BABHTA 1 ROUND 1

Q1.1) z = a + ib , where a and b $\in$ R and i = $\sqrt{-1}$.
If f(z) = z^2 , calculate f(3 + 4i)

Answer in form a + ib, where a and b $\in$ Z

Q1.2) ABCD is a square of side a and [BC] is produced to E so that $|CE| = \frac{1}{2}|BC|$.
Calculate $|AE|^2 - |DE|^2$ in terms of a.

BABHTA 2 ROUND 2

Q2.1) In how many ways can three of the letters A, B , C, a , b , c, d be arranged in a row, using one capital and two small letters, so that the capital letter is always first, if each letter is used only once?

Q2.2) The positive integers are arranged in a triangular grid as shown.

1						
2	3	4				
5	6	7	8	9		
10	11	12	13	14	15	16
.	.	.	.	.	.	.
.	.	.	.	.	.	. and so on

What is the fourth number in the 60^{th} row?

BABHTA 3 ROUND 3

Q3.1) Find the equation of the line through (1,4) which is parallel to the line $2x - y - 3 = 0$.
Answer in the form $ax + by + c = 0$,
where a , b and c $\in Z$.

Q3.2) If $x^2 + y^2 = 10$ and $xy = 3$ find all possible values of $x + y$.

BABHTA 4 ROUND 4

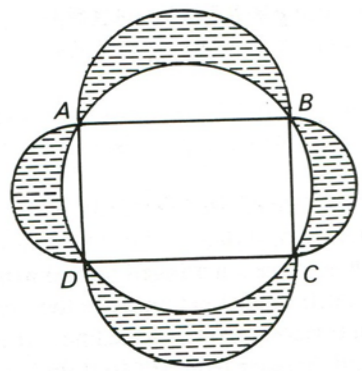
Q4.1) $x^4 - 48x + 28 = (x^2 + ax + 2)(x^2 + 4x + b)$.
Find the values of a and b, where a and b $\in$ R.

Q4.2) A triangle ABC has vertices A(18 , - 4), B(13 , 6), and C(x , 12).
The area of the ΔABC is 50 square units.
Find all possible points C in form (x , y).

BABHTA 5 ROUND 5

Q5.1) A club identifies each of its members with a unique three-digit number, the sum of whose digits is 24. How many such numbers exist?

Q5.2)



ABCD is a rectangle such that |AB| = 8 cm and |BC| = 6 cm. Semicircles are drawn on the four sides of the rectangle ABCD, each semicircle lying outside the rectangle. The circle passing through A, B , C and D determines four crescent shaped regions, one inside each semicircle as shown in the diagram.

Find the total area of the four crescent shaped regions in simplest form.

BABHTA 6 ROUND 6

Q6.1) Solve for a and b :

$\log(8x^3+4x^2-2x-1) = \log(2x-1) + 2\log(ax+b)$.

Q6.2) A dice rests on a table. Sam can see three faces and a total of 9 spots. Pat, who is on the opposite side of the table, can see three faces and 15 spots.
How many spots are on the top face of the dice?

BABHTA 7 ROUND 7

Q7.1) f and g are functions such that $f(x) = ax + 5$ and $g(x) = cx - 3$, where a and c $\in$ N. If $f(g(x)) = g(f(x))$ calculate the values of a and c.

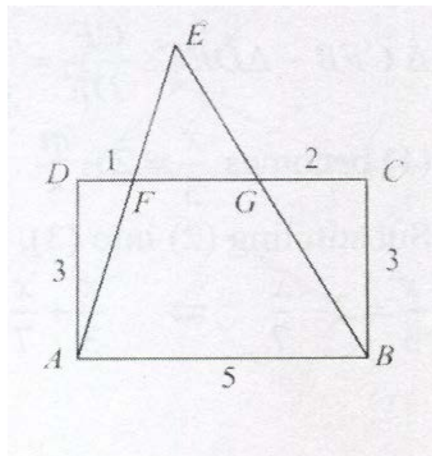
Q7. 2) Given two different two-digit positive integers whose product is 2016.

Find the greatest possible sum of the four digits.

Q7.3) In the expansion $(x + 3y + \frac{z}{4})^3$ find the coefficient of yz^2.

Answer in form $\frac{a}{b}$, where a and b $\in$ N.

Q7. 4) In the rectangle ABCD. $|AB| = 5$ and $|BC| = 3$.
Points F and G are on [CD] so that $|DF| = 1$ and $|GC| = 2$.
AF and BG intersect at E.
Calculate the area of the triangle AEB.



Answer in form $\frac{a}{b}$, where a and b $\in$ N.

