Pre-Leaving Certificate

Mathematics – P1

Ordinary Level

Marking Scheme

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale Label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 3, 5	0, 3, 4, 5	0, 2, 3, 4, 5	
10 mark scales	0, 10	0, 5, 10	0, 5, 7, 10	0, 3, 7, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 7, 10, 15	0, 5, 9, 12, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- **incorrect response**
- **correct response**

B-scales (three categories)

- **response of no substantial merit**
- **partially correct response**
- **correct response**

C-scales (four categories)

- **response of no substantial merit**
- **response with some merit**
- **almost correct response**
- **correct response**

D-scales (five categories)

- **response of no substantial merit**
- **response with some merit**
- **response about half-right**
- **almost correct response**
- **correct response**

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in scale 10C, 9 marks may be awarded.

Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section

Question 1 **(25 marks)**

(a) Expand the brackets and simplify $(5x-4)(x+1)$

$$5x^2+5x-4x-4$$

$$5x^2+x-4$$

5C [0, 3, 4, 5]

Low Partial Credit: (3 marks) – Any term correctly multiplied
High Partial Credit: (4 marks) – correctly expanded but not simplified or error in simplification

(b) Express r in terms of the other variables in the equation $V=\pi r^2 h$

$$\frac{v}{\pi h}=r^2$$

$$\sqrt{\frac{v}{\pi h}}=r$$

10C [0, 5, 7, 10]

Low Partial Credit: (5 marks) – attempt to transpose any part of formula
High Partial Credit: (7 marks) – r^2 correctly isolated but fails to get last line correct

(c) Sarah tried to solve the equation as follows. She made a mistake on the second line.

$$\frac{w+4}{3}-\frac{2w-1}{5}=1$$

$$5w+20-6w-3=15$$

$$17-w=15$$

$$w=2$$

Circle her mistake and explain what she did wrong. Provide the correct answer to the question also.

She should have gotten $+3$ when multiplying -3 by -1.

The rest of the question should be……

$$5w + 20 - 6w + 3 = 15$$

$$-w + 23 = 15$$

$$w = 23 - 15$$

$$w = 8$$

10C [0, 5, 7, 10]

Low Partial Credit: (5 marks) – correct error identified OR any work of merit towards doing question properly
High Partial Credit: (7 marks) – minor error in correct solution or solution correct but error in question not identified

Question 2 **(25 marks)**

(a) A hostel website gives the cost of staying for three nights in a hotel in Copenhagen as 3112.5 Danish kroner.

(i) Find the cost of the trip, in euro, if €1 is 7.5 Danish Kroner.

$$3112.5 \div 7.5$$

$$€415$$

5B [0, 3, 5]

Partial Credit – (3 marks) – multiplies instead of divides

(ii) The cost of the trip includes a 3.5% service charge for the hostel website. Find, in euro, how much the hotel will get for the transaction, correct to the nearest cent.

$$415 = 103.5\%$$

$$415 \div 103.5\% = 400.96618$$

So, the hotel gets €400.97 while the website gets a service fee of €14.03

10C [0, 5, 7, 10]

Low Partial Credit: (5 marks) – identifies that $€415 = 103.5\%$ or finds 3.5% of 415
High Partial Credit: (7 marks) – Finds how much the website gets without finding the fee the hotel gets

(b) A group of four friends, Jason, Philip, Nathalie and Ciara decide to book this deal. On the first night however, Jason loses his wallet which had €238 in it. Philip, Nathalie and Ciara

decide to loan him the €238 in the ratio 2:5:7 respectively. How much does Jason receive from each person.

$$2 + 5 + 7 = 14$$

$$\frac{238}{14} = 17$$

$(17)(2) = 34$ from Philip

$(17)(5) = 85$ from Nathalie

$(17)(7) = 119$ from Ciara

10C [0, 5, 7, 10]

Low Partial Credit: (5 marks) – indication of addition of ratio or attempt to divide 238 by 14
High Partial Credit: (7 marks) – fails to find how much each of 3 people give Jason

Question 3 **(25 marks)**

(a) (i) Given $z = -3 + i$, write down $\bar{z}$, the complex conjugate of *z*.

$$\bar{z} = -3 - i$$

5B [0, 3, 5]

Partial Credit – (3 marks) – changes sign of real part even if imaginary sign has been changed

(ii) Evaluate $z + \bar{z}$.

