Peter's Problem 2020 -Solutions

Problem No. 1

A car travels 24 860 km. The driver rotated the 5 tyres (spare wheel included) so that each tyre got equal wear. How many kilometres did each tyre travel?

No. 1 Solution

The 4 wheels travel a total of 4x24860km = 99440km.
This is covered by the 5 tyres so each tyre travels $\frac{99440}{5} = 19888$ km.

Problem No. 2

Using a certain code two of the words GLARE, LARGE and REGAL are written as 52314 and 41532.
(i) What is the third word which is not written in the code?
(ii) What word would be represented by the code 21325?

No. 2 Solution

G L A R E
L A R G E
R E G A L
5 2 3 1 4
4 1 5 3 2
In the final column 4 and 2 correspond to E, E and L and so they must be E and L. That means REGAL is one of the words used.
Therefore, matching REGAL, R = 5 or 4; E = 2 or 1; G = 3 or 5; A = 1 or 3; L = 4 or 2.
Since E is not in the first column and cannot be 4 it must be 2.
Therefore L = 4, R = 5, G = 3, A = 1.

(i) The third word is GLARE = 34152 which is not written in the code.
(ii) The code 21325 represents EAGER.

Problem No. 3

Log on to the website www.engineersweek.ie.
Click on the tab **'Get Involved'**, select **'Secondary Schools'** and view the video *'Women in Engineering'*.
How many engineers were named in the video?

No. 3 Solution

There are 5 engineers named in the video.

Problem No. 4

A 24-hour digital clock shows 6 numbers for hours, minutes and seconds.
How many times in any 24 hours do all 6 digits change at the same time?

No. 4 Solution

The hour digits change when the minute and second digits move from 59:59 to 00:00. This happens every hour but only one hour digit changes until 09 changes to 10; again when 19 changes to 20; and again when 23 changes to 00.
This means that there are 3 times when all 6 digits change at the same time.

Problem No. 5

An electricity meter reads 58273. All digits are different. How many more units of electricity will have been used when the other 5 digits are first displayed together?

No. 5 Solution

The other 5 digits are 0 1 4 6 9. The smallest change will occur when the 5 changes to a 6. Thus the 8 must change at least through 9 to 0. The remaining digits to be used are 1, 4 and 9. The smallest increase occurs when they appear as 149.
Then 58273 changes to 60149.
The increase in the units used is 60149 – 58273 = 1876.

There are 1876 more units used.

Problem No. 6

If the density of gold is 19.3 tonnes per m^3 and the price of gold is €44 per gram, find the value of a cube of gold the edges of which are 5cm long.

No. 6 Solution

$1m^3 = 1\,000\,000cm^3$; 19.3tonnes = 19 300 000g. Therefore $1cm^3 = 19.3g$.
The value is 19.3x44 = €849.20 per cm^3.

A Cube of 5x5x5 = $125cm^3$ has value 125x849.2 = €106 150.00.

Problem No. 7

If you were to count the digits in the page numbers in a book, starting at Page 1, on what page would you be when you reached the 2020th digit?

No. 7 Solution

Pages 1 to 9: One digit per page for 9 pages: total 9
Pages 10 to 99: Two digits per page for 90 pages: total 180
Total so far: 189. Remaining count: 2020 – 189 = 1831. This count will be due to 3-digit pages since 900 three-digit pages are possible giving a total of 2700 which is too high.
Then, 1831 ÷ 3 = 610 with Remainder 1.
Thus, 99 + 610 = 709 pages counted on the 2019th digit, so the 2020th digit falls on Page 710.
Answer: You would be on Page 710.

Problem No. 8

For how many integers, n, is $\frac{n}{100-n}$ also an integer?

