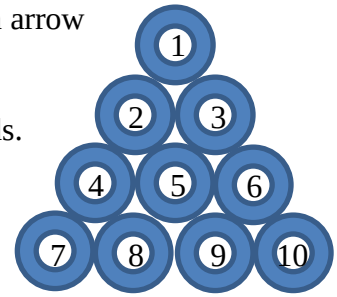
Peter's Problem 2019

Problem No. 1

Ten identical coins are arranged in a triangle as shown in the diagram as an arrow pointing upwards.



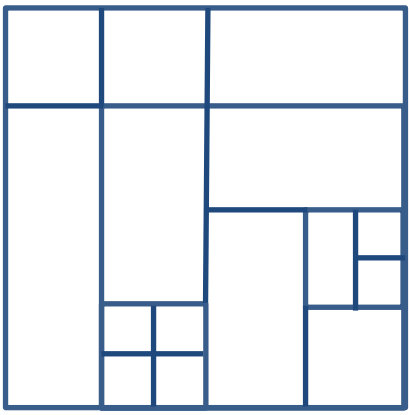
(i) Move 3 of the coins to new positions so that the arrow points downwards.
Draw a diagram (with numbers) of the coins in their new positions.
Highlight the numbers for the 3 coins which you have moved.

A further row of 5 coins is added to the given diagram to form a triangle of 15 coins in 5 rows.

(ii) What is the minimum number of coins that must now be moved into new positions in order to reverse the direction of the arrow?

Problem No. 2

How many rectangles can be found in the given diagram?



Problem No. 3

Beginning with 4, insert the mathematical signs +, –, × and ÷ along with brackets as appropriate between each of the digits so that this expression is correct:

4 6 5 3 8 7 9 2 1 = 58

Note: Brackets may be placed before the '4' or after the '1' if required.

Problem No. 4

Replace the asterisks with the digits, 1, 2, 3, 4, 5, 6, 7, 8 and 9, one for each, so that the mathematical statement of subtractions is correct.

$$*** - *** - *** = 100$$

Problem No. 5
Each of the symbols represents a number. The addition totals for each row and column are given, except one.
Find the number represented by the * symbol.

20	27	20	17	
Θ	Θ	Θ	¥	29
¥	η	¥	%	*
%	η	%	%	13
¥	¥	¥	Θ	23

Problem No. 6
Log onto https://nrich.maths.org/5612 (Method in Multiplying Madness?)

(i) Multiply 253 by 41 using the "Multiplying with Lines" method, showing your work.
(ii) Write down two advantages and two disadvantages of using this method of multiplication.

Problem No. 7
Write down any 3 digits and write down the same 3 digits again in the same order to make a 6-digit number. e.g. 123123 or 389389 or 306306, etc.

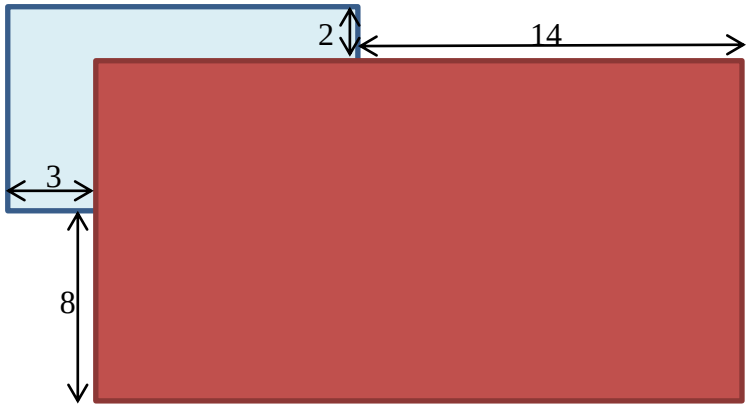
Explain why 7, 11 and 13 all divide evenly into all numbers of this form.

Problem No. 8
In a triangle, one angle is twice the size of another and one angle is three times the size of another.

List the measure in degrees of all of the smallest angles which are possible.