$$z + \bar{z} = -3 + i - 3 - i$$

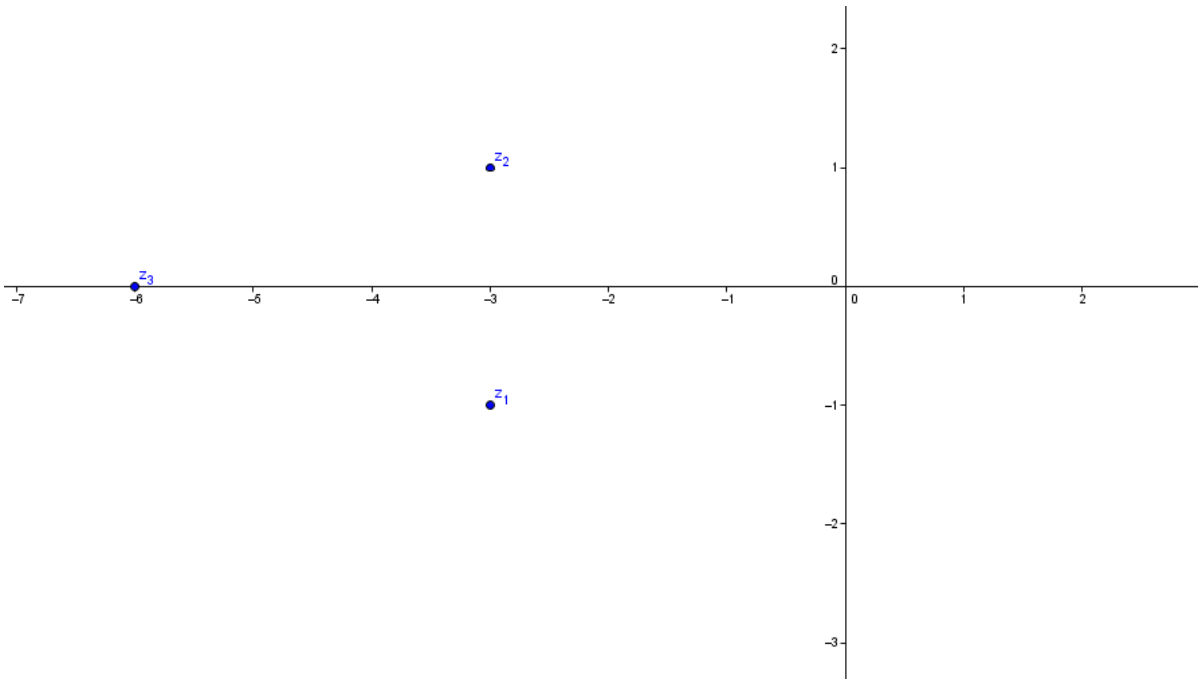
$$z + \bar{z} = -6$$

5B [0, 3, 5]

Partial Credit – (3 marks) – error in addition – use their answer to (i)

(iii) Show the three points z, $\bar{z}$ and $z + \bar{z}$ on an Argand Diagram.

$$\text{Let } z_1 = z, z_2 = \bar{z} \text{ and } z_3 = z + \bar{z}$$



5B [0, 3, 5]

Partial Credit – (3 marks) – Scales indicated or shows any knowledge of Argand diagram or at least one point correctly plotted

(b) Find $\frac{z}{1+3i}$, in the form $a + bi$.

$$\frac{-3+i}{1+3i}$$

$$\frac{-3+i}{1+3i} x \frac{1-3i}{1-3i}$$

$$\frac{-3+9i+i+3}{1+9}$$

$$\frac{10i}{10}$$

$$0 + i$$

10C [0, 5, 7, 10]

Low Partial Credit: (5 marks) – z correctly subbed in or shows any knowledge of multiplication by conjugate
High Partial Credit: (7 marks) – minor error in work or fails to simplify correctly

Question 4 **(25 marks)**

(a) Dots were used to create a pattern. The first three shapes in the pattern are shown.

Shape 1 **Shape 2** **Shape 3**

The number of dots used in each shape is recorded in the table.

Shape (S)	1	2	3
Number of Dots (N)	6	8	10

(i) Draw the next two shapes in the sequence

5B [0, 3, 5]

Partial Credit – (3 marks) – One shape correctly drawn

(ii) Síle thinks this is a Geometric pattern. Is she correct? Explain your answer fully.

