PETER'S PROBLEM 14 SOLUTION

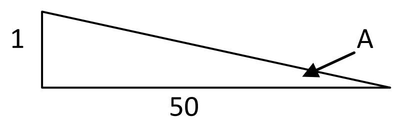
NOTE: The following solution is the outline of the solution **expected from students** who had completed the Junior Certificate - i.e., the most efficient incline would be used for its entire length.

General criteria

Rest areas must be included at various intervals. We can find the length of the incline (sloped pedestrian walk surface) from top to bottom and insert the rest areas as required.

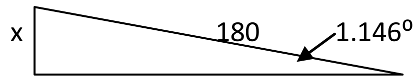
The rest areas classified as level may in fact be tilted with a ratio 1:50.

The rest areas are 180 cm square.



Tan A = $\frac{1}{50}$

=> A = 1.146^{o} correct to 3 d.p.

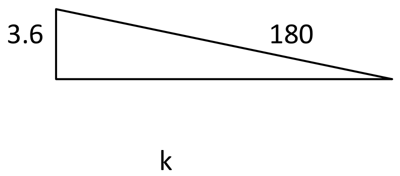


Sin 1.146^{o} = $\frac{x}{180}$

x = 180 (.0200) = 3.6 cm correct to 3 d.p.

=>Taking advantage of this, each compulsory rest area allows us to reduce the required height of the incline by 3.6cm per rest area. It is prudent to leave the rest areas at either end of the incline level at zero gradient.

We also note that our accuracy criteria allows us to take the horizontal length of the rest area to be 180 cm (=1.8m) as shown.



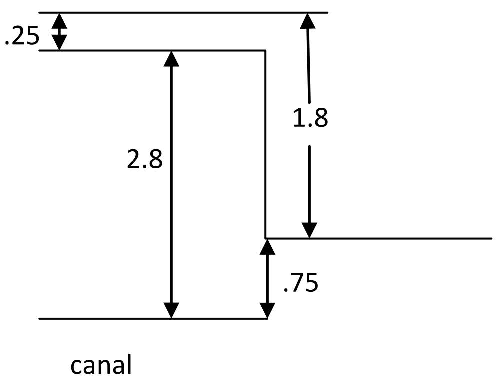
$180^2 = k^2 + 3.6^2$

$k^2 = 180^2 - 3.6^2 =$

$k = \sqrt{32387.04} = 179.9639 = 1.799639$m

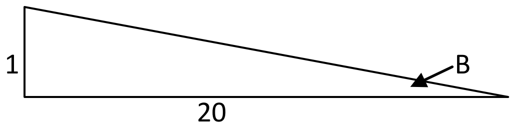
= 1.80 m correct to 3 d.p.

The height of the bridge above the ground is found to be 1.8 metres as shown:



There are 3 recommended maximum slopes for the incline – 1:20, 1:15 and 1:20.
(NB for the purposes of this question, intermediate ratios were deemed inappropriate)

Case 1 Ratio 1:20



Tan B $= \frac{1}{20}$

B $= \text{Tan}^{-1} \frac{1}{20}$ $= 2.862^{o}$ correct to 3 d.p.

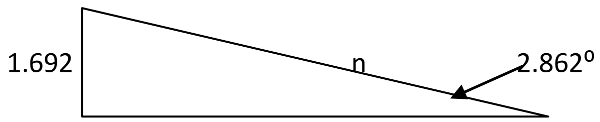
As the height is 1.8metres

- ⇨ No. of rest areas is $\frac{1.8}{.5}$ = 3.6 => 3 rest areas and 4 inclines.
- ⇨ A reduction of 3(3.6cm) is allowed in the required height = 10.8cm
- ⇨ The required height is 180 – 10.8 cm =169.2 cm (=1.692 m)

Check: Would two rest areas be sufficient?

180 - 2(3.6) = 172.8 = 1.728 m

1.728 ÷ .5 = 3.456 => 3 rest areas are required so two rest areas are not sufficient.



Sin $2.862^{o} = \frac{1.692}{n}$

n $= \frac{1.692}{\sin 2.862^{0}}$ = 33.887 m correct to 3 d.p.

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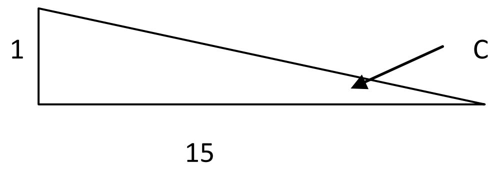
=> The length of the incline from the ground to the top (including rest areas) is

33.887 + 3(1.8) = 33.887 +5.4 = 39.287m.