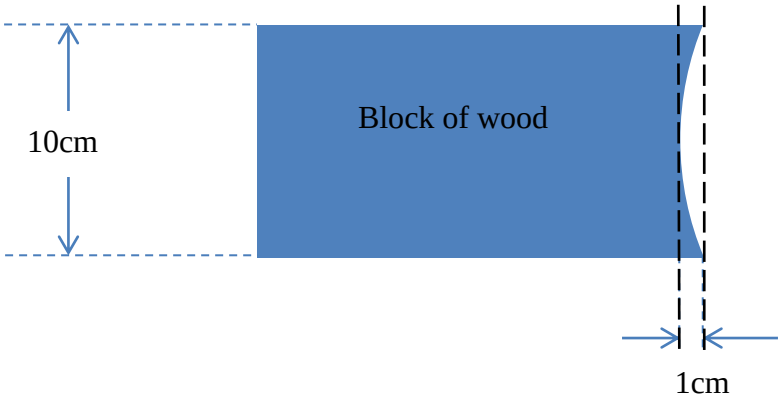
Problem No. 9

One rectangular sheet of paper is placed on top of another as shown.
Given that the dimensions of the larger sheet are double those of the smaller sheet, find the total area, in cm^2, covered by the sheets in the position shown. [Measurements are given in cm].



Problem No. 10

A circular cutting blade has been used to form the curve on a rectangular piece of wood as shown on the plan view given.
Using the dimensions given, find the radius of the cutting blade (in cm).



Problem No. 11
Log onto www.steps.ie

(i) From the 12-15 section, list three types of engineers engaged in the production of chocolate and three functions of engineers in the presentation of the finished product.

(ii) From the 16-18 section, list the 6 problems that engineers of the future will have to solve.

Problem No. 12
A chef made 4 mixes of orange squash and water in the following ratios
Orange squash : Water = 1:2, 1:3, 1:4 and 1:5.
The 4 mixes were placed in jars marked A, B, C and D respectively.

The chef then removed *k* ml from each of the jars marked A and D and poured each into a jar labelled X. She also removed *k* ml from each of the jars marked B and C and poured each into a jar labelled Y.

(i) Which of the jars, X or Y, contains the higher concentration of orange squash?
(ii) Why?

Problem No. 13
A prisoner was helping a contractor to repair the flat roof on his cell when he noticed that the edges of the roof were flush with the outer surface of the walls of the cell.

(i) If the cell measured 4m by 5·5m internally and the roof had an area of $38{\cdot}5m^2$, calculate the width of the wall, in cm, if all of the walls were the same thickness.

(ii) If the prisoner wishes to escape by tunnelling through the wall at an average rate of 1·3cm per day, on what date should he begin the tunnel if he is to escape on March 2th, in time for the beginning of Engineers Week, 2019?

Problem No. 14
Choose any two natural numbers each less than 10.
Select one of the numbers and add 1, multiply by 5, add 1 again, double your answer and subtract 1.
Add the second number to your answer, add 2, double again, subtract 8 and halve this number and write down your answer.

Write down the rule that identifies from the answer the two numbers selected at the start.

Problem No. 15
How many Natural numbers less than 1000 have the same remainder when divided by 3, 4 and 5?

Problem No. 16

A student began to count on the fingers of his left hand. He started by calling the thumb 1, the first finger 2, middle finger 3, ring finger 4, little finger 5, then reversed direction calling the ring finger 6, middle finger 7, first finger 8, thumb 9, first finger 10 and so on.

(i) On which finger did he reach 50?
(ii) On which finger would he stop if he were to count to 2019?

Problem No. 17

A railway depot is fitted with a track for trainee engine drivers. It contains two freight carriers, A and B, and an engine, E. The track is circular in shape with two sidings as shown in the diagram. It has a tunnel, T, which is large enough for the engine to pass through but not the freight carriages. At the start of the training exercise the engine is placed at the entrance to a siding (X) with the carriages A and B next to Tunnel North in the order shown in the first diagram. The trainee's task is to use the engine to bring the carriages and the engine to new positions as shown in the second diagram, with the carriages next to Tunnel South in the order shown and the engine at the entrance to Siding South, Y.
In order to push or pull a carrier the driver has to hitch the engine onto it. When finished moving that carrier the driver has to unhitch the engine from it.

