Foireann Mata 2019 : an Chraobh
(Team Maths Final 2019)

Babhta 1 Round 1

Q1.1 The line $12x + 5y = 60$ forms a triangle with the x and y axes. Calculate the sum of the lengths of the altitudes of the triangle?

Answer in the form $\frac{a}{b}$, where a and b $\in$ N in lowest terms.

Q1.2 Given that $\text{Tan}(A) + \text{Tan}(B) = a$
$\text{Cot}(A) + \text{Cot}(B) = b$

and $\text{Tan}(A + B) = c$

Write c in terms of a and b.

Answer in simplest fractional form in terms of a and b.

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Babhta 2 Round 2

Q2.1 P, Q and R are the points (3, 4) , (7, -2) and (- 2 , -1) respectively. Find the equation of the median through R of the triangle PQR.

Answer in form ax + by + c = 0 , where a , b and c $\in$ Z.

Q2.2 Find the slope of the tangent to the curve $y = x(x - 2)^2 + 5$ at the point (3,8).

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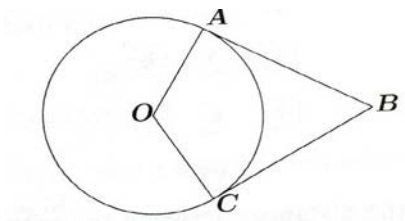
Babhta 3 Round 3

Q3.1 The triangle ABC is equilateral with sides tangent to the circle with centre O and radius $\sqrt{3}$.
Find the area of the quadrilateral ABCO.
Answer in simplest surd form



Q3.2 f is a function such that f(f(x)) = f (x + 2) – 3
for all integers x.
Given that f(1) = 4 and f(4) = 3 calculate the value of f(5) ?

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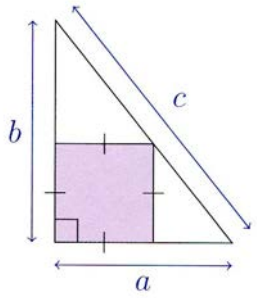
Babhta 4 Round 4

Q4.1 A square is drawn in the corner of a right angled triangle with sides of lengths a ,b and c as shown.

Write the ratio of the unshaded area to the shaded area as a fraction, in simplest form, in terms of a,b and c.



Q4.2 A sequence of fractions obeys the following rule : given any two successive terms **p** and **q** of the sequence, the next term is obtained by dividing their product **p.q** by their sum **p + q**.

Given that the first two terms are $\frac{1}{2}$ and $\frac{1}{3}$, find the value of the tenth term.

Answer in form $\frac{a}{b}$, where a and b $\in$ N in lowest terms.

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Babhta 5 Round 5

Q5.1 A fair die is rolled six times. Find the probability of rolling at least a five at least five times.

Answer in the form $\frac{a}{b}$, where a and b $\in$ N in lowest terms.

Q5.2 Given that

$x^3 + y^3 = 5$ and

$x^2 + y^2 = 3$, find possible values of x + y.

Answers in non-decimal form.

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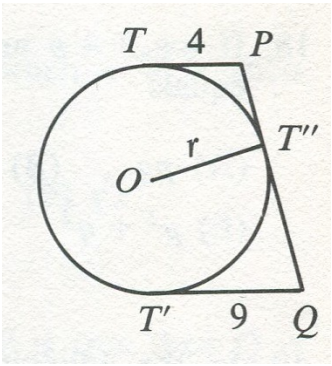
Babhta 6 Round 6

Q6.1 A certain number consists of three digits, which are in arithmetical progression.
The number divided by the sum of its digits is equal to 26, but if 198 is added to the number the digits are reversed.
What is the number?

Q6.2 In the diagram TP and $T'Q$ are parallel tangents to a circle of radius r, where T and T' are the points of tangency.
$PT''Q$ is a third tangent with T''as point of tangency.



If |TP| = 4 cm and |T'Q| = 9cm , find the length of the radius, r.

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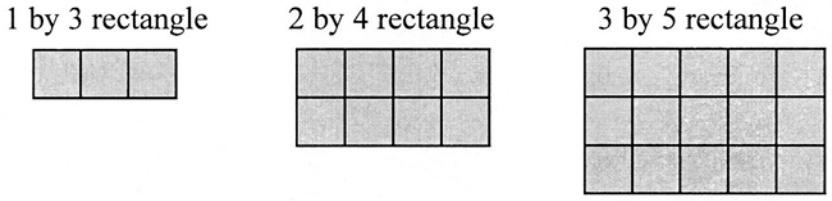
Babhta 7 Round 7

Q7.1 A unit square is a square whose sides are 1 unit long.
Rectangles can be formed by joining unit squares.
For the three examples shown below, their perimeters are, respectively, 8 units, 12 units, and 16 units.
The number of line segments of unit length that are **NOT** part of their perimeters are, respectively 2, 10 , and 22.