Case 2 Ratio 1:15



Tan C = $\frac{1}{15}$

C = $\tan^{-1}\frac{1}{15}$ = 3.814^{o} correct to 3 d.p.

As the height is 1.8metres

- ⇨ No. of rest areas is $\frac{1.8}{.333}$ = 5.4 => 5 rest areas and 6 inclines.
- ⇨ A reduction of 5(3.6cm) is allowed in the required height = 18cm
- ⇨ The required height is 180 – 18 cm =162 cm (=1.62 m)

Check: Would four rest areas be sufficient?

180 - 4(3.6) = 180 - 14.4 = 165.6 cm = 1.656 m

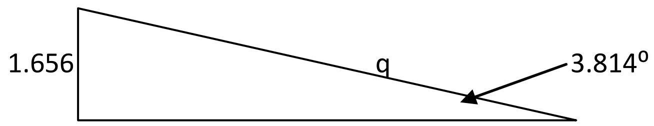
1.656 ÷ .333 = 4.97 => 4 rest areas and 5 inclines are sufficient.

Check: Would three rest areas be sufficient?

180 – 3(3.6) =180 - 10.8 = 169.2 cm (1.692 m)

1.692 ÷ .333 = 5.08 => 5 rest areas are required so 3 rest areas are insufficient.

Therefore we must consider 4 rest areas and 3 inclines.



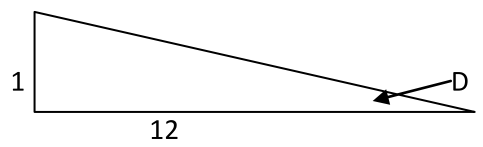
Sin 3.814^{o} = $\frac{1.656}{q}$

q = $\frac{1.656}{\sin 3.814^{0}}$ = 24.896 m correct to 3 d.p.

The length of the incline from the ground to the top (including rest areas) is

24.896 + 4(1.8) =24.896 + 7.2 = 32.096 m

Case 3: Ratio 1:12



$\text{Tan } D = \frac{1}{12}$

$D = \text{Tan}^{-1}\frac{1}{12} = 4.764^{o}$ correct to 3 d.p.

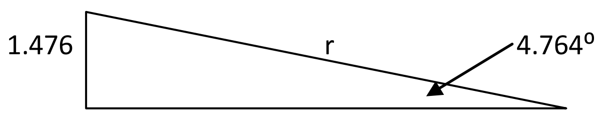
As the height is 1.8metres

- ⇨ No. of rest areas is $\frac{1.8}{.166}$ = 10.84 => 10 rest areas
- ⇨ A reduction of 10(3.6cm) is allowed in the required height = 36cm
- ⇨ The required height is 180 – 36 cm =144 cm (=1.44 m)

Check: Would nine rest areas be sufficient?
180 - 9(3.6) = 180-32.4 = 147.6 cm = 1.476 m
1.476 ÷ .166 = 8.89
This suggests that maybe 8 rest areas may be enough.

Check: Would 8 rest areas be sufficient?
=> A reduction of 8(3.6cm) is allowed in the required height (= 28.8cm).
=> The required height is 180 - 28.8 = 151.2 cm (= 1.512 m)
1.512 ÷ .166 = 9.108 => 9 rest areas are required.
So the use of eight rest areas must be discounted.

Therefore we must consider 9 rest areas and 10 inclines.



$\sin 4.764^{o} = \frac{1.476}{r}$

$r = \frac{1.476}{\sin 4.764^{0}}$ = 17.772 m correct to 3 d.p.

- ⇨ The length of the incline from the ground to the top (including rest areas) is

17.772 + 9 (1.8) =17.772 + 16.2 = **33.972 m**

Applying a ratio 1:20 gives an incline measuring 39.287 m

Applying a ratio 1:15 gives an incline measuring 32.096 m

Applying a ratio 1:12 gives an incline measuring 33.972 m

Therefore the most efficient ratio to use is 1:15

The width of the bridge is uniform at 1.8 m.

The length of the incline on both sides is 2 X 32.096 = 64.192 m

The area of both inclines is 64.192 X 1.8 = 115.55 m^2 correct to 2 d.p.