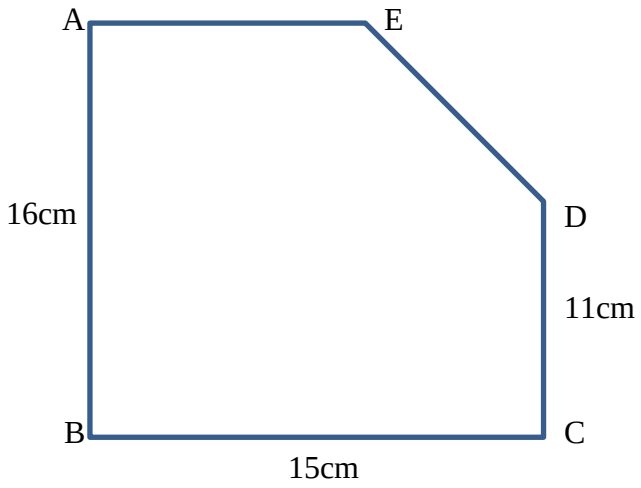
No. 8 Solution

Let $\frac{n}{100-n}$ = k, an integer.
We can isolate n:
n = k(100-n);
n = 100k –kn;
n+kn = 100k;
n(k+1) = 100k
So we get $n = \frac{100k}{k+1}$.
Since (k+1) cannot divide into k to give an integer result (k+1) must divide into 100.
There are 18 divisors of 100 (the values of (k+1)) which are ±1, ±2, ±4, ±5, ±10, ±20, ±25, ±50, ±100.
This means that there are 18 integers for which the result is also an integer.

The 18 values of k are 0, 1, 3, 4, 9, 19, 24, 49, 99, -101, -51, -26, -21, -11, -6, -5, -3, -2.
The 18 values of n are 0, 50, 75, 80, 90, 95, 96, 98, 99, 101, 102, 104, 105, 110, 120, 125, 150, 200.

Problem No. 9

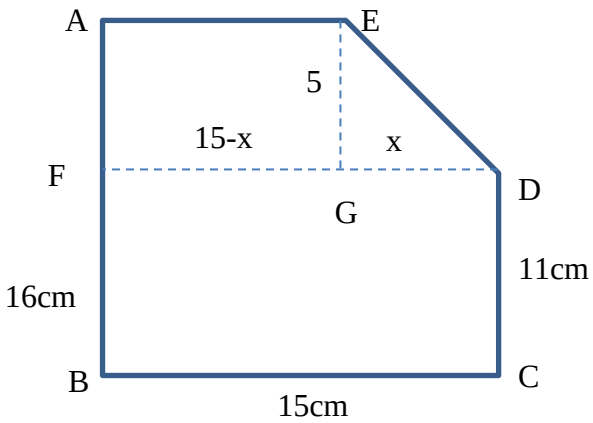
The diagram shows a pentagon ABCDE. The angles at A, B and C are right angles.
The lengths |AE| and |ED| are equal.



Find the area of the pentagon in the form $a\frac{b}{c}$.

No. 9 Solution

Draw DF parallel to BC to meet AB at F.
Draw EG perpendicular to FD to meet FD at G.
|EG| = 16 – 11 = 5.
Let |GD| = x. Then |AE| = |FG| = 15-x.



By Pythagoras Theorem $|ED|^2 = x^2 + 5^2 = x^2 + 25$.
$|AE|^2 = |ED|^2$
$(15-x)^2 = x^2 + 25$
$225 + x^2 - 30x = x^2 + 25$
$30x = 200$ so $x = \frac{200}{30} = \frac{20}{3} = |GD|$
and $|FG| = 15 - x = 15 - \frac{20}{3} = \frac{25}{3}$.

Area of ABCDE = area rectangle FBCD + area rectangle FGEA + area triangle EGD
$= (11)(15) + (5)\left(\frac{25}{3}\right) + \frac{1}{2}(5)(\frac{20}{3})$
Area of pentagon = $223\frac{1}{3}$ cm^2.

Problem No. 10

Show that if p is a prime number greater than 3, then $p^2 - 1$ is always divisible by 24 with no remainder?

No. 10 Solution

$p^2 - 1 = (p-1)(p+1)$ and $(p-1)$, p, $(p+1)$ are three consecutive numbers.
Since p is an odd prime, both $p-1$ and $p+1$ are even numbers which are both multiples of 2.
Since $(p-1)$ and $(p+1)$ are consecutive even numbers, one of them must also be a multiple of 4 so their product contains a multiple of 2(4) = 8.
Since p is odd and not a multiple of 3, one of $(p-1)$ or $(p+1)$ must be a multiple of 3.
Therefore the product is a multiple of 8x3 = 24.
Therefore p^2-1 is always divisible by 24 with no remainder.