If a 50 by 100 rectangle is formed with unit squares, how many line segments would **NOT** be part of the perimeter?

Q7.2 The Binomial $(x+y)^9$ is expanded in decreasing powers of x.
The second and third terms have equal values when evaluated at x = p and y = q, where p and q are positive numbers whose sum is one.
Find the value of p.

Answer in simplest form $\frac{a}{b}$, where a and b $\in$ N.

Q7.3 At the start of the year Joan borrowed €25 000 at an APR of 8%. She wishes to repay in twelve equal instalments over twelve years, with the first repayment one year after she takes out the loan.
Calculate, to the nearest euro, the amount of each repayment.

Q7.4 Given that (2 + 3i) is a root of $3z^3 - 13z^2 + 43z - 13 = 0$, find the other two roots.

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Babhta 8 Round 8

Q8.1 Simplify

$$\frac{1}{2\sqrt{1}+1\sqrt{2}}+\frac{1}{3\sqrt{2}+2\sqrt{3}}+\frac{1}{4\sqrt{3}+3\sqrt{4}}+.....+\frac{1}{100\sqrt{99}+99\sqrt{100}}$$

Answer in the form $\frac{a}{b}$, where a and b $\in$ N in lowest terms.

Q8.2) In a popular TV game if a contestant spins €1 on the Big Wheel, she wins €1000 and gets 2 bonus spins in which she has a $\frac{1}{20}$ chance of winning an additional €10 000 and a $\frac{1}{10}$ chance of winning an additional €5 000. Given that the contestant has a $\frac{1}{10}$ chance of spinning €1, what is the expected value of the contestant's winnings?

Q8.3 Two positive integers p and q satisfy the equation $p^2q = 2016$.

How many different values of p are possible?

Q8.4 Solve for x and y : $2x + 3y = 2$ and $2x^2 + 3xy - 3y^2 = \frac{2}{3}$

Answer in the form (x,y), where x and y $\in$ Q in lowest terms.

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Tiebreak Final 19

T1) Solve the equation $2019x^2 - 2019^{2019} = 0$.

Answers in form a^b, a $\in$ Z and b $\in$ N.

T2) What is the probability that a five card poker hand contains exactly two aces?

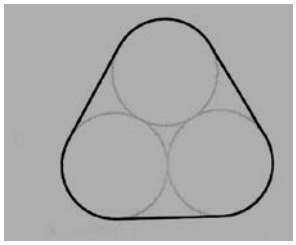
Answer correct to 5 decimal places

T3) Write in the form ax + by + c = 0 when x = $\frac{3 - t}{1 + t}$ and y = $\frac{5t}{1+ t}$,
where a, b and c $\in$ Z.

T4) Find the values of x $\in$ R for which $|x^2 - 3x| = 4$.

T5) Three cylindrical pipes of radius 1 meter are held together by a tight metal band as shown.
The length of the metal band is a + $b\pi$, where a and b $\in$ N.

Find the value of a + b.



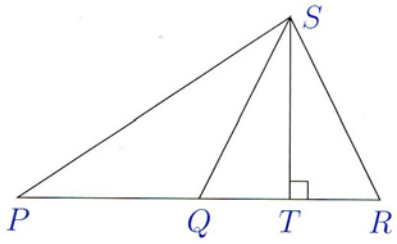
T6) Calculate the sum of the units digits of the sum $26^{26} + 33^{33} + 45^{45}$.

T7) Find the value of $\frac{1}{\log_2(36)} + \frac{1}{\log_3(36)}$ in simplest form.

T8) In the triangle |PQ| = |SQ| = |SR| = |QR|
and |<STR| = 90^0.

Calculate the ratio |PS| : |TR| .

Answer in form $a\sqrt{b}$: 1, a, b $\in$ N



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ANSWER KEY FINAL 19

Q1.1	$\frac{281}{13}$	Q1.2	$\frac{ab}{b-a}$
Q2.1	$2x - 7y - 3 = 0$	Q2.2	7
Q3.1	$3\sqrt{3}$	Q3.2	12
Q4.1	$\frac{c^2}{2ab}$	Q4.2	$\frac{1}{144}$
Q5.1	$\frac{13}{729}$	Q5.2	2 and $-1 \pm \sqrt{6}$
Q6.1	234	Q6.2	6
Q7.1	9850	Q7.2	$\frac{4}{5}$
Q7.3	€3317	Q7.4	$\frac{1}{3},\ 2 \pm 3i$
Q8.1	$\frac{9}{10}$	Q8.2	€200
Q8.3	6	Q8.4	$(\frac{1}{2},\frac{1}{3}), (3,-\frac{4}{3})$

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TIEBREAK

T1	$\pm 2019^{1009}$	T2	0.03993
T3	5x +4y - 15 = 0	T4	4 AND – 1
T5	8	T6	14
T7	$\frac{1}{2}$	T8	$2\sqrt{3} : 1$

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