APPLIED MATHEMATICS
Response from the Irish Mathematics Teachers Association to the NCCA *Draft Background Paper*, 2014

Introduction
It is good to see that, after suggestions for a long time that Applied Mathematics was due for review, the subject is being given thorough consideration. This submission attempts to set the review in historical context and to discuss IMTA members' reactions to the possibilities advanced in the NCCA's *Draft Background Paper*.

History and Context[1]
Since the start of the 1960s – that period of major reform of mathematics courses in many countries – the *Mathematics* courses in Ireland have undergone several revisions, including two major reformulations (the introduction of "modern mathematics" in the 1960s and the Project Maths initiative in the 2000s). During this period, *Applied Mathematics* has received only comparatively minor revisions. Thus, it is – at it was in 1960 and for many years before that – basically a course in mechanics, reflecting what some people consider to be a rather narrow conception of "applied mathematics."

However, **major changes have been considered at system level on several occasions**. In the late 1960s, the idea of bringing together Mathematics and Applied Mathematics into a single and broader course was discussed, but was not accepted. In 1974, 1984 and 1994, suggestions were made for major revisions to the Applied Mathematics course. Typically, students would have taken two modules from four or five; two modules (say, Mechanics I and Mechanics II) would have covered the existing Applied Mathematics course, allowing the subject to be continued in its present form for those who so wished, while other modules could cover (say) probability/statistics and programming. **In each case, the initiatives did not lead to change**. According to reports, the initial work done in the present cycle of consideration pointed to a similar modular model. As well as these system level initiatives, individuals have put forward interesting suggestions, but have done so outside structures that lead to change. One such example is that advanced in a guest lecture at Waterford Institute of Technology by Maurice O'Reilly (2002), cited in the NCCA's *Draft Background Paper*. There may be other instances of attempted major reform, or imaginative contributions to possible discussions, that have not been traced during the compilation of this response.

While no major reforms were implemented, the syllabus has been adjusted from time to time. Details are recorded in relevant versions of *Rules and Programme for Secondary Schools*. However, the lack of major reform means that **the syllabus documentation retains the style customary before the mid-1970s;** it is limited to a specification of content.[2] As a result, there is no "paper trail" – or at least no easily accessible paper trail – that can indicate the intended aims of the syllabus, and hence no easy way of establishing definitively if such aims were or are being achieved. For such syllabuses, custom and practice – together with

[1] This section draws on documentation from the IMTA, the NCCA and the Computers in Education Society of Ireland, on personal communications to writers of this report, and on a final-year undergraduate project produced by Brian Gallagher at Trinity College in 2013.

[2] The Intermediate Certificate Mathematics course introduced in 1973 was the first Mathematics course to be preceded by a preamble outlining "objectives" – in modern parlance, general aims rather than learning outcomes.

consideration of examination papers – help to indicate the scope and define the implementation. While custom and practice can drift away from intention, **personal communications and anecdotal evidence point to much implementation through a genuinely problem-solving approach**, though probably somewhat more topic-centred than is now familiar via PISA and Project Maths (see for example O'Reilly [2002] and also the responses from IMTA members as discussed below). The NCCA's *Draft Background Paper* cites instances where students reported a focus on learning to address the examination questions rather than on learning the subject in all its fullness. The culture of learning to the test in a points-driven setting, together with the lack of timetabled time typically accorded to the subject and the fact that some students address the subject substantially on their own, may account for some of the quoted responses.

New features of the present proposals are the incorporation of a "core" containing pure mathematics topics and the positioning of the (perhaps renamed) subject more formally as the culmination of a pathway towards third-level study of mathematics-related subjects. **It can be argued that such features could give students a better preparation for third level, and – via the range of suggestion options – may also widen the subject's appeal.** The approach is imaginative and of course very well intended; **however, there are problems at both theoretical and practical levels**. They are discussed below.