No Síle is incorrect as this is an arithmetic (linear) pattern. Each term is 2 more than the previous term – it is increasing by the same amount each time. To be a geometric pattern, each term would need to be a multiple of the previous term.

5B [0, 3, 5]

Partial Credit – (3 marks) – any indication of knowing what a geometric OR arithmetic (linear) pattern is

(b) (i) Find an expression, T_n, for the number of dots in any shape of the sequence.

$$T_n = a + (n-1)d$$

$$a = 6 \;\; d = 2$$

$$T_n = 6 + (n-1)(2)$$

$$T_n = 6 + 2n - 2$$

$$T_n = 4 + 2n$$

10C [0, 5, 7, 10]

Low Partial Credit: (5 marks) – Correct formula with either *a* or *d* identified or knowledge of what *a* and/or *d* is
High Partial Credit: (7 marks) – Minor error in work leading to incorrect expression

NB Student may arrive at correct answer without using formula

(ii) Hence, or otherwise, find the number of dots required to make Shape 156.

$$T_{156} = 4 + 2(156)$$

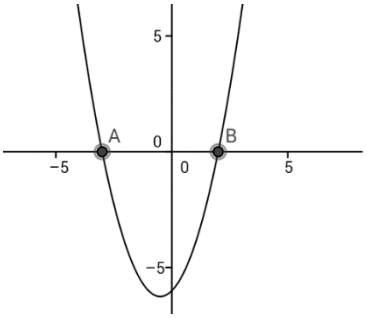
$$T_{156} = 4 + 312$$

$$T_{156} = 316$$

5B [0, 3, 5]

Partial Credit – (3 marks) – uses their answer to (i) with a substitution of 156 even if incorrect

Question 5
(25 marks)



(a) Part of the graph of the function $y = x^2 + x - 6$ is shown.

(i) Calculate the coordinates of points A and B.

$$x^2 + x - 6 = 0$$

$$(x + 3)(x - 2) = 0$$

$$x = -3 \text{ OR } x = 2$$

$$A(-3,0) \text{ and } B(2,0)$$

10C [0, 3, 7, 8, 10]

Low Partial Credit: (3 marks) – Multiple errors in solution or equates function to 0 or mention that A or B is a root
Mid Partial Credit: (7 marks) – 1 error in factorising or solving
High Partial Credit: (8 marks) – correctly solved but each answer not associated with correct point

(ii) Find the derivative of the function $y = x^2 + x - 6$

$$\frac{dy}{dx} = 2x + 1$$

5B [0, 3, 5]

Partial Credit – (3 marks) – any term correctly differentiated

(iii) Sketch the graph of the derivative of the function on the graph

5B [0, 3, 5]

Partial Credit – (3 marks) – any straight line drawn on original graph

(b) Explain why the derivative of the function $y = x^2 + x - 6$ has no turning points

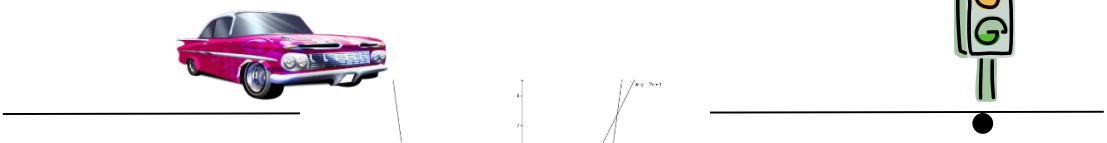
Because it is a straight line. The derivative of a quadratic function is a straight line and has no turning points.

5B [0, 3, 5]

Partial Credit – (3 marks) – reference to straight line but not entirely correct

Question 6 **(25 marks)**

A car begins to slow down at H in order to stop at a red light at R.



H

R

The distance of the car from H, after *t* seconds, is given by $s = 11t - 2t^2$, where *s* is in metres.

(i) Find the speed of the car as it passes H.

$$\frac{ds}{dt} = 11 - 4t$$

$$11 - 4(0) = 11m/s$$

5C [0, 3, 4, 5]

Low Partial Credit – (3 marks) expression correctly differentiated
High Partial Credit – (4 marks) $t = 0$ in derivative with a minor error

(ii) Find the time taken to stop

$$11 - 4t = 0$$

$$11 = 4t$$

$$t = {}^{11}/_{4} = 2.75\ seconds$$

5C [0, 3, 4, 5]

Low Partial Credit – (3 marks) – lets derivative = 0
High Partial Credit – (4 marks) - $11 = 4t$ but with further transposition error