There are 4 rest areas (1 at either end of each incline). The area is 4 X 1.8 X1.8 = 12.96 m^2

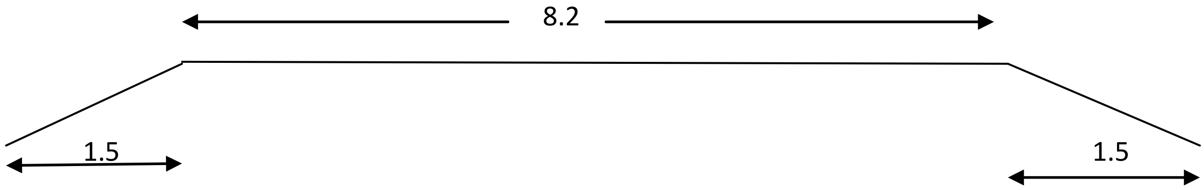
The length of the bridge span over the water is 8.2 + 2(1.5) = 11.2 m

The area of the span over the water is 11.2 X 1.8 = 20.16 m^2 correct to 2 d.p.

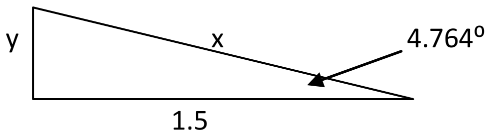
The total area is thus **148.67 m^2 correct to 2 d.p.**

Part Two

The span over the water is flat and the incline approaches the rest area at the top of each incline as shown.



The length of this short incline allows us to use the ratio 1:12 (as it is shorter than 2m) i.e., an angle of 4.764^0.



$\cos 4.764^0 = \frac{1.5}{x}$

$x = \frac{1.5}{\cos 4.764^0}$ = 1.505 m correct to 3 d.p.

and $1.505^2 = y^2 + 1.5^2$

- ⇨ $y^2 = 1.505^2 - 1.5^2 = .015025$
- ⇨ $y = \sqrt{.015025}$ = .12257 m = 12.26cm correct to 2 d.p.
- ⇨ This incline is 1.505 m long and reduces the height required for the incline parallel to the canal to 180-12.26 = 167.74 cm = 1.677m. correct to 3 d.p.

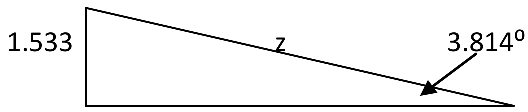
The number of rest areas is 4

- ⇨ A reduction of 4(3.6 cm) = 14.4cm is allowed from the required height of 167.74 cm.
- ⇨ The required height is now 167.74 – 14.4 = 153.34 cm (=1.533 m correct to 3 d.p.)

Check if 3 rest areas are sufficient
167.74 – 3(3.6) = 167.74 – 10.8 = 156.94cm (= 1.569 m correct to 3 d.p.)
1.569 ÷.333 = 4.79 => 4 rest areas are required => 3 rest areas not sufficient.
Therefore we must consider 4 rest areas and 5 inclines.

- ⇨ The most efficient ratio to apply is still 1:15 and an angle of 3.814^0.

The length of the incline from the ground to the top is



$\sin 3.814^0 = \frac{1.533}{z}$

$z = \frac{1.533}{\sin 3.814^0}$ = 23.046 m correct to 3 d.p.

The lengths of the long inclines are 23.046 + 4(1.8) = 23.0146 + 7.2 = 30.246 m

The overall area of the bridge is now

The 2 inclines parallel to the water 2 X 30.246 X 1.8 = 108.88566 = 108.89 m^2 correct to 2 d.p.

The 4 rest areas 4 X1.8 X 1.8 = 12.96 m^2.

The short slopes at either end of the span 2 X 1.505 X 1.8 = 5.42 m^2 correct to 2 d.p.

The flat span across the water 8.2 X 1.8 = 14.76 m^2.

The total area is now = 142.03 m^2

The difference is 148.67 - 142.03 = 6.64 m^2 correct to 2 d.p.

