IMTA

NUI Galway
OÉ Gaillimh

08 – March – 2014

Foireann Mata 2014 (An Chraobh) Team Maths 2014 (Final)

Team Math Final – 2014

Saturday March 8th, 2014

IMTA

NUI Galway
OÉ Gaillimh

08 – March – 2014

Foireann Mata 2014 (An Chraobh) Team Maths 2014 (Final)

BABHTA 1 **ROUND 1**

1) The curve $y = 17(x + 3)(x - 4)$ intersects the x-axis at A and B. Find the length of the line segment $[AB]$.

2) Let $(1 + i)^3 = x + iy$, where x and $y \in \mathbb{R}$ and where $i = \sqrt{-1}$. Find the value of $x + y$.

IMTA

NUI Galway
OÉ Gaillimh

08 – March – 2014

Foireann Mata 2014 (An Chraobh) Team Maths 2014 (Final)

BABHTA 2 **ROUND 2**

1) Find all the values of B, where $0 < B < \pi$, such that
$6\text{Cos}^2(B) - \text{Sin}(B) = 4$

Answers in terms of π.

2) Find the distance from (5, 8) to the nearest point on the circle

$$2x^2 + 2y^2 + 4x - 8y = 6$$

Answer in the form $a\sqrt{b}$ where a and $b \in \mathbb{N}$.

IMTA

NUI Galway
OÉ Gaillimh

08 – March – 2014

Foireann Mata 2014 (An Chraobh) Team Maths 2014 (Final)

BABHTA 3 **ROUND 3**

1) Find two numbers a and b such that

$$(a-b) : (a+b) : ab \;=\; 1 : 5 : 24\,, \quad b \neq 0$$

2) In the equation below, calculate the value of $x + y$.

$3^{2x} - 2^{2y} = 17$, where x and $y \in \mathbb{N}$.

IMTA

NUI Galway
OÉ Gaillimh

08 – March – 2014

Foireann Mata 2014 (An Chraobh) Team Maths 2014 (Final)

BABHTA 4 **ROUND 4**

1) The lengths of the sides of a right- angled triangle are in geometric progression and the shortest side has a length of 2 cm.
What is the length of the hypotenuse in simplest surd form?

Answer in the form $a + \sqrt{b}$, where a and $b \in \mathbb{N}$.

2) Three straight lines l, m and n have slope $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, respectively.
All three lines have the same y-intercept.
If the sum of the x-intercepts of the three lines is 36, calculate the value of the y-intercept.

IMTA

NUI Galway
OÉ Gaillimh

08 – March – 2014

Foireann Mata 2014 (An Chraobh) Team Maths 2014 (Final)

BABHTA 5 **ROUND 5**

1) Find the smallest positive integer, a, such that 2014 is a term of the following arithmetic sequence:

$$a,\ a+7,\ a+14,\ a+21,\ \ldots\ldots\ldots$$

2) We are given three boxes as follows:

Box A has 10 light bulbs, of which 4 are defective.
Box B has 6 light bulbs, of which 1 is defective.
Box C has 8 light bulbs, of which 3 are defective.

If we draw a box at random and then draw a bulb at random, what is the probability that the bulb is defective?

Answer in the form $\frac{a}{b}$, where a and $b \in \mathbb{N}$.

IMTA

NUI Galway
OÉ Gaillimh

08 – March – 2014

Foireann Mata 2014 (An Chraobh) Team Maths 2014 (Final)

BABHTA 6 **ROUND 6**

1) Solve the following system of equations for x , y and z:

$$xy + xz = 8 - x^2$$
$$xy + yz = 12 - y^2$$
$$yz + zx = -4 - z^2$$

2) In a triangle, two of its angles, A and B are acute.
If $\tan(A) = \frac{3}{4}$ and $\sin(B) = \frac{5}{13}$, find the cosine of the third angle, i.e. $\cos(C)$, in the form $\frac{a}{b}$, where a and $b \in \mathbb{Z}$.

IMTA

NUI Galway
OÉ Gaillimh

08 – March – 2014

Foireann Mata 2014 (An Chraobh) Team Maths 2014 (Final)

BABHTA 7 **ROUND 7**

1) A square and an equilateral triangle have the same perimeter.
Find the ratio of the area of the triangle to the area of the square.
Answer in the form $a\sqrt{b} : c$, where a, b and $c \in \mathbb{N}$.

2) $f(x) = (1+x) + (1+x)^2 + (1+x)^3 + \cdots + (1+x)^{100}$
What is the coefficient of x^{98}?

3)

"1"

"10" "2"

"9" "3"

"8" "4"

"7" "5"

"6"

Ten people form a circle. Each person calls out a whole number. Then each person computes the average of the numbers of the two people *adjacent to him / her in the circle* and writes down that average. The figure above shows the average of the two neighbouring numbers for each person (not the original number the person picked).
What number was called by the person who wrote down the average 6?

4) The student lockers at a certain college are numbered consecutively, beginning with locker number 1. The plastic digits used to number the lockers cost eight cents each.
For example, it costs eight cents to label locker number 8 and sixteen cents to label locker number 14. If it costs €555·92 to label all lockers, how many student lockers are there at the college?