(iii) The car stops exactly at *R*. Find the distance from *H* to *R*.

$$s = 11({}^{11}/_{4}) - 2\left(\frac{11}{4}\right)^2$$

$$s = \frac{121}{4} - 2\left(\frac{121}{16}\right)$$

$$s = \frac{242 - 121}{8}$$

$$s = \frac{121}{8} = 15.125\ metres$$

10C [0, 3, 7, 8, 10]

Low Partial Credit: (3 marks) – use of $t = 2.75$ (or equivalent) in answer
Mid Partial Credit: (7 marks) – minor error**s** in calculations
High Partial Credit: (8 marks) – minor error in calculation

(iv) Find the acceleration of the car

$$\frac{ds}{dt} = 11 - 4t$$

$$\frac{d^2s}{dt^2} = -4\ m/s^2$$

5C [0, 3, 4, 5]

Low Partial Credit – (3 marks) – mentions 2nd derivative
High Partial Credit – (4 marks) – minor error in derivative (no minus sign) – incorrect units

Section B	**Contexts and Applications**	**150 marks**

Answer **all three** questions from this section

Question 7 **(50 marks)**

(a) Wind turbines, such as those shown, are used to generate power.

In theory, the power that could be generated by a wind turbine is modelled using the equation

$$T = 20{,}000w^3$$

where *T* is the theoretical power generated, in watts and *w* is the speed of the wind, in metres per second.

(i) Using this equation, what is the theoretical power generated by a wind turbine if the wind speed is 7.3 m/s?

$$T = 20{,}000(7.3)^3$$

$$T = 7{,}780{,}340 \text{ watts}$$

5B [0, 3, 5]

Low Partial Credit – (3 marks) – error in calculation but 7.3 subbed in for *w*

(ii) In practice however, the actual power generated by a wind turbine is only 40% of the theoretical power. If A is the actual power generated, in watts, write an equation for A in terms of w.

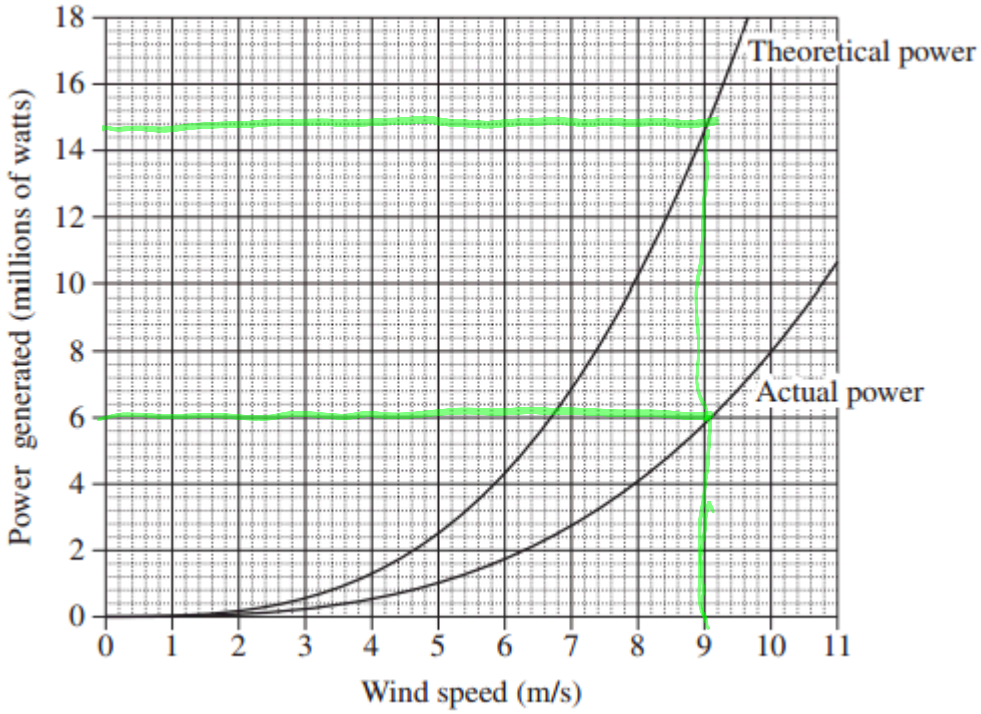
$$A = 40\%(20{,}000w^3)$$

$$A = 8{,}000(w^3)$$

5C [0, 3, 4, 5]

Low Partial Credit – (3 marks) – theoretical power formula used
High Partial Credit – (4 marks) – minor error in calculation

(b) The graph shown on the next page shows both the theoretical power generated and the actual power generated by a particular wind turbine.



