Foireann Mata 2013 Comórtas Réigiúnach
(Team Maths 2013 Regional Round)

Babhta 1 <u>Round 1</u>

1) Expand $(1 - x^2)(1 + x^3)$

2) The point (4, 3) is reflected in the x-axis to a point P. Then P is reflected in the y-axis to a point Q. What is the sum of the coordinates of Q?

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Babhta 2 <u>Round 2</u>

1) Find the radius of the circle which cuts the x-axis at $x = 2$ and $x = 4$ and the y-axis is a tangent to the circle.

2) Find the values of z^3 when $z^6 + 2z^3 + 2 = 0$
Answer in the form $x + iy$ where x and $y \in \mathbb{R}$

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Babhta 3 Round 3

1) A bag has 3 red and k white marbles. Two marbles are chosen at random from the bag. If the probability that the two marbles are the same colour is $\frac{1}{2}$, find the value of k, if $k > 1$.

2) In the subtraction below some of the digits have been replaced by letters.

$$\begin{array}{r} A4B7C \\ 5D8E6 \\ \hline 28499 \end{array}$$

Find the numerical value of A + B + C + D + E.

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Babhta 4 Round 4

1) If $f(x) = ax^2 + bx + c$ and the points (4, -9), (-1, - 4) and (0,-1) lie on the curve $y = f(x)$, calculate the product **abc.**

2) If $\tan(2\theta) = \frac{5}{12}$, calculate all possible values of $\sin(\theta)$.

Answers in the form $\frac{a}{\sqrt{b}}$, where a and b $\in \mathbb{Z}$

Babhta 5 <u>Round 5</u>

1) Using the usual notation find the area of the triangle ABC where $a = 3{\cdot}34$, $b = 5{\cdot}62$ and $c = 3{\cdot}84$.
Answer correct to 2 decimal places.

2) A piece of wire 12 m long is cut into two pieces, one of which is bent to form a square and the other to form a circle. Calculate, in terms of π, the length of the side of the square when the combined areas of the two figures is a minimum. Answer as a fraction.

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Babhta 6 <u>Round 6</u>

1) Two players compete in a tennis match of up to 3 sets. The match ends when a player wins two sets.
If player A has a probability of 60% in winning a set, what is this player's probability of winning the match?

Answer correct to 3 decimal places.

2) Find the accurate value of $\int_0^{\frac{\pi}{2}} \frac{Sin(x)dx}{1+4Cos(x)}$

Babhta 7 Round 7

1) In how many ways can a pigeon fancier separate his 10 pigeons into a group of 4 and a group of 6 if he has to keep a certain two of the pigeons in separate groups?

2) Components made by a certain process have a thickness which is normally distributed about a mean of 3 cm with a standard deviation 0·03 cm. A component is considered defective if its thickness is greater than 3·05 cm or less than 2·95 cm. What percentage of the components are defective? Answer correct to 1 decimal place.

3) If $\sin(x) + \sin^2(x) = 1$, find the value of $\cos^2(x) + \cos^4(x)$.

4) Find the derivative of $\ln\left(\frac{4x^2 - 1}{3x + 1}\right)$ when $x = 1$.
Answer in simplest form $\frac{a}{b}$, where a and b $\in \mathbb{Z}$

Babhta 8 Round 8

1) If $(3x + 2 - 3y)(2x - 3 + 2y) = 0$ and $x + y \geq 2$ find the value of $x - y$.
Answer in the simplest form $\frac{a}{b}$ where a and b $\in \mathbb{Z}$.

2) How many 6-digit numbers start and / or end with an even digit? (A six digit number cannot begin with 0)

3) Three fair dice are tossed once. What is the probability that the faces show three consecutive integers?
Answer in simplest form $\frac{a}{b}$ where a and b $\in \mathbb{N}$.

4) Two candles of equal length are lit at 10 pm. Each candle burns at a different constant rate.
One candle takes 6 hours to burn out; the other takes 3 hours.
At what time is one candle exactly twice as long as the other?

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Tiebreak

1) A total of twenty eight handshakes were exchanged at the end of a party. Assuming that everyone shook hands with everyone else at the party, how many people attended the party?

2) Find the coordinates of the point where the tangent to the curve $y = x^3 - 2x + 1$ at (2, 5) meets the curve for the second time.

3) Find $\int e^{\frac{\pi}{2}-t}dt$

4) Find the value of $(4^{-1} - 3^{-1})^{-1}$ in its simplest form.

5) In a special sort of lottery a player may buy a ticket on which he selects 10 different integers from the range 1 to 100 (inclusive). 10 numbers are then drawn at random. The player wins if his ticket contains none of the numbers drawn.
What is the probability that his ticket wins? Answer correct to 4 decimal places.

6) If $\dfrac{a}{a+2b} = -3$ calculate the value of $\dfrac{a}{b}$, giving the answer as a fraction in simplest form.

7) A large watermelon weighs 20kg, with 98% of its weight being water. It is left to stand in the sun, and some of the water evaporates so that now only 95% of its weight is water. What does it now weigh, if only the water content changes?

8) The roots of $x^3 + 4x^2 - 7x - 10 = 0$ are -5 , -1 and 2.
What are the roots of $(x-3)^3 + 4(x-3)^2 - 7(x-3) - 10 = 0$?

9) p, q and r are consecutive odd integers. If $p < q < r$ find the value of

$$p^2 - 2q^2 + r^2$$

10) Suppose five days before the day after tomorrow was Wednesday. What day of the week was yesterday?

11) Solve for x : $\dfrac{e^{2x}}{2} - e^x - 12 = 0$. Answer correct to 3 decimal places.

12) My house number is the lowest number on the street that, when divided by 2, 3 , 4 , 5 or 6 , will always leave a remainder of 1.
However when divided by 11 there is no remainder.
What is my house number?

13) I bought two watches, an expensive one and a cheap one. The expensive one cost €200 more than the cheap one. I spent €220 for both. How much did I pay for the cheap watch?

14) In a knock-out tennis tournament all players who entered took part. (there were no walkovers). In all there were 39 matches played before the outright winner emerged.
How many players entered the competition?

Answers:

Round 1	Q1	$1+x^3-x^2-x^5$	Round 2	Q1	3
	Q2	-7		Q2	$-1+i, -1-i$
Round 3	Q1	6	Round 4	Q1	2
	Q2	28		Q2	$\pm\frac{5}{\sqrt{26}},\ \pm\frac{1}{\sqrt{26}}$
Round 5	Q1	6.25	Round 6	Q1	.648
	Q2	$\frac{12}{4+\pi}$		Q2	$\frac{1}{4}\ln(5)$
Round 7	Q1	112	Round 8	Q1	$-\frac{2}{3}$
	Q2	9.5%		Q2	650 000
	Q3	1		Q3	$\frac{1}{9}$
	Q4	$\frac{23}{12}$		Q4	12 midnight

Tiebreak	Q1	8
	Q2	(-4,-55)
	Q3	$-e^{\frac{\pi}{2}-t}+c$
	Q4	-12
	Q5	0.3305
	Q6	$-\frac{3}{2}$
	Q7	8kg
	Q8	-2, 2 , 5
	Q9	8
	Q10	Friday
	Q11	1.792
	Q12	121
	Q13	€10
	Q14	40