

# Final Exam in 5<sup>th</sup> Year HL Maths

## Instructions

This final exam contains multiple choice questions on the topics studied during 5<sup>th</sup> Year in HL Maths and is worth **80%** of your end-of-year mark.

It can be attempted **ONCE** only.

There are **TWENTY QUESTIONS** in total. Questions are worth **ONE or TWO MARKS**.

There is an overall time limit of **2 HOURS** for the entire final exam.

**THE ORDER IN WHICH ANSWER OPTIONS APPEAR ON THIS QUESTION SHEET MAY NOT BE THE SAME AS THE ORDER IN WHICH THEY APPEAR ON THE ANSWER SHEET (IN MS FORMS).**

Don't forget to click **SUBMIT** at the end.

I hope each of you has a nice summer break. Take care of yourselves and families. And for the exam, good luck!

## Question 1 (TWO marks)

One solution of the equation  $4x^3 - 8x^2 + kx + 2 = 0$  is  $\frac{1}{2}$ .

What is the value of  $k$ ?

Select the correct option.

$$k = -1$$

$$k = 0$$

$$k = \frac{1}{2}$$

$$k = 1$$

## Question 2 (TWO marks)

Find the range of real values of  $x$  for which  $\frac{3x-2}{x-5} \leq 5$ .

Select the correct option.

Option A:  $x > 5, x \leq \frac{23}{2}$

Option B:  $x \geq 5, x \leq \frac{23}{2}$

Option C:  $x < 5, x \geq \frac{23}{2}$

Option D:  $x \leq 5, x \geq \frac{23}{2}$

**Question 3 (ONE mark)**

Solve the equation  $\sqrt{2x - 1} + \sqrt{x - 1} = 5$ .

Select the correct option.

$x = 5$  or  $x = 145$

$x = 5$

$x = 145$

none of the other three options is correct

**Question 4 (ONE mark)**

Find the value of the term which is independent of  $x$  in the expansion of  $\left(x - \frac{2}{x}\right)^6$ .

Select the correct option.

120

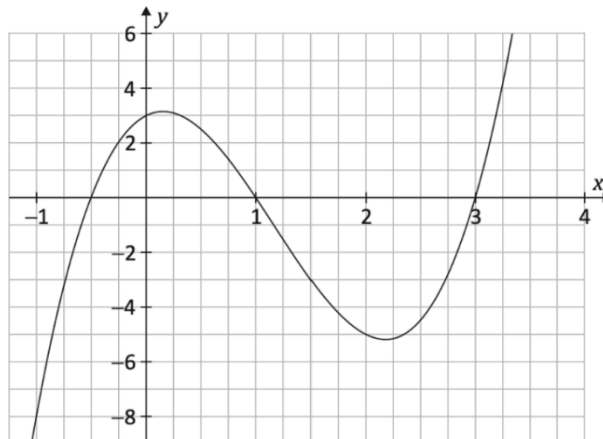
-120

160

-160

**Question 5 (TWO marks)**

The diagram shown shows part of the graph of a cubic function  $f$  whose output is given as  $f(x) = ax^3 + bx^2 + cx + d$ . By considering the graph shown, identify which of the expressions below is equal to  $f(x)$ .



Select the correct option.

$2x^3 - 7x^2 + 2x + 3$

$x^3 - 3.5x^2 + x + 3$

$2x^3 - 3.5x^2 + x + 1.5$

$x^3 - 3.5x^2 + x + 1.5$

**Question 6 (ONE mark)**

The expression  $(2\sqrt{3} - 3\sqrt{x})^2$  is expanded and simplified.

The result is rational.

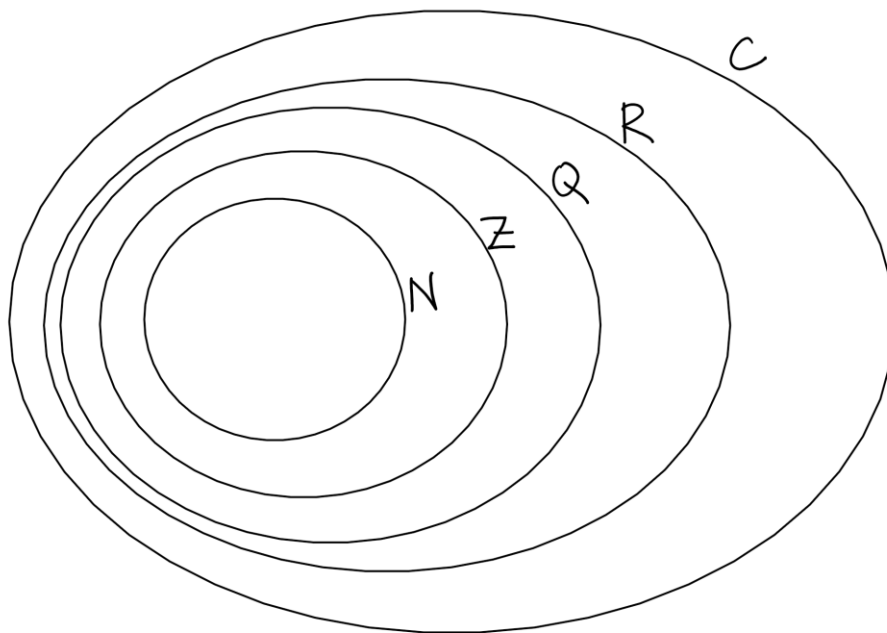
Which of the values of  $x$  below would produce a result that is rational?