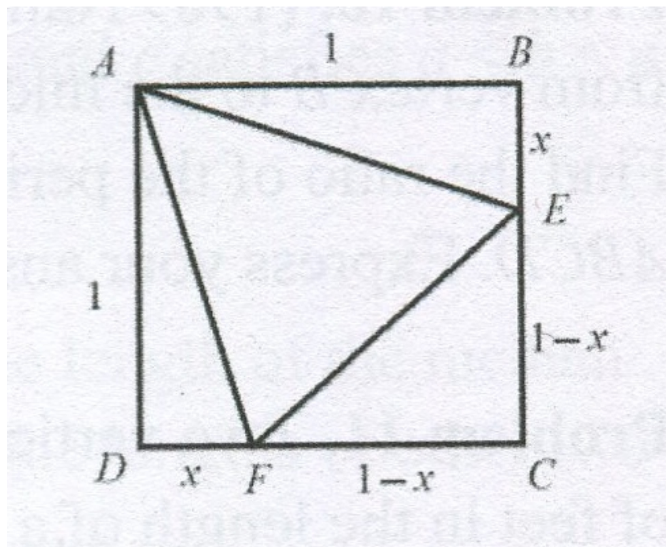
BABHTA 8 ROUND 8

Q8.1) If $0^0 \leq x \leq 180^0$ and $0^0 \leq y \leq 180^0$ find the pairs of angles, in the form (x,y),which satisfy the equations :

$$\text{Sin}(x+y) = +\frac{1}{\sqrt{2}}$$

$$\text{Cos}(2x) = -\frac{1}{2}$$

Q8.2) In the diagram an equilateral triangle is inscribed in a square of side 1 unit. From the diagram use the value of x to find the area of the triangle in form $a\sqrt{b} - c$, where a , b and c $\in$ N.



Q8.3) The coordinates of the vertices of a cyclic quadrilateral are A(2 , 5) , B(- 4 , 0) and D(7 , 0).
If the angles at B and D are right angles calculate the coordinates of vertex C.

Q8.4) Solve for x : $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$.

TIEBREAK

T1) The price of coat was reduced by 40% and the new price was then reduced by a further 40% during a sale.
By what percentage was the original price reduced?

T2) The sides of a triangle ABC are 10, 24 and 26 cm long.
A rectangle, with width 3cm, has an area equal to that of the triangle.
Find the perimeter of the rectangle, in cms.

T3) If $0^{o} \leq x \leq 360^{0}$ find the values of x for which
$3\text{Sin}^2(x) - \text{Sin}(x)\text{Cos}(x) - 4\text{Cos}^2(x) = 0$.
Answers to nearest degree.

T 4) Find the common ratio of the geometric sequence where the first term exceeds the third by 60 and the sum of the first two terms is 48.

Answer in form $\frac{a}{b}$, where a and b $\in$ Z.

T5) Find the points of intersection of the circles:

$x^2 + y^2 + 4x - 2y - 5 = 0$ and $x^2 + y^2 + 2x - 7 = 0$

Answers in coordinate form (x , y).

T6) A train travelling at 60 km/h takes 15 seconds to pass a certain point.
How long is the train?

T7) A complex number in the form a + ib , when multiplied by its conjugate a – ib yields 13, where a and b $\in$ R.
Find the numerical value of $a^4 + 2a^2b^2 + b^4$.

T8) Write in simplest form:

$$((x+1)^2(x-1)^2)((x^2+1)^2)$$

T9) The members of a Team Maths 2017 team are chosen at random from a class of twelve boys and fifteen girls.
Find the probability that the team consists of two boys and two girls.
Answer in simplest form $\frac{a}{b}$, where a and b $\in$**N.**

T10) Simplify $$\frac{\dfrac{x}{1+x}+\dfrac{1-x}{x}}{\dfrac{x}{1+x}-\dfrac{1-x}{x}}$$

Answer in simplest form as a fraction in terms of x.

T11) Find the perimeter of a right-angled triangle whose hypotenuse is 2 units and whose area is 1 square unit.

Answer in form $a+b\sqrt{c}$, where a , b and c $\in$ N.

T12) (m , 3) and (1 , m) are two points on the line l. The slope of l is m. Given that m > 0 , find the value of m in surd form.

T13) The point with coordinates (pCos(A) , pSin(A)) lies on the circle $x^2+y^2=16$.
Find the value of p, where p > 0.