IMTA

NUI Galway
OÉ Gaillimh

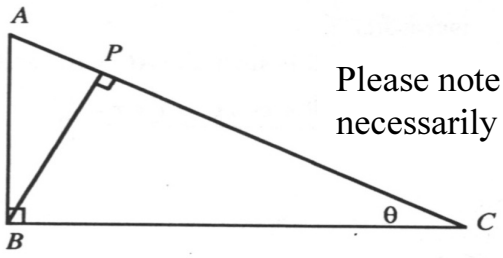
08 – March – 2014

Foireann Mata 2014 (An Chraobh) Team Maths 2014 (Final)

BABHTA 8 ROUND 8

1) In the right-angled triangle ABC, $|AC| = 4$, $|BP| = 1$ and $BP \perp AC$.

Find all possible values of the angle θ, in degrees.



Please note: This diagram is not necessarily drawn to scale.

2) The curve $y = ax^2 + bx + 1$ passes through the point (1, 2).
For what values of a does the curve intersect the x-axis at two distinct points?
Answer in simplest surd form.

3) If $x^2 + x = 1$, calculate the value of the expression

$$x^3 + 2x^2 + 2014.$$

4) On Wednesday, Jackie drove from home to work at an average speed of 70 km/h and arrived 1 minute late. On Thursday, she left home at the same time as on Wednesday and took the same route. This time she drove at an average speed 75 km/h and arrived 1 minute early.
What distance did she have to travel to work?

IMTA

NUI Galway
OÉ Gaillimh

08 – March – 2014

Foireann Mata 2014 (An Chraobh) Team Maths 2014 (Final)

ANSWER KEY

ROUND 1	Q 1 $\boxed{7}$	Q2 $\boxed{0}$		
ROUND 2	Q1 $\boxed{\frac{\pi}{6}, \frac{5\pi}{6}}$	Q2 $\boxed{4\sqrt{2}}$		
ROUND 3	Q1 $\boxed{a = 12 \\ b = 8}$	Q2 $\boxed{5}$		
ROUND 4	Q1 $\boxed{1 + \sqrt{5}}$	Q2 $\boxed{-4}$		
ROUND 5	Q1 $\boxed{5}$	Q2 $\boxed{\frac{113}{360}}$		
ROUND 6	Q1 $\boxed{2, 3, -1 \text{ and } -2, -3, 1}$	Q2 $\boxed{-\frac{33}{65}}$		
ROUND 7	Q1 $\boxed{4\sqrt{3} : 9}$	Q2 $\boxed{5050}$	Q3 $\boxed{1}$	Q4 $\boxed{2014}$
ROUND 8	Q1 $\boxed{15°, 75°}$	Q2 $\boxed{a > 3 + 2\sqrt{2} \text{ and } a < 3 - 2\sqrt{2}}$	Q3 $\boxed{2015}$	Q4 $\boxed{35 \text{ km}}$

IMTA

NUI Galway
OÉ Gaillimh

08 – March – 2014

Foireann Mata 2014 (An Chraobh) Team Maths 2014 (Final)

Scoilt **Tiebreak**

1) Susie has 138 identical cubes. She uses these to build a solid cube, using as many of the 138 cubes as possible. How many cubes will remain unused?

2) For how many values of $n \in \mathbb{Z}$, would right-angled triangles with sides of length $\sqrt{n+1}$, $\sqrt{n+2}$ and $\sqrt{n+3}$ exist?

3) What is the largest possible area of a rectangle whose perimeter is 36 cm?

4) If p% of q is k, what is q% of p?

5) The year 2013 contained four digits whose values are consecutive integers.
How many years after 2013 must we wait for this event to occur again?

6) Two different digits are selected at random from the digits 1 through 9. If the sum of the digits is even, find the probability that both digits are odd.
Answer in simplest form $\frac{a}{b}$, where a and $b \in \mathbb{N}$.

7) Suppose the waist measurements of 800 young people are normally distributed with a mean of 66 cm and standard deviation 5 cm. Find the number of young people with waists greater than or equal to 72 cm.

Please **turn over** to continue ➔

8) Let $\cos(2A) = -\frac{7}{9}$ and $0 < A < \frac{\pi}{2}$. Find the value of $\cos(A)$

9) Find the numerical value of

$$\sin\left(\frac{\pi}{6}\right) + \sin^2\left(\frac{\pi}{6}\right) + \sin^3\left(\frac{\pi}{6}\right) + \ \ldots\ldots\ldots$$

10) The solutions to the equation $\sqrt[4]{x} = \frac{12}{7 - \sqrt[4]{x}}$ are A and B.
Calculate the value of $A + B$.

11) If $a + b = 1$ and $a^2 + b^2 = 2$, calculate the value of $a^3 + b^3$.

Answer in the form $\frac{a}{b}$, where a and $b \in \mathbb{N}$.

12) Calculate the value of the term independent of x in the expansion of

$$\left(x^2 - \frac{2}{x^2}\right)^8.$$

13) Find the value of x when $4^{20} + 4^{20} = 2^x$.

14) Find the equation of the line through $(-1, 5)$ which is perpendicular to the line $y + 5x - 1 = 0$.
Answer in the form $ax + by + c = 0$, where a, b and $c \in \mathbb{N}$.

IMTA

NUI Galway
OÉ Gaillimh

08 – March – 2014

Foireann Mata 2014 (An Chraobh) Team Maths 2014 (Final)

Tiebreak Answers

Q1	13
Q2	1
Q3	81
Q4	k
Q5	18
Q6	$\frac{5}{8}$
Q7	92
Q8	$\frac{1}{3}$
Q9	1
Q10	337
Q11	$\frac{5}{2}$
Q12	1120
Q13	41
Q14	Any correct solution