(i) Using the graph, or otherwise, find the difference between the theoretical power and the actual power generated when the wind speed is 9 m/s.

<u>Using Graphs</u> Theoretical power when wind speed is 9 m/s = 14.8 million watts

Actual power when wind speed is 9 m/s = 6 million watts

Using Algebra

$$T = 20{,}000(9^3)$$

$$T = 20{,}000(729)$$

$$T = 14{,}580{,}000 = 14.58\ million\ watts$$

$$A = 8000(9^3)$$

$$A = 8000(729)$$

$$A = 5{,}832{,}000 = 5.832\ million\ watts$$

Differnce using Graphs = $14.8 - 6 = 8.8\ million\ watts$

Difference using Algebra = $14.58 - 5.832 = 8.748\ million\ watts$

10C [0, 3, 7, 8, 10]

Low Partial Credit: (3 marks) – attempt to find relevant points on graph – any indication – or any attempt to sub relevant points into a relevant formula
Mid Partial Credit: (7 marks) – both correct but outside tolerance of ± 0.2 million watts if using graph
High Partial Credit: (8 marks) – one completely correct/within tolerance or difference not found (results not subtracted)

(ii) A particular wind farm requires **at least** 4.4 million watts of actual power in order to be self-sufficient. What is the minimum wind speed required for the farm to be self-sufficient?

$$A = 8000(w^3)$$

$$4.4\ million = 4{,}400{,}000$$

$$4{,}400{,}000 = 8000(w^3)$$

$$\frac{4400000}{8000} = w^3$$

$$550 = w^3$$

$$\sqrt[3]{550} = w$$

$$w = 8.19322$$

So, anything over 8.19322….. m/s will allow the farm to be self-sufficient.

10C [0, 3, 7, 8, 10]

Low Partial Credit: (3 marks) – any work of merit using correct relevant formula with some substitution
Mid Partial Credit: (7 marks) – arrives at $550 = w^3$ but trouble with cube
High Partial Credit: (8 marks) – error with handling of units i.e 4.4 not treated as 4.4 million leading to incorrect answer

(c) A more accurate formula to calculate the power (P) generated by a wind turbine is

$$P = 0.61 \times \pi \times r^2 \times w^3$$

where *r* is the length of each blade, in metres *w* is the speed of the wind, in metres per second.

(i) Find the power generated by a wind turbine with a blade length of 43 metres, operating at a wind speed of 8 m/s, correct to two significant figures.

$$P = (0.61)(\pi)(43)^2(8)^3$$

$$P = 1{,}814{,}205.9202$$

$$p = 1{,}800{,}000 = 1.8\,million$$

10C* [0, 3, 7, 8, 10]

Low Partial Credit: (3 marks) – any work of merit using correct relevant formula with some substitution
Mid Partial Credit: (7 marks) – 1 incorrect value subbed into formula
High Partial Credit: (8 marks) – all values correctly subbed in leading to incorrect answer

(ii) Using the formula above, what would be the percentage increase in the power generated, to two decimal places, by the same wind turbine of blade length 43 metres, if the wind speed increased by 10%? You may write the figure for the new power generated correct to the nearest whole number.

