Babhta 1 Round 1

Q1.1 In how many ways can the letters of the word **COURAGE** be arranged, if the vowels are always to occupy the odd positions.
Answer in non-factorial form.

Q1.2 The inscribed circle of a triangle ABC touches [BC], [CA] and [AB] at X , Y and Z respectively.
|BC| = a cm, |CA| = b cm , and |AB| = c cm,
|AY| = x cm, |BZ| = y cm and |CX| = z cm.
Write x in terms of a , b and c.

Babhta 2 Round 2

Q2.1 Find the equations of the lines which pass through the point (- 2, 1) and make an angle of 45^0 with the line 3x + y + 5 =0.
Answers in form ax + by + c = o ,where a , b and c $\in Z$

Q2.2 x and y are integers such that $x^2+y^2=29$.
Calculate the maximum value of |x - y|

Babhta 3 Round 3

Q3.1 Find the coordinates of the image of the point (6, 0) after a 30^0 rotation anticlockwise about the origin.

Answer as a coordinate in simplest surd form.

Q3.2 f is a function and $f(x) = \frac{6x + 1}{3x - 2}$

Find the inverse function, $f^{-1}(x)$ in terms of x.

Answer in simplest form.

Babhta 4 Round 4

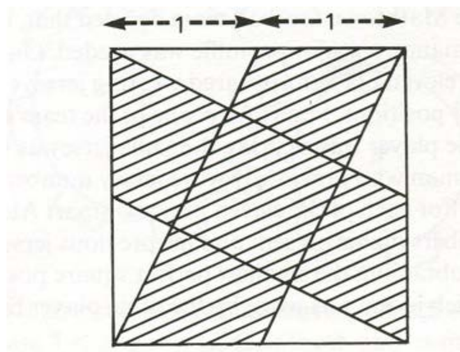
Q4.1 A man borrowed €20000 at 5% compound interest. The principal and interest are to be repaid in 20 equal annual instalments, with the first repayment one year after he takes out the loan. Calculate the amount of each instalment.
Answer to nearest euro.

Q4.2 If $z = \mathrm{Cos}(\theta) + i\mathrm{Sin}(\theta)$, write $\dfrac{1-\dfrac{1}{z^2}}{1+\dfrac{1}{z^2}}$

in simplest trig. form

Babhta 5 Round 5

Q5.1



In the diagram the square has a side 2 cm long, Line segments are drawn joining its vertices to the midpoints of the sides as shown in the diagram to form a cross. Find the area of the shaded cross.

Answer in form $\frac{\mathbf{a}}{\mathbf{b}}$, where a and b $\in N$.

Q5.2 Jane and Sarah each throw a dice.

What is the probability that Jane's throw is higher than Sarah's?

Answer in simplest form $\frac{\mathbf{a}}{\mathbf{b}}$, where a and b $\in N$.

Babhta 6 Round 6

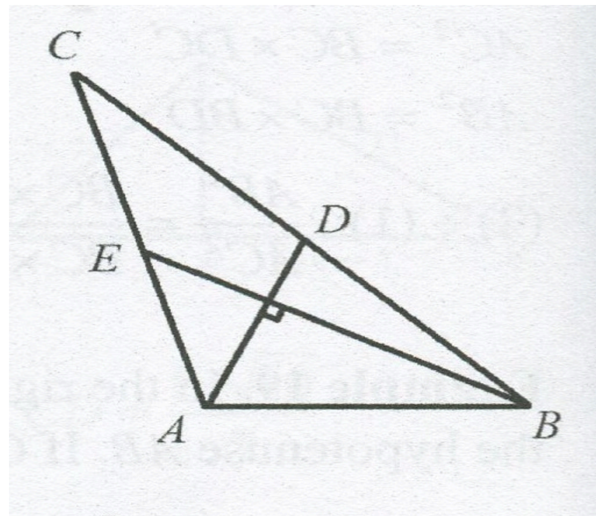
Q6.1 When a two-digit number is added to another two-digit number which has the same digits in reverse, the sum is a perfect square.
Find the sum of all such two-digit numbers less than 50.

Q6.2 Let $f(x) = \log_b(x)$ and let $g(x) = x^2 - 4x + 4$.
Given that $f(g(x)) = g(f(x)) = 0$ has exactly one solution and $b > 1$, find the value of b.

Answer in form $\sqrt{a}$, where $a \in N$.

Babhta 7 Round 7

Q7.1 In the triangle ABC , |BC| = 30 , |AC| = 20. [AD] and [BE] are two medians as shown in diagram and [AD] is perpendicular to [BE].
Find the length of [AB].
Answer in form $a\sqrt{b}$,
where a and b $\in N$.