Addendum 1

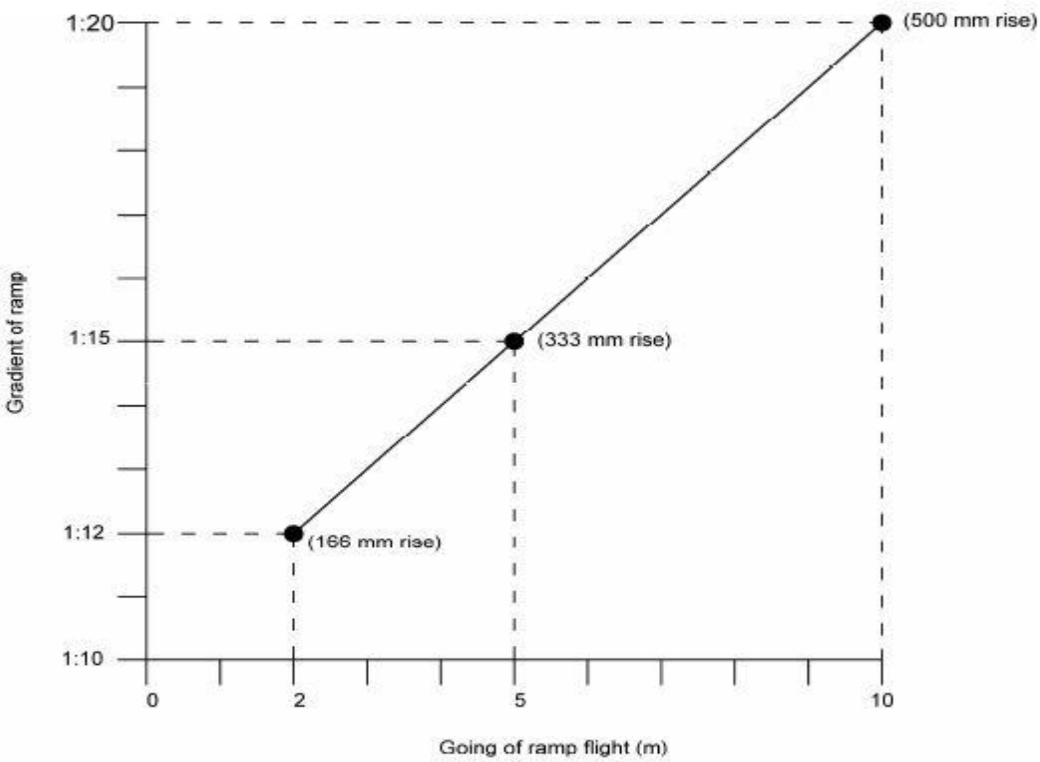
The Department of the Environment, Community and Local Government Building Regulations 2010, Part M:
http://www.environ.ie/en/Publications/DevelopmentandHousing/BuildingStandards/FileDownLoad,24773,en.pdf

Table 1 Limits for ramp gradients and lengths

Maximum going of a flight	Maximum gradient	Maximum rise
Not exceeding 10 m	1:20	500 mm
Not exceeding 5 m	1:15	333 mm
Not exceeding 2 m	1:12	166 mm

NOTE: For goings between 2 m and 10 m, it is acceptable to interpolate between the maximum gradients (refer to Diagram 3).

Diagram 3 Relationship between ramp gradient and the maximum going of a flight



NOTE: Where space permits, the gentlest possible gradient should be chosen i.e. the shallowest possible gradient between 1:12 and 1:20

Addendum 2

Combinations of Gradients:

For Part I

The best combination is for 5 rest areas and 6 inclines as follows:

Rise 180 – 5(3.6) = 162 cm = 1.62 m

4 inclines at 1:15 give a rise of 4(.333) = 1.332m. The incline is $\frac{1.332}{Sin3.814^0} = 20.025$ m correct to 3 d.p.

1 incline at 1:12 gives a rise of .166 m. The incline is $\frac{.166}{Sin4.764^0}$ = 1.999 m correct to 3 d.p.

1 incline at 1:12 gives a rise of .122 m. The incline is $\frac{.122}{Sin4.764^0}$ = 1.469 m correct to 3 d.p.

The length of the incline from ground to top then is 20.025 + 1.999 + 1.469 + 5(1.8) = 32.493 m.

The area of the bridge is

	2x33.493x1.8	= 116.97 m^2 correct to 2 d.p.
+	4x1.8x1.8	= 12.96 m^2
+	11.2x1.8	= 20.16 m^2

The total area is 150.09 m^2 correct to 2 d.p.

Thus 148.67 m^2 is the smallest area.

For Part II

The best combination is for 5 rest areas and 6 inclines as follows:

3 inclines at 1:15 give a rise of 3(.333) = .999 m. The incline is $\frac{.999}{Sin3.814^0}$ = 15.019 m correct to 3 d.p.

