TEAM MATHS 18 , REGIONAL ROUND , January 26, 2018

Round 1

Q1.1) Find both square roots of $-7-24i$, where $i = \sqrt{-1}$.
Answers in form $a + ib$, where a and b $\in$ Z.

Q1.2) What is numerical value of the units digit of the following sum ?

$$13^{841} + 17^{508} + 24^{617}$$

Round 2

Q2.1) When $(1 - \frac{1}{a})^6$ is expanded , find the sum of the coefficients of the three highest powers of $\frac{1}{a}$.

Q2.2) A square and an equilateral triangle have equal perimeters. The area of the triangle is $9\sqrt{3}$ cm^2.Find , in cms, the length of the diagonal of the square.
Answer in form $\frac{a\sqrt{b}}{c}$, where a , b and c $\in$ N.

Round 3

Q3.1) Find the range of real values of x for which $(x + 8)(x - 3) < 3x$

Q3.2) For what values of p and q is the expression $x^4 + 6x^3 + 13x^2 + px + q$ a perfect square?

Round 4

Q4.1) In a given triangle ABC , M is the midpoint of [BC] .
Write $|AB|^2 + |AC|^2$ in terms of |AM| and |MB|.

Q4.2) Let f be a function such that $f(x-1) = 2x^2-10x+3$.
Find the value of f(x) in terms of x

Round 5

Q5.1) Two circles, radii 7 cm and 9 cm, cut each other.
Given that their centres are 11 cm apart find the length of the common chord.
Answer correct to 1 decimal place.

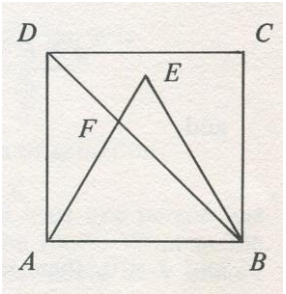
Q5.2) Find the equation of the line which passes through the point (- 5, 4) such that the portion of the line between the axes is divided by the point, **from x-axis to y-axis** in the ratio 1 : 2.

Answer in form ax + by + c = 0, where a , b and c $\in$ Z.

Round 6

Q6.1) Given that $(\sqrt{3}+\sqrt{2})^2 = 5+2\sqrt{6}$ and $(\sqrt{3}-\sqrt{2})^2 = 5-2\sqrt{6}$ find the values of x for which $(5+2\sqrt{6})^{\sin(x)} + (5-2\sqrt{6})^{\sin(x)} = 2\sqrt{3}$, where $0^0 \le x \le 360^0$

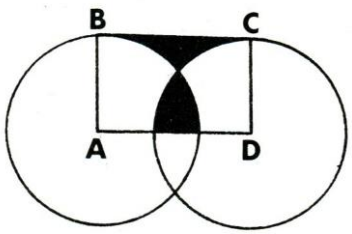
Q6.2) Vertex E of an equilateral triangle ABE is in the interior of a square ABCD, and F is the point of intersection of the diagonal [BD] and the line segment [AE].
Given that $|AB| = \sqrt{1+\sqrt{3}}$ calculate the area of the triangle ABF.
Answer in form $\frac{\sqrt{a}}{b}$, where a and b $\in$ N.



Round 7

Q7.1) The lengths of the medians of a right-angled triangle which are drawn from the vertices of the acute angles are 5 cm and $\sqrt{40}$ cm.
Calculate the length of the hypotenuse of the triangle.
Answer in form $a\sqrt{b}$, where a and b $\in$ N.

Q7.2) In the diagram are two congruent circles,



with centres A and D. Radii [AB] and [CD] intersect the tangent [BC].
Given that the shaded regions have equal areas and |AB| = 1, find the area of the quadrilateral ABCD.

Q7.3) Simplify $\dfrac{12}{3+\sqrt{5}+2\sqrt{2}}$

Answer in simplest surd form.

Q7.4) For how many values of **a** do the equations

$$x^2 + ax + 1 = 0 \text{ and } x^2 - x - a = 0$$

have a common real solution?