Q7.2 The numbers 49, 29, 9 , 40 , 22 , 15 , 53 , 33, 13 , 47 are grouped in pairs so that the sum of each pair is the same. Which number is paired with 15?

Q7.3 Five lamps A , B , C , D , E, are arranged in a row, each with its own switch. Initially, all five lamps are off. Jill starts pressing switches in the following order :
A, B, C, D, E, A, B, C, D, E, A, ……until she has pressed 2017 switches, counting repetitions. Assuming that each press turns a lamp off if it is on, or on if it is off , which lamps are finally on?

Q7.4 Calculate the value of the constant a if the coefficient of x^3 in the expansion $(a+2x)^4$ is 160.

Babhta 8 Round 8

Q8.1 The points A(-3 , - 9) and B(9 , -4) are the endpoints of a diameter of a circle.
This circle touches the x-axis at T.
Find the ratio of the length of [TA] to the length of [TB].

Answer in simplest form a : b, where a and b $\in$ N.

Q8.2 What is the product of the positive integers m and n such that m > n and

$$\frac{1}{m} + \frac{1}{n} + \frac{1}{mn} = 1?$$

Q8.3 An express train, travelling at uniform speed, leaves station A at 3 pm and reaches station B at 6 pm. A slower train, travelling at uniform speed, leaves station B at 1.30 pm and arrives at station A at 6pm.
At what time do they meet?

Q8.4 Solve for x and y :

$$x^3 + y^3 = 91$$
$$x^2y + xy^2 = 84$$

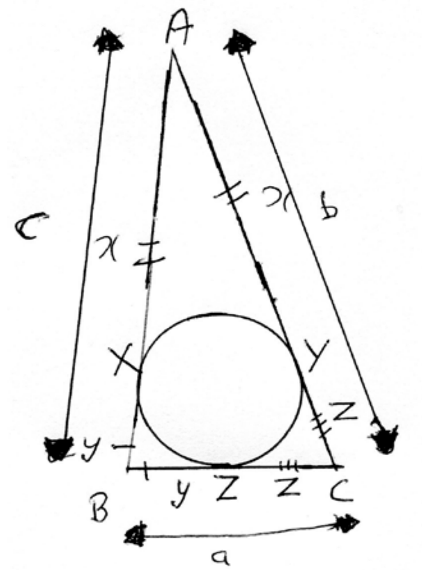
Answer in coordinate form (x , y).

TEAM Maths : Answer key final17

Round 1	Q1.1	144	Q1.2	x= $\frac{1}{2}(b + c - a)$
Round 2	Q2.1	x + 2y =0 and 2x – y + 5 =0	Q2.2	7
Round 3	Q3.1	$(3\sqrt{3}, 3)$	Q3.2	$\frac{2x+1}{3x-6}$
Round 4	Q4.1	€1605	Q4.2	iTan(θ)
Round 5	Q5.1	$\frac{16}{5}$ cm^2	Q5.2	$\frac{5}{12}$
Round 6	Q6.1	114	Q6.2	$\sqrt{3}$
Round 7	Q7.1	$2\sqrt{65}$	Q7.2	47
	Q7.3	C, D and E	Q7.4	5
Round 8	Q8.1	3 : 2	Q8.2	mn =6
	Q8.3	4 : 12 PM	Q8.4	(3 , 4) and (4 , 3)

Round 1

Q1.1 COURAGE has 7 letters: 4 vowels and 3 consonants
Vowels can occupy the odd positions in 4! ways.
Consonants can occupy the even positions in 3! ways.
Number of arrangements = 4! x 3! = 144 .



Q1.2 |AX| = |AY| = x, |BX| = |BZ| = y, |CY| = |CZ| = z

2x + 2y + 2z = a + b +c, a = y + z,
2x + 2(y + z) = a + b + c ,
2x + 2 a = a + b + c , x = $\frac{1}{2}(\mathbf{b + c - a})$

Round 2

Q2.1 P(- 2 , 1) , slope = tan (45$^{\mathbf{o}}$) = 1

Any line through (- 2 , 1) has form , $\mathbf{l}$: y – 1 =m(x + 2).

3x + y + 5 = 0 has slope -3.

Tan (45$^{\mathbf{o}}$) = $\pm(\frac{\mathbf{m+3}}{\mathbf{1 -3m}}) = \mathbf{1}$, m + 3 = 1 – 3m, m = $-\frac{1}{2}$.

Or m +3 = -1 +3m , m = 2

Lines are x + 2y = 0 and 2x – y + 5 = 0.

Q2.2 $\mathbf{x^2 + y^2 = 29 \Rightarrow eg\ x = \pm 5, y = \pm 2}$.