Prior to such discussion, it is worth noting four general points:

- Even in the 1970s/80s, **provision of an Applied Mathematics / Mechanics course in national curricula seems to have been quite rare**. In particular, it may have occurred chiefly in countries the curricula of which were strongly influenced by that of England, where such a course – and the inclusion of some applied mathematics in the main Mathematics curricula – has a long history. (For other countries, some "applied mathematics" material may figure in their Physics syllabuses.)
- Applied Mathematics has played a different role in the Irish curriculum; in practice it has been, and is, a "minority" subject often taught by dedicated and enthusiastic teachers outside the main school timetable. This means that it is particularly reliant on teachers' voluntary effort. A feature of the response to the suggestions for change in 1994, and perhaps on other occasions, has been that **greatly altering the subject could significantly increase the demands on teachers who are already giving time voluntarily to the subject and their students; hence that, in practice, aiming to develop a tender plant could have the effect of killing it.**
- While there has been some curricular association between Mathematics and Computer Studies / Computer Science / Programming over the years, not everybody has been happy about attempts to bring in the latter areas via the Mathematics curriculum. The "Computer Studies Option" introduced in 1980 was technically a part of the Mathematics curriculum, but in practice was a freestanding unit. The Computers in Education Society of Ireland would have preferred to downplay any association with Mathematics, but accepted the "option" as, hopefully, an interim arrangement en route to the establishment of an independent course. (Such a course was introduced into the Junior cycle in 1985, but perhaps made little impact.) **Now, at a time at which a coding short course has been developed for the Junior cycle, and at which there are moves to introduce fuller Computer Science / Computational Thinking / Coding course(s), the placement of a programming-type module in a "Mathematics" course is perhaps particularly inappropriate.**

- The *Draft Background Paper* appears to **undervalue the usefulness – indeed, necessity – of students' developing procedural skills as well as conceptual understanding**. The current aims for the *Mathematics* syllabus, correctly in our opinions, include both aspects. Mindless practice may not be productive; however, reflective practice, with appropriate feedback, is essential in developing fluency. According to Tom Good (2014), writing in *Teachers College Record*, "the point is the relative importance of conceptual knowledge (in relation to practice), not the absence of procedural information and practice."[3]

Reactions to the present proposals

Scope of this Response

While it was not possible to conduct a survey of IMTA members' views, because of time constraints, members with an interest in the area were invited to send their comments through their branch and/or to members of the Council. **The summary below is therefore offered as an indication of members' views rather than as a policy statement from the IMTA; in particular, it highlights that respondents see a number of difficulties with the proposals.**

It can be noted that most respondents currently teach Applied Mathematics as well as Mathematics. This may indicate that they are very familiar with the subject in its present form and with the culture of teaching it. It may also point to the extent to which many non-responding IMTA members do not see Applied Mathematics as their professional area. However, many members (both those that are and those that are not teachers of Applied Mathematics) may have made personal submissions to the NCCA. At least one branch committee decided that it was in no position for formulate a Branch-level response, but emailed its members encouraging them to provide their own feedback. Additionally, members who do teach Applied Mathematics may have already provided feedback through our sister organisation, the IAMTA.

Title of the Subject

Respondents are strongly in favour of **retaining the present title of "Applied Mathematics."** They are resisting the notion of renaming the course "Further Mathematics," as they perceive this as a move away from problem solving and a move towards more mathematical content. (See General Concerns below.)

Structure of the Syllabus

As regards the structure of the syllabus, **the "core and options" model described on page 34 of the document is very unpopular with respondents**, and it is felt that this structure would be detrimental to the subject. (Again, see General Concerns below.)

- However, if a "core and options" model is introduced, perhaps there should be criteria for the combinations of options, so that students get an appropriate blend of topics.

Removing Ordinary level would appear **elitist**. Surely every subject should be accessible to all students.

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[3] Good, T., *Teachers College Record* Volume 116 Number 1, 2014, p. - http://www.tcrecord.org ID Number: 17289, Date Accessed: 1/3/2015 4:57:45 AM

Suggestions for Development

Respondents would **welcome the subject incorporating more areas of applied mathematics apart from mechanics**. However, they think that every effort should be made to avoid making the subject content heavy, as it should be built around a problem-solving approach.

With regard to the **present content**, several points emerged from the responses. (These can be read in the light of the IMTA's document *Project Maths and the Irish Maths Teachers Association* (2013),[4] which expressed concern at the loss of material on vectors and some calculus from the Mathematics syllabus.)