Round 8

Q8.1) Solve simultaneously for x and y :

$$x^2 - xy + y^2 = 7$$
$$x - xy + y = -1$$

Answer in form (x , y) , where x and y Ï N

Q8.2) For what positive values of m is the line $x + y = \sqrt{2m}$ a tangent to the circle $x^2 + y^2 = m$?

Q8.3) Given that q is an acute angle and $\text{Sin}(\frac{q}{2}) = \sqrt{\frac{x - 1}{2x}}$, find the value of $\text{Tan}(q)$ in terms of x.

Q8.4) Two equal parallel chords are drawn 8 cm apart in a circle of radius 8 cm. Calculate the area of the part of the circle that lies between the chords.

Answer in form $a\sqrt{b} + \frac{c\pi}{d}$, where a , b , c and d $\in$ N.

TIEBREAK

T1) Calculate the value of $\frac{e^x - e^{-x}}{e^x + e^{-x}}$, where e = 2.718 and x = 0.378.

Answer correct to four significant figures.

T2) L(- 1, 0) , M(3, 7) and N(5, -2) are the midpoints of the sides [BC], [CA] and [AB] respectively of the triangle ABC.
Find the equation of [AB].

Answer in form ax + by + c = o, where a, b and c $\in$ Z.

T3) For $0^O \leq A \leq 360^O$, find the values of A for which 3Cos(2A) + Sin(A) = 1
Answers to nearest degree.

T4) Given xy = b and $\frac{1}{x^2} + \frac{1}{y^2} = a$, calculate the value of $(x + y)^2$ in terms of a and b.

T5) In how many ways can the letters of the word **ORANGE** be arranged so that the three vowels must not come together?

T6) The line **y = mx + c** a tangent to the circle $x^2 + y^2 = a^2$.
Write **c** in terms of **a** and **m, where a , m** and **c** Î **R**

T7) If $x - y : x + y = 5 : 9$, find the numerical value of the ratio $x^2 - xy + y^2 : x^2 + 4y^2$.
Answer in form a : b

where a and b Î N

T8) Determine x such that $(2.4)^x = (5.3)^{x-1}$.

Answer correct to 3 decimal places.

T9) A circle of radius r is inscribed in a right angled isosceles triangle . and a circle of radius R is circumscribed about the triangle . Calculate the ratio R : r .

Answer in form $a + \sqrt{b} : 1$ where a and b Î N.

T10) Find , in degrees, the least positive angle x for which

$$(2^{\sin^2(x)})(2^{\cos^2(x)})(2^{\tan^2(x)}) = 2^2$$

T11) Find the value of A + B , where A and B Î Z for which

$$\frac{5x - 11}{2x^2 + x - 6} = \frac{A}{x + 2} + \frac{B}{2x - 3}.$$

Answer Key Team Maths 2018 Regional Round.

Round 1 Q1.1 3 – 4i , -3 + 4i Q1.2 8

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Round 2	Q2.1	10	Q2.2	$\frac{9\sqrt{2}}{2}$
Round 3	Q3.1	$-6 < X < 4$	Q3.2	p =12, q = 4
Round 4	Q4.1	$2\|AM\|^2 + 2\|MB\|^2$	Q4.2	$2x^2 - 6x - 5$
Round 5	Q5.1	11.4	Q5.2	$8x - 5y + 60 = 0$
Round 6	Q6.1	$30^0, 150^0, 210^0, 330^0$	Q6.2	$\frac{\sqrt{3}}{2}$
Round 7	Q7.1	$2\sqrt{13}$	Q7.2	$\frac{\pi}{2}$
	Q7.3	$1+\sqrt{2}+\sqrt{5}-\sqrt{10}$	Q7.4	1
Round 8	Q8.1	(2,3), (3,2),	Q8.2	ALL positive REALS
	Q8.3	$\sqrt{x^2-1}$	Q8.4	$32\sqrt{3} + \frac{64\pi}{3}$

Tiebreak

T1	0.3609	T2	$7x - 4y - 43 = 0$
T3	$42^0, 138^0, 210^0, 330^0$	T4	$ab^2 + 2b$
T5	576	T6	$\pm a\sqrt{1+m^2}$
T7	3 : 5	T8	2.105
T9	$1 + \sqrt{2} : 1$	T10	45^0
T11	2		