of all numbers divisible by 9 is always a multiple of 9.

Solution to No. 7

The following array shows all the odd digits in three columns. Each choice of three digits must total a multiple of 9. Thus we can have 117, 171, 711; 135, 153, 315, 351, 513, 531; 333; 999 for a total of 11 three-digit numbers with only odd digits which are multiples of 9.

1	1	1
3	3	3
5	5	5
7	7	7
9	9	9

Answer: 11

Problem No. 8

In a downtown café each table has 3 legs and each chair has 4 legs. There are 6 members of staff. There are 2 chairs at half of the tables and 4 chairs at the other half. At a certain time two thirds of the chairs are occupied by customers. Each person in the café has 2 legs. There are 582 legs altogether in the café. How many chairs does the café have?

Solution to No. 8

Let 2x be the number of tables. These have 3(2x) = 6x legs.
Then x is the number of half the tables. These have 2 chairs each which is 2x chairs. These chairs have 4(2)x legs = 8x legs.
Also, x tables have 4 chairs each which is 4x chairs. These chairs have 4(4)x = 16x legs.
The total number of chairs is 2x + 4x = 6x. Two-thirds of these are occupied. That is 2(6x)/3 which is 4x chairs are occupied.
A person at each of these chairs means that there are 2(4x) legs of people = 8x customers' legs.
In addition there are 2(6) = 12 legs of staff members.
The total number of legs is 6x + 8x + 16x + 8x + 12 = 582.
Therefore, 38x + 12 = 582 => 38x = 570 => x = 15.
There are 6x = x(15) = 90 chairs.
Answer: 90 chairs.

Problem No. 9

On a digital read-out on a calculator the digits 0 – 9 appear as follows using LED lights.

'Zero' (0) is shown using 6 LED lights; 'One' (1) is shown using 2 LED lights; 'Two' (2) is shown using 5 LED lights; etc, as illustrated.

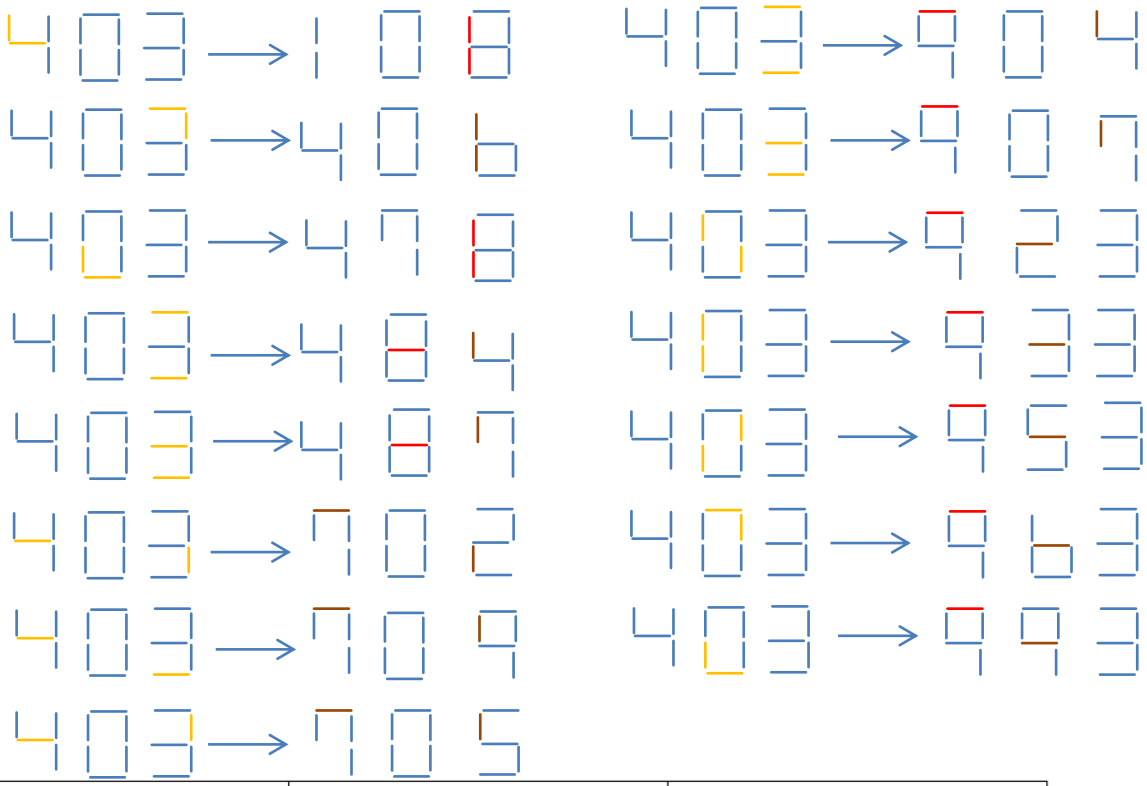
A fault occurs when a 3-digit number is keyed in. Any two of the LED lights malfunction and light up in the wrong place, occasionally causing a different 3-digit number to appear.