(i) How many '**hitch-unhitch the engine'** operations are required, at minimum. ie, how many times the engine has to be hitched to a carriage, some movement or movements carried out and then the engine unhitched from that carriage.

(ii) Describe, with the help of diagrams similar to the ones given, the moves required for the driver to move from the position shown in the first diagram to that in the second diagram giving an account of the 'hitch-unhitch the engine' operations.

[Notes:
1. From inside the cab the driver may operate the track siding-switch to allow a change to the rail from the circular track to the straight track.

2. There is no left-hand turn into the top siding travelling clockwise nor is there a right-hand turn into the lower siding travelling anti-clockwise.

3. For descriptive purposes the labels North Siding (SN), South Siding (SS), Track switch point entrance to Siding North (X) and Track switch point entrance to Siding South (Y), North side of Tunnel (TN) and South side of Tunnel (TS) may be used as in the third diagram.]

Diagram No. 1

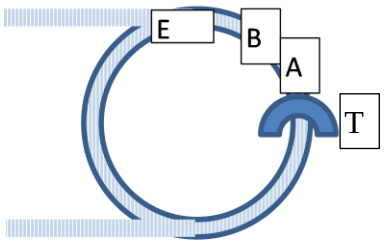


Diagram No. 2

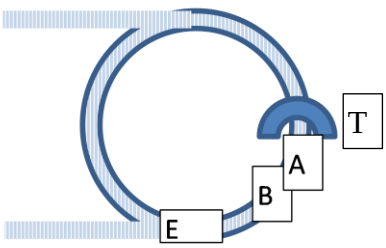
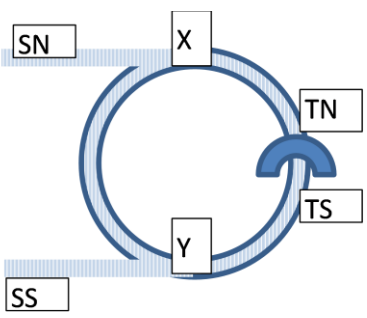


Diagram No. 3



Problem No. 18
Table-tennis balls are packaged in boxes of 8 and in boxes of 15.
Viz., an order for 38 balls may be filled without breaking open any box by sending 2 boxes of 15 and 1 box of 8 while an order for 37 balls cannot be filled without breaking open a box.

What is the maximum number of balls that cannot be bought without breaking open a box?

Problem No. 19
Three people – yourself and two friends – are faced with the problem of crossing a rope bridge which is only strong enough to hold any two at a time. Furthermore, a single special token used to open and close the gates at either end as people pass through must be brought by whichever person or group crosses the bridge. It must then be brought back before the next person or group can cross. Individually, the crossing times are 1 minute, 2 minutes and 10 minutes for yourself and your two friends, respectively.
Before you can cross the bridge a third friend arrives who also wishes to cross the bridge with you. You ask this third friend how long it takes them to cross the bridge if they were to cross on their own. When you are told you are happy that when the four of you cross as a group it will only add a further 4 minutes to the minimum crossing time that you had calculated for yourself and your two friends at the start.

What are the smallest and the greatest lengths of time (in whole numbers of minutes) that your third friend could have told you that he or she would take to individually cross the bridge?

Problem No. 20
Prove that every odd integer lies exactly halfway between a multiple of 3 and a multiple of 4.

Problem No. 21
Log on to www.math.harvard.edu/~knill/mathmovies/swf/maandpakettleaddition.html and view the Ma and Pa Kettle film (Length 2:15).

(i) What answer does Pa get when he divides 25 by 5?
(ii) What mathematical error does Pa make in his calculation?