



07 – March – 2015

Foireann Mata 2015 (An Chraobh)

Team Maths 2015 (Final)

Team Math Final – 2015

Saturday March 7th, 2015



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Team Maths 2015 (Final)

Babhta 1

Round 1

- 1) Given $c^x = 3$, what is the numerical value of $c^{4x} - 5$?

- 2) Given $2x^2 - 9x + 14 = a(x - 1)(x - 2) + b(x - 1) + c$,
find the numerical value of $a + b + c$.



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Babhata 2

Round 2

- 1) Let x be a positive real number.
Find the value of x when $x - 1$ is the reciprocal of $x + \frac{1}{2}$.
Answer in the form $\frac{a}{b}$, where a and $b \in \mathbb{Z}$.

- 2) The coordinates of three vertices of a parallelogram ABCD are:
A(-5, 1), B(2, 4) and C(1, 1).

Calculate the area of the parallelogram.



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Babhta 3

Round 3

- 1) For $0 < \theta \leq 2\pi$, find all the solutions to the equation $\sin^2 \theta = 1 - \cos \theta$

Answer in terms of π .

- 2) A line divides a circle of radius length 2 into two arcs.
If the length of the smaller arc is $\frac{\pi}{3}$, what is the area of the region bounded by the smaller arc and the line?

Answer in the form $\frac{\pi}{a} - b$, where a and $b \in \mathbb{Z}$.



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Babhta 4

Round 4

- 1) The quadratic expression $4x^2 + 11x + 3$ can be written in the form $a(x + b)^2 + c$.

Calculate, in its simplest form, the numerical value of $ab^2 + c$.

- 2) A coin is weighted so that $P(\text{head}) = \frac{2}{3}$ and $P(\text{tail}) = \frac{1}{3}$.
The coin is tossed once.

If a head appears, then a number is selected at random from the whole numbers 1 to 9.

If a tail appears, a number is selected at random from the whole numbers 1 to 5.

Find the probability that an even number appears.

Answer in its simplest form $\frac{a}{b}$, where a and $b \in \mathbf{N}$.



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Babhta 5

Round 5

- 1) Let a be a positive real number.
The curve $y = 1 - ax^2$ intersects the x and y axes at three points which form the vertices of an equilateral triangle.

Find the numerical value of a in its simplest form.

- 2) John enters a tall building. He has a decision to make: should he take the stairs or the lift.

If he decides to take the stairs, it would take him 20 seconds to walk up each flight of stairs.

If he decides to take the lift, he would have to wait 3 minutes for the lift to arrive, after which it would take 3 seconds to move up each floor.

Calculate the minimum number of floors for which taking the lift takes less time than taking the stairs.



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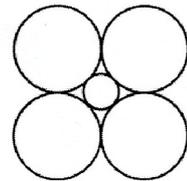
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Babhta 6

Round 6

- 1) Four circles of radius length 1 are touching each other and a smaller circle, as seen in the diagram.
What is the radius length of the smaller circle?



Answer in its simplest form $a\sqrt{b} + c$, where a, b and $c \in \mathbb{Z}$.

- 2) For a given arithmetic sequence, the ratio of the sum of the first m terms to the sum of the first n terms is $m^2 : n^2$. Find, in simplest form, the ratio of the m^{th} term to the n^{th} term in terms of m and n , where $m \neq n$.



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Babhta 7

Round 7

- 1) In the triangle ABC, using the usual notation, write the expression

$$\frac{4 \times (\text{Area } ABC)}{b^2 + c^2 - a^2}$$

as a single trigonometrical function in its simplest form.

- 2) An equilateral triangle is circumscribed about a circle of radius length 9.

Find the area of the triangle.

Answer in its simplest form $a\sqrt{b}$, where a and $b \in \mathbf{N}$.

- 3) Given $\sin x - \cos x = \frac{3}{5}$, find the numerical value of $\sin^3 x - \cos^3 x$

Answer in its simplest form $\frac{a}{b}$, where a and $b \in \mathbf{N}$.

- 4) I have a bag with 12 slips of paper in it. Some of the slips have a “2” written on them and the rest have a “7” written on them.

If the expected value of the number randomly drawn from the bag is 3.25, how many slips have a “2” written on them?



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Babhta 8

Round 8

- 1) Find the complex number z such that $\frac{2z-3i}{z+2} = -5 + i$

Answer in the form $a + bi$ where a and $b \in \mathbb{Q}$.

- 2) Given that $(a + c)(b + c)(a + b) \neq 0$ and $b^2 - a^2 = c^2 - b^2$ find the numerical value of

$$\frac{2}{a+c} - \frac{1}{b+c} - \frac{1}{a+b}$$

- 3) The lines $y = mx + 4$ and $y = 2mx + 3$ intersect at an angle $\tan^{-1}\left(\frac{1}{4}\right)$
Find all possible values for m .

Answers in the form $a + b\sqrt{c}$ where a, b and $c \in \mathbb{Q}$.

- 4) Calculate the range of values of x for which $3^{2x+1} + 3 < 3^{x+2} + 3^x$

TM 15 Final answers

Round 1	(q1) 76	(q2) 6
Round 2	(q1) $\frac{3}{2}$	(q2) 18
Round 3	(q1) $\frac{1}{2}\pi, \frac{3}{2}\pi, 2\pi$	(q2) $\frac{\pi}{3}-1$
Round 4	(q1) 3	(q2) $\frac{58}{135}$
Round 5	(q1) 3	(q2) 11
Round 6	(q1) $\sqrt{2}-1$	(q2) $2m-1 : 2n-1$
Round 7	(q1) $\tan A$	(q2) $243\sqrt{3}$
	(q3) $\frac{99}{125}$	(q4) 9
Round 8	(q1) $-\frac{3}{2} + \frac{1}{2}i$	(q2) 0
	(q3) $\pm 1 \pm \frac{\sqrt{2}}{2}$	(q4) $-1 < x < 1$