How many possible legitimate 3-digit numbers could now be displayed when 403 is keyed in?

Illustration: could appear but it is not a legitimate number.

Solution to No. 9

There are 15 possible 3-digit numbers which can be created.



108	406	478
484	487	702
705	709	904
907	923	933
953	963	993

Answer: 15

Problem No. 10

I must score 501 to win a darts game. I am allowed 3 throws of a dart at each visit to the oche. On the first visit I scored (treble 20, treble 19, double 18); on my second visit I scored (treble 20, treble 20, treble 5); on my third visit I scored (treble 15, 1, treble 20).
If I have to finish on a double with my final dart-throw give one example of the scores I can make on my next visit to the oche which will give me victory?

Solution to No. 10

Totals: 1st visit 153; 2nd visit 135; 3rd visit 106, Total so far: 394. Need to score 107 to win.
Answer: Example scores: treble 19, double 9, double 16 (57 + 18 + 32 = 107).

Problem No. 11

Each letter stands for a different digit. Find the number represented by CARE in the following addition problem

		E	R	R
+		E	R	R
C	A	R	E	

Solution to No. 11

th	h	t	u
	E	R	R
+	E	R	R
C	A	R	E

Two different results for R + R (ie. 2R = E in Units; 2R = R in Tens) means that there must have been a Carry Digit from the Units position.
Since R+R = R in the Tens column, then R must be 9, as 9+9+1 carried =19 and 9 is the only digit which returns itself in this manner.
Substituting for R in the Units column we see E must be 8 (9+9 = 18).
Substituting in the Hundreds column for E we get 8+8+ 1 carried = 17 which gives C=1 and A=7.
So, C =1, A= 7, R=9, E = 8.
Answer: CARE = 1798

Problem No. 12

Agfrud and Breol were hired at the same time by a technology company. They both started with a salary of €10 000 per year but with different special contracts. Every six months Agfrud's pay rose by €500 compared with that for the previous 6-month period. Every year, Breol's pay rose by €1 600 compared with that for the previous 12-month period.
Three years later, who had earned more and by how much?
Note: Contracts are made at the start of a year or period but only fully paid by the end of that year or period.

Solution to No. 12

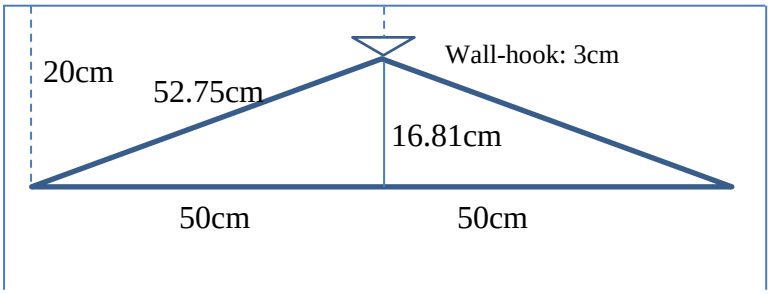
	Agfrud's earnings	Agfrud's total	Breol's earnings	Breol's total
Yr. 1: End of 1st 6 months	€5000	€5000		
End of Year 1	€5500	€10500	€10000	€10000
Yr. 2: End of 1st 6 months	€6000	€16500		
End of Year 2	€6500	€23000	€11600	€21600
Yr. 3: End of 1st 6 months	€7000	€30000		
End of Year 3	€7500	€37500	€13200	€34800

Answer: Agfrud has earned more after 3 years. The difference is €37500 - €34800 = €2700.

Problem No. 13

A householder is hanging a large painting using picture wire which does not stretch. The hooks on the frame of the painting are 1m apart and are inserted 20cm below the top of the frame. The householder adds 5.5% to the length of the wire between the hooks to allow it to hang from the middle onto the bottom of a wall-hook. The wall-hook is 3cm in height. How far, correct to the nearest mm, will the top of the wall-hook be below the top of the frame?