$$(8)(1.1) = 8.8$$

$$P = (0.61)(\pi)(43)^2(8.8)^3$$

$$P = 2{,}414{,}708.079$$

Increase in Power = $2{,}414{,}708.079 - 1{,}800{,}000 = 614{,}708$

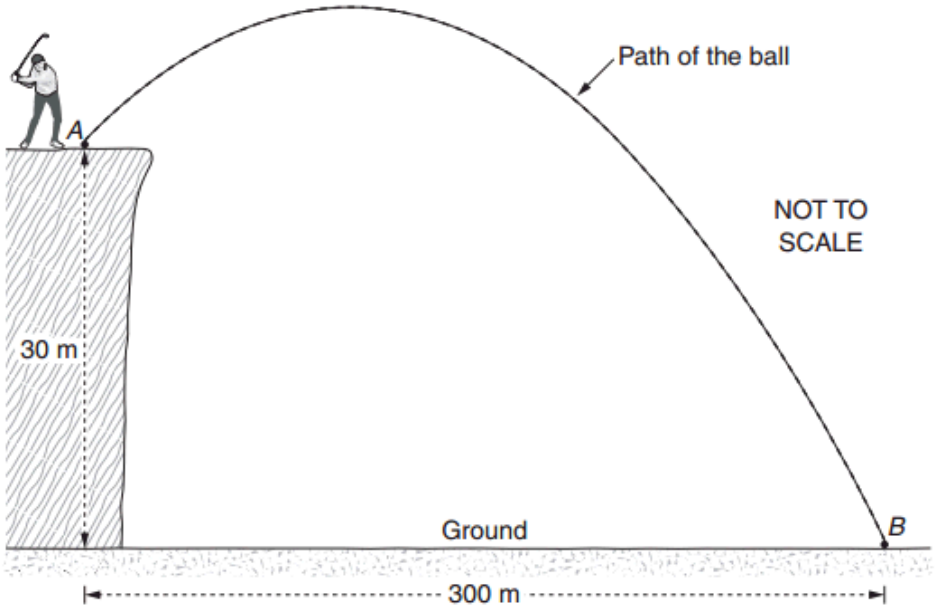
Percentage increase = $\frac{614708}{1800000} \times 100 = 34.15\%$

10C [0, 3, 7, 8, 10]

Low Partial Credit: (3 marks) – correct new wind speed found or any work of merit using correct relevant formula with some substitution
Mid Partial Credit: (7 marks) – all nos. correctly subbed in and new power figure found correctly but fails to finish
High Partial Credit: (8 marks) – attempt at finding percentage increase but mishandles figures

Question 8 **(60 marks)**

A golf ball is hit from point A to point B, which is on the ground as shown. Point A is 30 m above the ground and the horizontal distance from point A to point B is 300 m.



(a) The path of the golf ball is modelled using the equation

$$h = 30 + 0.2d - 0.001d^2$$

where h is the height of the golf ball above the ground in metres, and d is the horizontal distance of the golf ball from point A in metres.

(i) By using your calculator or otherwise, complete the table:

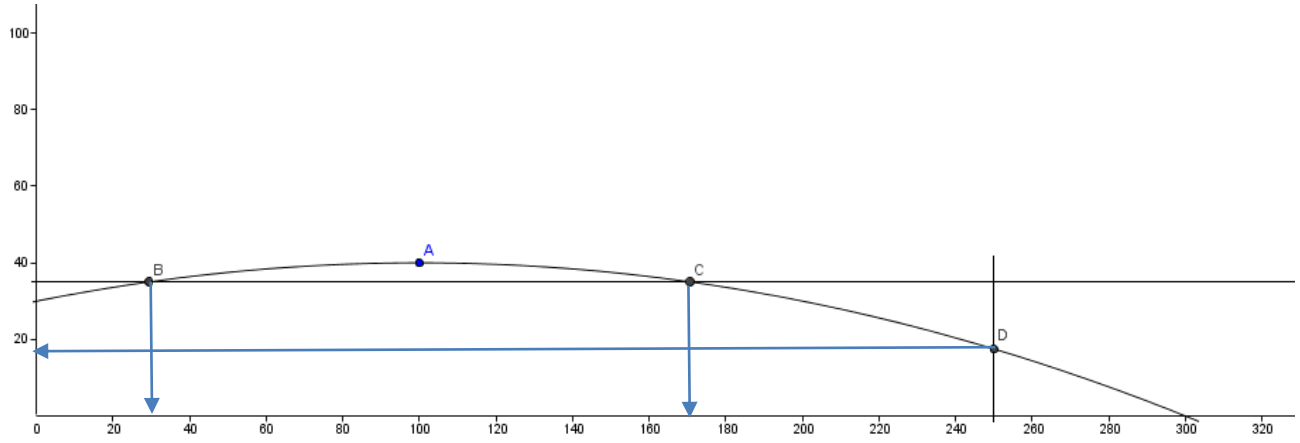
x	0	50	100	150	200	250	300
$h(x)$	30	37.5	40	37.5	30	17.5	0

10B [0, 5, 10]

Low Partial Credit: (5 marks) – at least 2 correct values in table

(ii) Using the values from your table, or otherwise, draw the graph of the function

$h = 30 + 0.2d - 0.001d^2$ in the grid below



10C [0, 3, 7, 8, 10]

Low Partial Credit: (3 marks) – scaled axes only
Mid Partial Credit: (7 marks) – at least 3 points correctly plotted (consistent with candidate's table)
High Partial Credit: (8 marks) – all points correctly plotted (consistent with candidate's table) but not connected accurately (to the eye)

(b) (i) From your graph, find the maximum height the ball reaches above the ground?