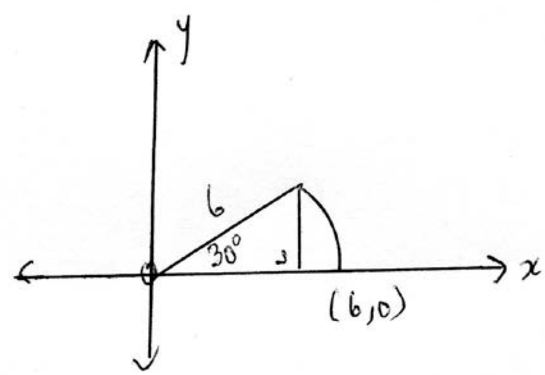
For max value take opposite signs. Max = |5 – (-2)| = |-5 -2| = 7

Round 3

Q3.1 $\mathbf{Sin(30^o)} = \frac{1}{2} = \frac{\mathbf{y}}{6}$, y = 3

$\mathbf{Cos(30^o)} = \frac{\sqrt{3}}{2} = \frac{\mathbf{x}}{6}$, x = $3\sqrt{3}$.

Point is $(3\sqrt{3}, 3)$



Q3.2 $f(x) = y = \dfrac{6x+1}{3x-2}$, $6x + 1 = 3xy - 2y$, $x = \dfrac{-2y-1}{6-3y} = \dfrac{2y+1}{3y-6}$.

$$f^{-1}(x) = \frac{2x+1}{3x-6}.$$

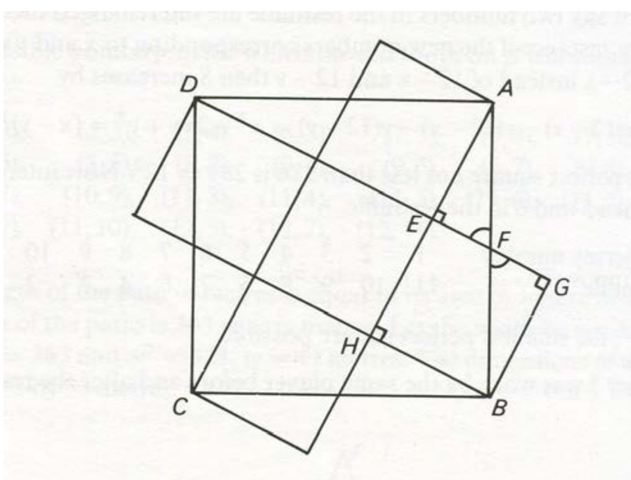
Round 4

Q4.1 A =€ $\dfrac{20000(1.05)^{20}(.05)}{(1.05)^{20}-1}$ = €1605 to nearest euro.

Q4.2 $$\frac{1-\frac{1}{z^2}}{1+\frac{1}{z^2}} = \frac{\frac{1}{z}(z-\frac{1}{z})}{\frac{1}{z}(z+\frac{1}{z})} = \frac{z-\frac{1}{z}}{z+\frac{1}{z}} = \frac{2iSin\theta}{2Cos\theta} = i\tan\theta$$

Round 5

Q5.1



Extend the cross as shown above. The triangles AEF and BGF are congruent. So BG = AE = BH and EGBH is a square. Also EF = GF = $\frac{1}{2}$EG.

Since the areas of triangles AEF and BGF are equal, we see that the area of the five small squares forming a cross equals the area of square ABCD, which is 4, and so each square has area $\frac{4}{5}$. Now triangle AEF has area $\frac{1}{4}$ of the area of a small square and so the four unshaded triangles in the original figure have total area $\frac{4}{5}$. Thus the area of the shaded cross is $4-\frac{4}{5}=\frac{16}{5}$.

Q5.2 Total number of outcomes = 6 x 6 = 36

	Jane	Sarah
Throw	2, 3,4, 5, 6	1
Throw	3,4,5,6	1,2
Throw	4,5,6	1,2,3
Throw	5,6	1,2,3,4
Throw	6	1,2,3,4,5

Number of outcomes where Jane's score is greater than Sarah's
= 5 + 4 + 3 + 2 +1 = 15

p(Jane > Sarah) = $\frac{15}{36} = \frac{5}{12}$

Round 6

Q6.1 Let the number 10a + b.

So 10a + b + 10b + a = n^2, for some n. , 11(a + b) = n^2

For a perfect square a + b = 11.

There are 8 possible pairs (2,9) , (9, 2). (8,3). (3,8), (4,7), (7,4), (6,5), (5,6).