Solution to No. 13



The length between the hooks is 1m. With 5.5% extra the length of the wire for the hanging is 105.5cm. The middle is attached to the wall-hook which allows $\frac{105.5}{2} = 52.75cm$ on each side between the picture-hooks and the wall-hook. We can form a right-angled triangle with base 50cm and hypotenuse 52.75cm. Using Pythagoras Theorem, $\sqrt{52.75^2 - 50^2} = \sqrt{282.5625} = 16.80959547$. Thus, we find that the wire has been raised by 16.81cm, correct to 2 decimal places. This will reach the bottom of the wall-hook. The wall-hook is 3cm high and so the top of the wall-hook is 16.81 + 3 = 19.81cm above the level of the picture hooks.
Since the picture hooks are 20cm from the top of the frame, the top of the frame will be 20 – 19.81 = 0.19cm above the top of the wall-hook. This is 1.9mm = 2mm correct to the nearest mm.

Answer: 2mm.

Problem No. 14

A rectangular garden is one metre longer than it is wide. A gardener has to cover its surface completely with weed barrier material which comes in rolls which are 1m wide. The gardener uses 110m from the roll. How wide is the garden?

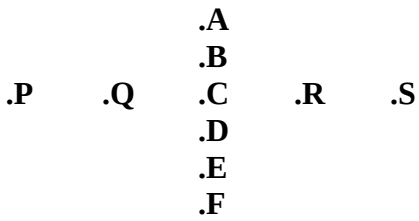
Solution to No. 14

The area of the weed barrier material used is1x110m^2. This is the area of the garden.
Let W metres be the width of the garden.
Then (W)(W+1) = 110 leading to W^2 + W – 110 = 0. Hence, (W – 10)(W + 11) = 0 and W = 10 taking the positive result only.
The garden is 10m wide.

Answer: 10m

Problem No. 15

The points A, B, C, D, E, F are collinear and the points P, Q, C, R, S are collinear, as shown. How many different triangles may be formed by joining the points as vertices?



Solution to No. 15

Using point A as one vertex and any two points from the centre row of 5 points as the other two vertices of a triangle will allow 10 triangles to be formed.
Similarly, we may form 10 triangles for each vertex B, D, E, F. That is, 50 triangles in total.
Using point P as one vertex and any two points from the centre column, except point C, as the other two vertices (ie using 5 of the points) will allow 10 triangles to be formed. Similarly, we may form 10 triangles for each of the vertices Q, R, S. That is, a further 40 triangles.
The total number of triangles is 50 + 40 = 90 triangles.
Answer: 90

Alternative solution:
An alphabetical list:
All triangles using vertex A: ABP, ABQ, ABR, ABS, ACP, ACQ, ACR, ACS, ADP, ADQ, ADR, ADS, AEP, AEQ, AER, AES, AFP, AFQ, AFR, AFS, APQ, APR, APS, AQR, AQS, ARS – 26 TRIANGLES;

All triangles using vertex B (but not A): BCP, BCQ, BCR, BCS, BDP, BDQ, BDR, BDS, BEP, BEQ, BER, BES, BFP, BFQ, BFR, BFS, BPQ, BPR, BPS, BQR, BQS, BRS – 22 TRIANGLES;

All triangles using vertex C (but not A nor B): CDP, CDQ, CDR, CDS, CEP, CEQ, CER, CES, CFP, CFQ, CFR, CFS – 12 TRIANGLES;

All triangles using vertex D (but not A nor B nor C): DEP, DEQ, DER, DES, DFP, DFQ, DFR, DFS, DPQ, DPR, DPS, DQR, DQS, DRS – 14 TRIANGLES;

All triangles using Vertex E (but not A, B, C, nor D): EFP, EFQ, EFR, EFS, EPQ, EPR, EPS, EQR, EQS, ERS – 10 TRIANGLES;

All triangles using vertex F (but not A, B, C, D nor E): FPQ, FPR, FPS, FQR, FQS, FRS – 6 TRIANGLES.

Total number of triangles: 26 + 22 + 12 + 14 + 10 + 6 = 90

Answer: 90 triangles.

Problem No. 16

All of the students in a school are to be lined up in rows for the St. Patrick's Day parade.
If they line up in rows of 3 there is one left over.
If they line up in rows of 4, two students are left over.
If they line up in rows of 5, three students are left over.
If they line up in rows of 6, four students are left over.
If they line up in rows of 7, no one is left over.
What is the smallest number of students which could be in the school?