Maximum height = 40 metres

5B [0, 3, 5]

Partial Credit – (3 marks) – answer outside tolerance ($\pm 2\ m$) or some indication on graph of understanding maximum height or answer using formula. Answer read directly from table (i.e with no work in this question) merits 0 marks.

(ii) There are two occasions when the golf ball is at a height of 35 metres. What horizontal distance does the ball travel in the period between these two occasions? Show your work on your graph

170 – 30 = 140 metres

5B [0, 3, 5]

Partial Credit – (3 marks) – at least one correct value found or some indication on graph of understanding the process involved or values not subtracted

(iii) What is the height of the ball above the ground when it still has to travel a horizontal distance of 50 metres to hit the ground at point B? Show work on your graph.

Approximately between 17 and 18 metres (Actual answer is 17.5 using graphing package or algebra)

- Accept any answer between 17-18

5B [0, 3, 5]

Partial Credit – (3 marks) – some indication on graph of understanding the process involved – using algebra warrants full marks

(c) (i) You calculated the maximum height of the graph in part (b) (i) already. Find the value of *d* which gives the maximum height.

$$\frac{dh}{dd} = 0.2 - 0.002d$$

$$0.2 - 0.002d = 0$$

$$d = 100$$

10C [0, 3, 7, 8, 10]

Low Partial Credit: (3 marks) – any term differentiated correctly or any mention of derivative
Mid Partial Credit: (7 marks) – expression correctly differentiated
High Partial Credit: (8 marks) – minor error in equating derivative to 0

(ii) Hence find the maximum height.

$$h = 30 + (0.2)(100) - (0.001)(100)^2$$

$$h = 30 + 20 - 10$$

$$h = 40\,metres$$

5B [0, 3, 5]

Partial Credit – (3 marks) – their answer to (i) subbed in to original function but not arriving at final answer

(iii) Explain fully what this maximum **point** means in the context of the question, i.e. explain the meaning of the *x* and *y* values found above in parts (c) (i) and (ii).

When the golf ball has travelled a horizontal distance of 100*m* it has reached its maximum height of 40*m* above ground.

5B [0, 3, 5]

Partial Credit – (3 marks) – one value correctly contextualised only

(iv) The graph of $h(d)$ is decreasing for all values of *x* shown. State whether you believe this to be true or false, explaining in detail why. You should use differentiation in your answer.

This is not true as can be seen from the graph alone. Between the *x*-values of 0-100 the graph is clearly increasing. This can be seen using the second derivate test.

$$\frac{dh}{dd} = 0.2 - 0.002d$$

$$\frac{d^2h}{dd^2} = -0.002 < 0$$

$\frac{d^2h}{dd^2} < 0$ means the curve is increasing in parts at least, so the graph of *h(d)* is definitely not decreasing for all values of *x* shown.

5C [0, 3, 4, 5]

Low Partial Credit – (3 marks) – False or 'not true' stated with no other work or reference to 2^{nd} derivative
High Partial Credit – (4 marks) – 2^{nd} derivative found but incorrect interpretation