Alternative solution: A prime > 3 is an odd number of the form (6n-1) or (6n+1) since the only other odd numbers of this form are 6n+3 = 3(2n+1), for n>0, which is not prime.
Take p= (6n-1). Then $p^2 - 1 = (6n-1)^2 - 1 = 36n^2 - 12n + 1 - 1 = 36n^2 - 12n = 12n(3n-1)$.
Cases: For n even (>0), n is a multiple of 2 so 12n is a multiple of 24.
For n odd, 3n-1 is even, and so 12(3n-1) is a multiple of 24.
Similarly, for p = 6n+1: $p^2 - 1 = (6n+1)^2 - 1 = 36n^2 + 12n + 1 - 1 = 36n^2 + 12n = 12n(3n+1)$.
Cases: For n even (>0), 12n is a multiple of 24; For n odd, 3n+1 is even and so 12(3n+1) is a multiple of 24.
Therefore p^2-1 is always divisible by 24 with no remainder.

Problem No. 11

You have 10 stacks of 10 gold coins. All of the coins in one of these stacks are counterfeit; all the other coins are not. A real coin weighs 10 grams. A counterfeit coin weighs 11 grams. You have a modern digital scale that provides an accurate readout.

How can the fake pile be identified in one weighing?

No. 11 Solution

Label the piles from 1 to 10. Take one coin from pile No. 1, two coins from pile No. 2, three coins from pile No. 3 and so on until finally taking ten coins from pile No. 10.
Altogether there are 1 + 2 + 3 + … + 10 = 55 coins.
Place these 55 coins on the digital scales.
If all the coins were real the total weight would be 550g. However, since one pile contains fake coins the weighing will show a number higher than 550 by the number of coins taken from the fake pile. E.g., if pile No. 2 contained fake coins then the weight shown would be 552g.

Problem No. 12

How many 3- digit numbers are there where the sum of the three digits is 10?

No. 12 Solution

The combinations of digits and the number of rearrangements possible in each case is
019 – 4 118 – 3 226 – 3 334 - 3
028 – 4 127 – 6 235 - 6
037 – 4 136 – 6 244 - 3
046 – 4 145 - 6
055 - 2
Total: 4(4) + 4(6) + 4(3) + 2 = 54

Problem No. 13

Peter cycles a journey on a Greenway at an average speed of 12 km per hour. He returns to his starting point at an average speed of 20 km per hour with the wind at his back. What is his average speed for both journeys?

No. 13 Solution

Let d = the distance travelled on the Greenway so the time taken is $\frac{d}{12}$.
The time taken to return the same distance is $\frac{d}{20}$.
Let a = the average speed for the both journeys. Then the time taken is $\frac{2d}{a}$.
Therefore $\frac{d}{12} + \frac{d}{20} = \frac{2d}{a}$. Multiply both sides of the equation by $\frac{60a}{d}$;
This gives 5a + 3a = 120
That is 8a = 120
a = 15
His average speed for both journeys is 15 km/hr.

Problem No. 14

Each of the symbols represents a number. The addition totals for each row and column are given, except one.

Find the number represented by the ? symbol.

Σ	Π	Π	Σ	26
λ	Π	Σ	λ	25
Ω	Ω	Ω	Ω	?
λ	Π	Σ	Π	28
21	32	22	24	

No. 14 Solution

From Row 1: $2\Sigma + 2\,\Pi = 26$; therefore $\Sigma + \Pi = 13$; [A]
From Row 2: $\Sigma + \Pi + 2\,\lambda = 25$; therefore $13 + 2\,\lambda = 25$ so $2\,\lambda = 12$; therefore $\lambda = 6$.
From Row 4: $\lambda + \Sigma + 2\,\Pi = 28$; which is $6 + \Sigma + 2\,\Pi = 28$ so $\Sigma + 2\,\Pi = 22$; [C]
Taking [A] from [C] we get $\Pi = 9$; and so from [A] $\Sigma + 9 = 13$; therefore $\Sigma = 4$;
From Column 2: $3\Pi + \Omega = 32$ so $27 + \Omega = 32$; therefore $\Omega = 5$.

From Row 3: $4\,\Omega = ?$; that is $4(5) = ?$ so the value of ? is 20.

Problem No. 15

Choose any 3-digit number where there is a difference of more than 1 between the first and third digits.
Reverse the order of all 3 digits in your number and subtract the smaller number from the greater number.
Reverse the order of all 3 digits in your answer and add these two numbers.
The answer is always 1089.
Why?