Select the correct option.

- 4
- 9
- 27
- 81

**Question 7 (ONE mark)**

Consider the Venn diagram below showing the complex set, the real set, the rational set, the integer set and the natural set.



The number  $a + bi$  (where  $a, b \in R$ ) is an element of the region  $R \setminus Q$ .

Which pair of values for  $a, b$  below would result in  $a + bi$  being in  $R \setminus Q$ ?

Select the correct option.

- $a = 3.5, b = 0$
- $a = \pi, b = -2$
- $a = 1.666666 \dots, b = \sqrt{2}$
- $a = \sqrt{3}, b = 0$

**Question 8 (ONE mark)**

You wish to prove the truth of a statement  $P(n)$ , using induction, for all natural numbers  $n$  greater than or equal to 10.

The first case that you need to prove the truth of is ...

Select the correct option.

$P(1)$

$P(2)$

$P(3)$

none of the other three options is correct

**Question 9 (ONE mark)**

An incomplete conclusion to a proof by induction is given as follows:

$P(4)$  true.

If  $P(k)$  true,  $k$  is natural and greater than or equal to 4, then  $P(k + 1)$  true.

Therefore, by induction,  $P(n)$  true \_\_\_\_\_.

Select the correct option to correctly complete the above conclusion.

for all natural numbers  $n$

for all natural numbers  $n$  which are greater than or equal to 4

for all natural numbers  $n$  which are greater than 4

none of the other three options is correct

**Question 10 (ONE mark)**

The cubic function  $f$  is graphed as shown.

This function is ...

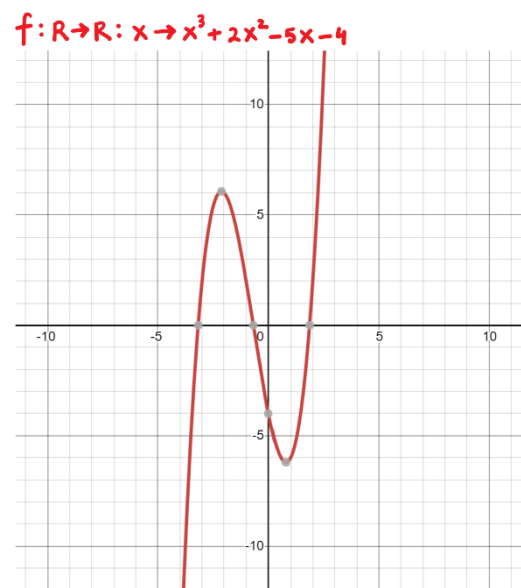
Select the correct option.

injective only

surjective only

bijjective

none of the other three options is correct



**Question 11 (TWO marks)**

Solve the equation

$$\log_e(2x+3) + \log_e(x-2) = 2 \log_e(x+4).$$

The solution is ...

Select the correct option.

$$x = 2$$

$$x = \frac{1 + \sqrt{21}}{2}$$

$$x = 7$$

$$x = 11$$

**Question 12 (ONE mark)**

The complex number  $z = 3 \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$ .

What is the value of  $z^6$ ?

Select the correct option.

$$0 - 18i$$

$$0 - 729i$$

$$\text{approximately } 17.94 + 1.48i$$

$$\text{approximately } 726.54 + 59.89i$$

**Question 13 (ONE mark)**

Consider the quadratic equation in  $x$  below, where  $a \in R$ .

$$ax^2 + x + a = 0$$

Select the statement below which correctly describes the solution set for this equation.

The two solutions are definitely real and are conjugates of each other.

The two solutions are definitely real and are not conjugates of each other.

The two solutions are definitely not real and are conjugates of each other.