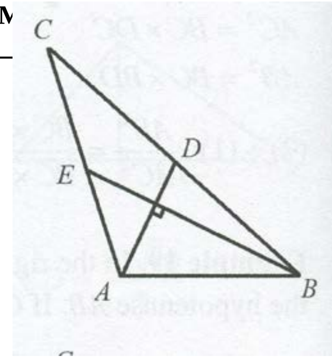
Sum of numbers < 50 is 29 + 38 + 47 = 114

Q6.2 f(x) = $\log_b(x)$, g(x) = $x^2 - 4x + 4 = (x-2)^2$

f(g(x)) = 0. So $f(x-2)^2 = \log_b(x-2)^2 = 2\log_b(x-2) = 0$
x -2 = $b^0 = 1$, x = 3

g(f(x)) = 0. So g($\log_b(x)) = (\log_b(x) - 2)^2 = 0$

$\log_b(x) = 2$, $x = b^2 = 3$, $b = +\sqrt{3}$



Round 7

Q7.1

By Pythagoras theorem

$4m^2 + n^2 = 15^2$, $4n^2 + m^2 = 10^2 = 100$, $5m^2 + 5n^2 = 325$, $m^2 + n^2 = 65$,

$|AB|^2 = 4m^2 + 4n^2 = 4x65$, $|AB| = 2\sqrt{65}.$

Q7.2 Arrange the numbers in ascending order : 9, 13, 15, 22, 29, 33, 40, 47, 49, 53

9 + 53 = 62, 13 + 49 = 62 etc
So 15 + 47 = 62. ANSWER 47.

Q7.3 At start lamps A , B , C , D and E are off
Then 1st to 5^{th} time all 5 lamps are on
Then 6^{th} to 10^{th} time all 5 lamps are off
Then 11th to 15th times all 5 lamps are on

Continuing the pattern 2011th to 2015^{th} time all lamps are on.
So tuning the switches 2016 and 2017 times lamps **A and B** are **off**
and so **C , D** and **E** are **on.**

Q7.4 $(a + 2x)^4 = a^4 + 4a^3(2x) + 6a^2(4x^2) + 4a(8x^3) + 16x^4$

32a = 160 , a = 5

Round 8

Q8.1 A(-3 , -9) , B(9, - 4) , Centre C $(3,-\frac{13}{2})$

Radius $=\sqrt{(3+3)^2+(-\frac{13}{2}+9)^2}=\frac{13}{2}$

Equation of Circle : $(x-3)^2+(y+\frac{13}{2})^2=\frac{169}{4}$

ie $x^2+y^2-6x+13y+9=0$

Circle cuts x-axis (y =0) , giving $x^2-6x+9=0\Rightarrow(x-3)^2=0$

So x = 3 , y = 0. So T(3, 0) , A(-3 , -9) , B(9, - 4)

|AT| = $\sqrt{36+81}=\sqrt{117}=3\sqrt{13}$
|TB| = $\sqrt{36+16}=\sqrt{52}=2\sqrt{13}$

So ratio |TA| : |TB| = 3 : 2

Q8.2 $\frac{1}{m}+\frac{1}{n}+\frac{1}{mn}=1\Rightarrow$ n + m + 1 = 0 $\Rightarrow$ mn – m – 1 = 0

Adding 2 to each side

mn – m – n +1 = 2 , (m – 1)(n – 1) = 2 x 1

m – 1 = 2 , m = 1 and n – 1 = 1 , n = 2, since m > n.
So mn = 6

Q8.3 Train 1 (from A to B) takes 3 hours
Train 2 (from B to A) takes 4.5 hours
Let d be distance between the stations

Train 1 speed = $\frac{d}{3}$ km/h

Train 2 speed = $\frac{d}{4.5}$ km/h = $\frac{2d}{9}$ km/h

Let trains meet x hours after 3 pm

In x hours train 1 has $\frac{xd}{3}$ travelled km and train 2 has travelled $\frac{(x+\frac{3}{2})(2d)}{9}$ km.

When they meet $\frac{xd}{3}+\frac{(x+\frac{3}{2})(2d)}{9}=d$

$\Rightarrow \frac{x}{3}+\frac{(x+\frac{3}{2})(2)}{9}=1 \Rightarrow$ 3x +2x+ 3 = 9, 5x = 6 , x = 6.2 hours after 3 pm

i.e 1 hour 12 minutes after 3 pm at 4.12 pm

Q8.4

$x^3+y^3=91$

$x^2y+xy^2=84$ and $\Rightarrow 3x^2y+3xy^2=252$

Adding $x^3+3x^2y+3xy^2+y^3=343$,
$(x+y)^3=343$, x + y = 7 , y = 7 – x .

Substituting $x^2y+xy^2=84 \Rightarrow$ xy(x + y) = 84, xy(7) = 84, xy =12.

So x(7 – x) = 12 , $x^2-7x+12=0$, x = 3 or 4 and y = 4 or 3.

Answers (4, 3) and (3, 4)