No. 15 Solution

Choose digits a, b, c with c>a+1.

	c	b	a
Subtract	a	b	c
	(c-1) –a	(10+(b-1)) –b	(10+a)-c.
=	c-a-1	9	10 + a –c
Reverse	10+a-c	9	c-a-1

Then c –a -1 +(10+a-c)= 9 for the first and third digit sums;
So we get 900 + 180 + 9 = 1089 in every case when c>a+1.

Problem No. 16

What is the next letter in the list A, C, G, M….?

No. 16 Solution

Assign numbers 1 to 26 to the letters of the alphabet.
The letters A, C, G, M follow the sequence 1, 3, 7, 13.
ie 1, 1+2, 1+2 +4, 1+2+4+6.
The next letter corresponds to 1+2+4+6+8 = 21 which is U.

Problem No. 17

I have two types of square tiles – one of side 1cm long and the other of side 2cm long. What is the area of the smallest square that I can form using equal numbers of each of the types of tile?

No. 17 Solution

Since there are equal number of both tiles they can be matched 1-to-1. Together one big and one small tile will cover $2^2\text{cm}^2 + 1^2\text{cm}^2 = 5\text{cm}^2$.
Let n be the number of each type of tile. The area covered will be $5n$ cm^2 and also a square. Thus the area will be square number which is a multiple of 5; ie. 25, 100, 225, …
For an area of 25cm^2, $n = 5$ so 5 of each is required with a side of 5cm. But using 5 large tiles requires a side of at least 6cm.
For an area of 100cm^2, n = 20 so 20 of each is required. 20 of the small tiles form the same shape as 5 large tiles so a square can be formed with the shape of 25 large tiles in a 5x5 grid with a side of 10cm.
Thus, the area of the smallest square that I can form is 100cm^2.

Problem No. 18

A brother and sister have asked their parents to put money aside for them for Christmas. Their parents said that they would put money aside for them every Saturday from when they returned to school on August 26th 2015 until they would go shopping on December 8th 2015.

They offer them the choice of saving €1 for the first week, €2 the second week, €3 the third week and so on for each of the children
or, 1 cent the first week, 2 cents the second week, 4 cents the third week and so on for the children to share equally.
The children chose the first option.
(i) How much did each child get for their Christmas shopping on December 8th?
(ii) How much more would have been saved for each of them if they had chosen the other option? Give the answer correct to the nearest 5 cent.

No. 18 Solution

There are 15 Saturdays between the two dates given.
In the first choice the amount saved for each child will be (in euro) 1 + 2 + 3 + 4 + …+ 15.
This totals to €120 for each child so €240 has been saved.
(i) Each child receives €120 for Christmas.

(ii) In the second choice the amount saved will be (in cent)
1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 + 1024 + 2048 + 4096 + 8192 + 16384.
This totals €327.67.
The extra money saved is €327.67 - €240 = €87.67 which is shared between the two children so €87.67/2 = €43.835 = €43.85, to the nearest 5c, more would have been saved for each of them.
So answer (ii) €43.85.

Problem No. 19

I want to buy a car in Northern Ireland and I respond to the following advertisement:

> FOR SALE: 2015 (Feb) Ford Focus 2.0, jet black, Titanium 145ps, automatic transmission, petrol engine, 5 door hatchback, 56789 miles. One careful owner. £10,500. No offers.

Vehicle Registration Tax (VRT) must be paid.

Use the internet to calculate
(a) the percentage rate of VRT quoted
(b) the Current OMSP (Open Market Selling Price)
(c) the amount of VRT calculated
(d) the net cost of the car in Euro, correct to the nearest euro,
given that the exchange rate is €1 = 89p Sterling.

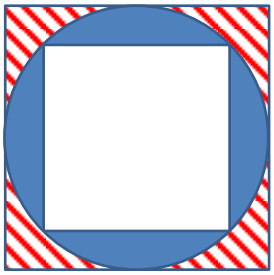
No. 19 Solution

Find the Revenue VRT Calculator at https://www.ros.ie/evrt-enquiry/vrtenquiry.html?execution=e1s2
Enter the details to get a quote.
The answers will correspond to the following:
(a) the percentage rate of VRT quoted = 30%
(b) the Current OMSP (Open Market Selling Price): May vary by date of query.
(c) the amount of VRT calculated : Varies according to the Current OMSP.
(d) the net cost of the car in Euro, correct to the nearest euro,
given that the exchange rate is €1 = 89p Sterling.: Varies according to the Current OMSP.