The two solutions are definitely not real and are not conjugates of each other.

The two solutions might be real and are conjugates of each other.

The two solutions might be real and are not conjugates of each other.

**Question 14 (ONE mark)**

What is the argument of the number  $-5 - 5\sqrt{3}i$ ?

Select the correct option.

- 150 degrees
- 120 degrees
- 60 degrees
- 30 degrees
- 60 degrees
- 120 degrees

**Questions 15 (ONE mark)**

If  $\tan A = \frac{t}{2}$ , for  $0^\circ \leq A \leq 90^\circ$ , then the correct expression for  $\cos A$  in terms of  $t$  is ...

Select the correct option.

$$\frac{2}{\sqrt{t^2+4}}$$

$$\frac{t}{\sqrt{t^2+4}}$$

$$t^2 + 4$$

$$t^2 - 4$$

**Question 16 (TWO marks)**

Which of the following sets of values is the solution set to the equation below?

$$2\cos 3A = -1, 0^\circ \leq A \leq \pi.$$

Select the correct option.

$$\frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$$

$$\frac{\pi}{3}, \frac{2\pi}{3}$$

$$\frac{2\pi}{9}, \frac{5\pi}{9}, \frac{8\pi}{9}$$

$$\frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$$

**Question 17 (ONE mark)**

In a triangle  $ABC$ ,  $|AB| = 6\text{cm}$  and  $|BC| = 8\text{cm}$ .

The area of the triangle  $ABC$  is  $12\text{ cm}^2$ .

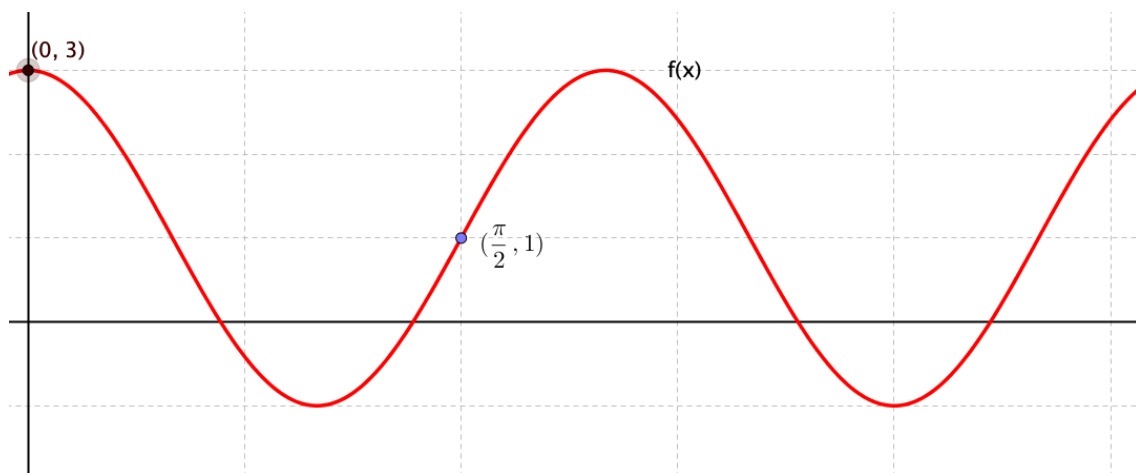
Which of the following are the two possible values of  $|\angle ABC|$ ?

Select the correct option.

- $30^\circ, 90^\circ$
- $30^\circ, 150^\circ$
- $60^\circ, 120^\circ$
- $45^\circ, 135^\circ$

**Question 18 (ONE mark)**

The diagram below shows two points on the graph of the function  $y = f(x)$ .



The equation of this function is ...

Select the correct option.

- $f(x) = 3\cos 2x + 1$
- $f(x) = 2\cos 3x + 2$
- $f(x) = 2\cos 3x + 1$
- $f(x) = 3\cos 2x - 1$

**Question 19 (ONE mark)**

In a triangle  $ABC$ ,  $|AB| = 4$  cm,  $|AC| = 3$  cm and  $|\angle ABC| = 44^\circ$ .

Two possible values of  $|\angle ACB|$  are ...

Select the correct option.

68.1°, 111.9°

22.4°, 157.6°

44°, 136°

67.9°, 112.1°

**Question 20 (ONE mark)**

In a triangle  $PQR$ ,  $|PR| = 7$  cm,  $|PQ| = 6.8$  cm and  $|RQ| = 9$  cm.

The measure of the smallest angle in this triangle is ...

Select the correct option.

81°

50°

48°

36°

**END OF EXAM**

**THERE ARE NO FURTHER QUESTIONS**