Answer key , Round 1 2017

Round 1	Q1.1)	$-7 + 24i$	Q1.2)	$2a^2$
Round 2	Q2.1)	36	Q2.2)	3485
Round 3	Q3.1)	$2x - y + 2 = 0$	Q3.2)	± 4
Round 4	Q4.1)	$a = -4$, $b = 14$	Q4.2)	(20, 12) and (0,12)
Round 5	Q5.1)	10	Q5.2)	48 cm^2
Round 6	Q6.1)	$a=2$, $b=1$	Q6.2)	5
Round 7	Q7.1)	$a = 1$, $c = 1$	Q7.2)	20
	Q7.3)	$\frac{9}{16}$	Q7.4)	$\frac{25}{2}$
Round 8	Q8.1)	$(60^0, 75^0)$ and $(120^0, 15^0)$	Q8.2)	$2\sqrt{3} - 3$
	Q8.3)	(1, -6)	Q8.4)	2

Tiebreak

T1) 64% T2) 86 T3) $53^0, 233^0, 135^0, 315^0$

T4) $-\frac{1}{4}$ T5) (-3,-2) and (1,2) T6) 250m

T7) 169 T8) $(x^4 - 1)^2$ or $x^8 - 2x^4 + 1$

T9) $\frac{77}{195}$ T10) $\frac{1}{2x^2 - 1}$

T11) $2 + 2\sqrt{2}$ T12) $\sqrt{3}$ T13) 4

TEAM MATHS 2017 Solutions

Round 1

Q1.1 f(z) = z^2 = $(3+4i)^2$ = 9 + 24i -16 = -7 + 24i.

Q1.2 $|AE|^2 = a^2 + (\frac{3}{2}a)^2 = \frac{13a^2}{4}$

$|DE|^2 = a^2 + (\frac{1}{2}a)^2 = \frac{5a^2}{4}$

$|AE|^2 - |DE|^2 = 2a^2$

Round 2

Q2.1 Number of ways = 3 x 4 x 3 = 36

Q2.2 Last number in nth row = n^2.

Last number in 59^{th} row = 59^2 = 3481.
First number in 60^{th} row is 3482.
Therefore 4^{th} number in 60^{th} row is 3485.

Round 3

Q3.1 Any line parallel to 2x – y – 3 =0 has form l:2x = y – k = 0.
(1,4) is on l $\Rightarrow$2- 4 +k = 0. So k = 2.

l: 2x – y + 2 = 0.

Q3.2 $x^2 + y^2 = 10$ and x y = 3.
So 2xy = 6.
$x^2 + 2xy + y^2 = 16$. i.e $(x+y)^2 = 16 \Rightarrow x + y = \pm 4$.

Round 4

Q4.1 $x^4 - 48x + 28 = (x^2 + ax + 2)(x^2 + 4x + b)$
$= x^4 + (4+a)x^3 + (4a + b + 2)x^2 + (ab + 8)x + 2b$

4 + a = 0 , a = - 4, 2b = 28, b = 14.

Q4.2 A(18 , -4) , B(13 , 6) , C(x, 12)

Area of triangle ABC = $\frac{1}{2}\left|18(6 - 12) + 13(12 + 4) + x(-4 - 6)\right| = 50$

$\Rightarrow \frac{1}{2}\left|100 - 10x\right| = 50$

$\Rightarrow 100 - 10x = \pm 100$, x = 0 or 20.

So C(20 , 12) or C(0 , 12)

Round 5

Q5.1 24 = 9 + 9 + 6 , giving 699, 969, 996
24 = 9 + 8 +7 , giving 987, 978, 897, 879, 798, 789
24 = 8 + 8 +8 , giving 888

TOTAL 3 + 6 +1 = 10

Q5.2 Area of rectangle = 8 x 6 = 48 cm^2

Area of circle = $\pi(5)^2 = 25\pi \text{ cm}^2$

Area of semicircles = $2(\frac{1}{2})\pi(4)^2 + 2(\frac{1}{2})\pi(3)^2 = 25\pi \text{ cm}^2$

Area of crescents = $25\pi - (25\pi - 48) = 48 \text{ cm}^2$

Round 6

Q6.1 $\log(8x^3 + 4x^2 - 2x - 1) = \log(2x - 1) + 2\log(ax + b)$

$= \log(2x - 1)(ax + b)^2$

$= \log(2a^2x^3 + (4ab - a^2)x^2 + (2b^2 - 2ab)x - b^2)$

$2a^2 = 8, a = 2$ since $2x - 1 > 0 \Rightarrow x > \frac{1}{2}$

$4ab - a^2 = 4 \Rightarrow 8b - 4 = 4$, b = 1

So a = 2 and b = 1

Q6.2 Let there be x spots on top face. Let the sides Sam sees have x , y and z spots.
So x + y +z = 9
The opposite sides of a dice ad up to 7.

So Pat see x , 7 – y and 7 – z spots.
Therefore x + 7 - y + 7 – z = 15
x – y – z = 1 and x + y + z = 9
2x = 10, x = 5 , top face has 5 spots.