3 inclines at 1:12 give a rise of 3(.166) = .498 m. The incline is $\frac{.498}{Sin4.764^0}$= 5.996 m correct to 3 d.p.

The length of the incline from ground to top then is 15.019 + 5.996 + 5(1.8) = 30.015 m.

The area of the bridge is

	2x30.015x1.8	= 108.05 m^2 correct to 2 d.p.
+	4x1.8x1.8	= 12.96 m^2
+	2x1.505x1.8	= 5.42 m^2
+	8.2x1.8	= 14.76 m^2

The total area is 141.19 m^2 correct to 2 d.p.

The difference is 148.67 – 141.19 = 7.48 m^2 correct to 2 d.p.

Addendum 3

Interpolated Gradients

The option of interpolating gradients could be used (from Diagram 1 of Part M). An interpolation formula may be established to give **Gradient = (100-Rise)/1000 where Rise is in cm.**

Interpolated gradients for Part I: The results are shown on Table I.

In these tables ratio is used rather than angles as a basis of calculations of length of Going.

Part I: Interpolated gradients Calculation of area for various ramp designs using interpolated gradients								
Rests	3	4	5	6	7	8	9	10
Flights	4	5	6	7	8	9	10	11
Rise required = 180 – (3.6)x(No. Rests) Rise per flight = Rise required/No. Flights								
Rise (cm)	169.2	165.6	162	158.4	154.8	151.2	147.6	144
Rise per flight (cm)	42.3	33.12	27	22.63	19.35	16.8	14.76	13.09
Max gradient (1/m)	1:17.34	1:14.96	1:13.7	1:12.93	1:12.4	1:12.02	1:11.74	1:11.51
Going cm (Rise x m)	2933.93	2477.38	2219.4	2048.11	1919.52	1817.42	1732.82	1657.44
Flight (cm) (Pythagoras)	2938.8	2482.91	2225.3	2054.23	1925.75	1823.7	1739.09	1663.68
Length of the ramp inclined section = Flight length + (No. Rests)x(180)								
Incline (cm)	3478.80	3202.91	**3125.3**	3134.23	3185.75	3263.7	3359.09	3463.68
Total length of the bridge = 2(Incline length) + 1840 Bridge Area = (Bridge length)x(180)								
Bridge (cm)			8090.6					
Bridge Area (cm^2)			1456308					
Bridge Area (m^2 (2d.p.))			**145.63**					

Table I

We do not need to calculate the other areas since the minimum length of incline gives the minimum area.

In this case there are 5 rest areas and 6 flights with a gradient of 1:13.7.

The minimum area of the surface is 145.63 m^2 under these conditions.

Interpolated gradients for Part II: Shown on Table II.

Part II Interpolated gradients Calculation of area for various ramp designs using interpolated gradients								
Rests	3	4	5	6	7	8	9	10
Flights	4	5	6	7	8	9	10	11
Rise required = 167.5 – (3.6)x(No. Rests) Rise per flight = Rise required/No. Flights								
Rise (cm)	156.7	153.1	149.5	145.9	142.3	138.7	135.1	131.5
Rise per flight (cm)	39.18	30.62	24.92	20.84	17.79	15.41	13.51	11.95
Max gradient (1/m)	1:16.5	1:14.42	1:13.32	1:12.64	1:12.17	1:12	1:12	1:12
Going cm (Rise x m)	2585.55	2207.7	1991.34	1844.18	1731.79	1664.4	1621.2	1578
Flight (cm) (Pythagoras)	2590.29	2213	1996.94	1849.94	1737.63	1670.17	1626.82	1583.47
Length of the ramp inclined section = Flight length + (No. Rests)x(180)								
Incline (cm)	3130.29	2933	**2896.94**	2929.94	2997.63	3110.17	3246.82	3383.47
Total length of the bridge = 2(Incline length) + 4(180) + 2(150.5) + 820 = 2(Incline) + 1841 Bridge Area = (Bridge length)x(180)								
Bridge (cm)			7634.88					
Bridge Area (cm^2)			1374278					
Bridge Area (m^2 (2d.p.))			**137.43**					

Table II

Using 5 rest areas and 6 flights the rise required is 149.5cm (see Table II). The interpolated gradient is 1:13.32. **This gives a final area of 137.43 m^2.**

The reduction then is 145.63 (from Table I) – 137.43 = **8.2 m^2** for interpolated gradients.