- There appears to be some consensus around retaining the material examined in questions 1 to 5; it has been noted that there is plenty of scope to increase the use of vectors and differential calculus with most of these topics. To make room for new topics, it is suggested that Rigid Body Motion and Hydrostatics be removed, as the former is considered unduly abstract and the latter has proven to be unpopular over the years.
- Another recurring comment is that the material examined in question 6 should be split into two distinct topics, Circular Motion and Simple Harmonic Motion. If this were to be done, there would again be plenty of scope for including more calculus and vectors.
- Many respondents have commented on the need for revising Statics. If this were done properly, vectors could be particularly emphasised here.

General Concerns

It appears that **there is an attempt to squeeze three separate subjects, "Applied Mathematics," "Further Mathematics" and "Computer Science," into one subject**. This would not be a good idea, as it is unlikely to address any of these disciplines satisfactorily.

- Respondents are not opposed to the creation of "Further Mathematics" or "Computer Science," but are concerned that Applied Mathematics as a distinct discipline would be lost to facilitate the introduction of these subjects.
- Moreover, respondents indicated that Computer Science should be a subject on its own, not made a part of another subject. Certainly what is proposed in the background document in relation to computer science would not do anything to help mitigate its current absence in the curriculum. It could be detrimental to have such an important subject addressed in this way. There is a fear that the content would be diluted rather than embraced in a comprehensive fashion.

More generally, the *Draft Background Paper* could be interpreted as **offering Applied Mathematics as a sacrifice to fix problems that exist elsewhere** in the Leaving Certificate system.

- There is strong feedback that the document is incorrect in its claim that the current syllabus is not teaching problem solving. Many respondents feel that this is the one subject where students engage with problem solving successfully; the subject does not lend itself to a rote learning approach. Many of the issues raised in the *Draft*

[4] The document can be accessed at http://www.imta.ie/IMTA%20PM_1%20Doc.pdf.

Background Paper are more to do with the Project Maths syllabus and mathematics education in general than the subject of Applied Mathematics.

- In particular, the creation of a topic-heavy core would take away from rather than adding to the problem-solving element of the subject as it stands; it would damage teachers' ability to encourage, promote and engage in problem solving in the classroom.

Teachers are nervous that **any new syllabus would be ill defined**. It is possible to spend months teaching the topic of statics and still omit aspects of it. It was emphasised that there is still plenty of opportunity for there to be new and challenging problems within clearly defined specific syllabus content.

Practical Difficulties

In addition to the theoretical or conceptual difficulties noted above, some practical difficulties can also be noted. They include the following.

- If the subject is to become more "mainstream," catering for a larger cohort of students, how will it be timetabled?
- If it does play the role of "Further Mathematics" and becomes a standard part of the pathway to mathematics-related courses at third level, what subject are students likely to drop so that they still typically take seven subjects in the Leaving Certificate? Are there implications for meeting matriculation requirements?
- Is there a danger that third-level institutions will start to require the amended subject for entry to mathematics-related courses before the school system is in a position to provide it satisfactorily?
- Who will teach the subject, especially if new content is introduced?
- How will adequate CPD – obviously of vital importance if the syllabus is much altered – be provided for the teachers?

Conclusion

The current Applied Mathematics course is taught by a small band of dedicated teachers who often work outside the ordinary school timetable in order to offer the subject to their students. As indicated by our respondents, these teachers see the subject as providing excellent opportunities for problem solving and for enhancing students' readiness and maturity for undertaking third-level mathematics courses. Previous attempts to alter the basic structure of the course did not lead to change. This may indicate that the present version works well, albeit for a limited cohort of students, and also that radical change might have the effect of killing the subject rather than developing it – especially with regard to developing it with a core-and-options structure as envisaged in the *Draft Background Paper*.

However, our respondents do see value in broadening the content so as to include other areas of applied mathematics as well as mechanics – provided that these areas can be addressed through a problem-solving approach. They note that extra attention could be paid to the use of calculus and vectors (two areas that have been regarded as missing out in the revised Mathematics syllabuses) in the context of addressing the existing or redeveloped content.

The Association offers the NCCA all good wishes for its ongoing work in mathematics education.