Round 7

Q7.1 f(x) = ax + 5 , g(x) = cx - 3 , where a and c $\in$ N.

f(g(x)) = f (cx – 3 = a(cx – 3) + 5 = acx – 3a +5

g(f(x)) = g(ax +5) = c(ax +5) - 3 = acx +5c -3

f(g(x)) = g(f(x)) $\Rightarrow$ acx – 3a + 5 = acx + 5c -3 $\Rightarrow -3a + 5 = 5c - 3$

$\Rightarrow c = \frac{-3a+8}{5} > 0$ since c $\in$ N. $\Rightarrow$ - 3a + 8 > 0

$$\Rightarrow a < \frac{8}{3} = 2\frac{2}{3}$$

Possible values for a are 1 and 2.

When a = 1 , c = 1 and when a = 2, $c = \frac{2}{5} \notin$ N.

Answers a = 1 , c = 1.

Q7.2 2016 = $2^5 \text{x } 3^2 \text{x } 7$
= 21 x 96, sum of digits = 18
= 63 x 32 , sum of digits = 14
= 42 x 48, sum of digits = 18
= 56 x 36, sum of digits = 20

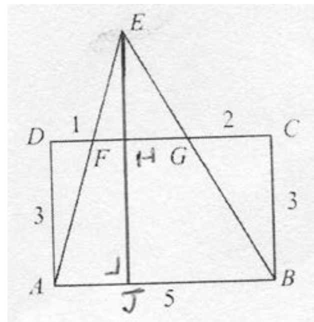
Greatest sum of digits is 20.

Q7.3 $((x + 3y) + \frac{z}{4}) = (x + 3y)^3 + 3(x + 3y)^2(\frac{z}{4}) + 3(x + 3y)(\frac{z^2}{16}) + \frac{z^3}{16}$

Taking relevant term $3(x + 3y)(\frac{z^2}{16}) = \frac{3xz^2}{16} + \frac{9yz^2}{16}$

Co-efficient of yz^2 is $\frac{9}{16}$.

Q7.4 Drop [EJ] perpendicular to [AB]. Let [EJ] cut [CD] at H.



Δ's EFG and AEB similar $\Rightarrow \frac{|FG|}{|AB|} = \frac{|EF|}{|EA|} = \frac{2}{5}$.

Δ's EFH and EAJ similar $\Rightarrow \frac{|EF|}{|EA|} = \frac{2}{5} = \frac{|EH|}{|EJ|} = \frac{|EH|}{|EH+3|}$

$\Rightarrow 5|EH| = 2|EH| + 6,\ 3|EH| = 6,\ |EH| = 2$

$\Rightarrow |EJ| = 5$

Area of triangle AEB = $\frac{1}{2}(5)(5) = \frac{25}{2}$ sq. units.

Round 8

Q8.1 Sin (x + y) = $\frac{1}{\sqrt{2}} \Rightarrow x + y = 45^0 \text{ or } 135^0$

Cos (2x) = $-\frac{1}{2} \Rightarrow 2x = 120^0 \text{ or } 240^0 \Rightarrow x = 60^0 \text{ or } 120^0$.

So y = $15^0 \text{ or } 75^0$.

Q8.2 $\text{side}^2 = 1 + x^2 = 2(1 - x)^2$

$1 + x^2 = 2 - 4x + 2x^2 \Rightarrow x^2 - 4x + 1 = 0$

$$\Rightarrow x = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$$

$$\Rightarrow x = 2 - \sqrt{3}, \text{ since } x<1$$

$side^2 = 1 + (2 - \sqrt{3})^2 = 8 - 4\sqrt{3}$

Area of triangle = $\frac{1}{2}(8 - 4\sqrt{3})\text{Sin }(60^{0)}=2\sqrt{3} - 3$ sq.units

Q8.3 Slope of AB = $\frac{5 - 0}{2+4} = \frac{5}{6} \Rightarrow$ Slope of BC = $-\frac{6}{5}$

Eqn of BC is $y - 0 = -\frac{6}{5}(x + 4)$

i.e 5y = - 6x - 24 i.e 6x +5y = -24 (i)

Slope of AD = $\frac{5-0}{2-7}$ = - 1 $\Rightarrow$slope of CD = 1

Eqn of CD is y = x – 7 (ii)

Substituting (i) $\Rightarrow$6x + 5(x – 7) = -24 , 6x + 5x - 35 = - 24
11x = 11 , x = 1 , y = - 6.

So C(1 , -6)

Q8.4 $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$

Squaring x =7 + $2\sqrt{(x+7)(x+2)}$ + x +2 = 6x+13

$2\sqrt{(x+7)(x+2)}$ = 4x + 4, $x^2 + 9x + 14 = 4x^2 + 8x + 4$,

$3x^2 - x - 10 = 0$, (3x + 5)(x – 2) = 0, x = $\frac{5}{3}$ or 2.

Checking x = $\frac{5}{3}$ invalid. So x = 2 only.