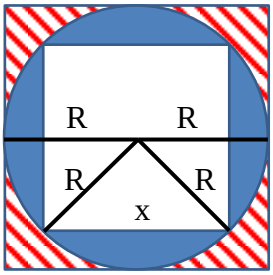
Note: The Statistical Code for speeding up the VRT Calculator entries is 41086331.

Problem No. 20

In the following diagram which is bigger, Area 1, the striped area between the circle and the outer square or Area 2, the solid coloured area between the circle and the inner square?



No. 20 Solution



Let R = radius of the circle. The length of the side of the larger square is 2R and the area of the larger square is $4R^2$.

Let x = length of the side of the smaller square.
Then, by Pythagoras Theorem $x^2 = R^2 + R^2$ so $x^2 = 2R^2$.
The area of the smaller square is $x^2 = 2R^2$.

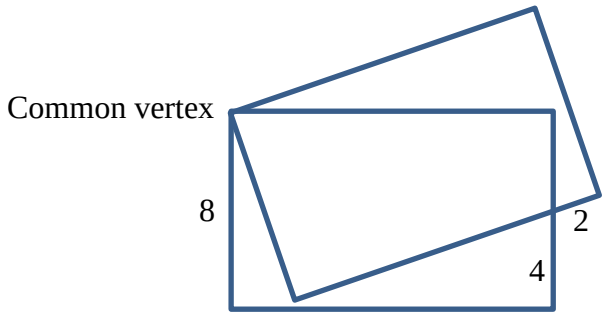
The area of the circle is πR^2.

Area 1, between the circle and the outer square = $4R^2 - \pi R^2 = R^2(4-\pi) = 0.858R^2$.
Area 2, between the inner square and the circle = $\pi R^2 - 2R^2 = R^2(\pi-2) = 1.141R^2$.

Therefore Area 2 is bigger.

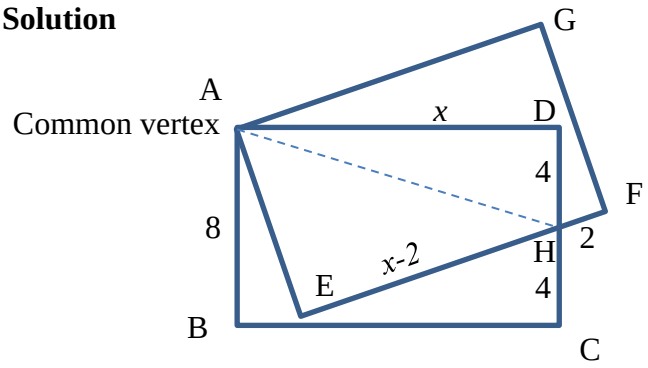
Problem No. 21

Two congruent rectangles have a common vertex and overlap, as shown in the diagram.



Find the area of the drawing that is not common to both rectangles.
Note: All measurements are in cm.

No. 21 Solution



Label the vertices. Join AH to form two right-angled triangles AEH and ADH which have a common hypotenuse AH.
Let |AD| = x. Then |EF| = x and |EH| = x-2.
Since |AB| = |CD| = 8 then |HD| = 4. Also |AB| = |AE| = 8.
By Pythagoras Theorem we have $|AH|^2 = |AE|^2 + (x-2)^2 = x^2 + |DH|^2$;
$8^2 + x^2 - 4x + 4 = x^2 + 4^2$

$68 - 4x = 16$
$4x = 52$
$x = 13$

Then, area triangle ADH = $\frac{(13)(4)}{2} = 26$; area triangle AEH = $\frac{(8)(11)}{2} = 44$.
The common area = area AEHD = 26 + 44 = 70.
Area rectangle ABCD = area rectangle AEFG = (8)(13) = 104.
The required area = (area ABCD – area AEHD) + (area AEFG – area AEHD)
= (104 – 70) + (104 – 70)
= 34 + 34
= 68
Answer: The total area not common to both rectangles is 68cm^2.