Solution to No. 16

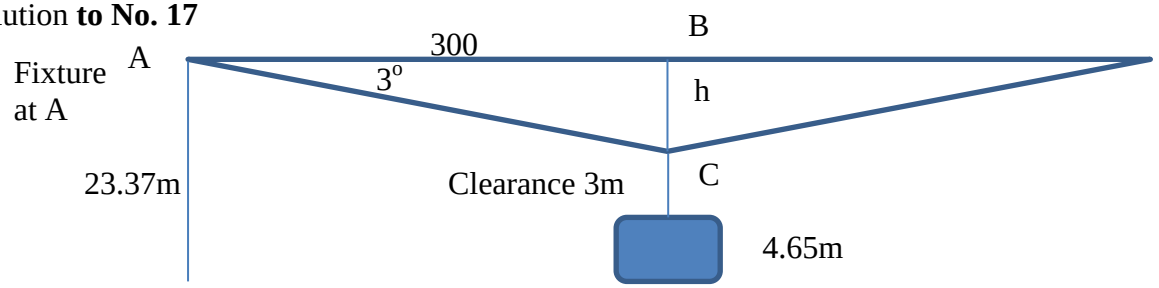
The number of students must be a multiple of 7.
Multiples of 7 which leave a remainder of 1 after division by 3 are: 7,28, 49, 70 … of the form $21n + 7$.
Of these, the number must be even and leave a remainder of 2 on division by 4. These are:
70, 154, 238, … of the form $84n + 70$.
Of these, the number must leave a remainder of 3 on division by 5. These are:
238, 658, 1078, …. Of the form $420n + 238$.
Each of these also leaves a remainder of 4 on division by 6.
Therefore the smallest number of students which could be in the school is 238.

Answer: 238 in the school. [3.79+1; 4.59+2; 5.47+3; 6.39+4; 7.34]

Problem No. 17

A 600m long tightrope is suspended over an Irish city street. It is fixed at both ends at equal heights. The weight of the tightrope walker causes the rope to sag by 3^o from the horizontal as measured in a direct line from the bottom of the walker's feet to a fixture point at either end when the walker is at the middle of the walk. A minimum of 3m clearance is required for buses or trucks which pass underneath. What is the minimum height for the placement of the fixtures at the ends? Give the answer in metres correct to the nearest cm. Regulations for vehicle heights may be found on the website of the Department of Transport, Tourism and Sport.

Solution **to No. 17**



In the right-angled triangle ABC the base, AB, is 300m and the angle at A is 3^o. The height, h, then is $300\text{Tan}3^o = 15.722$m. This is the amount of sag at the centre. The regulation for vehicle heights gives the maximum truck height as 4.65m. With a clearance of 3m we then have the height of the fixtures as 4.65 + 3 + 15.722 = 23.372m = 23.37m to the nearest cm.
Answer: 23.37m

Problem No. 18

A 4-digit number is a perfect square. When I add 1 to each of the digits, another 4-digit perfect square is formed.
Which is the smaller of the perfect squares?

Solution to No. 18

Let the square number be $a^2 = xyzw$. This can be written as $100x + 100y + 10z + w$. Then we also have the number $(x+1)(y+1)(z+1)(w+1)$ which is also a perfect square, say b^2. We can write this new number as $b^2 = 1000x + 1000 + 100y + 100 + 10z + 10 + w + 1$ which is $1000x + 100y + 10z + w + 1111$. That is, $b^2 = a^2 + 1111$ which gives $b^2 - a^2 = 1111$.
Factoring, we get $(b - a)(b + a) = 1111 = (11)(101)$.
Solving

$$\begin{array}{rl} b - a = & 11 \\ \underline{b + a = 101} & \\ 2b \quad = 112 & \end{array}$$

=> b = 56 and a = 45. Hence, the smaller number is $a^2 = 45^2 = 2025$.

Ans: 2025 (45^2; other number $3136 = 56^2$)

Problem No. 19

Insert each of the digits 2-9 in each of the empty boxes below so that the three rows across and the three columns down form correct arithmetic sentences. ALL OPERATIONS ARE FROM TOP TO BOTTOM AND LEFT TO RIGHT.

What is the value of K + L, when both K and L are whole numbers?

	X		÷		=15
+		÷		X	
(	-	1)	X		=24
-		+		÷	
	-		+		=K
=2		=15		=L	

Solution to No. 19

5	X	9	÷	3	15
+		÷		X	
(4	-	1)	X	8	24
-		+		÷	
7	-	6	+	2	K=3
=2		15		L=12	

Answer: K + L = 15

Problem No. 20

Sixty €1 coins are placed side by side in a straight row. Every second coin is then replaced by a 50 cent coin. Every third coin is then replaced by a 20 cent coin. Every fourth coin is then replaced by a 10 cent coin.
What is the value of all the coins in the row now?

Solution to No. 20

Let each position containing €1 be numbered in turn from 1 – 60.
Starting from the last instruction we have the following:
Each position whose number is divisible by 4 contains 10 cent. That is, positions 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60. There are 15 positions giving 15x10 cent pieces = €1.50.

