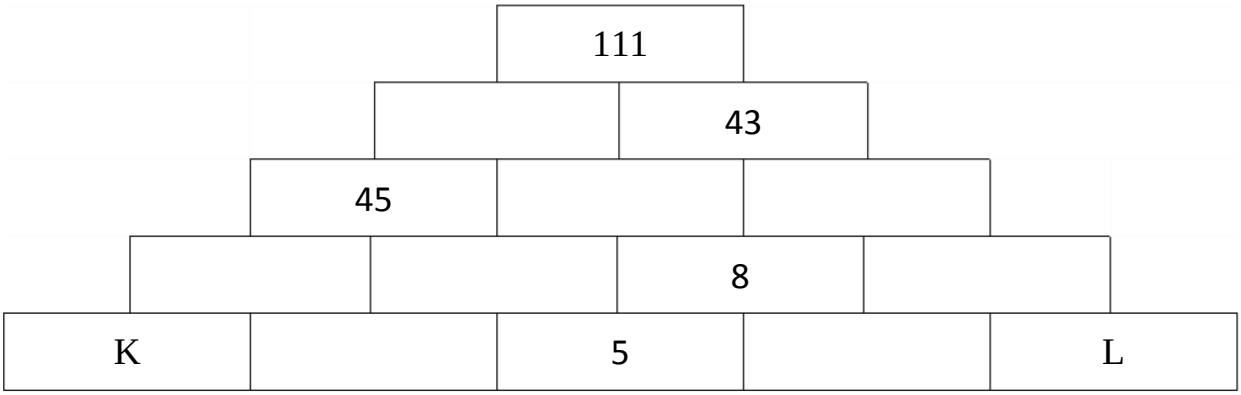
Peter's Problem 2021

Problem No. 1

In the pyramid, each rectangle is to be filled with the sum of the two numbers immediately below it.



What is the value of K+L?

Problem No. 2

Insert one of the words 'seven', 'eight', 'nine', 'ten', or 'eleven' into this sentence to make it true

The letter 'e' appears _____times in this sentence.

Problem No. 3

Imagine you are in a queue to the airport check-in desk. If two thirds of the queue is in front of you and three tenths of the queue is behind you, how many people are in the queue?

Problem No. 4

Any whole number containing 3 or more digits can be divided by 4 if the last two digits form a number that is divisible by 4. Why?

Problem No. 5

What is the smallest number of integer kilogram weights needed to weigh every whole kilogram between 1kg and 40kg?

Problem No. 6

The scale on a map of Ireland is 1:2 000 000.
How many square centimetres are represented on the map by the province of Munster, correct to the nearest square centimetre?

Problem No. 7

Find the values of a, b, c and d such that the 4-digit number

$$abcd \times 4 = dcba$$

Problem No. 8

What is the largest 3 digit number that can be divided evenly by 6, 7 and 8?

Problem No. 9

Use each of the given numbers only once and with any mathematical symbol to amount to a total to 439.

Given numbers: 50, 25, 9, 1, 4, 7

Problem No. 10

Five couples attend a dinner party. Each person shakes hands with each of the other people but not with their own partner.
How many handshakes are made?

Problem No. 11

A Garda is chasing a thief on foot. The thief has a 27 step head-start and takes 8 steps to the Garda's 5. However, the Garda's stride is longer and two of the Garda's steps equals five of the thief's.

How many steps does the Garda take in order to catch the thief?

Problem No. 12

There is a string of 33 pearls worth €65 000. The biggest and most expensive pearl is in the central spot. The cheapest pearls are at the ends. From one end the value of the pearls increases by €100, uniformly, up to and including the largest pearl; from the other end, they increase in value by €150 each, up to and including the big pearl.

What is the value of the big pearl?

Problem No. 13

The diagram below shows the odometer reading above the trip meter reading on a car. The odometer cannot be changed while the trip meter may be reset at any time. When either meter shows all 9s, the next number shown is all 0s.

The diagram shows the first 4 digits of each meter are the same.

Odometer	12345.6
Trip meter	123.4

Assuming that the trip meter has not been reset, how far has the car travelled when the first 4 digits are again the same on both meters?

Problem No. 14

19, 4, 15, 11, are the first four terms of sequence of numbers where each term is the difference between the previous two terms.

(i) What is the 2021st term?

(ii) What is the sum of the first 2021 terms?

Problem No. 15

In a cipher code (using a Caesar shift-cipher) the numbers 0 to 25 are assigned to the letters from A to Z respectively. I received a coded message where 3 has been added to the number assigned to the letter in the message.

The message I received was

L W V V
L A W L
P H V V
H Y H Q

What number may be found from the message written in this code?

Problem No. 16

Imagine a wire lies tightly around the equatorial circumference of the moon. The length of the wire is increased by 1 metre and raised by an equal amount all around the equator.

Which of the following is the largest ball that would fit under the longer wire – a golf ball, a sliotar, a football or a basketball?

Problem No. 17

N is a positive whole number such that 3N has exactly 5 positive factors including itself and 1.

How many factors does 5N have?

Problem No. 18

A 4-number combination lock uses the digits 1-9 without repetition as follows

The second digit is prime.
The sum of the third and fourth exceeds 10
The second and fourth digit differ by one
The first is 3 greater than the third
Exactly one digit is square.

What is the combination that will open the lock?

Describe fully how you arrived at your answer.

Problem No. 19

The sum of all the internal angles of an n-sided polygon (with no reflex angles) measures 1260 degrees.

What is the value of n?

Problem No. 20

On a 9-hole golf course the hole lengths are 150m, 300m, 250m, 325m, 275m, 350m, 225m, 400m, 425m.

A golfer can learn to play only two strokes of different lengths which are played to exactly those lengths whenever played. If every stroke the golfer takes goes directly in line with the hole, what two lengths should the golfer learn in order to minimise the number of strokes needed to complete all 9 holes?

Problem No. 21
Number Search

1	2	1	1	8	0	5	4	7
2	9	9	7	9	2	1	0	7
4	1	0	2	5	8	5	3	4
7	6	7	0	3	2	4	2	3
1	6	8	0	6	2	4	0	1
9	6	5	9	4	3	5	7	6

ACROSS

a) 7^5-1

b) The year of the birth of William Rowan Hamilton

c) This year, Peter's Problem Competition begins on January 7th and all entries must be submitted by March 5th.
The number of days which may be taken to complete the challenge

d) The smaller non unit factor of 2021

e) The speed of light in km per second, to the nearest kilometre

f) $9(1 + 2\times7) - 6 \times 5 - 4^3 + 26$

DOWN

a) The larger non unit factor of 2021

b) The value of the Roman Numeral MDCLXVI

c) The number of lead in the Periodic Table of Elements

d) 8! (That is, 8 Factorial)

e) The flight distance in km between airports whose codes are DUB and SYD, correct to 3 significant figures.

f) 1024 to the power of $\frac{2}{5}$

Cross out the answer to each question as it is found on the grid. Each answer must eliminate at least one number not already eliminated.

What is the sum of the numbers not used in the Number Search?