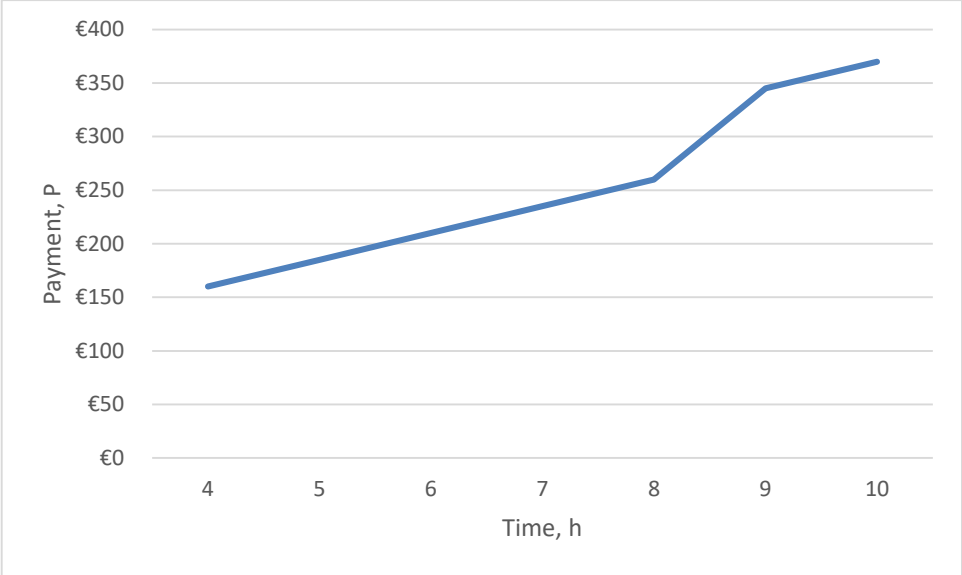
Question 9 **(40 marks)**

Emma is employing Ian to build a deck at her house. She provides all the building material. She pays Ian €*P* for the number of hours, h, that he works. She also pays for Ian's travel to her home each day. Ian works for 8 hours each day. He knows the deck will take more than 4 hours to build. To help Emma know how much she can expect to pay, Ian provides the following table:

Number of hours worked (h)	***Payment (P)***
4	€160
5	€185
6	€210

7	€235
8	€260
9	€345
10	€370

(a) On the grid below, plot a graph showing the payment required for the number of hours worked, plotting time, in hours, along the *x* axis.



10C [0, 3, 7, 8, 10]

Low Partial Credit: (3 marks) – scaled axes only (both x *and* y)
Mid Partial Credit: (7 marks) – all points plotted correctly but points not connected or minor errors in plotting at most 2 points
High Partial Credit: (8 marks) – axes reversed or minor error in plotting 1 point/joining points

(b) (i) How much does Ian charge for his travel each day?

$$345 - 260 = €85 - €25 = €60$$

5C [0, 3, 4, 5]

Low Partial Credit – (3 marks) – any work of merit – step in right direction
High Partial Credit – (4 marks) – forgets to subtract €25

(ii) Explain why the graph rises more steeply after 8 hours.

Because the rate of change increases from 25 to 60 as we are moving from the end of day 1 to the start of day 2 after 8 hours.

5B [0, 3, 5]

Low Partial Credit – (3 marks) – any work of merit in the right direction

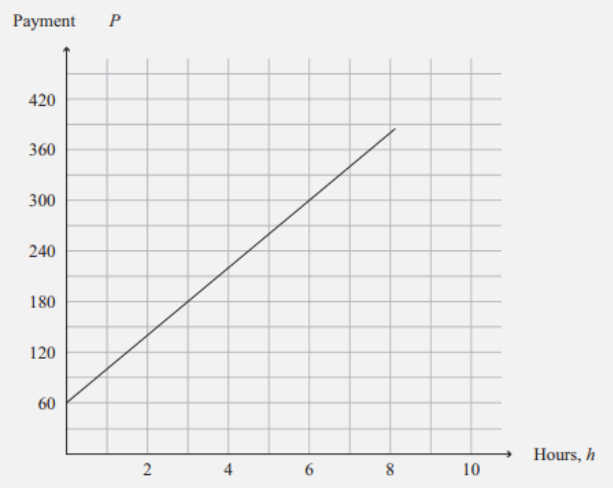
(iii) What would Emma expect to pay if the work took 30 hours? Explain how you arrived at your answer.

Ian will work for 8 hours per day Day 1 = €260 – 8 hours work Day 2 = €260 – 16 hours work Day 3 = €260 – 24 hours work (6 hours work left) Day 4 = $€60 + 6(25) = €210$ Total = €990 for 30 hours of work

10C [0, 3, 7, 8, 10]

Low Partial Credit: (3 marks) – any work of merit in the right direction
Mid Partial Credit: (7 marks) – major error or two minor errors
High Partial Credit: (8 marks) – minor error

(c) (i) Another builder gives Emma a graph, showing the amount she would charge for 8 hours work.



Give the rule for the payment that this builder would receive for the first 8 hours that she worked.

$60 + 40t$, where t represents the number of hours worked.

5B [0, 3, 5]

Low Partial Credit – (3 marks) – any work in the right direction

(ii) Which builder is better value for the work Emma needs done, assuming the work will take 8 hours to complete?

Builder 1 = €260 Builder 2 = $60 + 40(8) = €380$ Builder 1 is better value

5C [0, 3, 5]

Low Partial Credit – (3 marks) – any work in the right direction