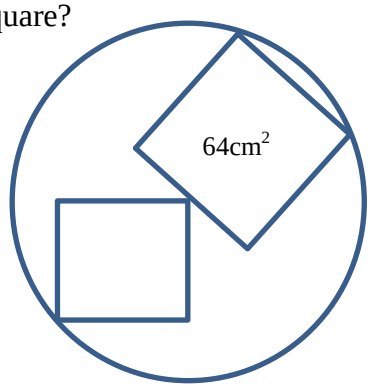
Each position number divisible by 3 and not already filled above contains a 20 cent piece. That is, positions 3, 6, 9, 15, 18, 21, 27, 30, 33, 39, 42, 45, 51, 54, 57. There are 15 positions giving 15x20 cent pieces = €3.00.
Each position number divisible by 2 and not already filled above contains a 50 cent piece. That is, positions 2, 10, 14, 18, 22, 26, 34, 38, 46, 58. There are 10 positions giving 10x50 cent pieces = €5.00.
So far, 15 + 15 + 10 = 40 positions have been accounted for. The remaining 60-40 = 20 positions each contain €1. This gives €20.

The total value now is €20.00+ €5.00 + €3.00 + €1.50 = €29.50

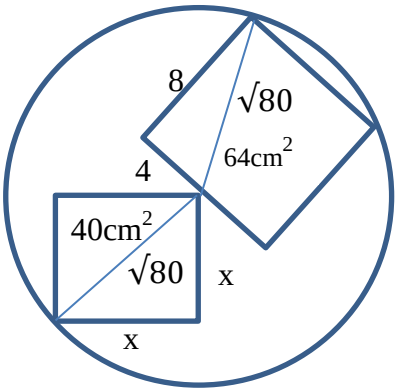
Answer: €29.50

Problem No. 21

Two squares touch at the centre of a circle (as shown). If the larger square has area $64cm^2$ what is the area of the smaller square?



Solution to No. 21



The side of the large square has length 8cm.
The centre is at the middle of the side and so
the right-angled triangle has sides 4 and 8.
The hypotenuse is $\sqrt{80}$ which is also the radius length.
In the small square the radius is the diagonal which is $\sqrt{80}$.
Let the side of the small square be x cm and its area is x^2 cm^2.
Then the right-angled triangle has sides x, x and $\sqrt{80}$.
Therefore $x^2 + x^2 = (\sqrt{80})^2$
This implies $2x^2 = 80$ and therefore $x^2 = 40$ which is the area of the small square.

Answer: $40cm^2$

Problem No. 22

A cheetah runs at a speed of 81 km. per hour and a snail moves at 25 hours per km. How long, in hours minutes and seconds, would it take the snail to cover the same distance as the cheetah covers in 18 seconds.
Show, in detail on the Long Answers sheet how you arrived at your answer.

Solution to No. 22

81km/hr = 81000/3600 m/sec. For 18 seconds the cheetah travels 18(81000)/3600 = 405m.
The snail will travel 405m in 25(405/1000) = 10.125hrs.
This is 10hrs 7mins 30seconds.

Ans: 10 hr 7min 30sec.

Problem No. 23

Thirty-one teams apply to enter a competition. The organisers agree to split the teams into two divisions where each team plays each other team in their division once only and no team in the other division. There are 45 more games played in the larger division than in the smaller one. How many teams are in the smaller division?

Solution to No. 23

Suppose the two divisions had 15 teams and 16 teams respectively. Then there would be 15 more games in the group of 16.
Suppose next that the two divisions had 14 teams and 17 teams respectively. The three extra teams in the group of 17 would have to play 16 extra for the 17^{th} team, 15 extra for the 16^{th} team (since the 17^{th} team has played them already) and, similarly, 14 extra for the 15^{th} team giving a total of 45 extra games for this group.
Therefore, there are 14 teams in the smaller group.

Answer: 14 teams.

Problem No. 24

Visit the STEPS Engineers Ireland website (Schools section)
https://www.engineersireland.ie/Schools/Explore-Engineering/What-do-engineers-do/Great-feats-of-engineering
and in the article concerning Great Feats of Engineering find the answer to the following question:

What did the National Academy of Engineering once call 'the greatest engineering achievement of the 20^{th} century'

Solution to No. 24

Navigate to
https://www.engineersireland.ie/Schools/Explore-Engineering/What-do-engineers-do/Great-feats-of-engineering
and read the article 'Great Feats of Engineering'.
Item No. 4, under the heading 'Modern Age 1500-present', has the relevant reference which is to Electrification.

Answer: Electrification